Chapter 9
Collapse Prediction and Safety of Masonry Arches

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Abstract Masonry structures without mortar or with mortar of low quality are used in several infrastructures, like bridges and retaining walls Unilateral contact plays a crucial role in their stability. Limit analysis and nonexistence of solution are related to the creation of collapse mechanisms. Open source and freely available software can be used for the analysis of such structures, usually with an acceptable for post-disaster, emergency situations. Numerical results related to a recently collapsed masonry bridge demonstrate the usage of the proposed method.

Keywords Limit analysis · Masonry arches · Collapse prediction · Stability of structures · Solvability of unilateral contact problems

9.1 Introduction

Masonry arch bridges and walls are traditional structures, consisting of stone blocks and mortar. The ability of the material and the joints to transfer compressive loading, and their inability to accept reliably tensile loading are optimally exploited. The modern approach to structural analysis of masonry arches is based on limit analysis concepts or unilateral contact mechanics, after suitable simplifications. These concepts can be used for the assessment of limit load and collapse modes or existing structures after a natural disaster.

Masonry arches are examples of structures where form meets mechanical function: stones and mortar, if it exists, of relatively high strength in compression and

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limited, unreliable, practically zero strength in tension are used in such a way so that self-weight keeps them stable in place and the whole structure in a functional state. Life loading is usually a small quantity in classical heavy structures, therefore this disturbance can be accommodated. If not, for example due to an accidental loading during a disaster, hinges develop due to unilateral contact separation, the degree of indeterminacy is reduced and finally a mechanism of collapse develops.

The main purpose of this contribution is to demonstrate a different approach to static analysis of structures subjected to extreme loading cases and facilitate the usage of these techniques by indicating suitable software packages which can be used for a first and quick assessment.

9.2 Modelling and Limit Analysis of Masonry Structures

Modelling of a structure with unilateral interfaces is based on concrete tools of non-smooth and contact mechanics (Demyanov et al. 1996; Mistakidis and Stavroulakis 1998; Leftheris et al. 2006; Bolzon 2017). For rigid body or linearly deformable structures, the structural analysis problem can be transformed in known forms of mathematical programming problems, like quadratic optimization with inequality constraints or complementarity problems. Stability of a multi-block structure is based on the ability of unilateral contact joints to transfer compressive loading from the one part to the other and finally to the supports. The peculiarity of the unilateral contact mechanism is the different behaviour in compression and tension. For the whole structure self-weight in cooperation with the shape of the structure usually stabilizes the system. Any additional load could potentially lead to collapse, since the total loading may become prohibitive for the structure. In other words the arising contact problem has no solution.

For the simplest case of frictionless contact, the structural analysis problem can be formulated as a potential energy minimization which includes the unilateral contact inequality constraint (non-penetration). This is given by the relation:

$$\begin{gathered}\min_{\mathrm{u}}(\tfrac{1}{2}\boldsymbol{u}^T\boldsymbol{k}\boldsymbol{u}-\boldsymbol{P}^T\boldsymbol{u})\\ \boldsymbol{N}_n\boldsymbol{u}-\boldsymbol{g}\le 0\end{gathered} \tag{9.1}$$

For the quadratic minimization problem, the Karush-Kuhn-Tucker (KKT) optimality conditions lead to the linear complementarity problem (LCP) of relations.

$$\begin{gathered}\boldsymbol{K}\boldsymbol{u}+\boldsymbol{N}_n^T\boldsymbol{r}_n=\boldsymbol{P}_o+\lambda\boldsymbol{P}\\ \boldsymbol{N}_n\boldsymbol{u}-\boldsymbol{g}\le 0\\ \boldsymbol{r}_n\ge 0\\ (\boldsymbol{N}_n\boldsymbol{u}-\boldsymbol{g})^T\boldsymbol{r}_n=0\end{gathered} \tag{9.2}$$

The first equation expresses the equilibrium of the discretized unilateral contact problem without friction, where **K** is the stiffness matrix and **u** the displacement vector. $\mathbf{P}_o$ denotes the self-weight of the structure and **P** represents the live load vector, multiplied by a scalar load multiplier λ. $\mathbf{N}_n$ is an appropriate geometric transformation matrix and vector **g** contains the initial gaps for the description of the unilateral contact joints. The next relations represent the constraints of the unilateral contact problem for the whole discretized structure. For the consideration of the constraints, the vector $\mathbf{r}_n$ representing Lagrange multipliers is used to depict contact pressure. The problem described above is a non-smooth parametric linear complementarity problem (LCP) parametrized by the one-dimensional load parameter λ. Values for solutions in the interval $0 \leq \lambda \leq \lambda_{failure}$ are investigated.

One of the first approaches to masonry arches using parallels to contact and plasticity is given in the classical monograph of Heyman (1982). Analytical studies have been published in several classical references which will not be given here. Numerical approaches which lead to easily solvable models suitable for quick evaluation purposes, are based on rigid blocks with unilateral contact (Gilbert and Melbourne 1994; Livesley 1978; Melbourne and Gilbert 1995; Ferris and Tin-Loi 2001; Orduna and Lourenço 2005; Gilbert et al. 2006; Portioli et al. 2013). The initial model has been extended in order to cover sliding between adjacent blocks, multiple spans and multiple arch rings spandrel walls as well as masonry with finite strength. Another approach constitutes the usage of the discrete element method, see for instance Caliò et al. (2010).

The most general approach, which incorporates the previously mentioned ones, is the usage of finite element models for deformable bodies with unilateral contact interfaces (Drosopoulos et al. 2006, 2008). Solvability of the underlying unilateral contact problem and suppression of possible rigid body motions due to the absence of classical bilateral supports and the inactive contact interfaces, corresponds to collapse of the structure under given loadings (Stavroulakis et al. 1991). The ability to have an automatic and relatively quick model of the structure in the actual geometric shape using scanners, provides us with additional strength related with the evaluation of structures in deformed, partially damaged condition (Stavroulaki et al. 2016).

It should be noted that accidental loadings due to natural disasters have been studied in a few works with results that indicate the importance of computational modeling for the remaining strength evaluation of existing structures. For instance, analysis of the influence of flooding on the collapse analysis of masonry arh bridges indicates that the load-carrying capacity of a fully flooded arch bridge backfilled with cohesionless fill could typically be reduced by a factor of 1.6–1.8, or even more in specific circumstances, as it has been shown in Hulet et al. (2006). Flood and post-flood performance of historic stone bridges has been investigated in Drdácký and Slížková (2007). Discrete elements are used in Liu et al. (2018), Fukumoto et al. (2014) and Quezada et al. (2019), for the evaluation of various accidental scenarios.

9.3 Computational Tools

A general approach to consider multi-block structures with unilateral interaction is the usage of finite element or boundary element nonlinear contact analysis. Especially the usage of linear complementarity techniques and the relation with solvability investigations gives us invaluable insight into the mechanics of these structures and a rigor mathematical framework to work. Every available general purpose finite element program able to solve contact problems is, in principle, suitable for this study.

The simplified approach of rigid blocks and the transformation of the collapse analysis into a linear programming program can be used for a quick assessment of existing, mainly two-dimensional structures like bridges and vaults. This approach has been used in Ring software, which is freely available for use. http://www.shef.ac.uk/ring, (see Gilbert 2001; Gilbert and Melbourne 1994).

For three-dimensional problems the software LiABlock_3D is available. Within it, structures are represented as 3D assemblages of rigid blocks interacting at no-tension, frictional contact interfaces. The mathematical programming problems arising are solved by Mosek optimization software, while collapse load and failure mechanism are plotted (Cascini et al. 2018).

Finally two recent software packages are mentioned, which nevertheless have not been thoroughly tested till now (Chiozzi et al. 2015; Galassi and Tempesta 2019).

9.4 Results Related to Keritis Bridge

Keritis bridge, which is located in Chania, Crete, Greece, was constructed in 1912. It was used by the residents for more than 100 years and it played a crucial role in the communication of the local population during historical periods and wars. Today, it is considered to be a monument. Unfortunatelly this structure collapsed on February 25, 2019, during an extremmely high flooding.

Keritis bridge has 3 arched spans. The geometry and dimensions of the bridge are presented in Fig. 9.1. The out of plane width of the bridge is equal to 8 m. The material properties of the structure, which must be used for its structural evaluation, have been estimated and shown in Table 9.1. For the fill material, the angle of internal friction and cohesion are received equal to 37° and 10 KN/m^2, respectively. For the asphalt material over the fill, the angle of load distribution is 26.60° and the density is 18 KN/m^3. Elasticity modulus and Poisson's ratio have been estimated from similar structures, due to lack of information.

For the determination of the ultimate response of Keritis bridge, the limit analysis software RING was used. As shown in Fig. 9.2, the arches which comprise the main structural system, the fill over the arch and the abutments are simulated within the software.

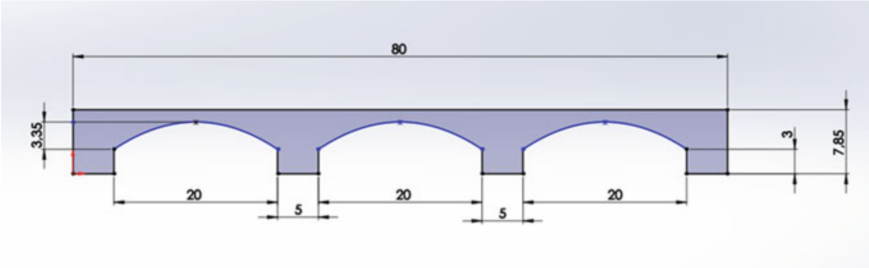

Fig. 9.1 Geometry and dimensions (m) of Keritis bridge

Table 9.1 Estimated material properties of Keritis bridge

Part of the structure	Density (KN/m^3)	Compressive strength (MPa)	Friction coefficient
Arch	19.60	5.00	0.60
Fill	19.00		0.60

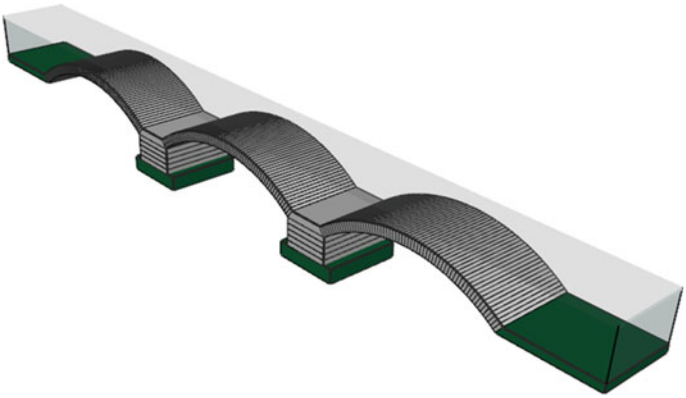

Fig. 9.2 Keritis bridge designed in RING software

To evaluate the influence of the self-weight and vehicle loading on the bridge, first each arch is independently loaded. Then, the total bridge is numerically tested. For every simulation, the ultimate safety factor λ, defined as the ratio of the ultimate load over the applied load, is calculated. For the determination of the influence of the vehicle loading, several scenarios for the positioning of more than one vehicle on each arch of the bridge are investigated.

When the first arch is considered and the self-weight loading is examined, five trial positions of a unit force are tested and the corresponding safety factors are estimated, as shown in Fig. 9.3 and Table 9.2.

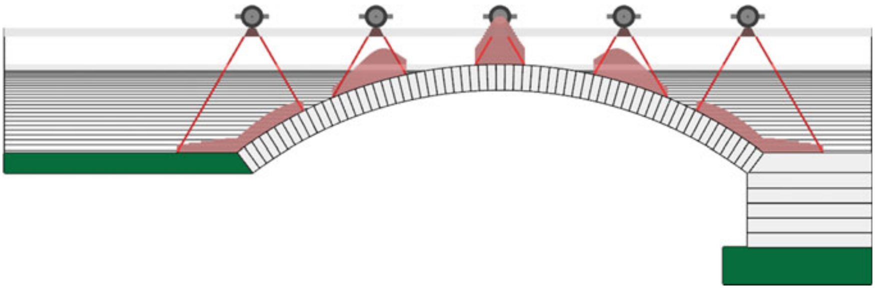

Fig. 9.3 Trial unit force positions for the first arch of the bridge

Table 9.2 Influence of self-weight loading on the strength of the first arch

Load position (m)	Safety factor λ
0	6.03E+03
5	603
10	384
15	970
20	3.1E+04

To investigate the real vehicle loading, several scenarios for the positioning of more than one vehicle shown in Table 9.3, are investigated. The length for each vehicle is considered equal to 12.3 m. A number of vehicles densely placed along the length of the bridge have been assumed.

Similar results for the second and third arches and for self-weight and vehicle loading, are given in Tables 9.4, 9.5, 9.6, and 9.7.

When the overall bridge is considered, the safety factors which are obtained for several trial positions of unit force along the spans are shown in Table 9.8. From the given safety factor values is shown that the structure is safe for self-weight loading.

When only one vehicle loading is applied to several positions of the bridge, the corresponding safety factors which are obtained, are shown in Table 9.9.

Table 9.3 Influence of vehicle loading on the strength of the first arch

Load position (m)	Safety factor λ
0.0 12.5	3.88
−9.0 3.5 16.0	5.81
−5.545 6.955 19.455	11.70

Table 9.4 Influence of self-weight loading on the strength of the second arch

Load position (m)	Safety factor λ
0	6.03E+03
5	603
10	384
15	603
20	6.03E+03

Table 9.5 Influence of vehicle loading on the strength of the second arch

Load position (m)	Safety factor λ
0.0 12.5	3.87
−9.0 3.5 16.0	5.81
−5.545 6.955 19.455	11.70

Table 9.6 Influence of self-weight loading on the strength of the third arch

Load position (m)	Safety factor λ
0	3.1E+04
5	970
10	384
15	603
20	6.03E+03

Table 9.7 Influence of vehicle loading on the strength of the third arch

Load position (m)	Safety factor λ
0.0 12.5	12.7
−9.0 3.5 16.0	6.39
−5.545 6.955 19.455	15.2

When more than one vehicle loading is considered on the bridge as shown in Fig. 9.4, the safety factor shown in Table 9.10 is obtained. This is an example of a severe load case, used to indicate the load bearing capacity of the bridge for an extreme loading scenario.

Table 9.8 Influence of self-weight loading on the strength of overall bridge

Load position (m)	Safety factor λ
0	1.71E+05
10	6.26E+04
20	1.71E+05
30	5.87E+04
40	5.87E+04
50	1.71E+05
60	6.26E+04
70	1.71E+05

Table 9.9 Influence of self-weight loading on the strength of overall bridge: one load case, see Fig. 9.3

Load position (m)	Safety factor λ
0	138
10	91.9
20	257
30	97.5
40	99
50	138
60	91.9
70	275

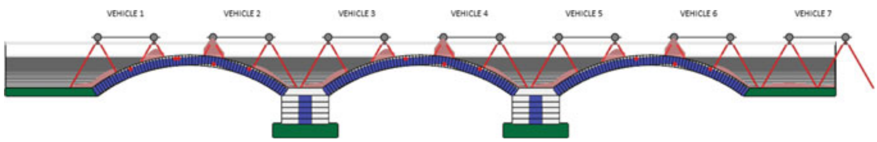

Fig. 9.4 Load case of seven vehicle forces applied along the length of the overall bridge

Table 9.10 Influence of self-weight loading on the strength of overall bridge: multiple loadings case, see Fig. 9.4

Load position (m)	Safety factor λ
0	93
12.5	
25	
37.5	
50	
62.5	
75	

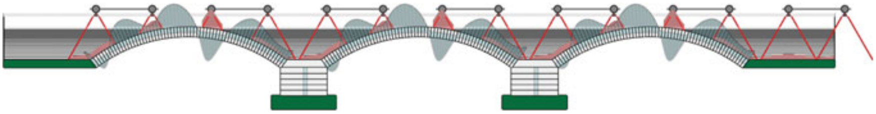

Fig. 9.5 Distribution of moments along the length of the bridge

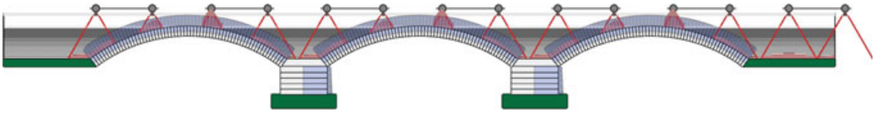

Fig. 9.6 Distribution of axial forces along the length of the bridge

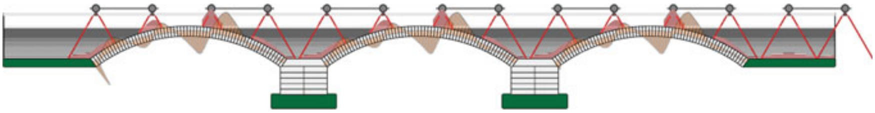

Fig. 9.7 Distribution of shear forces along the length of the bridge

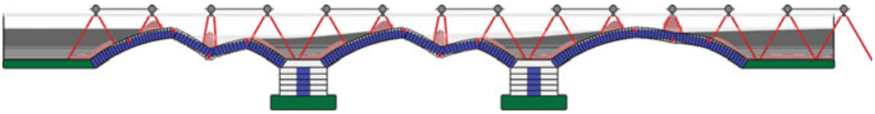

Fig. 9.8 Collapse mechanism and plastic hinges distribution obtained for the ultimate limit state

To finalize this investigation, the ultimate limit state of the bridge has been determined, by considering: $(\lambda) \times$ (Applied load) = (ultimate limit load), as the loading of the bridge. Figures 9.5, 9.6, 9.7, and 9.8 show the moment, axial and shear distribution along the length of the bridge, as well as the collapse mechanism, which are obtained for this ultimate limit load.

9.5 Conclusions

Methods which can be used to numerically assess the structural performance of masonry bridges and walls, under static and dynamic loading, have been presented in this article. Emphasis is posed on simplified limit analysis, which can be performed by openly available software. More detailed studies, including the application of non-linear finite element analysis and numerical homogenization can be found in the literature.

Extension to other structures can be found in the literature, e.g. Tralli et al. (2014). Using these methods collapse mechanisms and ultimate loads can be estimated quickly. Input of geometrical data can be automatized, by using modern photogrammetry techniques, for instance terrestrial laser scanners (Stavroulaki et al. 2016; Riveiro et al. 2016).

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