



# Influence of User-Defined Parameters Using Stochastic Subspace Identification (SSI)

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**Abstract.** Operational Modal Analysis (OMA) is one of the most used technique to study structures under environmental excitations, for the purpose of structural health monitoring, acceptance test and model updating. In OMA, the modal parameters are obtained only from the measured data using environmental vibrations as unknown input (e.g. wind load, micro-tremors, traffic) and without any artificial excitations applied on the structure. One of the advantages of OMA technique is the possibility to test large-scale structures, which are impossible to test by using artificial excitations, and to provide a modal model under operating conditions, meaning within true boundary conditions, actual forces and vibration levels. Other advantages of OMA are the velocity and cheapness to make the tests, and the possibility to detect close-spaced modal shapes. One of the most used methods in OMA is the Stochastic Subspace Identification (SSI). It relies on an elegant mathematical framework and robust linear algebra tools to identify the state-space matrix from raw data. As a result, non-linear optimization problems are avoided. Moreover, the use of well-known tools from numerical linear algebra, such as Singular Value Decomposition and LQ Decomposition, leads to a numerically very efficient implementation. In order to obtain accurate modal parameters estimations, some user-defined parameters need to be properly set. In this paper, Data-Driven Stochastic Subspace Identification (DD-SSI) method and its sensitivity to two user-defined parameters are investigated. These parameters are, namely, the number of block rows in Hankel matrix and the selection of the length of the data acquired and used in the identification process. In order to establish a standardization on the use of these parameters for reliable system parameters identification, a sensitivity analysis has been conducted on real scale building vibration data.

**Keywords:** OMA · SSI · Damping evaluation

## 1 Introduction

In recent years, the dynamic identification of civil structures is becoming increasingly important especially for existing buildings, because it allows to study the behaviour of a system under real boundary conditions. The dynamic identification is applied in

several applications, such as structural health monitoring [1–4], structural control [5–9], system and damage identification [10–12], acceptance tests and model updating of FE models [13].

Dynamic identification consists in evaluating modal parameters of the system examined, such as natural frequencies, modal shapes and damping ratios, by using experimental data obtained during tests.

Basically, the dynamic identification can be performed by means of two different approaches. The first one is the so called Experimental Modal Analysis (EMA) in which the identification of modal parameter is evaluated by applying a measured input on the system and measuring its response. The input can be applied by using impulse hammers, mechanical exciters and shaking tables [14–16]. One of the disadvantages of this technique is that the excitation generated by using impulse hammers or shakers is too small for the case of large-scale structures with low frequency range, so not all the structure could be excited.

The second approach, developed in last years, is the Operational Modal Analysis (OMA) in which the modal parameters are obtained only from the measured data using environmental vibrations as unknown input (i.e. wind load, micro-tremors, traffic), without any artificial excitations applied on the structure [17].

The OMA method provides a modal model under operating conditions, with true boundary conditions, actual forces and vibration levels. Another advantage of OMA is that it does not interfere with the use of the structure, so it can be used during tests. The algorithms used in OMA assume that the input forces are stochastic in nature. This is often the case for civil engineering structures like buildings, towers, bridges and offshore structures, which are mainly loaded by environmental random forces.

This paper is focused on one of the most used OMA method, namely Data-driven Stochastic Subspace Identification (SSI). This method allows to evaluate all modal parameters (frequencies, mode shapes and damping ratios) also in the case of closely spaced modes, overcoming the limitations of other OMA methods (e.g. Peak-Peaking method).

In recent years, a lot of researchers have analysed the main aspects of SSI methods and the applicability to different case studies. In [18], the SSI method has been used for model updating of a FE model of a cable-stayed footbridge. It has been applied on identification of a turbine blade in [19] and on an eleven spans concrete bridge subjected to weak environmental excitations in [20].

The identification results depend on some user-defined parameters, i.e. the number of block row  $i$  of the Hankel matrix, the model order  $n$ , the length of the acquired signal used in the identification process. In [21], the influence of the model order and the number of block rows on the accuracy and precision of modal parameter estimates has been investigated by using the SSI method, whereas in [22] a formula for determining the minimum value of the number of block row  $i$  of the Hankel matrix is derived. This formula is based on the lowest frequency of the system under test and on the sampling frequency of the acquired signals.

In this paper, two of the most important aspects of the SSI method have been investigated: the influence of the choice of the number of block rows  $i$  in Hankel matrix and the selection of the length of the data acquired using Stochastic Subspace Identification method (SSI Data-driven). Particularly, both aspects have been studied simultaneously in the identification process results.

The aim is to find an optimal value of these parameters in order to increase accuracy in the identification process and to reduce the computational burden. This last feature becomes important when OMA techniques are used for early detection of structural faults or into a continuous automated structural monitoring framework [23, 24].

In the first part of this paper, the SSI method and stabilization diagram for the evaluation of the real mode of vibration of the structure are explained. Then, a case study of a reinforced concrete building sited in Alcamo (Sicily, Italy) is introduced, on which the SSI method is used for the dynamical identification and the results are compared to those obtained by means of Enhanced Frequency Domain Decomposition Method (EFDD) [25, 26]. After a description of the structure, of the acquisition system and the data processing, the influence of the number of block rows  $i$  in Hankel matrix and of the length of the signal used in dynamic identification using SSI method are studied. Then, the results of the dynamical identification using SSI and EFDD methods are compared. Finally, some conclusions are drawn.

## 2 Data-Driven Stochastic Subspace Identification Method

Stochastic Subspace Identification algorithms have become very common in system identification in recent years. These techniques are very appealing because they are based on robust linear algebra instruments and on an elegant mathematical framework to identify the state-space matrix from the raw data. Moreover, the use of numerical linear algebra operations, like Singular Value Decomposition (SVD) and LQ decomposition, leads to a numerically efficient realization, so non-linear optimization problems are not necessary.

The dynamic behaviour of a structure can be expressed by the second order differential equations of motion expressed by:

$$[\bar{M}]\{\ddot{x}(t)\} + [\bar{C}]\{\dot{x}(t)\} + [\bar{K}]\{x(t)\} = \{f(t)\} \quad (1)$$

where  $\{\ddot{x}(t)\}$ ,  $\{\dot{x}(t)\}$ ,  $\{x(t)\}$  are the vectors of acceleration, velocity and displacement, respectively;  $[\bar{M}]$ ,  $[\bar{C}]$ ,  $[\bar{K}]$  denote the mass, damping and stiffness matrices,  $\{f(t)\}$  is the forcing vector.

State-space models allow to convert the second order problem, governed by Eq. (1), into two first order problems, defined by the state equation and observation equation, obtained from the equation of motion by some mathematical manipulations.

Defining the state vector as [17]:

$$\{s(t)\} = \begin{Bmatrix} \{\dot{x}(t)\} \\ \{x(t)\} \end{Bmatrix} \quad (2)$$

the state equation and the observation equation in discrete-time space model, in the case of OMA, can be written as:

$$\{s_{k+1}\} = [A]\{s_k\} + \{w_k\} \tag{3}$$

$$\{y_k\} = [C]\{s_k\} + \{v_k\} \tag{4}$$

where  $\{s_k\}$  is the discrete-time state vector yielding the sampled displacement and velocities;  $\{y_k\}$  is the sampled output,  $[A]$  is the discrete state matrix,  $[C]$  is the discrete output influence matrix,  $\{w_k\}$  and  $\{v_k\}$  are two unknown stochastic processes.

The matrices  $[A]$  and  $[C]$  are defined as:

$$[A] = e^{[A_c]\Delta t} \quad [C] = \left[ [C_v] - [C_a][\bar{M}]^{-1}[\bar{C}][C_d] - [C_a][\bar{M}]^{-1}[\bar{K}] \right] \tag{5}$$

where  $[C_a]$ ,  $[C_v]$  and  $[C_d]$  are the output location matrices for acceleration, velocity and displacement, respectively, and  $[A_c]$  is defined as:

$$[A_c] = \begin{bmatrix} -[\bar{M}]^{-1}[\bar{C}] & -[\bar{M}]^{-1}[\bar{K}] \\ [I] & [0] \end{bmatrix} \tag{6}$$

The algorithm starts from a block Hankel matrix constructed directly from the measured data, having  $2i$  block rows and  $j$  columns ( $j \rightarrow \infty$ ), where  $j$  are the samples. The output data are scaled by a factor  $1/\sqrt{j}$  to be consistent with the definition of correlation. Therefore, the Hankel matrix is:

$$\{H_{0/2i-1}\} = \frac{1}{\sqrt{j}} \begin{bmatrix} \{y_0\} & \{y_1\} & \dots & \{y_{j-1}\} \\ \vdots & \vdots & \ddots & \vdots \\ \{y_{i-1}\} & \{y_i\} & \dots & \{y_{i+j-1}\} \\ \{y_i\} & \{y_{i+1}\} & \dots & \{y_{i+j-1}\} \\ \vdots & \vdots & \ddots & \vdots \\ \{y_{2i-1}\} & \{y_{2i}\} & \dots & \{y_{2i+j-2}\} \end{bmatrix} \tag{7}$$

Starting from the Hankel matrix, the state matrix  $[A]$  and the output matrix  $[C]$  are determined through some mathematical operations detailed in [27].

The Eigen-Value Decomposition of  $[A]$  provides the modal parameters of the system:

$$[A] = [\Psi][\Lambda][\Psi]^{-1} \tag{8}$$

where the  $m$ -th column  $[\Psi_m]$  of  $[\Psi]$  is the  $m$ -th eigenvector of  $[A]$  and  $[\Lambda]$  collects the eigenvalues  $\mu$  of  $[A]$ . Conversely, the correspondent  $m$ -th mode shape of the system can be obtained from matrix  $[C]$  as:

$$[\Phi_m] = [C][\Psi_m] \tag{9}$$

Natural frequencies, damped frequencies and damping ratios of the continuous-time system are obtained from the eigenvalues, after their conversion from discrete time to continuous time, using the following formulas:

$$f_m = \frac{|\lambda_m|}{2\pi} \quad f_{d,m} = \frac{\text{Im}(\lambda_m)}{2\pi} \quad \xi_m = -\frac{\text{Re}(\lambda_m)}{|\lambda_m|} \quad (10)$$

with:

$$\lambda_m = \frac{\ln(\mu_m)}{dt} \quad (11)$$

The SSI method results depend on the number of block row  $i$  of the Hankel matrix, the length of the acquired signals, and consequently the number of samples  $j$ , used in identification process and the maximum model order  $n$ .

The number of block rows  $i$  determines the size of the Hankel matrix and consequently influences the evaluation of the state matrix  $[A]$ . Very low values of  $i$  produce a low redundancy of the measured response of the system on one hand; on the other hand, a too large value generates the occurrence of bias on the evaluation of modal parameters due to the presence of cluster of alignment of poles in the stabilization diagram and a significant increase of the computational time.

The length of the acquired signal, and consequently the number of samples  $j$  used in the identification process, is also very important, since it affects the number of columns of the Hankel matrix and consequently the evaluation of matrix  $[A]$ . From a statistical point of view, the number of sample points  $j$  can be infinite, but in real case a sufficiently high number of samples must be acquired, depending on the properties of the system.

The model order  $n$ , which is theoretically two-time the number of degrees of freedom of the system, is another user-defined parameter that can be set at the beginning of the identification process.

In practical applications, since the number of degrees of freedom of the system is unknown, a conservative approach is the over-specification of the model order  $n$ , which is set large enough to ensure the identification of all physical modes. The over-specification of the model order introduces spurious poles (noise modes or mathematical modes) that must be separated from the physical ones using the stabilization diagram, which shows the poles obtained for different model orders as a function of their frequencies. A very high value of model order should be avoided, since it may produce a difficult evaluation of the real poles of the system.

By tracking the evolution of the poles for increasing model orders, the physical modes can be identified from alignments of stable poles, since the spurious mathematical modes tend to be more scattered and typically do not stabilized [17]. The alignments of stable poles can start at lower or higher values of the model order, depending on the level of excitation of the modes, so a very low value of the model order should be avoided, too.

Only the poles which satisfy assigned user-defined stabilization criteria are labelled as stable.

In all analysis done, the following stability criteria have been used:

$$\left(\frac{|f_n - f_{n+1}|}{f_n}\right) < 0.01; \left(\frac{|\zeta_n - \zeta_{n+1}|}{\zeta_n}\right) < 0.05; MAC(\{\Phi_n\}, \{\Phi_{n+1}\}) > 0.95 \quad (12)$$

where  $f_n, f_{n+1}, \zeta_n, \zeta_{n+1}, \{\Phi_n\}, \{\Phi_{n+1}\}$  indicate natural frequencies, damping ratios and mode shapes obtained at order  $n$  and  $n + 1$ , respectively, while MAC is the Modal Assurance Criterion (bounded between 0 and 1) defined as [28]:

$$MAC(\{\Phi_i\}, \{\Phi_k\}) = \frac{|\{\Phi_i\}, \{\Phi_k\}|^2}{(\{\Phi_i\}^H \{\Phi_i\})(\{\Phi_k\}^H \{\Phi_k\})} \quad (13)$$

### 3 Case Study: Reinforced Concrete Building in Alcamo

The case study concerns a reinforced concrete building located in Alcamo (Sicily, Italy). It is composed by nine floors above ground and one basement floor, with an inter-storey height equal to 3,05 m, so the total height of the building is about 28 m.

The plant of the typical floor, although irregular, is about rectangular (ratio of the two side about 1:2) with a plant area of about 280 m<sup>2</sup>.

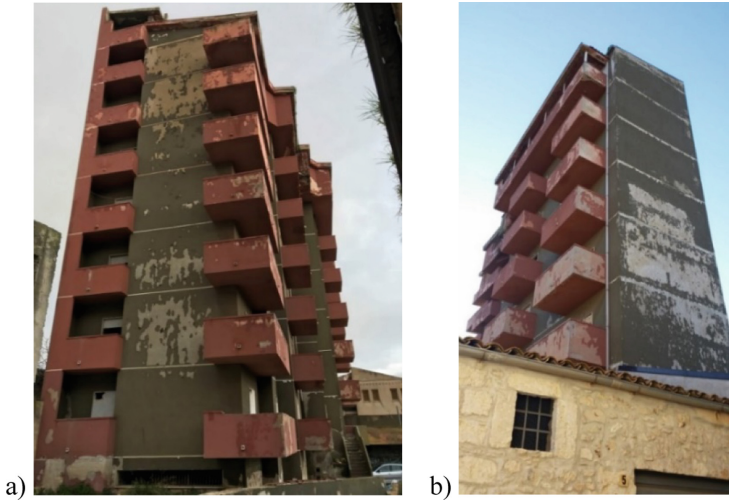
The bearing structure of the building is composed by reinforced concrete frames in both main directions. The building has a staircase, made by reinforced concrete and located in the central position of the deck, and an elevator shaft. Figure 1 shows two views of the building.

#### 3.1 Data Acquisition and Measurement Setup

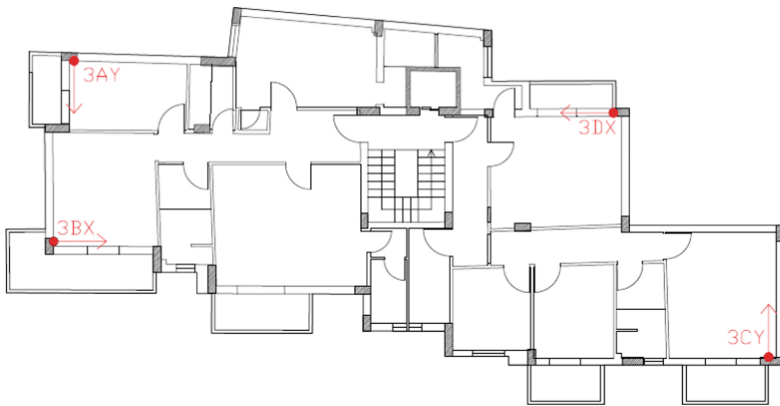
The location of the measurement sensors has been selected to clearly evaluate the first six mode shapes. The sensors have been positioned on 3th, 5th and 7th floors, in order to acquire the displacements in  $x$  and  $y$  directions and the rotations of each monitored floor. Figure 2 shows the location of the sensors for the 3th floor.

During the environmental vibrations test, 13 channels with 180 acquisitions of duration 50 s have been acquired, so the total duration of the test has been 9000 s (150 min). The acquired signals are sampled at 500 Hz using an application self-developed in LabView<sup>®</sup> environment.

The analogical signals from the accelerometers have been acquired, converted and saved on a hard disk to be processed. To avoid aliasing, Tukey window has been used to filter data in time domain. All the signals acquired have been filtered with low pass 4th order Butterworth filter, with cut-off frequency equal to 60 Hz.



**Fig. 1.** Pictures of the building under test.



**Fig. 2.** Position of the accelerometers at floor 3.

### 3.2 Identification of the Modal Parameters

Starting from the acquired signals, modal information of the building under test have been evaluated, for frequencies lower than 30 Hz.

The stabilization diagram is obtained for system order from 50 to 150, considering only the poles with damping ratios between 0 and 20%. The minimum order has been set to 50 since the building has eight floors, so in the hypothesis of infinite rigidity of the diaphragms, each one has three degrees of freedom, so 48 is the minimum value of the order to theoretically find all the 24 principal modes of the structure.

Figure 3 shows one of the stabilization diagrams obtained, in which stable poles are marked with black dots and un-stable poles with red dots. From this stabilization diagram, we can observe that for  $n \geq 130$ , all the alignments which correspond to the modes of vibration of the structure are identified, so  $n = 130$  can be considered as a good choice of model order of the system.

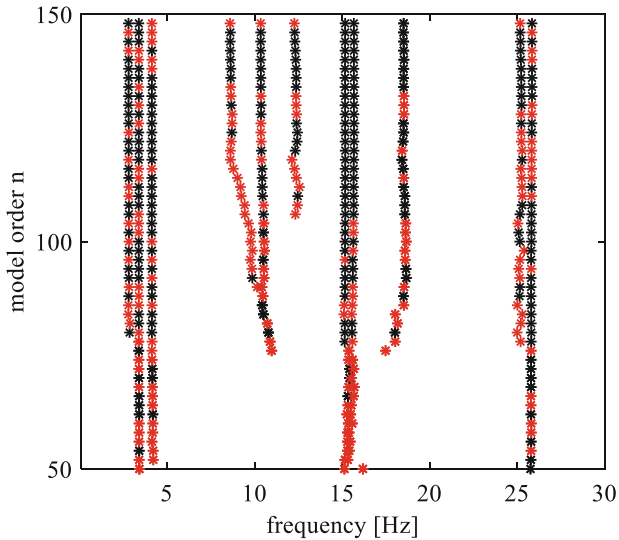


Fig. 3. Stabilization diagram obtained using SSI method.

### 3.3 Influence of the Number of Block Rows $i$ in Hankel Matrix and of the Length of the Signal on the SSI Method

In this section, two aspects of the SSI method have been investigated:

- the influence of the record length used in the SSI identification procedure for the evaluation of the modal parameters (natural frequencies, damping ratios);
- the influence of the number of block rows  $i$  of the Hankel Matrix on the evaluation of modal parameters (natural frequencies, damping ratios).

Since the aim of this paper is to study the influence of both user-defined parameters in the identification process, different lengths of the acquired signals have been used in the identification procedure (50 s, 100 s, 150 s, 200 s, 250 s, 300 s) and different values of the number of block rows  $i$  of the Hankel matrix have been set, from 80 to 300, using increment of 20.

In Table 1, considering all the different data lengths, mean natural frequencies and damping ratios, standard deviations and coefficients of variation, defined as the ratio between standard deviation and mean value, of the first eleven modes are reported for number of block rows  $i = 300$ . The results show that, in term of frequencies, the standard deviation and the variation coefficient are very small for almost all modes



identified, confirming the good accuracy on the evaluation of the natural frequency of the building. In term of damping, the findings confirm that the accuracy on the estimation of damping ratio is different between the identified modes.

In Fig. 4, the variation of the natural frequencies of the first six modes of vibration, evaluated as a function of the number of block row  $i$  and the length of the acquired data, are depicted. From this figure, it can be observed that for well excited modes, such as modes 2 and 3, the mean value of the frequencies (black line in Fig. 4) obtained for different acquisition lengths remains almost constant by varying  $i$ , whereas the interval of standard deviation (red lines) from mean decreases, ensuring a reduction of the uncertainties. Therefore, for low values of  $i$  only well excited modes can be detected using the SSI method, while to estimate the frequencies of not well excited modes (e.g. modes 4 and 6) it is necessary to increase the number of block rows. Specifically, for values of  $i < 140$  is not possible to identify all naturals frequencies of the building under consideration, thus for the identification of the model parameters  $i = 300$  has been set.

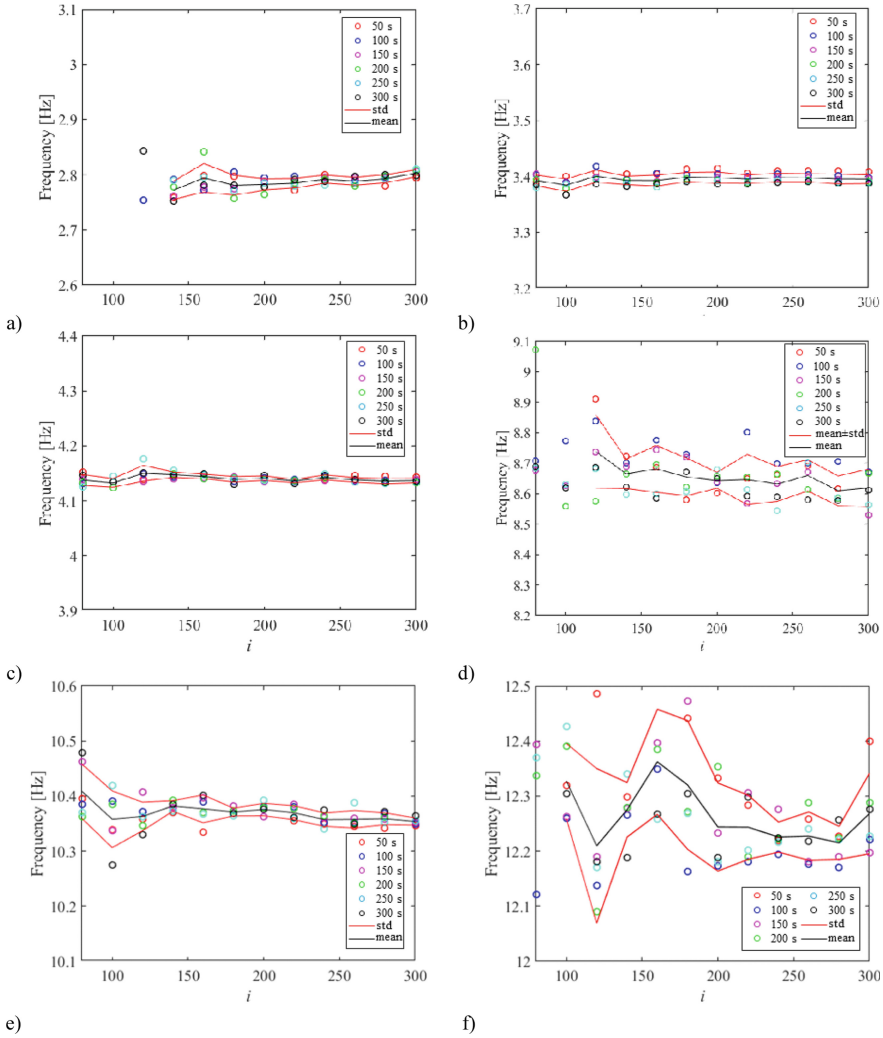
**Table 1.** Mean natural frequencies and damping ratios, standard deviations and coefficient of variation for  $i = 300$ .

Mode	Frequency			Damping ratio		
	Mean [Hz]	Standard deviation [Hz]	Coefficient of variation	Mean	Standard deviation	Coefficient of variation
1	2.802	0.0065	$2.32 \times 10^{-3}$	0.0279	0.00850	0.30466
2	3.395	0.0078	$2.29 \times 10^{-3}$	0.0043	0.00047	0.10930
3	4.137	0.0040	$9.67 \times 10^{-4}$	0.0070	0.00150	0.21428
4	8.618	0.0615	$7.13 \times 10^{-3}$	0.0424	0.00760	0.17924
5	10.368	0.0065	$6.26 \times 10^{-4}$	0.0119	0.00360	0.30252
6	12.268	0.0730	$5.95 \times 10^{-3}$	0.0426	0.01010	0.23709
7	15.179	0.0101	$6.65 \times 10^{-4}$	0.0036	0.00092	0.25556
8	15.662	0.0069	$4.41 \times 10^{-4}$	0.0061	0.00010	0.01639
9	18.499	0.0199	$1.07 \times 10^{-3}$	0.0123	0.00014	0.01138
10	25.229	0.0479	$1.89 \times 10^{-3}$	0.0145	0.00069	0.04759
11	25.842	0.0147	$5.69 \times 10^{-4}$	0.0014	0.00022	0.15714

From the exposed results, to obtain a good estimate of natural frequencies of the system, a record length at least of 50 s (140 times the first natural period of the structure) can be selected.

Figure 5 shows the variation of the damping ratios of the first 6 modes with the number of block rows  $i$  and the length of the acquisition data. The ranges of the latter parameters are the same used in the previous analysis.

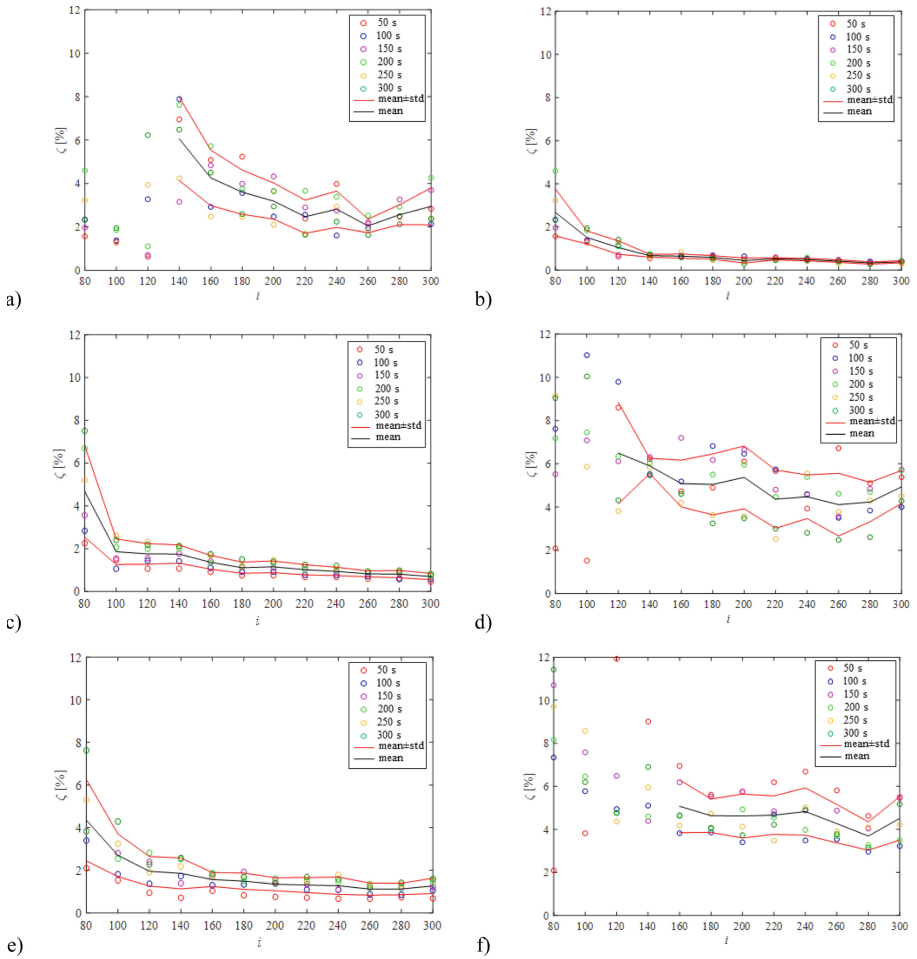
In this case, it can be observed that well excited modes have damping ratios almost independent from the length of the acquisition used and constant for values of  $i$  greater than 160. For not well excited modes, e.g. mode 4 and 6, the damping ratio is not easy to identify due to its variability throughout the examined domain.



**Fig. 4.** Natural frequency estimated for different value of  $i$  and different data length: (a) mode 1, (b) mode 2, (c) mode 3, (d) mode 4, (e) mode 5, (f) mode 6.

### 3.4 Comparison Between SSI Method and EFDD Method

In order to evaluate the accuracy of the SSI method in estimating the modal parameters, a comparison with the EFDD method has been performed. The natural frequencies and damping ratios obtained by using the two identification methods are listed in Table 2. Parameters obtained using EFDD method have been evaluated from the first singular value of the mean Power Spectral Density (PSD) matrix calculated for record length of 500 s and considering all 150 min of data acquisition. As concerns the frequencies, very good agreement between the two methods has been obtained, since relative errors



**Fig. 5.** Damping ratio trend for different length of the data and for  $i$  ranging from 80 to 300: (a) mode 1, (b) mode 2, (c) mode 3, (d) mode 4, (e) mode 5, (f) mode 6.

are less than 1% except for mode 4. Damping ratio values identified by using the two methods are quite different, pointing out the challenge to determine these parameters with different methods.

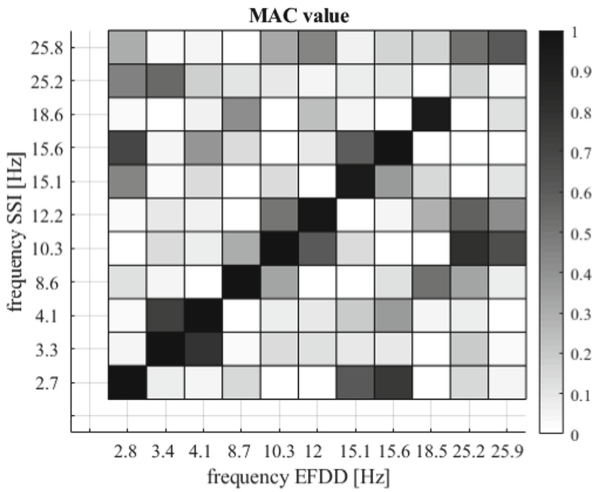
The mode shapes evaluated using SSI are the same of the ones obtained with EFDD, since MAC values are about 0.99 for all the first six modes, as shown in Fig. 6. These values confirm the good agreement between the two methods used.

Finally, the first six modes shapes are shown in Fig. 7. In all of them, black lines represent the un-deformed configuration, blue lines represent the displacements of the third floor, green lines represent the displacements of the fifth floor and red lines represent the displacements of the seventh floor.

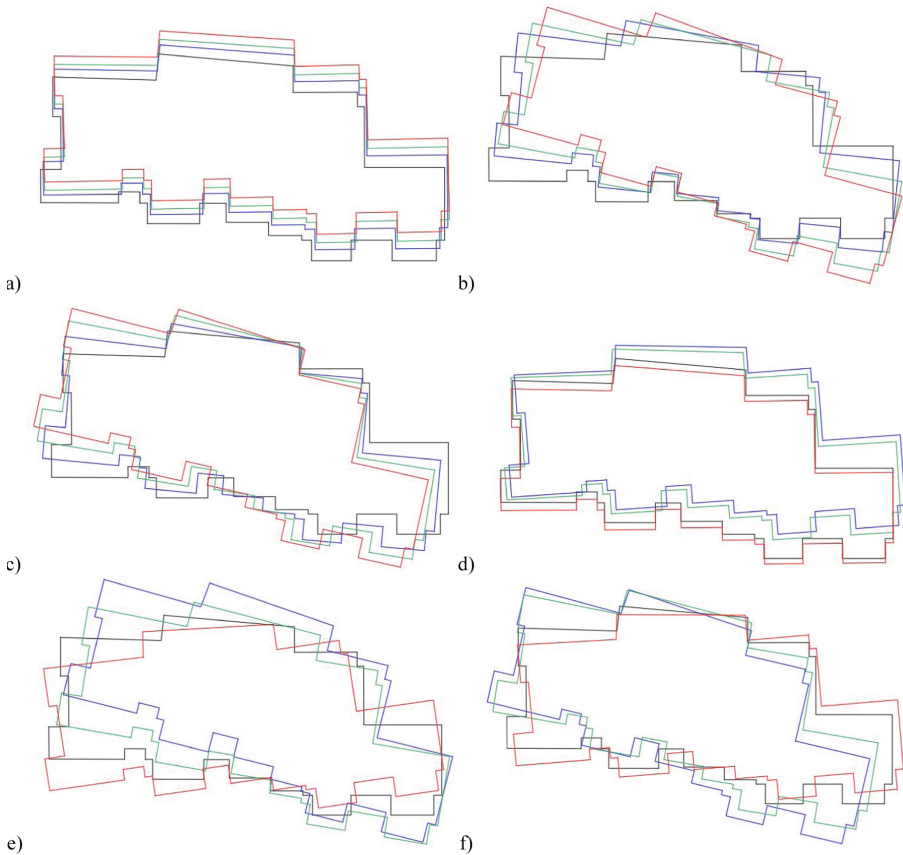
In particular, Fig. 7 illustrates the first and second modes, which are bending modes in  $x$  and  $y$  direction, respectively, the third and fourth mode, which are torsion mode and the second bending mode in  $y$  direction, the fifth and sixth mode of vibration, which are the second bending mode in  $x$  direction and the second torsion mode.

**Table 2.** Natural frequencies and damping ratios obtained using SSI and EFDD methods.

Mode	Frequency [Hz]		Damping ratio [%]	
	SSI	EFDD	SSI	EFDD
1	2.80	2.81	2.79	0.51
2	3.39	3.39	0.43	0.99
3	4.14	4.15	0.70	0.85
4	8.62	8.77	4.24	2.86
5	10.27	10.37	1.19	2.15
6	12.27	12.16	4.26	6.22
7	15.18	15.15	0.36	0.37
8	15.66	15.70	0.61	3.25
9	18.50	18.51	1.23	2.70
10	25.23	25.24	1.45	0.38
11	25.84	25.92	0.14	0.70



**Fig. 6.** MAC values for mode shapes between SSI and EFDD method.



**Fig. 7.** Mode shapes: (a) mode 1, (b) mode 2, (c) mode 3, (d) mode 4, (e) mode 5, (f) mode 6.

## 4 Conclusions

In this paper, the influence of the number of block rows  $i$  in Hankel matrix and the length of the acquired data in the use of Stochastic Subspace Identification method (SSI) are investigated considering a case study of a real scale reinforced concrete building located in Alcamo (Sicily, Italy).

The reported results have shown the high importance of the choice of user-defined parameters in the identification process especially for damping estimation.

The main results obtained can be listed below:

- not always one of the main hypothesis of OMA, regarding the assumption of white noise excitation of environmental vibration, is valid. For this reason, not all modes of the structure can be excited in the same manner and, consequently, not all the modal parameters can be computed with the same accuracy;

- to obtain a good estimate of natural frequencies of the system, a record length greater than 140 times the first natural period can be selected;
- increasing the number of block rows  $i$ , the damping ratios tend to an approximately constant value, in the case of well excited modes (e.g. modes 2 and 3), independently from the record length; for not well excited modes, the scatter between values evaluated for different record length remains quite high, independently of the value of  $i$  set;
- increasing the number of block rows  $i$  produces an improvement of the accuracy on the evaluation of modal parameters (natural frequencies and damping ratios);
- for well excited modes (e.g. modes 2 and 3) a good value of  $i$  can be 200, while a good choice of the system model order  $n$  could be 130.

Future developments will regard the study of the influence of the system properties (e.g. mass, stiffness, material) on the selection of user-defined parameters using the SSI method, for modal parameters evaluation, in order to establish a standard criterion valid for all type of system.

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