On the Modelling of the Hysteretic Behaviour of Wire Rope Isolators

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Abstract. Wire Rope Isolators are made of two parallel retaining plates, connected through metallic wire ropes. Due to their good performances as vibration isolators, and shock absorbers, these devices have been widely employed in industrial applications. The dynamic behaviour of Wire Rope Isolators is strongly affected by both geometric and material non-linearities, mainly due to the peculiar hysteretic bending behaviour of metallic ropes. In this work a typical approach to characterize the hysteretic behaviour of wire rope isolators, based on a semi-empirical phenomenological model, is compared to a different approach based on a beam-like description of the wire rope and on a nonlinear formulation of the cross sections cyclic bending behaviour. The hysteretic cross-sectional model is then implemented within a corotational beam finite element, to fully account for the geometric non-linearities which characterize the response of the device. The performance of the proposed models are assessed through a comparison with the results of a well documented experimental test.

Keywords: Wire rope dampers · Base isolation · Wire rope modelling

1 Introduction

Wire rope isolators (WRI) are made of two parallel retaining plates or bars, connected through a metallic cable (wire rope), see e.g. Fig. 1. The cable is often arranged in a nearly-helical shape, although different geometric configurations can also be adopted. Due to their good performances as vibration isolators and shock absorbers, the wire rope isolators have been widely employed in industrial applications to support equipment and secondary structures [20]. Applications of this technology to the seismic isolation of lightweight civil structures have also been envisaged and some research on this subject is recently surfacing in the literature (see e.g. [19,21]).

The dynamic behaviour of wire rope isolators is strongly affected by both geometric and material non-linearities. The latter are mainly due to the peculiar

A. Carcaterra et al. (Eds.): AIMETA 2019, LNME, pp. 1535–1542, 2020.
https://doi.org/10.1007/978-3-030-41057-5_124

hysteretic bending behaviour of metallic cables. Relative displacements between the wires of the cable, indeed, can occur during flexural vibrations, depending on the value of the vibration amplitude. These internal sliding phenomena are associated with frictional dissipation (see e.g. [7–10]), which makes the dynamic response of the device inherently non-linear and non-holonomic.

The typical approach adopted in the literature to characterize the hysteretic behaviour of wire rope isolators (see e.g. [17,20]) is based on semi-empirical phenomenological models. These models allow to characterize the response of the devices with respect to simple loading cases (typically along a set of three coordinate direction, i.e. the vertical, shear and roll directions, see also Fig. 1).

A different modelling approach is pursued in this work. The proposed model is based on a beam-like description of the wire rope and on a nonlinear formulation of the cross sections cyclic bending behaviour. At the cross-sectional level, the mechanical behaviour of the wire rope is described by extending the mechanical formulation for the hysteretic bending of stranded cables developed in [11], which has been recognized as adequate to represent the local behaviour mainly controlled by interwire sliding processes. The hysteretic cross-sectional model is then implemented within a corotational beam finite element to fully account for the geometric non-linearities which characterize the response of the device. The performance of the proposed models are assessed through comparisons with the results of a well documented experimental test of the literature.

2 Black Box Approaches for WRIs

The typical approach adopted in the literature to characterize the hysteretic behaviour of wire rope isolators (see e.g. [17,20]) is based on semi-empirical phenomenological models. These models, however, allow to characterize the response of the devices only with respect to simple loading cases (typically along a set of three coordinate direction, i.e. the vertical, shear and roll directions). Indeed, the study conducted by Demedriades et al. [3] shows that the hysteretic cycles in both shear and roll directions are symmetric, while the vertical direction has an asymmetric cycle, which marks a clear difference between compression and tensile behavior.

A survey of the literature on WRI shows as the Bouc-Wen (BW) model [2,4,15,22] is the favorite one for modeling (with some modifications made necessary as it will be explained in the following) these devices, since it is able to efficiently describe hysteretic systems and is mathematically easy to understand. A disadvantage of this model is that the model parameters need to be identified for each loading direction (see Fig. 1), which increases their number and the difficulty to associate them a physical meaning in terms of geometrical and mechanical properties of the specimen.

Moreover, the asymmetry of the behavior in the vertical mode will make the simple Bouc-Wen model not adequate. Hence, the common approach in literature is to introduce a modulating function F_2 that "weights" in different ways the output from a BW model, depending it is for an input that has one sign or the other.

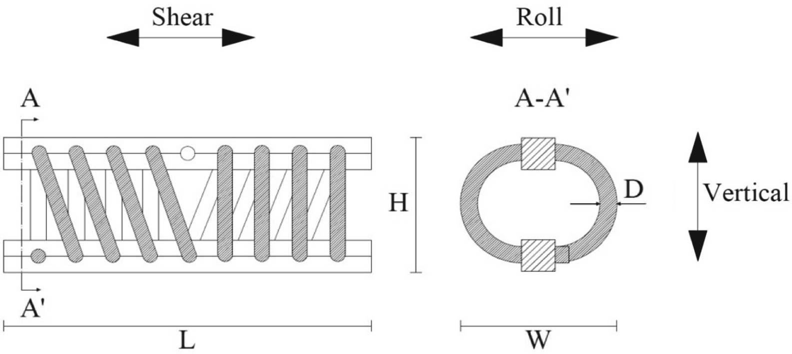

Fig. 1. Schematic representation of a wire rope isolator.

As an example, in this work a black box approach based on a five parameters BW model [16] and a power modulating function F_2 proposed by Ni et al. [17], has been adopted as a reference modeling option to be compared with a potentially more advanced mechanical model.

By denoting with F the restoring force provided by the WRI, with x the work-conjugated displacement of the WRI and with z the hidden variable of the hysteretic BW model, the adopted model can be expressed as:

$$F\left(x(t), z(t)\right) = F_2\left(x(t)\right) F_1\left(x(t), z(t)\right) \tag{1}$$

$$F_1\left(x(t), z(t)\right) = k_1 x(t) + (k_2 - k_1) x_0 z(t) \tag{2}$$

$$F_2\left(x(t)\right) = b^{cx(t)} \tag{3}$$

$$\dot{z}(t) = \frac{1}{x_0}\left(\dot{x}(t) - \sigma\left|\dot{x}(t)\right|\left|z(t)\right|^{n-1} z(t) + (\sigma - 1)\dot{x}(t)\left|z(t)\right|^{n}\right) \tag{4}$$

where a dot denotes derivation with respect to a time-like variable t, and $\{x_0, \sigma, n, k_1, k_2, b, c\}$ are the model parameters.

3 Mechanical Model of WRIs

Metallic wire ropes are structural elements whose construction process follows a strict hierarchy. Their internal structure can be described following a top-down approach. Restricting the attention to the wire rope in Fig. 2, and recalling that the approach can be easily generalized to other configurations, starting from the top of the hierarchy the following levels are identified: the rope itself, the strands and the wires. The internal structure of the rope is completely defined by the centerline and the orientation of the cross section of every element at each level.

On each element a local reference system is defined introducing on its centerline the Serret-Frenet moving frame. The strands are helically twisted and grouped in concentric layers to form the rope and the same process forms the strand from the wires. Accordingly, the centroidal line of each element is described as a cylindrical helix in the frame of reference of the respective upper-level element (see e.g. [5]).

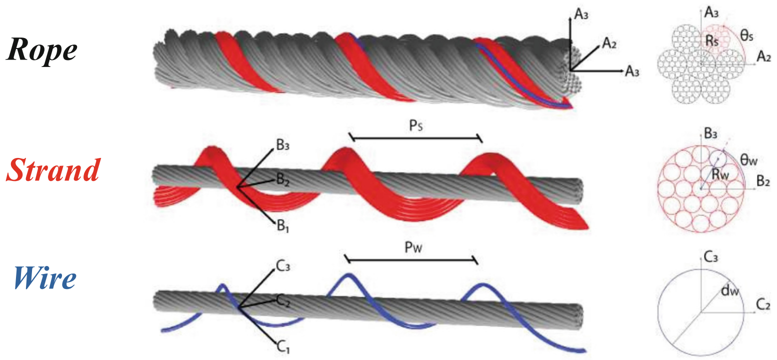

Fig. 2. Schematic representation of the internal structure of a metallic wire rope.

The axial-torsional response or wire ropes subjected to typical service load conditions can be idealized as being linear elastic and decoupled from the bending response. Several models have been proposed in the literature to characterized the stiffness of metallic strands and wire ropes starting from the knowledge of the geometry and mechanical properties of the constituents (see e.g. [6,13,18]). The bending behavior, on the other hand, is markedly non linear and non-holonomic due to the possible activation of the sticking/sliding frictional interfaces between the wires [7–11].

3.1 Moment-Curvature Law for the Rope Cross-Section

Whenever the rope is bent, an axial force gradient is generated along the length of the wires. This gradient makes the wires prone to sliding with respect to the neighbouring ones, and is counteracted by the tangential friction forces acting on the internal contact surfaces between the wires.

A typical moment-curvature curve of a metallic wire rope is depicted in Fig. 3 for a curvature that increases monotonically. At large values of the bending curvature $\chi = \chi(t)$ the bending problem is non-linear and controlled by the gross-sliding of the contact surfaces between the wires of the strand; the wires are said to be in a "full-slip" state with respect to the neighboring ones and the tangent bending stiffness of the strand reaches its minimum value EI_{min}.

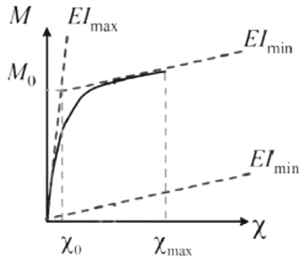

Fig. 3. Typical cross-sectional bending moment-curvature ($M-\chi$) diagram for a metallic wire rope.

At small values of the curvature, friction forces are large enough to overcome the axial force gradient along the wires, and all the wires are stuck together ("full-stick" state). In this state the cross section can be modeled according to a plane section hypothesis, leading to the maximum value of the bending stiffness, which is denoted as EI_{max}. A first estimate of the values of the stiffness EI_{min} and EI_{max}can be obtained as in [5].

As already pointed out in [10], it is handy to introduce a curvature value χ_0 that allows to define a bi-linear approximation to the moment-curvature curve; χ_0 will depend on the friction coefficient between the rope constituents and will be linearly dependent on the axial force T (see e.g. [8,10]). As a simplifying assumption, whose validity will be later assessed against experimental data, the dependence of χ_0 on T will be in the following disregarded.

Focusing on a rope section in tension the moment-curvature curve is symmetric and can be described, as proposed in [11], through the following five-parameter BW model:

$$M\left(\chi(t), z(t)\right) = EI_{min}\chi(t) + (EI_{max} - EI_{min})\chi_0 z(t) \tag{5}$$

$$\dot{z}(t) = \frac{1}{\chi_0}\left(\dot{\chi}(t) - \sigma\left|\dot{\chi}(t)\right|\left|z(t)\right|^{n-1} z(t) + (\sigma - 1)\dot{\chi}(t)\left|z(t)\right|^{n}\right) \tag{6}$$

The set of model parameters that describe the rope cross-section bending behaviour is then $\{\chi_0, \sigma, n, EI_{max}, EI_{min}\}$.

3.2 The Corotational Beam Element

Equations (5) and (6), along with a linear model for the axial-torsional response, fully define the relation between generalized stress and strain variables of the strand cross section whenever embedded in a plane Euler-Bernoulli beam. The proposed constitutive equations have been implemented within a corotational beam element previously developed in [5,12,14] to study the static and dynamic response of flexible structures, taking into account geometrical and material nonlinearities.

4 Numerical Applications

The modeling approaches described in the previous Sections have been applied to the study of a WRI device, similar to the one Fig. 1, tested by Balaji et al. [1]. The geometrical characteristic of the chosen WRI are (see Fig. 1): wire rope diameter $D = 12$ mm, number of coils $N = 12$, width of plate $W = 120$ mm, length of plate $L = 215$ mm, height of the device $H = 127$ mm. The parameters of both the black-box and mechanical models have been identified to match the experimental results reported in [1], and are listed in Tables 1 and 2.

Figures 4(a) and (b) show the numerical and experimental results for two different values of the maximum imposed displacements (±5 mm and ±10 mm) in the case of testing in direction "vertical" of Fig. 1. The parameters for both models were identified on the experimental cycle of ±5 mm amplitude, and used unaltered to predict the response for the lager amplitude cycle.

The outcomes from the black-box approach are in very good agreement with the experimental data. The limit of this approach, however, is that a set of parameters has to be identified for each basic deformation mode (i.e. roll, shear, vertical), which leads to errors in the case of complex loading scenarios that involve a combination of the basic deformation modes of the WRI.

The outcome form the mechanical model properly reproduces in tension the experimental response for the ±5 mm test, used in the calibration of this model (see Fig. 4(a)), but is not able to fully reproduce the response in compression. The reason has been traced not to geometrical effects at the global level, which are correctly captured by the corotational finite element formulation, but to the cross-sectional bending behaviour. Indeed, experimental results show that this depends on the axial force in a different way if the force is of tension or of compression. The outcome depicted in Fig. 4(b), for a cycle of larger amplitude (±10 mm), highlights the shortcomings of having considered χ_0 independent from the value of the axial force (both in tension and in compression). This hypothesis that assigns to χ_0 a role similar to that of the first yielding curvature in a bi-linear elastic-plastic moment-curvature law, leads to neglect the role of the axial force on the internal contact pressures. These, in turn, control the sticking-sliding transition of the wire-to-wire contacts, and hence both the dissipated energy and the value of the tangent stiffness.

Table 1. Identified values of the parameters for the black-box model.

k_1 [kg/mm]	k_2 [kg/mm]	x_0 [mm]	σ [-]	n [-]	b [-]	b [-]
8.5	100	0.2	3	1	1.4	0.1

Table 2. Identified values of the parameters for the mechanical model.

EI_{min} $[Nm^2]$	EI_{max} $[Nm^2]$	χ_0 $[m^{-1}]$	σ [-]	n [-]
4.9	37.5	0.09	1	1

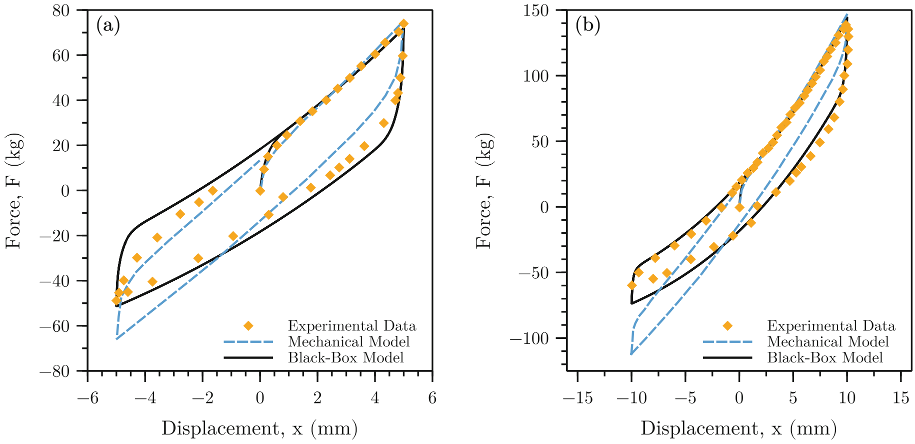

Fig. 4. Comparison between experimental and numerical results: (a) imposed displacement: ±5 mm; (b) imposed displacement ±10 mm. Experimental data are from [1].

5 Conclusions

In this work a typical semi-empirical phenomenological (black-box) model to characterize the hysteretic behaviour of wire rope isolators is compared to a different modeling approach, based on a beam-like description of the wire rope and on a nonlinear formulation of the cross sections cyclic bending behaviour.

The performance of the proposed models, assessed through a comparison with the results of a well documented experimental test, has highlighted a good agreement of the outcome from the phenomenological model with the experimental data. The limit of this approach being that it requires a different calibration for each loading direction (i.e. roll, vertical and shear direction).

The mechanical model, while being potentially more general, suffers in its present formulation from: (a) not being able to fully reproduce the response in compression and (b) not being able to fully account for the dependence of the dissipated energy and the value of the tangent stiffness on the value of the axial force. These last aspects are at present under development.

Acknowledgments. This work has been partially supported by MIUR (Italian Ministry of Education, University and Research) under the project "PRIN 2015–2018 Identification and monitoring of complex structural systems".

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