Determination of Dynamic Errors in Machines with Elastic Links

Yuri A. Semenov and Nadezhda S. Semenova

Abstract One of the important objectives in the design of machines is the reduction of dynamic errors caused by elastic vibrations of actuating mechanisms in a steady state and transient conditions. This objective is especially relevant in the production of high-performance machines, such as industrial robots, positioning stages, and others. The open kinematic structure of actuating mechanisms results in a significantly lower stiffness of the structure and greater dynamic loads, which in turn leads to intensive vibrations of operating elements in transient conditions. Furthermore, dynamic errors caused by free vibrations several times exceed static positioning errors of actuating mechanisms. Therefore, conventional methods of reducing dynamic errors with the use of flywheels, counterbalance mechanisms, shock absorber, dynamic dampers and other passive means do not always prove to be efficient. Instead feedback control systems have broader functional capabilities.

Keywords Chain system with the fixed end · Dynamic errors · Frequency equation · Fundamental mode

1 Introduction

In modern machines, gears remain the primary way of transmitting power. This is due to their small size, well-developed manufacturing technology, the ability to accurately provide the required gear ratio. However, it is known that gears are a source of internal vibration activity of the machine unit. The measure of this vibroactivity is the perturbing moments caused by the rigidity of the gearing, the kinematic error of the gear, and leading to dynamic errors. Under dynamic error understand deviations of laws of movement of links from their program values. The vibrations caused by them can lead to both the opening of the mating profiles of the teeth, and to the shifting of the lateral gaps between the teeth of the wheels. Dynamic loads arising in the drive, can thus be several times higher than the load from the moment of the

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resistance forces applied to the working body of the machine and determined from the strength calculation of the transmission.

2 The Building of a Machine Aggregate Simulation Model

Earlier machine aggregate simulations incorporated a model of a mechanism with rigid links [1]. However, in practice, structural elements of links and kinematic joints transform under the action of static and dynamic loads emerging in a steady state and in motion. As a result, laws of motion for a machine's operating elements differ from laws of motions imposed exclusively by engines. Therefore, one of the primary objectives of machine dynamics is identification of static and dynamic errors caused by the transformation of links and kinematic joints.

First, amplitudes of dynamic errors of a machine aggregate resulting from the flexibility of its bearings should be estimated [2–7]. Figure 1 shows a dynamic model of a machine aggregate. Gears 1 and 2 of a single-stage gear reducer and the actuating element 3 are set into rotation by the engine rotor 0. The figure depicts elements that are considered elastic; c_{01}, c_{12}, c_{23} are their stiffness; b_{01}, b_{12}, b_{23} are the damping coefficients; J_0, J_1, J_2, J_3 are the moments of inertia of the masses relative to their axes of rotation; M_0 and M_c are the driving torque and the drag torque respectively. The bending flexibility of shafts and the flexibility of the bearings shall be neglected.

The system under consideration has four degrees of freedom. Absolute rotation angles of the engine rotor q_0, gears $q_1 - q_2$ and the actuating element q_3 can be chosen as generalized coordinates. It should be more convenient to align these coordinates to the axis of the engine rotor, thus introducing new generalized coordinates:

$$\varphi_0 = q_0; \ \varphi_1 = q_1; \ \varphi_2 = i_{12} q_2; \ \varphi_3 = i_{12} q_3, \qquad (1)$$

where i_{12} is a transmission ratio of the gearing. Deformations of shafts and gears connected to driving wheels can be defined as:

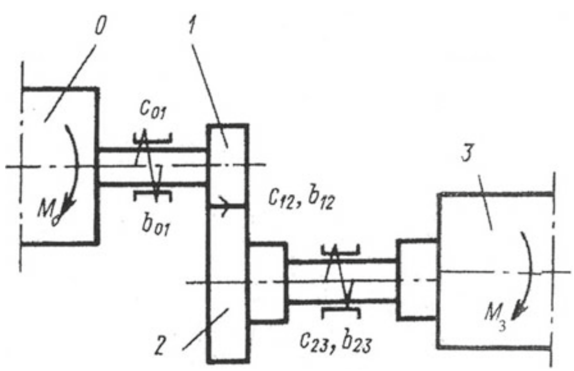

Fig. 1 A dynamic model of a machine aggregate

$$\theta_{01} = \varphi_1 - \varphi_0;\ \theta_{12} = r_{b2}q_2 - r_{b1}q_1 = r_{b1}(\varphi_2 - \varphi_1);\ \theta_{23} = i_{12}^{-1}(\varphi_3 - \varphi_2), \quad (2)$$

where r_{b1}, r_{b2} are radii of the base circles.

Next, kinetic and potential energies and the dissipative functions should be determined to generate Lagrangian equations of the second order.

The kinetic energy of the system is defined as:

$$T = \frac{1}{2}\sum_{s=0}^{3} J_s \dot{q}_s^2 = \frac{1}{2}\sum_{s=0}^{3} J_s \left(\frac{\dot{\varphi}_s}{i_{0,s}}\right)^2 = \frac{1}{2}\sum_{s=0}^{3} J_{s*}\dot{\varphi}_s^2 \quad (3)$$

where

$$J_{0*} = J_0;\ J_{1*} = J_1;\ J_{2*} = i_{12}^{-2}J_2;\ J_{3*} = i_{12}^{-2}J_3 \quad (4)$$

are the moments of inertia of links of the transmission device adduced to the rotation axis of the engine rotor.

The potential energy of the system is defined as:

$$\Pi = \frac{1}{2}\sum_{s=1}^{3} c_{s-1,s}\theta_{s-1,s}^2 = \frac{1}{2}\sum_{s=1}^{3} c_{s*}(\varphi_s - \varphi_{s-1})^2 \quad (5)$$

where

$$c_{1*} = c_{01};\ c_{2*} = r_{b1}^2 c_{12};\ c_{3*} = i_{12}^{-2}c_{23}. \quad (6)$$

The dissipative functions of system are defined as:

$$\Phi = \frac{1}{2}\sum_{s=1}^{3} b_{s-1,s}\dot{\theta}_{s-1,s}^2 = \frac{1}{2}\sum_{s=1}^{3} b_{s*}(\dot{\varphi}_s - \dot{\varphi}_{s-1})^2, \quad (7)$$

where

$$b_{1*} = b_{01};\ b_{2*} = r_{b1}^2 b_{12};\ b_{3*} = i_{12}^{-2}b_{23}. \quad (8)$$

External forces in a mechanical system are the driving torque M_0, applied to the engine rotor, and the drag torque M_c, applied to the actuating element. The elementary work of external forces in the virtual deformation of the system should be expressed as:

$$\delta W = M_0\delta q_0 + M_c\delta q_3. \quad (9)$$

The generalized drag force for the generalized coordinate φ_3 can be derived as: $M_3 = i_{12}^{-1}M_c$.

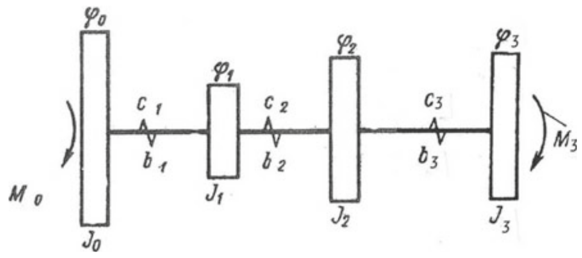

Fig. 2 The reduced model of a free chain system

By applying the following Lagrange equations:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\varphi}_s}\right) - \frac{\partial T}{\partial \varphi_s} = -\frac{\partial \Pi}{\partial \varphi_s} - \frac{\partial \Phi}{\partial \dot{\varphi}_s} + M_s, \tag{10}$$

it is apparent that ($s = 0, 1, 2, \ldots, 3$):

$$J_s\ddot{\varphi}_s + b_s(\dot{\varphi}_s - \dot{\varphi}_{s-1}) - b_{s+1}(\dot{\varphi}_{s+1} - \dot{\varphi}_s) + c_s(\varphi_s - \varphi_{s-1}) - c_{s+1}(\varphi_{s+1} - \varphi_s) = M_s. \tag{11}$$

An asterisk is omitted in the system parameters for clarity and $c_0 = b_0 = c_4 = b_4 = 0$, are applied.

The derived differential Eq. (11) describe motions in a transmission device as belonging to a system of perfectly rigid bodies interconnected in series by instantaneous elastic and dissipative elements. As each rigid body has one degree of freedom, such a one-dimensional model can be considered as a chain system. Figure 2 shows a four-mass oscillating system compatible with the differential Eq. (11).

It is possible to produce a set of equations for obtaining $\varphi_0 - \varphi_3$ and the driving torque when combining these equations with an engine performance equation M_0.

The equations of motion in a mechanical system can also be written in another way if the law of motion for the engine rotor is known. In this case, an equation can be easily derived from the equations of motion (11), where $s = 0$:

$$J_0\ddot{\varphi}_0 - b_1(\dot{\varphi}_1 - \dot{\varphi}_0) - c_1(\varphi_1 - \varphi_0) = M_0$$

Next, deformation coordinates should be considered that define the shifting of masses relative to the engine rotor:

$$\theta_s = \varphi_s - \varphi_0(t). \tag{12}$$

Finally, the equations of the system motion can be written as:

$$\left.\begin{array}{c} J_s\ddot{\theta}_s + b_s(\theta_s - \dot{\theta}_{s-1}) - b_{s+1}(\dot{\theta}_{s+1} - \dot{\theta}_s) + c_s(\theta_s - \theta_{s-1}) \\ -c_{s+1}(\theta_{s+1} - \theta_s) = M_s - J_s\ddot{\varphi}_0(t),\ \ s = 1, 2, 3, \\ J_0\ddot{\varphi}_0 - b_1\dot{\theta}_1 - c_1\theta_1 = M_0. \end{array}\right\} \tag{13}$$

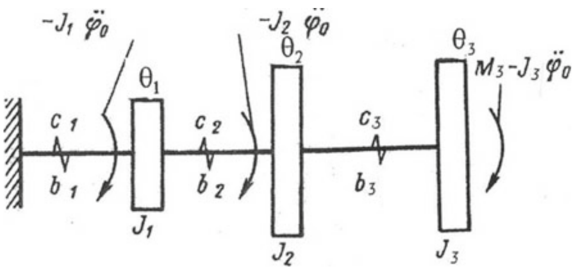

Fig. 3 The reduced model of a chain system with the fixed end

where $\theta_0 = \dot{\theta}_0 = 0,\ c_4 = b_4 = 0,\ M_1 = M_2 = 0.$ are taken into account.

Figure 3 demonstrates a chain oscillating system. Its equations of forced oscillations caused by the applied active inertia forces and the forces of moving space $-J_s\ddot{\varphi}_0(t)$ coincide with Eq. (13). The system depicted in Fig. 2 will be further called a free system, while the system in Fig. 3 will be called a system with the fixed left end [8–25].

3 Determination of Dynamic Errors in Transient Conditions

First, the equations of motion (13) should be presented in a matrix:

$$\begin{array}{c} J\ddot{\theta} + B\dot{\theta} + C\theta = M - J\ddot{\varphi}_0 \cdot 1, \\ J_0\ddot{\varphi}_0 - b_1\dot{\theta}_1 - c_1\theta_1 = M_0, \end{array}. \tag{14}$$

where $\theta = (\theta_1, \theta_2, \theta_3)^T$ is a three-dimensional column matrix, the generalized force matrix is $M = (0, 0, M_3)^T$, $1 = (1, \ldots, 1)^T$ is a single column, and three-dimensional symmetric matrices of dissipative and elastic system parameters are:

$$J = \begin{pmatrix} J_1 & 0 & 0 \\ 0 & J_2 & 0 \\ 0 & 0 & J_3 \end{pmatrix};\ B = \begin{pmatrix} b_1 + b_2 & -b_2 & 0 \\ -b_2 & b_2 + b_3 & -b_3 \\ 0 & -b_3 & b_3 \end{pmatrix};$$

$$C = \begin{pmatrix} c_1 + c_2 & -c_2 & 0 \\ -c_2 & c_2 + c_3 & -c_3 \\ 0 & -c_3 & c_3 \end{pmatrix}.$$

Next, based on the generalized coordinates of a mechanical system, principal coordinates can be calculated with the help of a linear conversion:

$$\theta = \sum_{m=1}^{3} h_m z_m. \tag{15}$$

As columns h_m, which are fundamental modes, are linearly independent, Eq. (15) establishes one-to-one correspondence between generalized coordinates θ_m and principal coordinates z_m.

By introducing (15) into the first matrix system Eq. (14), the following equation can be generated:

$$J\sum_{m=1}^{3} h_m \ddot{z}_m + B\sum_{m=1}^{3} h_m \dot{z}_m + C\sum_{m=1}^{3} h_m z_m = M - J\ddot{\varphi}_0 \cdot 1.$$

Next, the equation should be consecutively multiplied by fundamental modes h_s:

$$\sum_{m=1}^{3} (Jh_m)^T h_s \ddot{z}_m + \sum_{m=1}^{3} (Bh_m)^T h_s \dot{z}_m + \sum_{m=1}^{3} (Ch_m)^T h_s z_m = M^T h_s - (J \cdot 1) h_s \ddot{\varphi}_0. \tag{16}$$

Taking into consideration the mode orthogonality:

$$(Jh_m)^T h_s = 0; \quad (Ch_m)^T h_s = 0 \quad \text{if } s \neq m,$$

Eq. (16) can be written as follows:

$$\alpha_m \ddot{z}_m + \sum_{m=1}^{3} \beta_{ms} \dot{z}_s + \gamma_m z_m = Z_m - g_m \ddot{\varphi}_0 \quad (m = 1, 2, 3), \tag{17}$$

where

$$Z_m = M^T h_m, \quad g_m = (J \cdot 1)^T h_m = \sum_{r=1}^{3} J_r h_{mr}, \ \beta_{ms} = (Bh_m)^T h_s.$$

The coefficient $\alpha_m = (Jh_m)^T h_m$ is called a modal moment of inertia (from the English word *mode*), and a scalar $\gamma_m = (Ch_m)^T h_m$ is called modal stiffness, or stiffness adduced to the mode m.

A complete separation of variables in Eq. (17) may occur if the damping coefficients are zero or proportional to the respective stiffness ($b_m = \lambda c_m$), or masses. In this case, Eq. (17) can be defined as follows:

$$\alpha_m \ddot{z}_m + \beta_m \dot{z}_m + \gamma_m z_m = Z_m - g_m \ddot{\varphi}_0 \quad (m = 1, 2, 3). \tag{18}$$

Next, Eq. (18) should be presented in an operator form ($d()/dt \rightarrow p()$):

$$(\alpha_m p^2 + \beta_m p + \gamma_m) z_m = Z_m - g_m \ddot{\varphi}_0 \quad (m = 1, 2, 3). \tag{19}$$

Thus,

$$z_m = (\alpha_m p^2 + \beta_m p + \gamma_m)^{-1} Z_m - (\alpha_m p^2 + \beta_m p + \gamma_m)^{-1} g_m \ddot{\varphi}_0. \tag{20}$$

Now, based on the principal coordinates, initial coordinates can be computed. For $s = 1, 2, 3$, they are the following:

$$\theta_s = \sum_{m=1}^{3} h_{ms} z_m = \sum_{m=1}^{3} h_{ms} (\alpha_m p^2 + \beta_m p + \gamma_m)^{-1} Z_m$$
$$- \sum_{m=1}^{3} h_{ms} (\alpha_m p^2 + \beta_m p + \gamma_m)^{-1} g_m \ddot{\varphi}_0 = \sum_{r=1}^{3} e_{sr}(p) M_r - \sigma_s(p) \ddot{\varphi}_0, \tag{21}$$

where the operator of dynamic compliance linking the external moment with a deformation error is:

$$e_{sr}(p) = \sum_{m=1}^{3} \frac{h_{ms} h_{mr}}{\alpha_m p^2 + \beta_m p + \gamma_m}, \tag{22}$$

and the transfer function linking the kinematic force with a deformation error is:

$$\sigma_s(p) = \sum_{m=1}^{3} \frac{h_{ms} g_m}{\alpha_m p^2 + \beta_m p + \gamma_m}. \tag{23}$$

Assuming the engine rotor rotates steadily ($\dot{\varphi}_0 = \text{const}$), the drag torque $M_3 = -M_{30} + M_{31} \sin \nu t$ is applied to the latest mass of a chain system and the positional damping coefficient is $b_s = \psi c_s / 2\pi\nu$ ($\psi = 0.2 - 0.6$—is the absorption coefficient), a deformation error in a fixed end system in accordance with (21–22) $\ddot{\varphi}_0 = \text{const}$ may be defined as follows ($s = 1, 2, 3$):

$$\theta_s = \Delta_s + \tilde{\theta}_s = -M_{30} |e_{s3}(j0)| + M_{31} |e_{s3}(j\nu)| \sin\left[\nu t + \arg e_{s3}(j\nu)\right]$$
$$= -M_{30} \sum_{m=1}^{3} \frac{h_{ms} h_{m3}}{\gamma_m} + M_{31} \sum_{m=1}^{3} \frac{h_{ms} h_{m3}}{\gamma_m \sqrt{\left[1 - \left(\frac{\nu}{k_m}\right)^2\right]^2 + \left(\frac{\psi}{2\pi}\right)^2}} \sin(\nu t + \xi_m). \tag{24}$$

The equation includes $\text{tg}\xi_m = \frac{\psi/2\pi}{(\nu/k_m)^2 - 1}, \quad \gamma_m = k_m^2 \alpha_m$.

Next, deviations in the laws of motion $\varphi_s(t)$ from the program ones should be determined through identification of dynamic errors in the machine aggregate depicted in Fig. 1 under the following system parameters (see Fig. 3): $c_1 = 8 \times 10^5 \text{N} \cdot \text{m}$; $c_2 = 1.44 \times 10^6 \text{N} \cdot \text{m}$; $c_3 = 1.2 \times 10^6 \text{N} \cdot \text{m}$; $J_0 = 0.6\ \text{kg} \cdot \text{m}^2$; $J_1 = 2.9 \times 10^{-3}\ \text{kg} \cdot \text{m}^2$; $J_2 = 1.5 \times 10^{-2}\ \text{kg} \cdot \text{m}^2$; $J_3 = 2.7\ \text{kg} \cdot \text{m}^2$; $M_{30} = 10\ \text{N} \cdot \text{m}$; $M_{31} = 50\ \text{N} \cdot \text{m}$.

The frequency equation can be generated:

$$\begin{vmatrix} c_1 + c_2 - J_1 k^2 & -c_2 & 0 \\ -c_2 & c_2 + c_3 - J_2 k^2 & -c_3 \\ 0 & -c_3 & c_3 - J_3 k^2 \end{vmatrix}$$
$$= 0.00011745k^6 - 111443k^4 + 1.041333 \times 10^{13} k^2 - 1.3834 \times 10^{18} = 0,$$

that can be used to calculate fundamental frequencies: $k_1 = 364\ \text{s}^{-1}$; $k_2 = 10243\ \text{s}^{-1}$; $k_3 = 29048\ \text{s}^{-1}$ and fundamental modes:

$$h_1 = (1, 1, 1)^T;\ h_2 = (1.5553,\ 1.3442, -0.1437)^T;$$
$$h_3 = (2.219, -0.0057,\ 0.00007)^T.$$

Modal stiffness:

$$\gamma_1 = (Ch_1)^T h_1 = 8 \times 10^5\ \text{N} \cdot \text{m};\ \gamma_2 = (Ch_2)^T h_2 = 4.656289 \times 10^6\ \text{N} \cdot \text{m};$$
$$\gamma_3 = (Ch_3)^T h_3 = 1.106678 \times 10^6\ \text{N} \cdot \text{m}.$$

Static errors in principal coordinates $z_{m0} = -M_{30} h_{m3} / \gamma_m$, in particular $z_{10} = -0.27 \times 10^{-4}$, $z_{20} = 0.12 \times 10^{-7}$, $z_{30} = -0.68 \times 10^{-10}$, and dynamic errors in principal coordinates ($m = 1,\ 2,\ 3$):

$$\tilde{z}_1 = \frac{M_{31} h_{13}}{\gamma_1 \sqrt{\left[1 - \left(\frac{\nu}{k_1}\right)^2\right]^2 + \left(\frac{\psi}{2\pi}\right)^2}} \sin(\nu t + \xi_1) = 0.137 \times 10^{-2} \sin(350t - 0.681),$$

$$\tilde{z}_2 = \frac{M_{31} h_{23}}{\gamma_2 \sqrt{\left[1 - \left(\frac{\nu}{k_2}\right)^2\right]^2 + \left(\frac{\psi}{2\pi}\right)^2}} \sin(\nu t + \xi_2) = 0.613 \times 10^{-7} \sin(350t - 0.063),$$

$$\tilde{z}_3 = \frac{M_{31} h_{33}}{\gamma_3 \sqrt{\left[1 - \left(\frac{\nu}{k_3}\right)^2\right]^2 + \left(\frac{\psi}{2\pi}\right)^2}} \sin(\nu t + \xi_3) = 0.342 \times 10^{-10} \sin(350t - 0.063).$$

Thus, the static and dynamic errors are:

$$\begin{aligned}\theta_{10} &= h_{11}z_{10} + h_{21}z_{20} + h_{31}z_{30}, & \tilde{\theta}_1 &= h_{11}\tilde{z}_1 + h_{21}\tilde{z}_2 + h_{31}\tilde{z}_3,\\ \theta_{20} &= h_{12}z_{10} + h_{22}z_{20} + h_{32}z_{30}, & \tilde{\theta}_2 &= h_{12}\tilde{z}_1 + h_{22}\tilde{z}_2 + h_{32}\tilde{z}_3,\\ \theta_{30} &= h_{13}z_{10} + h_{23}z_{20} + h_{33}z_{30}. & \tilde{\theta}_3 &= h_{13}\tilde{z}_1 + h_{23}\tilde{z}_2 + h_{33}\tilde{z}_3.\end{aligned}$$

4 Determination of Dynamic Errors in Transient Conditions

Determination of deformation errors in transient processes is of great significance for the dynamic analysis of machines with program management (various machines, robot manipulators, multi-moving platforms and others), transient processes of which take a considerable amount of the operation time.

External moments in transient processes are assumed to be zero for clarity. It should not be complicated to consider these moments in the calculation as the equations of motion are presented in a linear set.

Eq. (21) at $M_r = 0$ $(r = 1, 2, 3)$ can generate the following $(s = 1, 2, 3)$:

$$\theta_s = -\sigma_s(p)\ddot{\varphi}_0 = -\sum_{m=1}^{n} \frac{g_m h_{ms}}{\alpha_m p^2 + \beta_m p + \gamma_m}\ddot{\varphi}_0 = -\sum_{m=1}^{n} \frac{\rho_s^{(m)}}{\tau_m^2 p^2 + 2\zeta_m \tau_m p + 1}\ddot{\varphi}_0, \tag{25}$$

where

$$g_m = (J \cdot 1)^T h_m = \sum_{r=1}^{3} J_r h_{mr}; \quad \rho_s^{(m)} = g_m h_{ms}/\gamma_m.$$

The Duhamel integral should be applied to determine dynamic errors:

$$\theta_s(t) = -\sum_{m=1}^{n} \frac{\rho_s^{(m)} k_m}{\sqrt{1-\xi_m^2}} \int_0^t e^{-\xi_m k_m (t-\xi)} \sin\left[\sqrt{1-\xi_m^2}\, k_m (t-\xi)\right] \varepsilon(\xi) d\xi \quad (s = 1, 2, 3). \tag{26}$$

It is evident that $\theta_s(t)$ represents free vibrations emerging in a system as a result of disturbances caused by accelerated motion. These vibrations, which continue after the positioning, are considered highly undesirable as they lead to oscillations in a machine's operating elements hindering proper operating processes.

In Eq. (26) the first summand is of the most significance. This is due to two reasons. First, the coefficients $\rho_s^{(m)}$ decline rapidly with a higher m. Second, the duration of a transient process usually exceeds by several times the longest free vibrations period of the system, that is T_1. The most significant summand in Eq. (26) is the first one

($m = 1$), that corresponds to damped vibrations in the first mode:

$$\theta_s(t) \approx -\frac{\rho_s^{(1)} k_1}{\sqrt{1-\xi_1^2}} \int\limits_0^t e^{-\xi_1 k_1 (t-\xi)} \sin[\sqrt{1-\xi_1^2} k_1 (t-\xi)]\, \varepsilon(\xi) d\xi \quad (s = 1, 2, .., 3). \tag{27}$$

A dynamic error of a gear $\theta_2(t)$ in a constant run-up and braking in a three-mass system with the fixed end (see Fig. 3) can be determined with the Duhamel integral:

$$\varepsilon(t) = \varepsilon_0 \eta(t) - 2\varepsilon_0 \eta(t - t_\mathrm{p}) + \varepsilon_0 \eta(t - t_\mathrm{T}),$$

where $\varepsilon_0 = 10\ \mathrm{s}^{-2}$ is the acceleration amplitude, $\eta(t)$ is the unit-step function, $t_\mathrm{p} = 3\, s$ is the run-up time, $t_\mathrm{T} = 2t_\mathrm{p}$ is the braking time. In the system under consideration

$$\begin{aligned}
g_1 &= J_1 h_{11} + J_2 h_{12} + J_3 h_{13} = 6.017\ \mathrm{kg\,m^2};\\
g_2 &= J_1 h_{21} + J_2 h_{22} + J_3 h_{23} = 0.0076\ \mathrm{kg\,m^2};\\
g_3 &= J_1 h_{31} + J_2 h_{32} + J_3 h_{33} = 0.948 \times 10^{-6}\ \mathrm{kg\,m^2};\\
\rho_2^{(1)} &= \frac{g_1 h_{12}}{\gamma_1} = 0.169 \times 10^{-4}\ \mathrm{s}^2;\\
\rho_2^{(2)} &= \frac{g_1 h_{22}}{\gamma_2} = 0.229 \times 10^{-8}\ \mathrm{s}^2;\\
\rho_3^{(2)} &= \frac{g_3 h_{32}}{\gamma_3} = -0.126 \times 10^{-10}\ \mathrm{s}^2, n_1 = 1.
\end{aligned}$$

Thus, the dynamic error in the transient process is:

$$\theta_2(t) \approx -\rho_1^{(2)} k_1 I_1(t),$$

where

$$I_1(t) = \begin{cases} \varepsilon_0 k_1^{-1}(e^{-n_1 t}\cos k_1 t - 1) & 0 \le t \le t,\\ \varepsilon_0 k_1^{-1}\left[1 + e^{-n_1 t}\cos k_1 t - 2e^{-n_1(t-t_\mathrm{p})}\cos k_1(t-t_\mathrm{p})\right] & t_\mathrm{p} \le t \le t_\mathrm{T},\\ \varepsilon_0 k_1^{-1}\left[e^{-n_1 t}\cos k_1 t - 2e^{-n_1(t-t_\mathrm{p})}\cos k_1(t-t_\mathrm{p}) \right. & \\ \left. \quad + e^{-n_1(t-t_\mathrm{T})}\cos k_1(t-t_\mathrm{T})\right] & t \ge t_\mathrm{T}. \end{cases}$$

Equation (27) demonstrates that amplitudes of dynamic errors with a predetermined terminal angular velocity (in the case of run-up and braking) or with a predetermined rotation angle (in the case of positioning) are inversely proportional to the time of the transient process and the square of the first fundamental frequency. Therefore, the reduction of errors in transient processes can be achieved through decreased

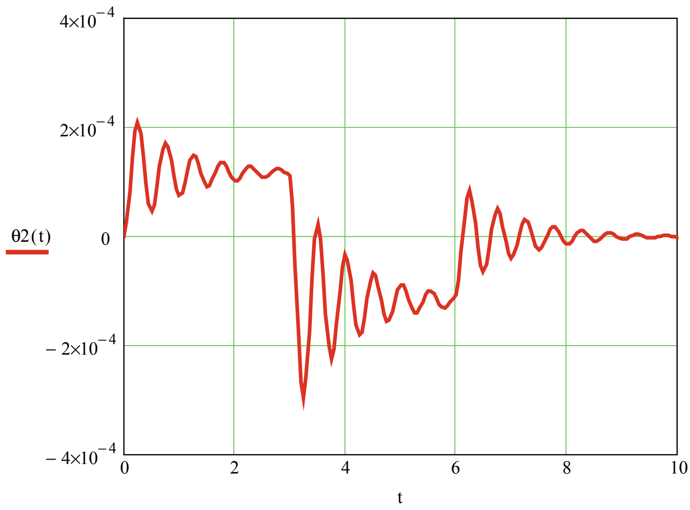

Fig. 4 The graph of the dynamic error that occurs in the machine during the run-up

masses or a greater stiffness of a transmission device and a longer transient process. Figure 4 shows the dependency graph $\theta_2(t)$.

In practice, implementation of these recommendations is limited. A greater stiffness of a transmission device is usually accompanied by greater reciprocating masses, thus failing to significantly increase the first fundamental frequency. A longer transient process results in a decreased machine performance, which is undesirable. In conclusion, it can be stated that the reduction of dynamic errors in contemporary machines is a complicated technical task.

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