

Designing Insurance Against Extreme Weather Risk: The Case of HuRLOs

Martin Boyer, Michèle Breton, and Pascal François

INTRODUCTION

Hurricanes are among the most catastrophic natural events. Even though the number of hurricane landfalls appears stable in the United States, with an average of 18 per decade since 1900 (NHC [2018\)](#page-30-0), they tend to be more and more costly. According to the National Hurricane Center (NHC), 13 out of the 18 hurricanes that caused more than US\$10 billion (infation-adjusted) of damage since 1900 occurred during the last 15 years despite a 10-year lull between 2006 and 2015. Table [5.1](#page-1-0) reports the hurricanes that caused the biggest fnancial damage in the United States since 1900.

M. Boyer \bullet P. François (\boxtimes)

M. Breton Department of Decision Sciences, HEC Montréal, Montreal, QC, Canada

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Department of Finance, HEC Montréal, Montreal, QC, Canada e-mail: martin.boyer@hec.ca[; pascal.francois@hec.ca](mailto:pascal.francois@hec.ca)

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Rank	Hurricane	Regions hit	Year	Category	Damage
1	Katrina	SE FL, LA, MS	2005	3	160
\overline{c}	Harvey	TX, LA	2017	$\overline{4}$	125
$\overline{3}$	Maria	PR, USVI	2017	$\overline{4}$	90
$\overline{4}$	Sandy	Mid-Atlantic & NE U.S.	2012	$\mathbf{1}$	70.2
5	Irma	FL	2017	$\overline{4}$	50
6	Andrew	SE FL, LA	1992	5	47.79
7	Ike	TX, LA	2008	$\overline{2}$	34.8
8	Ivan	AL, NW FL	2004	$\overline{3}$	27.06
9	Wilma	S FL	2005	3	24.32
10	Rita	SW LA, N TX	2005	$\overline{3}$	23.68
$_{11}$	Charley	SW FL	2004	$\overline{4}$	21.12
12	Hugo	SC, USVI, PR	1989	$\overline{4}$	18.09
13	Irene	Mid-Atlantic & NE U.S.	2011	$\mathbf{1}$	14.985
14	Frances	FL	2004	$\overline{2}$	12.936
15	Agnes	FL, NE U.S.	1972	1	12.516
16	Allison	N TX	2001	TS	11.815
17	Betsy	SE FL, SE LA	1965	3	11.152
18	Matthew	SE US	2016	1	10.3
19	Jeanne	FL	2004	$\overline{3}$	9.9
20	Camille	MS, SE LA, VA	1969	5	9.776
21	Floyd	Mid-Atlantic & NE U.S.	1999	\overline{c}	9.62
22	Georges	USVI, PR, FL, MS, AL	1998	\overline{c}	9.06
23	Fran	NC	1996	$\overline{3}$	7.9
24	Diane	N _C	1955	1	7.63
25	Opal	NW FL	1995	3	7.614
26	Alicia	N TX	1983	$\overline{3}$	7.47
27	Isabel	Mid-Atlantic	2003	$\mathfrak{2}$	7.37
28	Gustav	LA	2008	\overline{c}	6.96
29	Celia	TX	1970	$\overline{3}$	6.026
30	Frederic	AL, MS	1979	$\overline{3}$	5.712
31	Iniki	Kauai, HI	1992	$\overline{4}$	5.487
32	Long Island Express	NE US	1938	3	5.279
33	NC/VA 1944	Mid-Atlantic	1944	3	4.927
34	Carol	NE US	1954	3	4.198
35	Marilyn	USVI, PR	1995	$\overline{2}$	3.402

Table 5.1 Costliest mainland United States tropical cyclones, 1900–2017

Damage is expressed in US\$ billions after accounting for infation to 2017 dollars Source: National Hurricane Center. Category "TS" stands for tropical storm

Due to the growth of hurricane occurrence and the costly damages associated with them, important literature has been devoted to the assessment of hurricane risk and to their trajectories (see inter alia, Jewson and Hall [2007,](#page-30-1) Nakamura et al. [2015](#page-30-2), Kriesche et al. [2014,](#page-30-3) and the

references therein). Approaches to predict hurricane outcomes can be divided into two main streams: meteorological and probabilistic methods. Meteorological methods (see for instance Gray et al. [1992\)](#page-30-4) are based on complex models of natural phenomena, while probabilistic methods (static or dynamic) rather rely on historical frequencies (see Bove et al. [1998;](#page-29-0) Epstein [1985](#page-29-1); Vickery et al. [2000](#page-31-0); Jewson and Hall [2007](#page-30-1); Bonazzi et al. [2014](#page-29-2)). Both approaches aim at assessing the probability of catastrophic events in a given area.

The reason why so much energy has been devoted to predicting this natural phenomenon is that insurance against catastrophic risk, such as hurricanes, wind storms, and tsunamis, is an important concern for homeowners. The concern is especially latent for homeowners in the south-east United States, around the Gulf of Mexico, and in the western North-Pacific¹

The increase in concern is due in part to the increased concentration of insured risk in coastal regions vulnerable to climatic catastrophes,^{[2](#page-2-1)} leading to an increase in the cost of rebuilding communities, and to a reduction in the ability of the insurance industry to fnancially support such losses in these regions. The potential insurer insolvency risk associated with major climatic catastrophes creates an important entry barrier since newcomers in the hurricane and catastrophic insurance market must have large amounts of capital available. This entry barrier led Froot ([2001\)](#page-30-5) to conclude that the market for catastrophic risk suffers from supply restrictions that can be partly explained by the market power exerted by traditional reinsurers.

To supplement the traditional insurance market, fnancial instruments called Insurance Linked Securities have emerged in the 1990s and early 2000s (see Cummins and Barrieu [2013](#page-29-3)). Such instruments are also known as catastrophe options and catastrophe bonds, industry loss warranties, and sidecars, all of which are fnancially competitive when compared to traditional reinsurance (see Ramella and Madeiros [2007\)](#page-31-1).

¹ See the hurricane generation models of Hall and Jewson [\(2007](#page-30-6)) and Rumpf et al. ([2009\)](#page-31-2) for the case of the North Atlantic, and of Rumpf et al. ([2007](#page-31-3)) and of Yonekura and Hall ([2011\)](#page-31-4) for the case of the western North-Pacifc.

2According to Pielke et al. ([2008\)](#page-31-5) the hurricanes that landed in Miami in 1926 resulted in losses of 760 million dollars. If such a hurricane were to hit the Miami agglomeration today, the fnancial losses would amount to approximately 150 billion dollars (or 102 billion 2004-dollars according to Kunreuther and Michel-Kerjan [2009\)](#page-30-7). Hurricane Katrina, which hit New Orleans in 2005, caused damages estimated at 108 billion dollars according to the National Oceanic and Atmospheric Administration.

Moreover, and in contrast to traditional insurance contracts, they can be designed in a way to have very low moral hazard and credit risk (Ramella and Madeiros [2007\)](#page-31-1). Such interesting design features come, however, at the cost of increasing the instruments' basis risk (Doherty [1997\)](#page-30-8). One new instrument is the HuRLO (Hurricane Risk Landfall Option), launched in 2008 by Weather Risk Solutions (WRS), that allows investors to take positions on hurricane landfall in a similar way as in pari-mutuel frst-by-the-post horse race betting. While the interest in catastrophe bonds and other insurance-linked securities has been growing steadily (Cummins [2008](#page-29-4), [2012](#page-29-5)), the literature on the hurricane-risk market itself is not extensive. Using data from the Hurricane Futures Market, Kelly et al. ([2012\)](#page-30-9) study the traders' perception and the trading dynamics according to available information on hurricane risk. They conclude that relative-demand pricing is consistent with a Bayesian update of beliefs according to information released by various offcial meteorological centers.

The objective of this paper is to analyze the operation of the HuRLO market by modeling the decisions of rational risk-averse decision-makers who want to hedge against catastrophic losses. Using the HuRLO as a motivating example, Ou-Yang ([2010](#page-31-6)) and Ou-Yang and Doherty [\(2011\)](#page-31-7) compare pari-mutuel³ and traditional insurance for risk-averse expected utility maximizing hedgers. They compute the optimal dynamic hedge of a single agent in an economy where the decisions of the other players are assumed to be exogenous. Moreover, they examine the properties of the equilibrium on that market when agents and risks are symmetrical. They fnd that a pari-mutuel mechanism leads to under-insurance. They also fnd that a pari-mutuel setting can be advantageous when transaction costs of traditional insurance are high and when information asymmetry problems are rampant.

³The pari-mutuel mechanism was invented by Pierre Oller in 1865 in order to limit the proft of bookmarkers who were then controlling the betting industry in France. Since 2002, many investment banks have used a pari-mutuel mechanism for wagering on various economic statistics; odds on these statistics have been shown to be effcient forecasts of their future values (Gürkaynak and Wolfers [2006](#page-30-10)). The pari-mutuel market microstructure is analyzed by Lange and Economides [\(2005](#page-30-11)) who show the existence of a unique price equilibrium and fnd many advantages of pari-mutuel over the traditional exchange mechanism. A pari-mutuel auction system for capital markets is proposed by Baron and Lange [\(2007](#page-29-6)).

With respect to HuRLOs in particular, Wilks [\(2010](#page-31-8)) describes their market structure, and the mechanism and adaptive algorithm used to price the options. He shows that the proposed price adjustment mechanism converges rapidly to the market participants' beliefs about the outcome probabilities. Meyer et al. ([2008](#page-30-12), [2014](#page-30-13)) study the behavior of participants in an experiment of a simulated hypothetical hurricane season, during which they are allowed to trade in both primary and secondary HuRLO markets. Meyer et al. ([2008](#page-30-12), [2014\)](#page-30-13) look into the potential bias traditionally observed in pari-mutuel betting. They fnd that market prices converge to effcient levels and that biases are not signifcant at the aggregate level. A priori, it therefore seems that HuRLOs should be a perfect additional tool for hedging catastrophic loss in the Southeast United States, and in the state of Florida in particular.

Our contribution to the literature is twofold. We frst examine the effectiveness of pari-mutuel insurance, as Ou-Yang and Doherty ([2011](#page-31-7)), but in a more realistic setting, with dynamic trading, price updating, and strategic interactions between market participants. In addition, unlike Meyer et al. ([2008,](#page-30-12) [2014](#page-30-13)), our model involves agents characterized by concave utility functions acting optimally, albeit possibly with limited foresight.

The HuRLO market provides investors with the opportunity to hedge against, or speculate on, the risk that a specifc region in the Gulf of Mexico and on the East Coast of the United States will be the frst to be hit by a hurricane (or that no hurricane will make landfall in the continental United States) during a year. Many characteristics of HuRLOs distinguish them from traditional insurance, including the fact that the payment received doesn't depend on an individual's fnancial loss (or lack thereof), nor on the price paid for such protection. And, because of the pari-mutuel setting, HuRLOs have characteristics that distinguish them from standard derivatives such as: (1) the absence of counterparty and liquidity risk; (2) the absence of an underlying traded asset; and (3) a market-demand-based payoff function.

HuRLOs are interesting for both hedging and speculating purposes. On the hedging side, HuRLOs could be useful to agents (individuals, frms, or otherwise) that own assets in hurricane-prone and thus vulnerable areas. Speculators could also participate in that market by taking advantage of differences in market-based and objective landfall

probabilities. As a competitor for traditional reinsurance products, HuRLOs have important merits: since the risk is limited to the invested capital, no counterparty is needed to assume the position opposite to what the insurer/reinsurer desires, and there is essentially no need for a probability or a loss appraisal since the payoff depends on market-wide factors that all but eliminate adverse selection and moral hazard issues.

Despite such advantages and the HuRLOs' complementarity with other forms of natural catastrophe hedging instruments, the market for HuRLOs has not taken off. According to the Weather Risk Solution website,⁴ HuRLOs are not presently (2016–2019) available for trading.

Given the particular price formation mechanism in the HuRLOs market, one interesting question is the possible presence of strategic issues: when buying HuRLO for insurance purposes, should one place a single order, or buy options sequentially? Should one be the frst to trade, or wait to observe the trades of other players? To address this question, we study the behavior of many agents facing potential losses from hurricanes. Our model of the HuRLO market is dynamic, and we explicitly account both for the impact of individual players' decisions on the option price and for risk aversion in the face of catastrophic losses. As in Ou-Yang and Doherty ([2011](#page-31-7)), agents are nonstrategic investors who maximize their utility by assuming exogenous prices and stakes of other players. We model their behavior during simulated hurricane seasons to evaluate various investment strategies in terms of sequence and size of purchases. Our simulations reveal that the order type, sequence, and order packaging have a signifcant impact on the price paid, and on the number of traded options. Therefore, HuRLO contracts appear to be difficult to evaluate and to purchase optimally. This seriously questions the ability of the HuRLO market to act as an effective insurance mechanism.

The rest of the paper is organized as follows. Section "[Hurricane Risk](#page-6-0) [Landfall Options"](#page-6-0) gives details on the HuRLO product and market organization and on the price adjustment mechanism. Section "[A Simulation](#page-11-0) [Experiment](#page-11-0)" reports on the implementation and results of the simulation model. Section "[Recommendations and Public Policy Implications"](#page-21-0) elaborates on recommendations and public policy implications. Section ["Conclusion"](#page-23-0) concludes.

⁴www.weatherrisksolutions.com (last visited in January 2019).

Hurricane Risk Landfall Options

HuRLOs are binary options on the occurrence of hurricane landfall in various regions during a given hurricane season; 75 of these options are available: 74 correspond to a given county or area (thereafter identifed as counties), and the "null" option corresponds to the case where none of the 74 options received a payoff before the end of the hurricane season. When the National Hurricane Center (NHC) issues a hurricane warning because the hurricane is closing in on a specifc county, trading is suspended until the hurricane makes landfall or the immediate threat vanishes. Options are automatically "exercised" when a hurricane hits one county[.5](#page-6-1)

When this landfall occurs, holders of the winning option (corresponding to the hit county) receive a payoff (i.e., the option matures in-the-money) while holders of all the other options receive nothing (options mature outof-the-money). At the end of the season, the holders of the null option of all the series^{[6](#page-6-2)} that have not yet materialized receive a payoff, while all the other options are worthless (since risk pools are separated for different series, payoffs of the null option differ across series). When a hurricane hits two counties, it is considered as a second hurricane if the contact points are more than 150 nautical miles apart.

HuRLOs are priced to refect market demand. This contrasts with classical pari-mutuel settings where the price of a claim is constant and independent of the demand for a given position. When the outcome is realized, the total mutual reserve is shared equally among the owners of the winning claim, irrespective of the price they paid for their option. Thus, if at a given date and for a given series we observe the market price of option $k(\pi_k)$, the total mutual reserve (M) , and that m_k options of type k were purchased in the primary market, then the payoff of a stake if outcome *k* is realized (R_k) , the (decimal) odds of outcome $k(O_k)$, and the implied market probability (q_k) are given by⁷:

⁵To qualify, a hurricane must be identified as such by the NHC and must cause more than 1 million dollars damage according to EQECAT (now part of Corelogic).

⁶A new series of options is launched every time a new hurricane is identifed by the National Hurricane Center.

⁷A list of notations for all parameters and variables used in this paper is provided in Appendix 1.

$$
R_k = \frac{M}{m_k} = \frac{\sum_k \pi_k m_k}{m_k}
$$

$$
O_k = \frac{R_k}{\pi_k} = \frac{M}{\pi_k m_k}
$$

$$
q_k = \frac{\pi_k m_k}{\sum_k \pi_k m_k} = \frac{1}{O_k}.
$$

In a classic pari-mutuel setting, the corresponding R_k , O_k , and q_k values are obtained by fixing $\pi_k = c$ for all *k* so that $M = c \sum_k m_k$ and $q_k = m_k / \sum_k m_k$.

Each HuRLO series is "seeded" by a fnancial institution that buys an equal number of each option (say 1), at a price that refects the historical probabilities of the possible outcomes (see the Table [5.6](#page-26-0) in Appendix [2](#page-25-0) for a summary of the historical probabilities in the United States). As options are bought on the primary market, prices adjust dynamically to the collective trading of market participants, refecting the relative demand for the various options. As a result, when an order for a block of identical options is executed, the price of each option in the block is increasing, while the prices of all the other options are decreasing, refecting the increasing total relative demand for this option.

This dynamic adjustment mechanism is not considered by Ou-Yang and Doherty (2011) , who are solving a static optimization problem for a single agent, under perfect information on odds across all areas. Accordingly, they model the decision problem faced by an individual who "places his stake at the end of the wagering period after all other participants have placed their stakes." Assuming a stake of *x* dollars in option *k* when the mutual reserve is *M* and the total stakes on outcome *k* placed by other participants is M_k , the payoff to the agent becomes

$$
R_k(x) = x \frac{M+x}{M_k+x}
$$

if outcome *k* is realized. In other words, in a static world, all the mutual reserve is shared according to the amount wagered rather than to the number of options held, since the price of each option is constant so that $\pi_k = c$. This yields an analytical characterization of the optimal stake using frst-order optimality conditions.

Even under the assumption of perfect information, if the last player in a small market decided to invest *x* to buy a block of HuRLOs for a given county, all of these options would need a different price to refect the increasing demand, and these successive prices are needed to express the payoff as a function of *x*. If that is not the case, then one has to assume that an agent's demand is too small to infuence prices.

The adaptive algorithm to set HuRLO prices in "practice" is described by Horowitz et al. ([2012](#page-30-14)). The market price of each HuRLO is adjusted dynamically, each time a security is bought on the primary market. Thus, when a HuRLO of type *k* is bought, a smoothing parameter is used to increase its price and decrease the price of all the other HuRLOs. This adjustment ensures that the total of all HuRLO prices is equal to their (time-adjusted) nominal value, while maintaining the payoff \hat{R}_k as close as possible to this amount. Because the market for HuRLOs is operating during a signifcant time horizon, a capitalization factor is used to compensate participants for opportunity costs, rewarding early entries and penalizing late ones. More precisely, denote by *r* the annual rate refecting the time-value of money, by *c* the nominal value of the option, and by *t* the date, measured in years since the initialization of the market. An investor purchasing an option for π_k dollars at date *t* expects a payout in the neighborhood ce^{rt} if outcome k occurs at date t , so that the total of all option prices $\sum k \pi_k$ is equal to ce^{rt} . Consequently, if the market is in equilibrium, the number of options of all types should be approximately equal. This adjustment process is described in detail in Appendix [3.](#page-25-1)

Wilks ([2010](#page-31-8)) examines the behavior of market probabilities implied by option prices in the setting of the HuRLO market. Using simulations where the most favorably priced HuRLO is purchased, he shows that the pricing algorithm responds promptly to participants beliefs. He does not, however, provide any rationale for choosing dynamic market probabilities rather than the classical pari-mutuel setting where the price of claims is held constant.

The following two fgures illustrate the relative behavior of the two systems over time, measured in transactions, when participants in the market are buying the option with the highest expected payoff, according to their beliefs. In Fig. 5.1 , we assume that prices are constant, whereas we assume adaptive prices in Fig. [5.2](#page-10-0). We assume in these two fgures three possible outcomes whose initial probabilities of occurrence are given or believed to be (10%, 30%, 60%) for outcomes 1, 2, and 3, respectively. The initial mutual reserve is set to 900,000\$. During the frst 150 transactions,

Fig. 5.1 Market evolution as a function of transactions with *constant* prices. (Parameters are M_0 = \$900,000, p_0 = (0.1, 0.3, 0.6), p_{150} = (0.85, 0.1, 0.05))

Fig. 5.2 Market evolution as a function of transactions with adaptive prices. (Parameters are M_0 = \$900,000, p_0 = (0.1, 0.3, 0.6), p_{150} = (0.85, 0.1, 0.05))

outcome probabilities remain at (10%, 30%, 60%). Between transactions 150 and 151, public information is released, changing the participants' beliefs to (85%, 10%, 5%) for outcomes 1, 2 and 3 respectively.

This experiment makes it apparent that the adaptive price market (Fig. [5.2](#page-10-0)) is reacting much more promptly to changes in beliefs than the fxed-priced market (Fig. [5.1](#page-9-0)). After 300 transactions, the odds in the fxed-price market are still very far from the objective probabilities, while the market-based odds implied by the adaptive prices refect closely even dramatic changes in objective probabilities. This is particularly interesting for the smooth functioning of a climatic catastrophe market, where forecasts can change dramatically over a very short time period. Another advantage of the adaptive-pricing algorithm is the fact that, when prices adjust smoothly, option holders have a good idea of the payoff they will receive if they hold a winning option, and therefore of the amount of insurance they hold. That is not the case with classical fxed-priced parimutuel bets.

Let us now compare Fig. [5.2](#page-10-0) with Fig. [5.3](#page-12-0) where we examine the case of adaptive prices when orders are executed for blocks of 10 options. Since blocks of options are executed sequentially, prices update after each single purchase. This means that market participants do not know precisely the total cost of their order at the time it is placed. One can observe that, because market probabilities adjust quickly, block orders can have a signifcant impact on prices.

A Simulation Experiment

We now simulate and observe the behavior of players and the evolution of the HuRLO market during typical hurricane seasons. As in Ou-Yang and Doherty [\(2011](#page-31-7)), we assume that players do not anticipate the impact of their decisions on the decisions of other players and on the evolution of the fnal payoff. Accordingly, at a given decision date, players deciding about purchasing an option or a block of options only consider the current state of the system and evaluate the options' fnal payoff by assuming that they are making the last purchase on the primary market.

In our setting, players can buy options in three series of HuRLOs and can purchase options on the primary market at distinct moments during the hurricane year, depending on the number of hurricanes. The players involved in the market are risk-averse investors who may have vulnerable assets in various counties.

Fig. 5.3 Market evolution as a function of transactions with adaptive prices, when orders are placed in blocks of 10 options. (Parameters are $M_0 = $900,000$, $p_0 = (0.1, 0.3, 0.6), p_{150} = (0.85, 0.1, 0.05))$

This simulation model allows us to experiment with different investment strategies. These strategies are myopic, in the sense that players solve a static decision problem each time they have the possibility to buy an option. However, we give them repeated access to the market during the season, allowing them to increase their stake as the prices of options evolve. We conduct a large number of experiments using a representative data set.

Assumptions

A1: Probabilities of hurricane formation and landfall do not change during the season (recall that trading is suspended when a hurricane warning is issued).

A2: For each potential hurricane forming in the season, we distinguish two stages: (1) Before the hurricane has formed, and (2) When a hurricane is present in the Atlantic Basin but has not yet landed or vanished. We further assume that only one hurricane can be present in the Atlantic Basin at any given moment.

A3: All players have CRRA utility with the same Arrow-Pratt coefficient of relative risk aversion.

A4: In each successive phase of the hurricane season, transactions take place sequentially and market data is updated after each single purchase.

Hurricane formation is assumed to be governed by a Poisson process with intensity $\lambda = 3$ to match with the number of series that we are using.⁸ The probability p_k that a hurricane makes landfall in county k , given that one is present in the Atlantic Basin, is supposed to be known by market participants. Given the probability of hurricane formation and landfall (see Appendix [2\)](#page-25-0), players can compute at any decision stage the probability *Pjk* that an option *k* of series *j* will be the winning option (i.e., mature in-themoney). This probability depends on the remaining duration *d* of the hurricane season, on the number *h* of series that have already been executed, and on the actual presence $(l = 1)$ or absence $(l = 0)$ of a hurricane in the Atlantic Basin (details are given in Appendix [4](#page-27-0)). Using the initial state of the hurricane season, as given by historical landfall probabilities, and the seeder portfolio of options, one can then calculate the current market

⁸ In reality, it seems that *λ* is between 5 and 6. See [http://climateaudit.org/2007/01/14/](http://climateaudit.org/2007/01/14/more-evidence-that-hurricanes-are-the-result-of-a-poisson-process/) [more-evidence-that-hurricanes-are-the-result-of-a-poisson-process/](http://climateaudit.org/2007/01/14/more-evidence-that-hurricanes-are-the-result-of-a-poisson-process/) (last visited on February 22th, 2019).

prices, the total mutual reserve, and the number of options in the three HuRLO series, as a function of *d*, *h*, and *l*.

Players' utility is given by the concave CRRA function *U*(*W*) = *W*¹−*^γ* / (1−*γ*). We shall assume that players have a total utility that is equal to the expected utility of their fnal wealth in each of the *k* counties. In other words, for a given player i with current wealth W_i , the expected utility will be given by

$$
Y_i = \sum_{k=1}^{K} p_k \frac{(W_i - L_{ik})^{1-\gamma}}{1-\gamma} + p_0 \frac{(W_i)^{1-\gamma}}{1-\gamma}
$$

where L_{ik} is Player i 's expected loss if a hurricane makes landfall in county *k*.

Therefore, when considering the possibility to buy an option *k* in series *j* given the values of *d*, *h*, and *l*, a player observes the state of the market, that is, the total mutual reserve M_i , and vectors containing the number of each type of options m_j and market prices π_j in series *j*, along with the probability *Pjk*. Players also consider their wealth *Wi* and the number of each option *k* in series *j* they own. Because the market price vector π *j* applies only to the next option purchased, it is not possible to fnd an analytical expression for the optimal wager of Player *i*. However, it is easy to determine whether or not purchasing a single option *k* in series *j* would increase a player's expected utility, assuming this would be the last transaction in the market, by considering the marginal impact $\mu_{ijk}(\cdot)$, computed as the difference in expected utility for Player *i* due to the purchase of a single option k in series j (see Appendix [5](#page-28-0)).

A market simulation typically involves many players, each of whom has positive marginal impact *μijk* for many options *k* in many series *j*. Even if one agent assumes to be the only investor in the market, it may still be interesting to buy many options, and the order in which these purchases are made will infuence the total cost. The order in which purchases are made can also alter the composition of an investor's portfolio of options (because the *μijk*'s will change after each purchase).

In our experiments, we allow the players to use various specifc investment strategies, in order to assess the importance of strategic issues in the HuRLO market. These strategies are displayed in Table [5.2.](#page-15-0) Strategies S1 and S2 pertain to the size of the order, while strategies S3, S4, and S5 pertain to the choice among options. Accordingly, the purchase order of a

Strategy	Definition
	Player <i>i</i> places an order for a single option.
	Player <i>i</i> places an order for a block of options, updating μ_i after each purchase.
	Player <i>i</i> chooses randomly among all the options with positive μ_{ik} .
	Player <i>i</i> chooses the option with the highest (positive) μ_{ik} .
	Player <i>i</i> chooses the option with the lowest (positive) μ_{ik} .

Table 5.2 Set of possible strategies used by player *i*

single player *i* having computed the vector μ_i given the current state of the market is described by a strategy pair. On each transaction day, the order in which players are given access to the market is determined randomly, and players are offered the possibility to trade as long as they are interested.

The simulation consists of generating a large number of hurricane seasons, discretized in days. Players are given the opportunity to trade each day. The option prices, the mutual reserves, and the number of live options are updated, following each trade, using the algorithm presented in Appendix [3](#page-25-1). The simulation algorithm is detailed in Appendix [6](#page-29-7).

General Results from the Simulations

We present our observation of the evolution of the market under various scenarios about the strategies used by the players. We thus report representative results obtained with a model involving four players, four counties at risk for a hurricane landfall, and three option series. The HuRLO market is initialized with an initial mutual reserve of \$1,000,000 used to purchase an equal number of each option at prices set to what can be viewed as historical landfall probabilities. For the sake of the simulation, these landfall probabilities are set to $p_1 = 0.2$, $p_2 = 0.15$, $p_3 = 0.25$, $p_4 = 0.22$. The complement, which corresponds to the probability that a given hurricane does not make landfall is given by $p_0 = 0.18$. Parameter values are $c = 1000$, $r = 0$ and $\gamma = 0.5$ and a seeding fee of 3% is taken from the mutual reserve at a settlement date. Results are based on 200 repetitions of the simulation algorithm and are robust to changes in parameter values.

We conducted fve experiments using the same simulation data (200 trials) to assess whether the HuRLO-purchasing strategies have any signifcant impact on the market as a whole. In experiments E1 and E2 all players use the same strategy, while in experiments E3, E4, and E5, one of

Experiment	Definition
E_1	All players use strategies S_1 and S_4 .
E ₂	All players use strategies S_2 and S_4 .
E_3	Player 1 uses strategy S_1 and S_4 , the others use S_1 and S_3 .
E_4	Player 1 uses strategy S_1 and S_4 , the others use S_1 and S_5 .
E_{5}	Player 1 uses strategy S_2 and S_4 , the others use S_2 and S_5 .

Table 5.3 Set of experiments in trading

	Number of options			Total mutual reserve $(\$)$			
Experiment	Series 1	Series 2	Series 3	Series 1	Series ₂	Series 3	
E_1	120.2	104.2	143.9	25,810	21,467	27,381	
E ₂	676.1	655.6	508.4	122,337	128,852	84,491	
E_3	120.6	104.2	145.6	25,919	21,456	27,630	
E_4	122.4	103.1	143.1	26,315	21,249	27,266	
E_5	682.1	646.6	512.5	114,766	126,723	84,786	

Table 5.4 Number of options and total mutual reserve in the three series

the four players is using a strategy that differs from that of the others. Table [5.3](#page-16-0) presents the set of experiments and the strategies used by the players.

Table [5.4](#page-16-1) reports the average number of options and the average total mutual reserve, excluding the initial seeding capital, for each series according to the fve experiments, characterized by different joint strategies.

A frst obvious conclusion is that considerably more interest is generated when players are allowed to order their options by blocks (strategy *S*2) instead of separately (strategy S_1). Both the mutual reserve and the number of options are much larger in experiments E_2 and E_5 than in experiments E_1 , E_3 , and E_4 . This is surprising since players have the opportunity to buy options every day, and as many times as they want in a single day. We infer from these experiments that the way in which orders are placed is very important, not only for the decision-making player, but also for the activity of the market. In terms of investment and number of options, the global results do not differ much when players are using the same or different ordering strategies.

Table [5.5](#page-17-0) reports on the players' average utilities at the end of the hurricane season in the fve experiments. These payoffs are computed by

Experiment	Player 1	Player 2	Player 3	Player 4	
E_1	109	106	105	109	
E_2	404	359	334	327	
E_3	128	94	103	107	
E_4	143	92	105	90	
E_5	402	246	337	399	

Table 5.5 Players' utilities

assuming that all players have identical initial wealth and vulnerable properties in all counties. We see that the difference in utility enjoyed by players who are using a "better" strategy than the others can be signifcant. We also observe that utility is generally higher when players are trading blocks of options $(E_2 \text{ and } E_5)$.

From these experiments, we can conclude that, if one decides to participate in the HuRLO market, the timing and ordering of option purchases are important, even when all players are myopic.

An Illustrative Example

Figures [5.4](#page-18-0), [5.5](#page-19-0), and [5.6](#page-20-0) present the evolution of each of the fve option prices, of the mutual reserve, and of the number of options purchased over time for one of the trials in the simulation experiment. In the trial illustrated in Figs. [5.4](#page-18-0), [5.5,](#page-19-0) and [5.6](#page-20-0), three hurricanes are formed and land on days 113, 187 and 200. Three of the players are using strategy S_1-S_3 , while one of them is using strategy S_1-S_4 . In other words, we are showing the dynamics of one trial of experiment *E*3. Each day, players are offered, in turn, the possibility to buy a single option. When players are interested in more than one option, they choose randomly (Players 1, 2, 3) or they choose the one with the highest marginal impact (Player 4). The round of offers continues until no player is interested in buying options, which closes the trading day.

Four options (dashed lines) are associated with four different districts. The ffth one (straight line) is the null option. In this trial, three hurricanes are formed and land on days 113, 187 and 200, respectively. The top, middle, and bottom graphs correspond to option prices of the frst, second, and third series. The landfall probabilities are kept constant over the year at $p_0 = 0.18$, $p_1 = 0.2$, $p_2 = 0.15$, $p_3 = 0.25$, and $p_4 = 0.22$.

Fig. 5.4 Evolution of option prices over the season

Fig. 5.5 Size of the mutual reserve in the three series over the season

This example is representative of market evolution over time; we see that prices evolve according to the market demand, and that this demand depends on the time elapsed and on the realization of uncertain events (recall that the p_k 's remain constant over time). In particular, while activity in the null option market is regular, most "insurance" options are bought in phases where hurricanes are present in the Atlantic Basin. Obviously, the demand for such options in series 1 is higher than in series 2 and 3. Obviously, the demand for options of series 2 and 3 pick up after options of past series have been settled.

Computational Considerations

The results of our simulation experiments clearly show that the way purchase orders are processed makes a signifcant difference on the players' utilities and on the market activity. This indicates that strategic considerations should be important for a player wishing to participate in the HuRLO market.

However, one inescapable conclusion from our experiment is the diffculty of identifying an optimal way to purchase HuRLOs. Our simulation experiment involves myopic players, who do not anticipate the impact of their decisions on the decisions of other market participants and on the evolution of the fnal payoff, and who evaluate their payoff by assuming

that they are making the last purchase on the primary market. Even in that case, the information required to evaluate any purchasing strategy of Player *i* (that is, how many options of each kind to buy and the sequence in which the orders should be executed) is an observable state vector containing Player *i*'s current wealth, the conditional probability that a hurricane will make landfall in each county, the number of each options of each series owned by Player *i*, the number of each option of each series in the market, the total mutual reserve of each series, and the market price of each option in each series.

In a normal HuRLO market offering 3 series of 75 options (one per county plus the null option), purchase orders are vectors of dimension 225, with combinatorial possible sequences, while the state vector is of dimension 754. Finding the best response of a player to fxed strategies of other players, or fnding the equilibrium strategy in a market populated by rational farsighted players, may be a very difficult problem. The use of a simulation approach is probably the only way to gain some insight about the strategic issues present in the HuRLO market.

Recommendations, and Public Policy Implications

One interesting feature of the pari-mutuel approach to managing hurricane risk is that there seems to be very little demand for such market design. In particular, as pointed out by Ou-Yang and Doherty ([2011](#page-31-7)), pari-mutuel insurance has several merits with respect to insurance markets, including the possibility of becoming an interesting alternative to traditional insurance. Pari-mutuel insurance can be sought as an alternative when traditional insurance has high transaction costs, is too expensive, and is plagued with informational problems, or when it is simply not available. With respect to the Florida catastrophic and weather risk market, the development of such an alternative to traditional insurance products would appear to have a high potential. One could even imagine that the Florida market would be ready for the introduction of such a risk management tool that is neither plagued by moral hazard nor adverse selection problems. Moreover, in an active HuRLO market, insurers would bear no counterparty or default risk, and they do not need to invest in further loss and cost appraisals. That is why it was natural to think, in 2008 when such a market was introduced, that HuRLOs would perform well in Florida,

with 75 HuRLOs being possibly traded (74 coastal counties, plus the null). No transactions seem to have occurred on this market after 2009. The question is why would such a market fnd no traction in the fnancial and risk management world?

One possible reason for the absence of a market is that there were not enough "speculators" who were willing to take a position on there being no hurricane that would make a landfall so that natural hedgers could not be able to get enough return of their market positions in HuRLOs. This is similar to saying that the initial seed to lift the market (which was 5 million dollars in 2009) was not large enough to attract speculators and players with no stake in the Florida hurricane market to "buy" the null contract. Without enough speculators (or risk neutral investors) populating the HuRLO market, it is possible that this market fell into a "no-trade theorem" gap (see Milgrom and Stokey [1982](#page-30-15)) where prices were adjusting to new information in such a way that there was essentially no money to be made by entering a transaction. The "no-trade theorem" states that if all players are rational and all players receive the same information at the same time (in our case, hurricane trajectory), then a market that is designed to give an effcient equilibrium will fnd to have very little (or no) volume of transactions. It seems that the market for HuRLOs embedded all these conditions.

Another reason why the market for HuRLOs did not fnd much traction with entities exposed to hurricane risk in Florida is that positions on where a hurricane would make landfall could be done continuously up until the point where the option paid. In other pari-mutuel settings, such as horse races, betting stops at some time well before the state of the world is realized. If one was able to take positions on a horse while the race is going on, no one would have an incentive to "invest" in a horse before more information is learned throughout the race. This creates tension between risk management based on prior probability and risk management based on posterior probability. In the case of HuRLOs, the tension is made worse by betting against other players than the house (a similar discussion could be constructed around the game of roulette).

For the market to run appropriately, there needs to be important changes to the current structure to reduce both the "no-trade" problem and the tension between prior and posterior probabilities and beliefs. From a broader perspective, it seems to us that a central planner could play

a key role in this market by providing the necessary seed money for the market to be active. The central planner could then use the HuRLO market as a soft commitment device to pay only the seed amount, and no more, when a storm hits some coastal area. At the same time, the central planner could put together similar markets for similar events such as an earthquake in California, typhoons in the Pacifc and winter storms in Europe. With enough seed capital from some public source, new players, who have only indirect natural catastrophe exposures, could become interested in this pari-mutuel market. Another potential nudge on this market would be to force insurers who are involved in writing Hurricane risk insurance in Florida to hold a certain basket of such options in a way that would be similar the managing of carbon-emission trading schemes. Without a more capital intensive presence of the insurance industry or of local governments, there is little that pure speculators can do to see this market take off.

Conclusion

In this research, we examined one particular market design: Hurricane Risk Landfall Option (or HuRLOs), which seeks to become an alternative to traditional insurance and reinsurance contracts. These pari-mutuel markets for hurricane risk were launched in 2008 by Weather Risk Solutions, but never found enough players to make the market liquid or dynamic enough to effectively help with managing hurricane risk. One reason may be that HuRLOs are not pure pari-mutuel products since their payoffs do not depend directly on the amount wagered.

To assess why the market never took off, we investigated whether negative strategic issues were too detrimental to the market when a player decided to invest in HuRLOs. We showed that the order, sequence, and packaging of an order make a difference in the price paid, and in the number of options held by players. This highlights a major drawback of HuRLOs as an insurance product, that is, HuRLOs are very diffcult to evaluate and to purchase optimally. In addition, it is important to note that speculators are really needed for the HuRLO market to work. Indeed, if the only options bought correspond to counties where the hurricane risk is high, then there is a real possibility that properties in these counties will be underinsured. In the limit, if there is only one county where hurricanes can strike, then without speculators buying the null option, there is no

insurance at all since, in that scenario, what the investors will recover will be exactly what they put in the pool.

As a last remark, we would like to point out that we did not consider the choice between buying insurance or buying options in the sense that we assumed that the only available hedge against losses are the HuRLOs. Also, we did not examine in our simulations whether market participants would prefer to acquire other types of securities. We assumed that investors could only buy HuRLOs, and they would do so whenever it would increase their expected utility. The choice between traditional and parimutuel insurance may be simplifed somewhat if one assumes that the payoff of a winning option should be near its par value in a well-functioning market. However, the question of how and when to buy pari-mutuel insurance remains open.

Appendix 1: List of Parameters and Variables

Appendix 2: Hurricane Landfall Probabilities in the United States

For each of the 11 Atlantic regions of the United States, Table [5.6](#page-26-0) presents the probability that a named storm, a hurricane, or an intense hurricane will make landfall in a given year.

Under a Poisson distribution, the probability that at least one storm makes landfall is given by the complement probability that no storm makes landfall: $1 - P(0) = 1 - \exp(-x/d)$, where *x* is the number of named storms or hurricanes or intense hurricane to make landfall in that particular region over a span of *d* years according to historical records.

Appendix 3: Price Updating Algorithm

The market is seeded by the purchase of an equal number of each option (say 1) at a price that refects historical probabilities of the possible outcomes (see Appendix [2\)](#page-25-0). Each time an option of type *b* is bought, the market price vector of HuRLOs is updated using an adjustment factor *α* that ensures that "pricing probabilities" sum to 1 and that the payoff of the last option bought is close to the time-adjusted nominal value. Denote by $\beta = \exp(rt)$ the capitalization factor applied to account for the time elapsed since the initialization of the market, and by ξ_k the pricing probabilities, $k = 1, \ldots, K + 1$. The updating algorithm is the following:

$$
\xi_{k} \leftarrow \begin{cases} \xi_{b} + \alpha \xi_{b} \left(1 - \xi_{b} \right) \text{if } k = b, \\ \xi_{k} \left(1 - \alpha \xi_{k} \right) \text{otherwise}, \end{cases}
$$

$$
\pi_{k} \leftarrow \xi_{k} \beta c,
$$

$$
\alpha \leftarrow \begin{cases} 0 \text{ if } \frac{M}{m_{b}} > \beta c, \\ \frac{1}{m_{b} + \pi_{b}} \text{ if } \frac{M}{m_{b}} = \beta c, \\ A \text{ otherwise}, \end{cases}
$$

where m_k and M represent the number of type- k options and the total mutual reserve immediately before the transaction, and where

 $A > 1/(m_b + \pi_b)$. The effect of the smoothing constant *α* is to increase the pricing probability of option *b* and to reduce the pricing probabilities of the other options. When $M/m_b > \beta c$, option *b* is overpriced, and no update is made. When $M/m_b = \beta c$, the smoothing constant corrects for the dilution effect due to the additional claimer for outcome *b*. Otherwise, the option is underpriced, and the smoothing factor should be higher. In our implementation, we used $A = \max\{(M - c\beta m_b)(1 - \pi_b)/M, 1/(m_b + \pi_b)\},\$ which has the desirable properties of increasing with the imbalance, decreasing with the total mutual reserve, and decreasing with the pricing probability of option *b*.

APPENDIX 4: COMPUTATION of the Outcome Probabilities

The probability distribution of the number of new hurricanes that will form until the end of the season is denoted by ϕ_d , where *d* is the remaining time in the season. Assuming that hurricane formation is a Poisson process of intensity *λ*, the probability that *y* hurricanes will form until the end of the season is then given by:

$$
\phi_d(y) = \frac{(\lambda d)^y \exp(-\lambda d)}{y!}.
$$

The probability P_{ik} (*d*, *h*, *l*) that option *k* in series *j* will be the winning option depends on the remaining time in the season, on the number of series that have already been settled, and on the presence or absence of a hurricane. For $j = 1$ and $h = 0$, it is given by:

$$
P_{1k}(d,0,0) = \sum_{y=1}^{\infty} \phi_d(y) p_k \sum_{s=0}^{y-1} p_0^s, \text{for } k = 1,...,K,
$$

$$
P_{1k}(d,0,1) = p_k + p_0 \mathbb{E}\Big[P_{1k}(d+D,0,0)\Big], \text{for } k = 1,...,K,
$$

where *D* is a random variable representing the presence time of a hurricane. Here, an option *k* in the frst series will be executed if one hurricane or more is formed, and the frst one to land does so in county *k*. Clearly, P_{1k} (*d*, *h*, *l*) = 0 for $h \ge 1$. Similarly, for $j = 2$ and $h = 0$ we get:

$$
P_{2k}(d,0,0) = \sum_{y=2}^{\infty} \phi_d(y) p_k \sum_{s=0}^{y-2} C_{s+1}^1 p_0^s (1-p_0), \text{for } k = 1,...,K,
$$

$$
P_{2k}(d,0,1) = p_0 \mathbb{E} \Big[P_{2k}(d+D,0,0) \Big] + (1-p_0) \mathbb{E} \Big[P_{2k}(d+D,1,0) \Big], \text{for } k = 1,...,K,
$$

$$
P_{3k}(d,h,l) = P_{2k}(d,h-1,l), \text{for } h \ge 1.
$$

Notice that there is exactly one winning option in each series, so that

$$
P_{j0}(d,h,l) = 1 - \sum_{k=1}^{K} P_{jk}(d,h,l).
$$

Appendix 5: Computation of Marginal Impact

The computation of the marginal impact of buying an option is presented for the case of a single series $(J = 1)$. When more than one series are offered, the computation is done in a similar way, accounting for the fact that an option $k \neq 0$ in series *j* can only be a winning option if the null option is not the winning option in series $j - 1$.

If Player *i* does not buy option *b*, expected utility is given by:

$$
A=\sum_{k=0}^K P_k(d,h,l)U_i\bigg(W_i-L_{ik}+\frac{n_{ik}}{m_k}M\bigg),
$$

where $L_{i0} = 0$.

On the other hand, if Player *i* buys option *b*, expected utility is given by:

$$
B = \sum_{k=0, k \neq b}^{K} P_k (d, h, l) U_i \left(W_i - \pi_b - L_{ik} + \frac{n_{ik}}{m_k} M \right) + P_b (d, h, l) U_i \left(W_i - \pi_b - L_{ib} + (n_{ib} + 1) \frac{M + \pi_b}{m_b + 1} \right).
$$

The marginal value of investing in option *b* is therefore:

$$
\mu_{ib}
$$
 $(d,h,l,M,\pi,m,n_i,W_i) = B - A.$

Appendix 6: Simulation Algorithm

- 1. Read parameters and initialize the market using p , yielding M^0 and m^0 . Initialize the options held by all players to $n_{ik}^0 = 0$ for $I = 1,...,$ *I* and *k* = 0,…, *K*. Set *d* = 1, *h* = 0 and *l* = 0.
- 2. Using the hurricane model, generate hurricane dates, durations, and outcomes during the hurricane season.
- 3. Compute probabilities P_{ik} at (d, h, l) and select randomly an ordering *O* of the players.
- 4. For *I* = 1,…, *I*
	- a. Determine the purchase order for Player $O(i)$ according to her strategy,
	- b. For each transaction by player $O(i)$, update market variables,
	- c. When purchase order of Player $O(i)$ is completed, set $i = i + 1$.
- 5. If *d* = 0, stop. Otherwise, update *h* and *l* according to the hurricane scenario realization. Set $d = d - 1/365$ and go to 3].

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