Parameterized Complexity of Synthesizing b-Bounded (m, n)-T-Systems

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Abstract. Let $b \in \mathbb{N}^+$. Synthesis of pure b-bounded (m, n)-T-systems ((m,n)-Synthesis, for short) consists in deciding whether there exists for an input (A, m, n) of transition system A and integers $m, n \in \mathbb{N}$ a pure b-bounded Petri net N as follows: N's reachability graph is isomorphic to A, and each of N's places has at most m incoming and at most n outgoing transitions. In the event of a positive decision, N should be constructed. The problem is known to be NP-complete, and (m,n)-Synthesis parameterized by $m + n$ is in XP [14]. In this paper, we enhance our understanding of (m,n)-Synthesis from the viewpoint of parameterized complexity by showing that it is $W[1]$-hard when parameterized by $m + n$.

1 Introduction

Petri net *synthesis* consists in deciding whether there is a Petri net (PN, for short) that implements a given behavioral specification and in constructing such a net if it exists. Valid synthesis methods yield implementations that are correct by design. The possibility of finding effective or even efficient synthesis algorithms crucially depends on the specification and the searched net. This has been subject of research for many years: It is undecidable whether there is a P/T net implementing a pushdown- or a HMSC-language or whether there is a (pure) bounded P/T net implementing a modal transition systems (MTS, for short) [9,11]. If the specification is a deterministic pushdown-language or -graph, and the search net is a P/T-net, synthesis is decidable [4]. It is also decidable whether there is a b-bounded Petri net that implements an MTS [12]. If the specification is a transition system (TS, for short), and the searched net is a 1-bounded PN, synthesis is NP-complete [2], even if the TS is strongly restricted [15,16]. The synthesis of b-bounded PNs from TSs is NP-complete, even if the searched net is strongly restricted [13,14]. If the bound b is not fixed in advance, the synthesis of bounded PN from TSs is polynomial [1]. If the PN is additionally to be choice-free or a marked graph, even better procedures exist [5,7].

In this paper, we investigate an instance of PN synthesis that is called (m, n)-Synthesis. It consists in deciding whether there exists for an input (A, m, n) of TS A and integers $m, n \in \mathbb{N}$ a pure b-bounded Petri net N as follows: N's reachability graph is isomorphic to A, and each of N's places has at most m incoming

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and at most n outgoing transitions. The b-bounded (m, n)-T-systems generalize the notion of (weighted) T-systems [6,10] and adapt it to b-bounded PN. In [14], we have shown that (m, n)-SYNTHESIS is NP-complete. We have also argued that (m, n)-SYNTHESIS parameterized by $m + n$ belongs to the complexity class XP. Thus, the question arises whether this parameterization makes the problem fixed parameter tractable. In this paper, we answer this question negatively and show that (m, n)-SYNTHESIS parameterized by $m+n$ is $W[1]$-hard. The proof presents a parameterized reduction from REGULAR INDEPENDENT SET, which restricts the canonical $W[1]$-hard problem to regular graphs [8], to (m, n)-SYNTHESIS. This paper is organized as follows. Section 2 introduces necessary preliminary notions, Sect. 3 presents the $W[1]$-hardness result and Sect. 4 closes the paper.

2 Preliminaries

We assume that the reader is familiar with the concepts relating to fixed-parameter tractability, the standard notions relating to graphs and REGULAR INDEPENDENT SET, the canonical $W[1]$-hard problem restricted to regular graphs. Due to space restrictions, we omit some formal definitions and some proofs. See [8] for the definitions of relevant notions in parameterized complexity theory. In the remainder of this paper, if not stated explicitly otherwise, then $b \in \mathbb{N}^+$ is assumed to be arbitrary but fixed.

Transition Systems. A *transition system* (TS, for short) $A = (S, E, \delta, \iota)$ consists of a finite disjoint set S of states, E of events, a partial *transition function* $\delta : S \times E \to S$ and an *initial state* $\iota \in S$. A TS A is interpreted as edge-labeled directed graph, and every triple $\delta(s, e) = s'$ is considered an e-labeled edge $s \xrightarrow{e} s'$, called *transition*. An event e *occurs* at state s, denoted by $s \xrightarrow{e}$, if $\delta(s, e) = s'$ for some state s'. This notation is extended to words $w' = we$, $w \in E^*, e \in E$, by inductively defining $s \xrightarrow{\varepsilon} s$ for all $s \in S$ and $s \xrightarrow{w'} s''$ if and only if there is a state $s' \in S$ satisfying $s \xrightarrow{w} s'$ and $s' \xrightarrow{e} s''$. If $w \in E^*$, then $s \xrightarrow{w}$ denotes that there is a state $s' \in S$ such that $s \xrightarrow{w} s'$. If $e \in E$, then by $s_i \xrightarrow{(e)^b} s_{i+b}$ we denote that there are distinct states $s_i, s_{i+1}, \dots, s_{i+b-1}, s_{i+b} \in S$ such that $s_i \xrightarrow{a} s_{i+1} \dots s_{i+b-1} \xrightarrow{a} s_{i+b}$. We assume all TSs to be *reachable*: $\forall s \in S, \exists w \in E^* : s_0 \xrightarrow{w} s$.

***b*-Bounded Petri Nets.** A *b-bounded Petri net* (b-net, for short) $N = (P, T, f, M_0)$ consists of finite and disjoint sets of *places* P and *transitions* T, a (total) *flow function* $f : P \times T \to \{0, \dots, b\}^2$ and an *initial marking* $M_0 : P \to \{0, \dots, b\}$. If $f(p, t) = (m, n)$, then $f^-(p, t) = m$ and $f^+(p, t) = n$ define the *consuming* and the *producing* effect of t on p, respectively. The *preset* of a place p is defined by ${}^\bullet p = \{t \in T \mid f^+(p, t) > 0\}$ (transitions producing on p) and its *postset* is defined by $p^\bullet = \{t \in T \mid f^-(p, t) > 0\}$ (transitions consuming from p). Accordingly, the *preset* of a transition t is defined by ${}^\bullet t = \{p \in P \mid f^-(p, t) > 0\}$ (places from which t consumes) and its *postset* by

$t^\bullet = \{p \in P \mid f^+(p,t) > 0\}$ (places on which t produces). A b-net N is *pure* if $\forall (p,t) \in P \times T : f^-(p,t) = 0$ or $f^+(p,t) = 0$, that is, $\forall p \in P : {}^\bullet p \cap p^\bullet = \varnothing$. Let $m, n \in \mathbb{N}$. A b-net N is an *(m,n)-T-system* if $\forall p \in P : |{}^\bullet p| \leq m, |p^\bullet| \leq n$.
The firing rule of b-nets defines their behavior: A transition $t \in T$ can *fire* or *occur* in a marking $M : P \to \{0, \dots, b\}$, denoted by $M \xrightarrow{t}$, if $M(p) \geq f^-(p,t)$ and $M(p) - f^-(p,t) + f^+(p,t) \leq b$ for all places $p \in P$. The firing of t in marking M leads to the marking M' if $M'(p) = M(p) - f^-(p,t) + f^+(p,t)$ for all $p \in P$. This is denoted by $M \xrightarrow{t} M'$. Again, this notation extends to sequences $\sigma \in T^*$, and the *reachability set* $RS(N) = \{M \mid \exists \sigma \in T^* : M_0 \xrightarrow{\sigma} M\}$ contains N's reachable markings. The firing rule preserves N's *b-boundedness* by definition: $M(p) \leq b$ for all $p \in P$ and all $M \in RS(N)$. The *reachability graph* of N is the TS $A_N = (RS(N), T, \delta, M_0)$, such that for all $M, M' \in RS(N)$ and all $t \in T$ we define $\delta(M,t) = M'$ if and only if $M \xrightarrow{t} M'$.

b-Bounded Regions. To find a b-net N implementing a TS A, we want to synthesize N's components purely from the input A. Since A and A_N are to be isomorphic, A's events correspond to N's transitions. However, the notion of a *place* is not known for TSs. A *b-bounded region* R (region, for short) of a TS $A = (S, E, \delta, s_0)$ is a pair $R = (sp, sg)$ of *support* $sp : S \to \{0, \dots, b\}$ and *signature* $sg : E \to \{0, \dots, b\}^2$ such that for every edge $s \xrightarrow{e} s'$ of A holds $sp(s) \geq sg^-(e)$ and $sp(s') = sp(s) - sg^-(e) + sg^+(e)$. If $sg(e) = (m,n)$, then $sg^-(e) = m$ and $sg^+(e) = n$ define e's consuming and producing effect (concerning R), respectively.

A region (sp, sg) models a place p and the corresponding part of the flow function f: $sg^+(e)$ models $f^+(e)$, $sg^-(e)$ models $f^-(e)$ and $sp(s)$ models $M(p)$ in the marking $M \in RS(N)$ corresponding to $s \in S(A)$. The *preset* of R is defined by the *producing events* ${}^\bullet R = \{e \in E \mid sg^+(e) > 0\}$ and its *postset* by the *consuming events* $R^\bullet = \{e \in E \mid sg^-(e) > 0\}$. If $sg(e) = (0,0)$, then e is called *neutral*. The region R is *pure* if ${}^\bullet R \cap R^\bullet = \varnothing$. Let $\mathcal{R}$ be a set of regions of A, and let $e \in E$. By ${}^\bullet e_{\mathcal{R}} = \{(sp, sg) \in \mathcal{R} \mid sg^-(e) > 0\}$ and $e^\bullet_{\mathcal{R}} = \{(sp, sg) \in \mathcal{R} \mid sg^+(e) > 0\}$ we define the *preset* and *postset* of e (concerning $\mathcal{R}$), respectively. The set $\mathcal{R}$ defines the *synthesized b-net* $N_A^{\mathcal{R}} = (\mathcal{R}, E, f, M_0)$ with flow function $f((sp, sg), e) = sg(e)$ and initial marking $M_0((sp, sg)) = sp(s_0)$ for all $(sp, sg) \in \mathcal{R}, e \in E$. We emphasize again that a *region* R of $\mathcal{R}$ is a *place* of $N_A^{\mathcal{R}}$ with the preset ${}^\bullet R$ and the postset $R^\bullet$; every *event* $e \in E$ is a *transition* of $N_A^{\mathcal{R}}$ with preset ${}^\bullet e = {}^\bullet e_{\mathcal{R}}$ and postset $e^\bullet = e_{\mathcal{R}}{}^\bullet$. It is well known that $A_{N_A^{\mathcal{R}}}$ and A are isomorphic if and only if $\mathcal{R}$'s regions solve certain separation atoms [3], to be introduced next.

A pair (s, s') of distinct states of A defines a *state separation atom* (SSP atom, for short). A region $R = (sp, sg)$ *solves* (s, s') if $sp(s) \neq sp(s')$. The region R is to ensure that $N_A^{\mathcal{R}}$ contains at least one place R such that $M(R) \neq M'(R)$ for the markings M and M' corresponding to s and s', respectively. If there is a b-region that solves (s, s'), then s and s' are called *b-solvable* (solvable, for short). If every SSP atom of A is solvable, then A has the *b-state separation property* (SSP for short). If $e \in E$ and $s \in S$ such that e does not occur at s ($\neg s \xrightarrow{e}$), then the pair (e, s) is an *event state separation atom* (ESSP atom, for short). A b-region $R = (sp, sg)$ *solves* (e, s) if $sg^-(e) > sp(s)$ or $sp(s) - sg^-(e) + sg^+(e) > b$. The meaning of R is to ensure that there is at least one place R in $N_A^{\mathcal{R}}$ such that $\neg M \xrightarrow{e}$ for the marking M corresponding to s. If there is a region that solves (e, s), then e and s are called *b-solvable*; we also say e is solvable at s. If every ESSP atom of A is b-solvable, then A has the *b-event state separation property* (ESSP, for short).

A set $\mathcal{R}$ of regions of A is called *b-admissible* if for every of A's (E)SSP atoms there is a region R in $\mathcal{R}$ that solves it. The following lemma, borrowed from [3, p. 163], summarizes the connection between b-admissible sets of A and synthesis:

Lemma 1 ([3]). *A b-net N has a reachability graph isomorphic to a given TS A if and only if there is a b-admissible set* $\mathcal{R}$ *of A such that* $N = N_A^{\mathcal{R}}$.

We say a b-net N *solves* A if A_N and A are isomorphic. By Lemma 1, searching for a restricted b-net reduces to finding a b-admissible set of accordingly restricted regions. The following example illustrates this fact.

Example 1. Let $m, n \in \mathbb{N}$, A be a TS and $\mathcal{R}$ be a b-admissible set of pure regions of A. If every region $R \in \mathcal{R}$ satisfies $|{}^\bullet R| \leq m$ and $|R^\bullet| \leq n$, then $N_A^{\mathcal{R}}$ is a pure (m, n)-T-system solving A. In particular, if $b = 2$, then the TS $A = s_0 \xrightarrow{e_1} s_1 \xrightarrow{e_2} s_2$ has the following pure regions:

i	$sp_i(s_1)$	$sp_i(s_2)$	$sp_i(s_3)$	$sg_i(e_1)$	$sg_i(e_2)$	i	$sp_i(s_1)$	$sp_i(s_2)$	$sp_i(s_3)$	$sg_i(e_1)$	$sg_i(e_2)$
1	2	0	0	$(2,0)$	$(0,0)$	3	0	2	0	$(0,2)$	$(2,0)$
2	0	2	0	$(0,2)$	$(2,0)$	4	0	1	2	$(0,1)$	$(0,1)$

The set $\mathcal{R} = \{(sp_i, sg_i) \mid 1 \leq i \leq 4\}$ is 2-admissible. Since ${}^\bullet(sp_4, sg_4) = \{e_1, e_2\}$, the solving 2-net $N_A^{\mathcal{R}}$ is not a $(1,1)$-T-system. However, the set $\mathcal{R}' = \{(sp_i, sig_i) \mid 1 \leq i \leq 3\}$ is 2-admissible, and $N_A^{\mathcal{R}'}$ is a $(1,1)$-T-system solving A.

3 $W[1]$-Hardness Parameterized by $m + n$

This section is dedicated to the proof of our main result:

Theorem 1. (m, n)-Synthesis *parameterized by* $m + n$ *is* $W[1]$*-hard.*

The proof of Theorem 1 consists of a parameterized reduction of Regular Independent Set to (m, n)-Synthesis. Let (G, k) be an instance of Regular Independent Set. That is, $G = (V(G), E(G))$ is a graph with set of nodes $V(G) = \{v_1, \ldots, v_n\}$, set of edges $E(G) = \{a_1, \ldots, a_m\}$, and there is an integer $r \in \mathbb{N}$ such that for every node $v \in V(G)$ holds $|\{e \in E(G) \mid v \in e\}| = r$, and k is a positive integer. We reduce (G, k) to an instance $(A, 2rk + 20, 2rk + 20)$ of (m, n)-Synthesis, parameterized by $m + n$, such that G has a k-independent set if and only if A is solvable by a $(2rk + 20, 2rk + 20)$-T-system.

To represent G, the TS A has for every edge $a_i = \{v_{i,1}, v_{i,2}\}$, $i \in \{1, \ldots, m\}$, the following gadget G_i, which uses $a_i, v_{i,1}$ and $v_{i,2}$ as events:

$$g_{i,1} \xrightarrow{\delta_i^1} g_{i,2} \xrightarrow{\zeta_{i,1}^1} g_{i,3} \xrightarrow{(v_{i,1})^b} g_{i,b+3} \xrightarrow{\zeta_{i,2}^1} g_{i,b+4} \xrightarrow{(v_{i,2})^b} g_{i,2b+4} \xrightarrow{\zeta_{i,3}^1} g_{i,2b+5} \xrightarrow{a_i} g_{i,2b+6}$$

$$g_{i,2} \xrightarrow{\alpha_1} g_{i,2b+7}$$

Let $i \in \{1, \dots, m\}$. The proof of the *if*-direction bases on the idea to ensure that if A is solvable, then there is a pure region $R = (sp, sg)$ that satisfies the following conditions. Firstly, $sg(\alpha_1) = (b, 0)$, which implies $sp(g_{i,2}) = b$. Secondly, the producing effect of the node events is zero, that is, $sg^+(v_{i,1}) = sg^+(v_{i,2}) = 0$. Thirdly, the ζ-events are neutral, that is, $sg(\zeta_{i,1}^1) = sg(\zeta_{i,2}^1) = sg(\zeta_{i,3}^1) = (0, 0)$. As a result, the support value of $g_{i,2b+5}$ is given by $sp(g_{i,2b+5}) = b - b \cdot (sg^-(v_{i,1}) + sg^-(v_{i,2}))$. Moreover, if $sp(g_{i,2b+5}) < b$, then there is *exactly* one $e \in \{v_{i,1}, v_{i,2}\}$ such that $sig^-(e) > 0$. Otherwise we would have the contradiction $sp(g_{i,2b+5}) < 0$. Furthermore, the region R ensures that there are exactly rk edge events with a positive producing effect. That is, there are exactly rk indices $i_1, \dots, i_{rk} \in \{1, \dots, m\}$ such that $sg^+(a_{i_j}) > 0$ for all $j \in \{1, \dots, rk\}$. Since R is pure, this implies $sg^-(a_{i_j}) = 0$ for all $j \in \{1, \dots, rk\}$. Moreover, by $g_{i_j,2b+5} \xrightarrow{a_{i_j}} g_{i_j,2b+6}$, we obtain $sup(g_{i_j,2b+6}) = sup(g_{i_j,2b+5}) + sig^+(a_{i_j})$. This requires $sp(g_{i_j,2b+5}) < b$, and exactly one of $v_{i_j,1}$ and $v_{i_j,2}$ has a positive consuming effect. The region R ensures that there are *exactly* k node events $v_{\ell_1}, \dots, v_{\ell_k}$ with a positive consuming effect. Recall, for every node $v \in V(G)$ holds $|\{e \in E(G) \mid v \in e\}| = r$. Thus, if $v_{\ell_1}, \dots, v_{\ell_k}$ are not independent, then the number of edges which are adjacent to a node of $v_{\ell_1}, \dots, v_{\ell_k}$ is at most $rk - 1$. Since rk edge events have a positive producing effect, and each of it needs a consuming node, this is a contradiction. Consequently, the set $I = \{v \in V(G) \mid sg^-(v) > 0\}$ defines a k-independent set of G.

For the *only-if*-direction we show that if G has a k-independent set then there is a b-admissible set of regions $\mathcal{R}$ such that $|^\bullet R|, |R^\bullet| \leq 2rk + 20$ for all $R \in \mathcal{R}$. The major challenge here is to keep the number of consuming and producing events of solving regions smaller than the parameter. To do so, we exploit G's regularity and the δ- and ζ-events. In what follows, we prove the following lemma:

Lemma 2. *1. If A is solvable, then there is a region (sp, sg) such that the following conditions are true:*

(a) $sg(\alpha_1) = (b, 0)$ and $sg(\zeta_{i,1}^1) = \dots = sg(\zeta_{i,3}^1) = (0, 0)$ for all $i \in \{1, \dots, m\}$.

(b) If $e \in \{a_1, \dots, a_m\}$ then $sg^-(e) = 0$ and there are exactly rk events $a_{i_1}, \dots, a_{i_{rk}} \in \{a_1, \dots, a_m\}$ with $sg^+(a_{i_j}) > 0$, where $j \in \{1, \dots, rk\}$.

(c) If $e \in \{v_1, \dots, v_n\}$ then $sg^+(e) = 0$. Furthermore, there are exactly k events $v_{i_1}, \dots, v_{i_k} \in \{v_1, \dots, v_n\}$ with $sg^-(v_{i_j}) > 0$ for all $j \in \{1, \dots, k\}$.

2. If G has an independent set of size k then there is a b-admissible set $\mathcal{R}$ of A such that $|^\bullet R|, |R^\bullet| \leq 2rk + 20$ for all $R \in \mathcal{R}$.

3.1 The Proof of Lemma 2.1

This section introduces the gadgets that ensure Lemma 2.1. For now, we refrain from explaining in which way they are actually conjunct to build A. This conjunction is postponed to Sect. 3.2, which is dedicated to Lemma 2.2.. We let events $e \in E(A)$ occur b times in row to restrict their possible signature in advance:

Lemma 3. *Let A be a TS, and let $e \in E(A)$ be an event that occurs b times in a row: $s_1 \xrightarrow{(e)^b} s_{b+1} \in A$. For any pure region (sp, sg) of A with $sg^+(e) \neq sg^-(e)$ holds either $sg(e) = (1,0)$, $sp(s_1) = b$ and $sp(s_{b+1}) = 0$ or $sg(e) = (0,1)$, $sp(s_1) = 0$ and $sp(s_{b+1}) = b$.*

Proof. The claim follows by $b \geq sp(s_{b+1}) = sp(s_1) + b \cdot (sg^+(e) - sg^-(e)) \geq 0$.

The TS A has for $i \in \{1,2,3\}$ the following w-maker gadget X_i:

$$x_{i,1} \xrightarrow{\delta_i^{10}} x_{i,2} \xrightarrow{(\alpha)^b} x_{i,b+2} \xrightarrow{\zeta} x_{i,b+3} \xrightarrow{w_i} x_{i,b+4} \xrightarrow{(\alpha)^b} x_{i,2b+4}$$

If A is solvable, then there is a region $R = (sp, sg)$ that solves the atom $(\alpha, x_{1,b+3})$, that is, $sg^-(\alpha) > sp(x_{1,b+3})$ or $sp(x_{1,b+3}) - sg^-(\alpha) + sg^+(\alpha) > b$. This implies $sg(\alpha) \neq (0,0)$. Thus, by Lemma 3, we have $sg(\alpha) \in \{(1,0),(0,1)\}$. Since our arguments are symmetrically true for the case $sg(\alpha) = (0,1)$, we assume $sg(\alpha) = (1,0)$ and show that this implies a k-independent set of G.

Since R solves $(\alpha, x_{i,b+3})$, by $sg(\alpha) = (1,0)$, we conclude $sg^-(\alpha) > sp(x_{i,b+3}) = 0$. Moreover, by Lemma 3, we obtain $sp(x_{i,b+2}) = 0$ and $sp(x_{i,b+4}) = b$ for all $i \in \{1,2,3\}$. Furthermore, by $sp(x_{1,b+2}) = sp(x_{1,b+3}) = 0$, we get $sg(\zeta) = (0,0)$. By $sp(x_{i,b+2}) = 0$, this implies $sp(x_{i,b+3}) = 0$ for all $i \in \{2,3\}$. Finally, by $sp(x_{i,b+3}) = 0$ and $sp(x_{i,b+4}) = b$ for all $i \in \{1,2,3\}$, we get the three producing w-events w_1, w_2, w_3: $sg(w_1) = sg(w_2) = sg(w_3) = (0,b)$.

The TS A has for $i \in \{1,\dots,9\}$ a so called α-maker Y_i that uses w_1 and w_2 to manipulate the support of some states and provides the consuming α-event α_i:

$$y_{i,1} \xrightarrow{\delta_i^{11}} y_{i,2} \xrightarrow{w_1} y_{i,3} \xrightarrow{\alpha_i} y_{i,4} \xrightarrow{w_2} y_{i,5}$$

By $sg(w_1) = sg(w_2) = (0,b)$, we have $sp(y_{i,3}) = b$ and $sp(y_{i,4}) = 0$. This implies $sg(\alpha_i) = (b,0)$ for $i \in \{1,\dots,9\}$. The events $\alpha_1,\dots,\alpha_9$ are applied to manipulate the support of some states. For example, by $sg(\alpha_1) = (b,0)$ and $g_{i,2}\xrightarrow{\alpha_1}$, we have $sp(g_{i,2}) = b$ for all $i \in \{1,\dots,m\}$ as discussed before. The following β-makers also exemplify the functionality of the α-events.

The TS A has for every $i \in \{1,\dots,5\}$ the following β-maker Z_i that uses the events α_7 and α_8 to provide the producing β-event β_i:

$$z_{i,1} \xrightarrow{\delta_i^{12}} z_{i,2} \xrightarrow{\alpha_7} z_{i,3} \xrightarrow{\beta_i} z_{i,4} \xrightarrow{\alpha_8} z_{i,5}$$

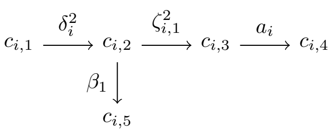
In particular, by $sg(\alpha_7) = sg(\alpha_8) = (b,0)$, we get $sp(z_{i,3}) = 0$ and $sp(z_{i,4}) = b$. This implies $sg(\beta_i) = (0,b)$ for all $i \in \{1,\dots,5\}$. Just like the α-events, the β-events serve to manipulate the support of some states.

In the remainder of this section, we first introduce the gadgets ensuring that $R = (sp, sg)$ selects exactly rk edge events $a_{i_1}, \dots, a_{i_{rk}}$ such that $sig^+(a_{i_j}) > 0$ for all $j \in \{1, \dots, rk\}$. Secondly, we introduce the gadgets that ensure that there are exactly k node events $v_{\ell_1}, \dots, v_{\ell_k}$ such that $sig^-(v_{\ell_j}) > 0$ for all $j \in \{1, \dots, k\}$. Similar to the already presented gadgets $G_1, \dots, G_m$, these gadgets apply ζ-events, that is, elements of the set $Z = \{\zeta^i_{j,\ell} \mid i, j, \ell \in \mathbb{N}\}$. For the region R, corresponding to Lemma 2.1, these events have to be neutral. For the proof of Lemma 2.2. they allow solving regions with small preset- and postset-cardinality. If $\zeta^i_{j,\ell} \in Z \cap E(A)$, that is, $\zeta^i_{j,\ell}$ actually occurs in A, then A has the following ζ-makers $\bigominus^i_{j,\ell}$ (left) and $\bigoplus^i_{j,\ell}$ (right). These gadgets ensure $\zeta^i_{j,\ell}$'s neutrality:

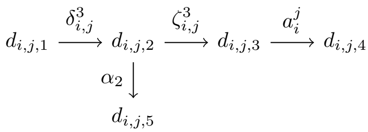
By $sg(w_3) = (0, b)$ and $sg(\alpha_9) = (b, 0)$, we get $sp(\ominus^i_{j,\ell,2}) = 0$ and $sp(\oplus^i_{j,\ell,2}) = b$. Moreover, by $0 = sp(\ominus^i_{j,\ell,2}) \geq sg^-(\zeta^i_{j,\ell})$, we obtain $sg^-(\zeta^i_{j,\ell}) = 0$. Finally, by $b \geq sp(\oplus^i_{j,\ell,4}) = sp(\oplus^i_{j,\ell,2}) - sg^-(\zeta^i_{j,\ell}) + sg^+(\zeta^i_{j,\ell})$, implying $b \geq b + sg^+(\zeta^i_{j,\ell})$, we get $sg^+(\zeta^i_{j,\ell}) = 0$.

So far, we have introduced A's gadgets that yield us the α-, β- and ζ-events with the following behavior: If $s \xrightarrow{\alpha}$, then $sp(s) = b$; if $s \xrightarrow{\beta}$, then $sp(s) = 0$; if $s \xrightarrow{\zeta} s'$, then $sp(s) = sp(s')$. These events are applied in the subsequently introduced gadgets, which collaborate to provide the announced behavior of A.

The TS A has for every edge event a_i, $i \in \{1, \dots, m\}$, exactly rk *edge copies* (*e*-copies, for short) $a^1_i, \dots, a^{rk}_i$. These copies are used to enable the announced selection of rk edge events $a_{i_1}, \dots, a_{i_{rk}}$. To achieve this goal, it is necessary that edge events do not consume and e-copies do not produce. The TS A has for every $i \in \{1, \dots, m\}$ an *edge noCon* C_i. This gadget ensures that a_i does *no*t *con*sume. Moreover, for all $i \in \{1, \dots, m\}$ and all $j \in \{1, \dots, rk\}$ it has an *e-copy noPro* $D_{i,j}$. This gadget guarantees that a^j_i does *no*t *pro*duce.



The edge noCon C_i.



The e-copy noPro $D_{i,j}$.

By $sg(\beta_1) = (0, b)$ and $sg(\zeta^2_{i,1}) = (0, 0)$, we have $sp(c_{i,3}) = 0$. Since $sp(c_{i,3}) \geq sg^-(a_i)$, this implies $sg^-(a_i) = 0$. Similarly, by $sg(\alpha_2) = (b, 0)$ and $sg(\zeta^3_{i,j}) = (0, 0)$, we obtain $sp(d_{i,j,3}) = b$. The region R is pure. Thus, if $sg^+(a^j_i) > 0$ then $sg^-(a^j_i) = 0$. This implies $sp(d_{i,j,4}) = b + sg^+(a^j_i) > b$, a contradiction. Hence, $sg^+(a^j_i) = 0$ is true.

The region R selects for every $j \in \{1, \dots, rk\}$ exactly one $i \in \{1, \dots, m\}$ such that the e-copy a^j_i has a positive consuming effect, that is, $sig^-(a^j_i) > 0$.

The other e-copies remain neutral. To achieve this, the TS A uses for every $j \in \{1, \dots, rk\}$ the *edge selector* F_j. The gadget F_j applies the events $a_1^j, \dots, a_m^j$, that is, the j-th copy of every edge event $a_1, \dots, a_m$. On F_j, every a_i^j occurs b times consecutively. Separated by ζ-events, these occurrences $(a_1^j)^b, \dots, (a_m^j)^b$ are placed in a sequence. We abridge $\ell = (m-1)(b+1)$ and define F_j:

$$
\begin{array}{l}
f_{j,1} \xrightarrow{\delta_j^4} f_{j,2} \xrightarrow{\zeta_{j,1}^4} f_{j,3} \xrightarrow{(a_1^j)^b} f_{j,b+3} \cdots f_{j,\ell} \xrightarrow{\zeta_{j,m}^4} f_{j,\ell+1} \xrightarrow{(a_m^j)^b} f_{j,m(b+1)+2} \xrightarrow{\zeta_{j,m}^4} f_{j,m(b+1)+3} \\
f_{j,2} \xrightarrow{\alpha_3} f_{j,m(b+1)+5} \qquad f_{j,m(b+1)+3} \xrightarrow{\beta_2} f_{j,m(b+1)+4}
\end{array}
$$

By $sg(\alpha_3) = (b, 0)$, we have $sp(f_{j,2}) = b$ and, by $sg(\beta_2) = (0, b)$, we have $sp(f_{j,m(b+1)+3}) = 0$. The ζ-events are neutral, and $sg^+(a_i^j) = 0$ for all $i \in \{1, \dots, m\}$. Thus, we obtain $0 = \sum_{i=1}^{m} b \cdot sg^-(a_i^j) < b$. Consequently, there is an $i \in \{1, \dots, m\}$ such that $sg^-(a_i^j) = 1$, and $sg^-(a_{i'}^j) = 0$ for all $i' \in \{1, \dots, m\} \setminus \{i\}$. The following edge connectors complete the set of A's gadgets that allow the selection of rk edges $a_{i_1}, \dots, a_{i_{rk}}$.

The TS A has for all $i \in \{1, \dots, m\}$ a so called *edge connector* H_i whose purpose is twofold. On the one hand, it ensures that the edge selectors never choose two consuming copies of the same edge event, that is, if $j \neq j'$, $sg^-(a_i^j) > 0$ and $sg^-(a_{i'}^{j'}) > 0$, then $i \neq i'$. On the other hand, $sg^+(a_i) > 0$ if and only if there is a $j \in \{1, \dots, rk\}$ such that $sg^-(a_i^j) > 0$. Since $F_1, \dots, F_{rk}$ select rk consuming edge copies, this picks out exactly rk edges $a_{i_1}, \dots, a_{i_{rk}}$ with a positive producing effect. The gadget H_i applies the event a_i and its rk copies. Separated by ζ-events, two sequences of a_i's copies $a_i^1, \dots, a_i^{rk}$, each of if it occurring b times consecutively, embrace the event a_i. For readability, we abridge $\ell = (b+1)rk + 2$ and define H_i as follows:

$$
\begin{array}{l}
h_{i,1} \xrightarrow{\delta_i^5} h_{i,2} \xrightarrow{\zeta_{i,1}^5} h_{i,3} \xrightarrow{(a_i^1)^b} h_{i,b+3} \cdots h_{i,\ell-2} \xrightarrow{\zeta_{i,rk}^5} h_{i,\ell-1} \xrightarrow{(a_i^{rk})^b} h_{i,\ell} \xrightarrow{\zeta_{i,rk+1}^5} h_{i,\ell+1} \xrightarrow{a_i} h_{i,\ell+2} \\
h_{i,2} \xrightarrow{\alpha_4} h_{i,2\ell+6} \\
h_{i,2\ell+5} \xleftarrow{(a_i^{rk})^b} h_{i,2\ell+4} \xleftarrow{\zeta_{i,2rk+2}^5} h_{i,2\ell+3} \cdots h_{i,\ell+b+3} \xleftarrow{(a_i^1)^b} h_{i,\ell+3} \xleftarrow{\zeta_{i,rk+2}^5} h_{i,\ell+2}
\end{array}
$$

By $sg(\alpha_4) = (b, 0)$ it is $sp(h_{i,2}) = b$. The ζ-events are neutral, and $sg^+(a_i^j) = 0$ for all $j \in \{1, \dots, rk\}$. Thus, it is $sp(h_{(b+1)rk+3}) = b - \sum_{j=1}^{rk} b \cdot sg^-(a_i^j)$, and, by $sp(h_{(b+1)rk+3}) \geq 0$, there is at most one $j \in \{1, \dots, rk\}$ such that $sg^-(a_i^j) > 0$. Consequently, two copies of the same edge event are never selected by the edge selectors. By Lemma 3, if $sg^-(a_i^j) > 0$, then $sg^-(a_i^j) = 1$. This implies $sp(h_{(b+1)rk+3}) = 0$. Furthermore, a_i^j occurs again b times in a row "after" the occurrence of a_i at $h_{i,(rk+j-1)(b+1)+5}$. This implies $sp(h_{i,(rk+j-1)(b+1)+5}) = b$. Since no edge copy produces, $sg(a_i) = (0, b)$ is immediately implied. Conversely, if $sg^+(a_i) > 0$, then $sp(h_{(b+1)rk+3}) < b$. Thus, by $sp(h_{(b+1)rk+3}) = b - \sum_{j=1}^{rk} b \cdot sg^-(a_i^j)$, there is a consuming copy of a_i. Consequently, $sg^+(a_i) > 0$ if and only if $sg(a_i) = (0, b)$ and there is exactly one $j \in \{1, \dots, rk\}$ such that $sg^-(a_i^j) = 1$.

So far we have argued that there are exactly rk distinct indices $i_1, \dots, i_{rk} \in \{1, \dots, m\}$ such that $sg(a_{i_1}) = \dots = sg(a_{i_{rk}}) = (0, b)$. Moreover, $sg(a_i) = (0, 0)$ for all $i \in \{1, \dots, m\} \setminus \{i_1, \dots, i_{rk}\}$. It remains to argue that these rk "edges" $a_{i_1}, \dots, a_{i_{rk}}$ are covered by exactly k "nodes". To achieve this goal, the TS A uses gadgets that work symmetrically to the ones used for the selection of the edges. So called *node noPros* ensure that the node events $v_1, \dots, v_n$ do *no*t *pro*duce. Moreover, the TS A applies for all $i \in \{1, \dots, n\}$ k *node-copies* $v_i^1, \dots, v_i^k$ and uses *n-copy noCons* to prevent them from consuming. Furthermore, *node selectors* force exactly k node copies to have a producing signature. The *node connectors* ensure that two copies of the same node are never selected and connects a producing node copy v_i^j with its, then consuming, node event v_i. Finally, exactly k nodes $v_{\ell_1}, \dots, v_{\ell_k}$ consume. Since these gadgets work symmetrically to the ones for the edges, we only briefly prove their functionality.

The TS A has for $i \in \{1, \dots, n\}$ the so called *node noPro* P_i (left hand side) and for $i \in \{1, \dots, n\}$ and $j \in \{1, \dots, k\}$ the so called *n-copy antiCon* $Q_{i,j}$ (right hand side) which are defined as follows:

$$
\begin{array}{ccccccc}
p_{i,1} & \xrightarrow{\delta_i^6} & p_{i,2} & \xrightarrow{\zeta_{i,1}^6} & p_{i,3} & \xrightarrow{v_i} & p_{i,4} \\
 & & \alpha_5 \downarrow & & & & \\
 & & p_{i,5} & & & &
\end{array}
\qquad
\begin{array}{ccccccc}
q_{i,j,1} & \xrightarrow{\delta_{i,j}^7} & q_{i,j,2} & \xrightarrow{\zeta_{i,j}^7} & q_{i,j,3} & \xrightarrow{v_i^j} & q_{i,j,4} \\
 & & \beta_3 \downarrow & & & & \\
 & & q_{i,j,5} & & & &
\end{array}
$$

By $sg(\alpha_5) = (b, 0)$ and $sg(\zeta_{i,1}^6) = (0, 0)$, we get $sp(p_{i,3}) = b$ which implies $sg^+(v_i) = 0$. Moreover, by $sg(\beta_3) = (0, b)$ and $sg(\zeta_{i,j}^7) = (0, 0)$, we get $sp(q_{i,j,3}) = 0$ which implies $sg^-(v_i^j) = 0$.

The TS A has for every $j \in \{1, \dots, k\}$ a *node selector* T_j. On T_j, separated by ζ-events, the j-th copy of every node event $v_1, \dots, v_n$ occurs b times in a row. We abridge $\ell = (n - 1)(b + 1) + 2$ and define T_j as follows:

$$
\begin{array}{ccccccccccccccc}
t_{j,1} & \xrightarrow{\delta_j^8} & t_{j,2} & \xrightarrow{\zeta_{j,1}^8} & t_{j,3} & \xrightarrow{(v_1^j)^b} & t_{j,b+3} & \cdots & t_{j,\ell} & \xrightarrow{\zeta_{j,n}^8} & t_{j,\ell+1} & \xrightarrow{(v_n^j)^b} & t_{j,n(b+1)+2} & \xrightarrow{\zeta_{j,n+1}^8} & t_{j,n(b+1)+3} \\
 & & \beta_4 \downarrow & & & & & & & & & & & & \downarrow \alpha_6 \\
 & & t_{j,n(b+1)+5} & & & & & & & & & & & & t_{j,n(b+1)+4}
\end{array}
$$

By $sg(\beta_4) = (0, b)$, $sg(\alpha_6) = (b, 0)$, the neutrality of the ζ-events and $sg^-(v_1^j) = \dots = sg^-(v_n^j) = 0$, we have $b = sp(t_{j,n(b+1)+3}) = \sum_{i=1}^{n} b \cdot sg^+(v_i^j) > sp(t_{j,3}) = 0$. Thus, there is exactly one producing j-th copy produces and others are neutral.

Finally, the TS A has for every $i \in \{1, \dots, n\}$ a so called *node connector* U_i that, among others, applies the β-event β_5, the k copies $v_i^1, \dots, v_i^k$ of v_i and the event v_i. We abridge $\ell = (k - 1)(b + 1) + 2$ and define U_i as follows:

$$
\begin{array}{ccccccccccccccc}
u_{i,1} & \xrightarrow{\delta_i^9} & u_{i,2} & \xrightarrow{\zeta_{i,1}^9} & u_{i,3} & \xrightarrow{(v_i^1)^b} & u_{i,b+3} & \cdots & u_{i,\ell} & \xrightarrow{\zeta_{i,k}^9} & u_{i,\ell+1} & \xrightarrow{(v_i^k)^b} & u_{i,k(b+1)+2} & \xrightarrow{\zeta_{i,k+1}^9} & u_{i,k(b+1)+3} \\
 & & \beta_5 \downarrow & & & & & & & & & & & & \downarrow v_i \\
 & & u_{i,k(b+1)+5} & & & & & & & & & & & & u_{i,k(b+1)+4}
\end{array}
$$

By $sg(\beta_5) = (0, b)$, the neutrality of the ζ-events and $sg^-(v_i^j) = 0$ for all $j \in \{1, \dots, k\}$, it holds $b \geq sp(u_{j,k(b+1)+3}) = \sum_{j=1}^{k} b \cdot sg^+(v_i^j)$. Thus, at most one node-copy v_i^j, $j \in \{0, \dots, k\}$, of v_i is not neutral. In particular, two copies of the same node are never selected by the node selectors. Moreover, if $sg^-(v_i) > 0$, then $sp(u_{j,k(b+1)+3}) > 0$. Consequently, if v_i consumes, then there is a producing copy v_i^j. Since there are at most k producing node copies, there are at most k consuming nodes $v_{\ell_1}, \dots, v_{\ell_k}$. Thus, the rk producing events $a_{i_1}, \dots, a_{i_{rk}}$ are "covered" by exactly k consuming events $v_{\ell_1}, \dots, v_{\ell_k}$. Altogether, this proves that $I = \{v \in V(G) \mid sg^-(v) > 0\}$ defines an independent set of size k of G.

3.2 The Proof of Lemma 2.2

Table 1. The gadgets of A and their corresponding γ-events.

Gadget	G_i	C_i	$D_{i,j}$	F_j	H_i	P_i	$Q_{i,j}$	T_j	U_i	X_i	Y_i	Z_i	$\bigoplus^i_{j,\ell}$	$\bigominus^i_{j,\ell}$
γ-event	γ_i^1	γ_i^2	$\gamma_{i,j}^3$	γ_j^4	γ_i^5	γ_i^6	$\gamma_{i,j}^7$	γ_j^8	γ_i^9	γ_i^{10}	γ_i^{11}	γ_i^{12}	$\gamma_{i,j,\ell}^{13}$	$\gamma_{i,j,\ell}^{14}$

The reduction merges the introduced gadgets to a directed labelled binary tree with initial state $\iota = g_{1,1}$. The resulting TS A consists of 14 blocks, cf. Figure 1. The TS A has for each of its gadgets a γ-event in accordance to Table 1. Using these events, the joining connects the "initial states" of the gadgets as follows:

$$g_{1,1} \xrightarrow{\gamma_1^1} \cdots \xrightarrow{\gamma_{m-1}^1} g_{m,1} \xrightarrow{\gamma_m^1} c_{1,1} \xrightarrow{\gamma_1^2} \cdots \xrightarrow{\gamma_{m-1}^2} c_{m,1} \xrightarrow{\gamma_m^2} d_{1,1,1} \xrightarrow{\gamma_{1,1}^3} \cdots \xrightarrow{\gamma_{1,rk-1}^3} d_{1,rk,1} \xrightarrow{\gamma_{1,rk}^3} d_{2,1,1}$$

$$d_{2,1,1} \xrightarrow{\gamma_{2,1}^3} \cdots$$

$$\ominus^9_{n,k+1,1} \xleftarrow{\gamma_{n,k}^9} \ominus^9_{n,k,1} \xleftarrow{\gamma_{n,k-1}^9} \cdots \xleftarrow{\gamma_{1,1}^4} f_{1,1} \xleftarrow{\gamma_{m,rk}^3} d_{m,rk,1} \xleftarrow{\gamma_{m,rk-1}^3} \cdots$$

The γ-events $\gamma_{i,j,\ell}^h$, where indices that are 0 are omitted, occur "lexicographically" ordered by $hij\ell$ in accordance to the canonical order on the natural numbers. This defines also an order on the gadgets and makes the conjunction unambiguous.

Due to space restrictions, most of the proof of Lemma 2.2. is omitted. However, the following lemma states the solvability of α and $v_1, \dots, v_n$ and exemplifies in which way A allows regions that respect the parameter.

Lemma 4. *If (G, k) is a yes-instance of* Regular Independent Set *then the events α and $v_1, \dots, v_n$ are solvable by regions that respect the parameter $4rk + 40$.*

Proof. For the sake of space restrictions, we implicitly define solving regions $R_i = (sp_i, sg_i)$ by $sp_i(\iota)$ and sg_i, to be seen in Table 2: The ι-column shows $sp(\iota)$. The event sets occur in the column in accordance to the signature of their elements.

For example, $sp_1(e) = (b,0)$ for $e \in \{\alpha_1, \dots, \alpha_9\}$. Moreover, if $e \in E(A)$ does not occur in any presented set corresponding to R_i, then $sg_i(e) = (0,0)$. In particular, all signatures get along with $(b,0), (1,0), (0,0), (0,1), (0,b)$. By $sp_i(s') = sp_i(s) - sg_i^-(e) + sp_i^+(e)$ for all $s \xrightarrow{e} s' \in A$, this defines R_i completely.

Solving α: Let $r \in \mathbb{N}^+$ such that every node of G has degree r, and let $I = \{v_{\ell_1}, \dots, v_{\ell_k}\}$ be a k-independent set of G. The nodes $v_{\ell_1}, \dots, v_{\ell_k}$ are independent, and each of it has exactly r adjacent edges. Thus, there are exactly rk edges $a_{i_1}, \dots, a_{i_{rk}} \in E(G)$ such that for all $a \in E(G)$ the following is true. If $a \in E_I = \{a_{i_1}, \dots, a_{i_{rk}}\}$, then $|a \cap I| = 1$ and otherwise $|a \cap I| = 0$. Using I and E_I, we define region R_1 in accordance to Table 2. If we follow the arguments for the proof of Lemma 2.1, then it is easy to see that R_1 is well defined and solves α at $x_{i,b+2}, x_{i,b+3}$ and $x_{i,2b+4}$ for all $i \in \{1,2,3\}$. Moreover, $|^\bullet R_1| \leq k(r+1) + 13$ and $|R_1^\bullet| \leq k(r+1) + 11$, cf. Table 2. Thus, the region R_1 respects the parameter. Notice that the latter is possible by grouping "similar" gadgets into blocks. For example, if the node noPros alternated with the node selectors $(P_1, T_1, \dots, P_n, T_n)$, then the number of consuming and producing γ-events would depend on $|V(G)|$ and would not respect the parameter. The region R_2 of Table 2 solves α at the remaining states of A and respects the parameter.

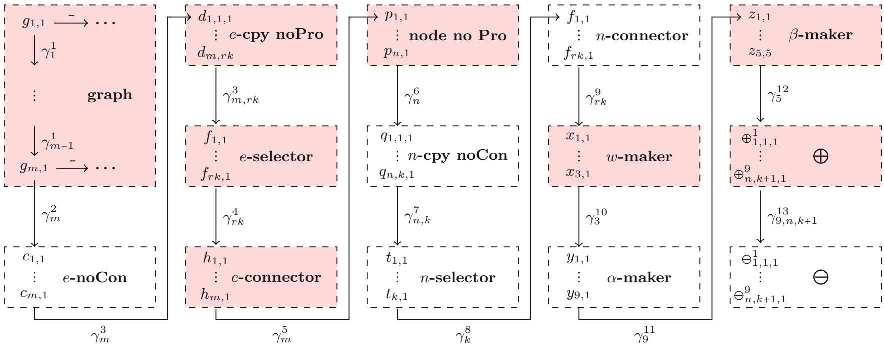


Fig. 1. The gadgets' conjunction to finally build A, consisting of "blocks" in accordance to similar "gadget-types". The red colored areas mark the gadgets whose initial states are mapped to b by R_1 (Table 2) solving $(\alpha, x_{i,b+2}), (\alpha, x_{i,b+3}), (\alpha, x_{i,2b+4})$ for all $i \in \{1,2,3\}$. (Color figure online)

Solving v_i, $i \in \{1, \dots, n\}$: The Region R_3 solves v_i at all states except the sinks of the affected ζ-events. The region R_4 solves v_i at these remaining sinks. The event v_i occurs in $G_{i_1}, \dots, G_{i_r}$, P_i and U_i. Thus, $|^\bullet R_3| \leq r+2$, $|^\bullet R_4| \leq r$, $|R_3^\bullet|$ and $|R_4^\bullet| \leq 1$.

Table 2. Implicitly defined regions of A that solve α and v_i for all $i \in \{1, \dots, n\}$.

R	ι	$(b, 0)$	$(1, 0)$	$(0, 1)$	$(0, b)$
R_1	b	$\{\alpha_1, \dots, \alpha_9\}$, $\{\gamma_m^2, \gamma_n^6, \gamma_3^{10}, \gamma_{9,n,k+1}^{13}\}$	$\{\alpha\} \cup I$, $\{a_{i_\ell}^\ell \mid 1 \leq \ell \leq rk\}$	$\{v_{\ell_i}^i \mid 1 \leq i \leq k\}$	$\{w_1, w_2, w_3, \beta_1, \dots, \beta_5\}$, E_I, $\{\gamma_m^3, \gamma_{rk}^9, \gamma_9^{11}\}$
R_2	0		$\{\alpha\}$		$\{\zeta, \delta_1^{10}, \delta_2^{10}, \delta_3^{10}\}$
R_3	0		$\{v_i\}$		$\{e \in E(A) \mid \xrightarrow{e} s \xrightarrow{v_i}\}$
R_4	0		$\{v_i\}$		$\{\delta_{i_1}^1, \dots, \delta_{i_r}^1, \delta_i^6, \delta_i^9\}$

4 Conclusion

In this paper, we enhance our understanding of synthesizing (m, n)-T-systems from the viewpoint of parameterized complexity. Although (m, n)-Synthesis parameterized by $m + n$ belongs to XP, we show that there is little hope that this parameterization puts the problem into FPT. Future work might consider the *occupancy number* o_N of a searched net N a parameter. Let $N = (P, T, f, M_0)$ be a pure b-net, and let RS be the set of N's reachable markings. The *occupancy number* o_p of a place $p \in P$ is defined by $o_p = \{M \in RS \mid M(p) > 0\}$, and $o_N = max\{o_p \mid p \in P\}$ defines the *occupancy number* of N. At first glance, this parameter seems promising, at least synthesis parameterized by o_N is in XP.

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