Chapter 5
Russian Mathematics Education After 1991

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Abstract The aim of the present chapter is to trace the recent history of Russian mathematics education, considered as part of the social history of Russia. Although by contrast with what has taken place in politics, in mathematics education there has never even been talk about "perestroika" or restructuring, let alone any radical changes, the changes that have taken place over the last 30 years have in fact been significant and are clearly connected, even if often not directly, with social-economic changes. This chapter will focus on the main aspects of mathematics education: the organization of the learning process, textbooks, exams, the preparation and professional development of teachers, and so forth.

Keywords Russia · Reform · Standards · Textbooks · Uniform State Exam · Teacher education · Gifted education

The Soviet Union officially collapsed in December 1991. Of the countries that came into being on its former territory, the largest is Russia or the Russian Federation, which will be the subject of the present chapter. In the subsequent almost 30 years of its existence, it has seen all manners of political as well as cultural changes. It does not follow, of course, that these changes were always accompanied by changes in the teaching of mathematics; the changes that took place in this sphere—and such changes undoubtedly did take place, even if they might not have seemed as dramatic as the political ones—were produced by a combination of factors, including both domestic developments in mathematics education and processes common to the whole world; what is clear, however, is that the development of mathematics education did not occur in isolation from what was taking place outside the doors of mathematics classrooms. The aim of the present chapter is to trace the recent history of Russian mathematics education, considered as part of the social history of Russia as a whole.

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Studying recent history is usually not very easy and not only because certain documents are inaccessible—we still have an enormous quantity of various publications at our disposal—but because these publications often exude a political fervor, which may infect their authors and cause them to distort the facts. Up until a certain time, the author of the present chapter himself took part in what was happening—as a teacher, as an author of textbooks, and even as an administrator—and fully acknowledges the fact that he must not elevate either his experience or his observations into absolutes, without, however, excluding them from consideration. To a certain extent, the goal of this chapter is precisely to collect various observations and opinions, whose collocation with official information might help to paint a true picture.

It should also be noted that, as far as the author of this chapter knows, there have hitherto been practically no attempts at a survey study of what has occurred in mathematics education in Russia over the past three decades. The closest approximation to such a study is M.I. Bashmakov's (2010) chapter, to which we will repeatedly refer in what follows.

It is necessary to voice yet another caveat: Russia is the largest country in the world in terms of area, and living conditions in it—and therefore also the conditions of teaching mathematics—are by no means everywhere uniform. Furthermore, it is evident that what is taking place in mathematics education in different parts of the country is not treated with the same level of detail in the press and in other types of publications. In writing about Russia below, the author will attempt not to confine himself to Moscow and St. Petersburg, but, undoubtedly, more will be said about them than about any other parts of the country.

Below, we will address various directions in the changes that have occurred, but clearly we must begin by discussing what mathematics education was like in Russia during the last years of the USSR.

1 Mathematics Education in the Last Years of the USSR

By the late 1970s and early 1980s, Soviet mathematics education, although formally preserving its main former features, in fact had become significantly different from what it had developed into during the 1930s–1940s (Karp 2010). Undoubtedly, the system was still absolutely centralized—curricula and textbooks were approved and endorsed in Moscow, at the Ministry of Education of the RSFSR (in keeping with uniform Soviet Union requirements). The streamlined vertical system of methodological direction—consisting of a specialist in mathematics education at the ministry, regional methodologists, and district methodologists—was supported by and was a part of a general vertical system of control, from the Ministry, to the regional Departments of Education, and further to district departments. Mathematical methodologists, working with departmental inspectors, were tasked with, as it was said, direction and control over methodological work, making sure that curricula were followed in the manner prescribed in Moscow and that the requisite results were consequently achieved.

In practice, however, in the 1980s, the system worked even less smoothly than it had originally when it was first established. In many respects, human resources had already been exhausted. A shortage of mathematics teachers had already begun, and in general, it had become necessary to soften the former severity (although, to be sure, the system could by no means have been characterized as "soft"). For all the virtues of the developed methodology, it could not provide the desired level of results; but if during the 1940s, along with fighting underachievement, it was deemed necessary to fight against "window dressing" and complacency, so that 15–20% of students in a class failing was not something unheard of (Karp 2010), now times had changed, and it became undesirable to have even 1–2% of students in a class fail. The constitution of the country, from 1977 on, guaranteed the provision of universal mandatory secondary education. A consequence of this was the development of what at the time was called "percentomania," in other words, the awarding of grades above what a student deserved according to existing norms, so that the class and the school, and thus also the district and the city, might achieve higher results. The measurement of results (above all, examinations) could turn into a yearslong virtually open racket. The author of this chapter remembers, for example, how beginning in the second half of the 1970s and for approximately 10 years the problems on final examinations in mathematics for mass-scale schools would become known in Leningrad (St. Petersburg) to both teachers and students several days before the examination. Again, drawing on my own experiences, I can recall hearing a talk at the Collegium of the Ministry (State Committee) by V.D. Shadrikov, one of the heads of Soviet education at the time, who spoke about the fact that no one actually knew what results had been achieved—the official figure was that virtually 99% of students in Russia learned elementary calculus. But how many learned it in reality? Even if it was only 50%, that was wonderful, but the information simply did not exist.

By the mid-1980s, the system had abandoned one of its main principles—the single textbook. Formerly, the whole country had been taught using the same textbook; now, as a result of a struggle among various influential groups following the abandonment of the so-called Kolmogorov reform (which to a certain extent paralleled New Math in the United States), it became necessary to conduct an open competition for textbooks and subsequently to allow for the parallel existence of several textbooks on the same subject for the same grade (Abramov 2010; Bashmakov 2010). In fact, a quarter of a century earlier, it had already become clear that the acknowledgment of certain differences among students was inevitable: there appeared schools with an advanced course in mathematics (Karp 2011), to which the Soviet educational system largely owed its high international reputation.

The Kolmogorov reform was an attempt to change the content of mathematics education—an attempt that was inevitable, since to continue forever with a course that had effectively taken shape even before the revolution of 1917 was impossible; but it was a reform whose most innovative components—for example, the introduction of elementary discrete mathematics—failed (Bunimovich 2011). The unified centralized system was not well suited for updating content (recall that classic Soviet textbooks—above all, the legendary textbooks of Kiselev—had come into the schools even before 1917 in the context of an open competitive struggle, which no internal review process could replace). It cannot be said, of course, that everything done by the reformers failed: despite the negative discussions in the press, including such leading communist party periodicals as *Kommunist*, and the denunciation of certain approaches employed by Kolmogorov, for example, set theoretical symbolism, much in the course survived; but nevertheless, the general view was that it was necessary to return to the tried and the old—by creating, as it were, an updated Kiselev (Karp and Werner 2011).

Attempts were also made to reform the manner in which classes were conducted: thus, V.F. Shatalov, a mathematics teacher from Donetsk who published several books (Shatalov 1979, 1980, 1987) that advocated the method of *supportive abstracts*—which allegedly gave incredible results—became popular in the late 1970s. Although Shatalov's arguments were harshly criticized (Dadayan et al. 1988) and the wave of enthusiasm for his methods gradually subsided, their very popularity was indicative and attested to a sense of incongruity in what was happening in mathematics education (which had traditionally and largely justifiably been considered a strong or even the strongest part of Soviet education) in the hope of finding some kind of miraculous approach that would remedy everything, without, however, really changing anything. We should also note that a sense of crisis grew with the initiation and development of Gorbachev's "Perestroika," which called for a reconsideration of the traditional Soviet system as a whole.

2 Reforms in Education

2.1 The General Situation

Reforms were viewed as changes not specifically of mathematics education but of the whole educational system. Plans for such reforms had already been drawn up relatively long before 1991. As early as 1988, the State Committee on Education had formed a temporary scientific research collective named "The Basic School" (and later called simply "The School"), which was directed by Eduard Dneprov—who later became Russia's minister of education—and which became a center for the preparation of reforms. Pertinent ideas were expressed relatively quickly, but their realization in the USSR kept getting postponed. But they were put to use when Russia became an independent state. Dneprov himself (Dneprov 1998) articulated the causes of the crisis quite clearly:

> The critical condition of the schools, which had become apparent already by the early 1980s, reflected an analogous condition of society and stemmed from the same basic cause—a crisis in the totalitarian regime, the exhaustion of resources for its development[1] (p. 36).

[1] All translations from Russian are by the author.

In his opinion, the reform had to be based on the following principles (pp. 46–52):

- Democratization of education (including destatization of the schools and decentralization of their administration)
- Pluralism of education (including its multistructurality, variability, and alternativeness)
- The people and national character of education
- Openness of education (in particular, emancipation from dogmas)
- Regionalization of education
- Humanization of education (the school must turn toward the child)
- Humanityzation of education (the school must pay more attention to humanities)
- Differentiation of education
- The developmental and practical character of education
- Continuity of education.

This and similar phraseology became the basis of legislative measures passed at the beginning of the Yeltsin period, including the Education Act of 1992 and the famous Decree No. 1 on Top Priority Measures for the Development of Education in the RSFSR (Russian Soviet Federative Socialist Republic), which was enacted in July 1991 and which by the very number in its title emphasized the fact that the government's problems of top priority were no longer in the sphere of the arms industry but in the area of education and spiritual development.

What, then, did all of this mean for mathematics education? Here, we must immediately acknowledge that school life was by no means regulated exclusively by the legislation that governed education but also simply by life itself—the unfolding economic crisis (not to say catastrophe) inevitably had an influence on the schools. Teachers, whose position had recently still been quite stable, even if they did not belong to the wealthiest sections of society, suddenly found themselves in a situation in which their salaries fell to fractions of what they had been previously and moreover were paid very irregularly. This could not but have an impact on teaching and teachers' professional ethics. A blow was dealt to parents as well, whose preoccupation with work also began to change (and therefore also their attitude toward schools and their opportunity to help children in their studies). Not the least significant role was also played by psychological factors—students and their parents saw that the former goal of becoming an engineer (for which one needed to be a good student in mathematics) was now quite questionable—engineers were losing their work and livelihood en masse. Completely different professions became prestigious. It must also not be forgotten that mass emigration began—many scientists left the country, whose influence on schools, in one way or another, had been considerable. Dneprov (1998, p. 106) especially notes that education reformers, by contrast with other reformers of the Gorbachev and subsequent periods, had a clear plan and program, and while others, having gotten the airplane off the ground, as it were, did not know where it would land, education reformers supposedly knew everything. The problem with this view, however, is that education is not an isolated airplane—consequently, there neither was nor could there have been any "pure" experiment to determine what would have happened if education had been changed according to Dneprov's plan, while everything else remained stable and sound. The overall effect from all the transformations (and not just directly in education) was not sufficiently considered, and it is naive to reduce matters to the incompetence of specific leaders—after all, Dneprov himself particularly praises Moscow mayor. Luzhkov, who, by contrast with other local politicians, found money for education (Dneprov 1998, pp. 152–155), ignoring Moscow's special position in the country.

But let us return to specifically educational issues. Even before the passing of the Education Act, which Dneprov regards as the starting date of the reform (Dneprov 1999, p. 13), there was a feeling that the pressure of prohibitions had weakened, and consequently all kinds of local experiments that affected the teaching of mathematics were beginning to take place, and people also simply sensed that there was less oversight and control.[2] The administrative component of the national reform that was probably most tangible to mathematics teachers—already in June 1993, under Minister Ye.V. Tkachenko, who had replaced Dneprov—was the appearance of a new basic teaching plan or, more precisely, different versions of a basic teaching plan (see, for example, Committee 1994). The corresponding Decree No. 237 from the Ministry of Education legalized the changes that had already started taking place and pushed them further. Embracing the principles of pluralism, the ministry ceased specifying a precise number of hours to be allocated to each subject in each classroom across the whole country. Certain areas of education were defined—social sciences, natural sciences, Russian language, mathematics, and others—and for each of them, the minimal possible number of hours per week was indicated. For example, for mathematics in the two upper grades, this number was 3 h. Along with mandatory "ministry" hours, there were also regional hours, allocated according to the decisions of the regional governments (for example, the government of St. Petersburg could decide that all of the city's students were required to study the history of St. Petersburg, while the government of some autonomous republic might make the republic's history a requirement, thus fulfilling the principle of attention to national character), and finally, hours allocated by each school itself.

In this way, schools all at once acquired the right radically to change the existing structure—it became possible to teach mathematics in the upper grades for 3 h, 5 h, 8 h, or even more. It became possible radically to reduce the teaching of physics, which was evidently connected with the teaching of mathematics, replacing it either with some kind of integrated course or with some subject that could be related to the natural sciences.

In principle, the newly acquired freedom could only have been a cause of rejoicing; but not sufficient thought had been given to how and why each school would make its specific choices, particularly given the numerous constraints that each

[2] See, for example, the article by Eidel'man (2007) with the characteristic title "The Year of Realized Utopias: Schools, Teachers, and Education Reformers in Russia in 1990," in which the author, copiously quoting other teachers, tells about the changes that occurred at that time (even though, despite her obvious admiration for what took place then, after quoting certain discussions from the period, she sometimes adds: "today one can detect a certain touch of madness in these words").

school was under, beginning with the fact that everything that a school had at its disposal (for example, its teaching staff) had been organized with another system in mind. Thus, if a school was short of a physics teacher, for example, the principal could decide that there was no need for this teacher. The choice of a teaching curriculum might stem from simple administrative convenience, rather than from any higher considerations. There was no mechanism for accommodating existing needs and opinions nor were there any mechanisms for gathering information, without which even the formation of such opinions became problematic (to be sure, what such mechanisms should be like is a difficult question and unlikely fully solved anywhere).

Schools actively competed for “good” children, inventing ways to attract them, or at least not to lose them, keeping them from transferring to other schools—because such transfers also became easier and freer. Consequently, schools often started making use of a “brand” that had earned a good reputation—an advanced course in mathematics. New “humanities” schools also appeared, which promised at the outset not to torment children with mathematics. Some schools offered something altogether unexpected, hoping to attract students by this means—the author of this chapter once visited a school that offered the study of the Coptic language (already in elementary school), which in the opinion of the school’s directors would guarantee genuine depth of preparation in the humanities. To repeat, such “specialized” schools and classes began appearing even somewhat before the law was passed—people felt that this was already possible.

There was less time to pay attention to children who were “not good,” especially since there was less enforcement of such attention, just as there was less enforcement in general, while the time that a teacher had available to spend hours with “weak” students—and probably the desire that a teacher had to do so—was often reduced to nothing: it was simpler to give a student the coveted passing grade of “3” (out of a possible 5). On the whole, one might say that both “good” and “bad” teachers acquired greater freedom to act as they pleased.

The following information—to a certain degree and with certain caveats—conveys an idea of the state of affairs that existed at the time.

2.2 Diagnostic Work in St. Petersburg in 1992

In October 1992, so-called diagnostic work was conducted in St. Petersburg in all tenth grades in two districts, in 49 schools altogether (here and below we will rely on the publication Committee 1994). The diagnostic work was based on materials of basic schools (that is, 9-year schools). It must be emphasized that the results of the work pertain to St. Petersburg, and there is no reason to infer that the results would have been identical everywhere else in the country; moreover, even the selection of two districts out of the twenty that existed at the time—a large and somewhat peripheral one (Kirovsky) and a small and central one (Dzerzhinsky, which was subsequently eliminated as a separate administrative unit)—can be subjected to criticism; for example, Dzerzhinsky District happened to have one of the best, if not the best, schools with an advanced course in mathematics, which drew children from the whole city and whose presence obviously considerably influenced the average figures. Even so, however, the data below help to understand the emerging picture. And although it is naturally impossible to rule out a certain amount of cheating, copying, and the like, nonetheless, we can assume that it was limited; a second diagnostic work was planned (but never carried out), so that if a teacher's own work was going to be judged, then it would be judged only on the basis of the changes that had occurred since the first diagnostic work, which merely determined the existing level of the classes, which had been formed only a short time beforehand; thus, teachers had no incentive to inflate the outcomes.

The diagnostic work consisted of two parts and was to be completed in 3 h. The first (main) part contained five groups of problems (on identical transformations, solving inequalities, solving systems of equations, studying graphs and functions, and word problems, respectively). Each group contained a problem A, worth a maximum of three points; a problem B, worth a maximum of six points; and a problem C, worth a maximum of nine points. Only one problem from each group was counted. The students themselves decided which problems to solve. Thus, a student could get a maximum of 45 points. For example, the fourth group contained the following problems:

(A) Construct a graph of the following function $y = x^2 - 6x + 5$ and find the coordinates of the points whose y-coordinate equals 5.
(B) Find the least value of the following function $y = x^4 + 5x^2 + 2$.

(C) Given the function $f(x) = \begin{cases} x-1, & \text{if} \quad x \le 0 \\ ax^2 + 2x + a, & \text{if} \quad 0 < x < 1 \\ x+2, & \text{if} \quad x \ge 1. \end{cases}$

Find all values of parameter a such that this function is an increasing function.

One could say that problems A corresponded to the standard requirements of general education schools; problems B corresponded to the standard requirements of schools with an advanced course in mathematics; and problems C corresponded to heightened requirements of such schools.

The second part consisted of ten problems (the first of them had three parts, that is, there were 12 questions in all), which were far simpler than the problems in the first part from a technical point of view but which required a certain capacity for reasoning—for example, in one of them, students were asked to give an example of an equation that has exactly four roots on the segment [−3, 5]. For each question (again, there were 12 in all), a student could get a maximum of three points.

This diagnostic work was not graded, but one could say that 15 points on the first part were equal to the highest grade of "5" at a general education school, while 9 points were equal to the lowest passing grade of "3" at such a school.

The outcomes showed an extreme stratification among the students. Out of 2175 people who took part in the diagnostic work, 952 (that is, 43%) got fewer than nine points on the first part, that is, were unable to complete the work in a way that met the minimal standard requirements of general education schools. Moreover, there existed entire classes in which no one got nine points. Meanwhile, the students' grades for the ninth grade, on the basis of which they were admitted to the tenth grade, were usually quite good (certainly not lower than "3"). On the other hand, 572 students (that is, 26.3% of those taking part in the diagnostic work) got 15 points or more, that is, would have gotten a grade of "5" by general education school standards. Seventy-five people or 3.4% of those taking part in the diagnostic work got 30 points and more (note that 65 of them attended the same school).

Two tables show the figures for schools of different types. Table 5.1 presents the results for the first part of the test. Table 5.2 – for the second (with a maximum score of 36). The results are quite similar.

As can be seen, very many students at that time left for schools that were specialized in one way or another (and to repeat, it can be assumed that in other districts of the city, and even more so in smaller cities, the corresponding figures would have been somewhat lower). Furthermore, even in humanities-oriented schools, the outcomes in mathematics turned out to be somewhat better than in general education schools. Note that even the technically simple problems in the second part often presented serious difficulties—instances in which students completed computational and algorithmic problems relatively well but demonstrated a complete incapacity for even the simplest reasoning were by no means a rarity.

Let us also give the figures separately for boys and girls (Table 5.3).

As can be easily seen, among those unable to complete the assignments, even by the standards of general education schools, girls were more numerous than boys.

Table 5.1 Results for schools of different types (first part)

School type	Number of students	Average score	Average grade (if it had been given)
General education	838	6.80	2.27
Humanities	634	9.54	3.21
Physics-mathematics	475	18.45	5
Total	2175	10.70	3.57

Table 5.2 Results for schools of different types (second part)

School type	Average score
General education	11.44
Humanities	13.73
Physics-mathematics	24.56
Total	15.62

Table 5.3 Results for boys and girls separately (first part)

School type	Boys	Girls
Got fewer than 9 points	375	577
Got 15 points or more	307	265
Got 30 and more points	56	19

This author does not know of any statistical figures of this type for previous years—in the USSR, people took little interest in such questions, automatically declaring and assuming equality between the sexes. However, informal teachers' opinion had regarded "slackers" as boys rather than girls—based on the reasoning that girls deviated to a lesser degree than boys from the requirements of the school. It may be argued that the changes stemmed from a change in public opinion: mathematics stopped being considered a subject necessary to everyone and in particular a subject necessary to girls (we might add that, based on our observations, the number of girls entering schools specializing in mathematics fell sharply). In general, as was noted in the study cited here, the interest of students and their parents in obtaining a high-quality education in mathematics was a crucial factor in their "educatedness." The old Soviet slogan—"If you don't know how to, we will teach you. If you don't want to, we will force you"—was disappearing into the past. Somewhat simplifying matters, we might say that those who "didn't know how" were not always taught, while those who "didn't want" were not only not forced, but often not even helped to begin to want.

2.3 From 1993 to 2000

Eduard Dneprov, who was the head of the ministry of reformers, believed that the reforms were fully realized only during his tenure:

> With the passage of the Education Act, the first, groundbreaking phase of the educational reforms ended. The reforms entered a new phase—the phase of their technological realization. But this phase was effectively aborted. Thanks to the efforts of "velvet restorers," the educational reforms were transformed into **pseudoreforms** with clear tendencies toward backsliding (Dneprov 1999, p.13).

Nonetheless, however one evaluates what occurred, it is clear that changes during this period continued (again, without immediately assessing what led away from the Soviet model and what led back to it and without in any way assuming that everything in the Soviet model was bad). In addition, the changes that Dneprov took pride in did not reach the schools immediately (as has already been said, even the new basic plans arrived after Dneprov retired). Consequently, even agreeing with Dneprov about the growth of backsliding tendencies, we can still view the period from 1991 to 2000 (the Yeltsin years) as a whole and study the processes that took place—by no means all at once—during this period.

Despite the tendency to refer to these years as "wild," which became entrenched in the propaganda of the subsequent period, it cannot be said that everything that happened in mathematics education then was unsound: just the opposite, the unfolding processes, as is clear from what has already been said, were very contradictory. New schools opened, new ideas were expressed, new textbooks and problem books were written, and teachers learned and experimented. At the same time, it is impossible not to note that, for example, the financing of education out of the state budget shrank by 48% between 1991 and 1999[3] (Dneprov 1999, p. 5), which inevitably had an impact on education. And even strictly educational measures were by no means always considered beneficial by everyone.

Below, we will examine certain important aspects of the mathematics education of that time, without deciding in advance whether they were reformist or counter-reformist in character.

3 The Content of Education and Standards

Mathematics was obviously not among those subjects in which educators needed to struggle especially hard against former Soviet ideology, reconstructing its whole content. Nonetheless, the question of the content of education became relevant in this field also. The Soviet model indicated the knowledge, abilities, and skills that had to be achieved by teaching. With the beginning of the period of reforms, such lists became unwelcome—they started being referred to as the "notorious ZUNs" (acronym for "*znaniya, umeniya, navyki*"—"knowledge, abilities, skills")—and it was explained that educators had to move away from meaningless drills, rote memorization of facts that no one needed, and the like. In the monograph by Stefanova et al. (2009), this position was formulated as follows:

> Reform of the education system in contemporary Russia is a way to overcome the crisis in the sphere of education, which used to be characterized by a high level of politicization of the learning process, an orientation toward conserving the ideals of the existing sociopolitical system, the determination of all aspects of the educational process by a prevailing subject-centered conception of education. Within the framework of this conception, the assimilation of subject (mathematical) content was seen as the main goal of education. As for the content of mathematics education, it was structured as a kind of model of scientific knowledge suited for both general education schools and higher educational institutions. (p. 181)

And further:

> At the contemporary stage of the development of society, the main goal of general education consists in the formation of a many-sidedly creative personality, capable of realizing its personal potential under dynamic socioeconomic condition both in its own interests and in the interests of society. (p. 182)

The idea of developmental teaching had been assimilated by Russian education at least since the time of Vygotsky (and in reality, much earlier), but it would be beneficial to be less general and to give it some kind of concrete form. All the same, the finding that almost half of the tenth graders in St. Petersburg were unable to meet standard requirements in a subject could not be considered a proof of the fact that they had benefited from a many-sided development.

[3] How this decrease should be understood given the radical changes in the purchasing power of the ruble is open to discussion.

The way out was found in the use of an expression that was new to Russian education: “standard.” The Federal Standard Law was passed already in 1992. It called for developing standards that would regulate education. The aforementioned Dneprov (1999) notes that, “in itself, the idea of mandatory standards [was] undoubtedly positive” (p. 19), since they were supposed to:

> describe a sphere of basic education in the interests of the child and of society, to stipulate the bounds of the greatest acceptable academic workload, to formulate mandatory requirements for school graduates. In addition, the standards were supposed to serve as a bulwark against pressure from the “methodological lobbies,” which ceaselessly strove infinitely to expand the content and limits of their academic subjects. (p. 19)

The outcome, however, was not what had been envisioned, in Dneprov’s opinion, and the developers of the standards were to blame. In his opinion, the standards were “implemented in accordance with the old ‘ZUN’ philosophy” (p. 21) and became a tool for the conservation of the old, the overloading of the students, and other evils.

The first drafts of standards began to appear as early as 1993. The draft prepared by the Institute of the General Education School at the Academy of Education (Institut 1993) did not significantly differ from the requirements that had already been formulated by the Ministry back in Soviet times (Ot Glavnogo upravleniya 1991). As in the older document, the requirements were presented at two levels. The first level was called the *level of possibilities*: “it describes the outcomes toward which students who study the general education course may aspire” (Institut 1993, p. 10). “The second [level] is the *level of mandatory preparation*. It describes the unquestionable minimum that all students must achieve and defines the lowest acceptable threshold for the outcomes of a mathematics education” (p. 10).

The level of mandatory preparation was formulated not only verbally but also through models of typical problems. For example, the section on equations at the high school level indicated the following:

> *The level of mandatory preparation is defined by the following requirements:* solving the simplest exponential, logarithmic, and trigonometric equations; using the interval method to solve simple rational inequalities. (p. 19)

And further, a small set of problems was given, the ability to solve which was considered mandatory. For example, the following problems on using the interval method were included:

> Solve using the interval method:
>
> a) $\frac{(x-1)(2x+3)}{4-x} \geq 0$, b) $\frac{x^2-4}{x+5} < 0.$(p.19)

Working teachers found special significance in the concluding sections, which discussed how the implementation of the standards was to be monitored and verified and, in general, how the teaching process was to be structured in light of the standards. The idea of differential learning and open requirements was variously advocated. It was emphasized that testing to ascertain that a desired level had been achieved had to take place at boundary stages, for example, upon completion of the basic school (ninth grade) and high school (eleventh grade), and that educators must not limit themselves to testing for the achievement of the mandatory level but that they also need to investigate how was achieved a higher level.

In the ensuing discussion, various critical views were expressed. It was noted that, for teachers, the normative document ought to consist not of standards but of programs and curricula (Timoschuk and Nozdracheva 1994); that it would be worthwhile to write multilevel problem books, which would themselves define the standards (Dubov 1994); that the teaching of probability theory needed to be substantially strengthened (Gnedenko and Gnedenko 1994); and much else. Professor Gladky (1994) of Moscow wrote about the legal illegitimacy of the draft—standards had to be approved on a competitive basis. Moreover, he went on, although the draft paid lip service to democratic phraseology, in fact it proposed the introduction of a uniform methodological system for the whole country—something unseen either under the tsars or under Soviet rule. This system itself he regarded as absurd:

> It needs to be pointed out that the principle of "free choice of the level of assimilation," proposed by the authors, is fundamentally flawed. This can be seen especially clearly when it is applied to the study of mathematics. Imagine that a child with good, but not exceptional abilities, at the age of ten carelessly chooses the lowest level, and the "democratic" teacher, paying heed to the instructions of our authors, does not attempt to influence the child's choice. If the child remains at the same level for two or three years, for the rest of his life he will never have any choice with regard to mathematics, except perhaps if he is lucky enough to find himself under especially favorable circumstances of some kind. This person will be spiritually robbed and many paths in life will remain permanently closed to him. This kind of "humanism" and "democratism" is in reality nothing other than a profound indifference to the child and the child's fate. (p. 7)

Along with common federal standards, regional standards were also published. Thus, for example, the Moscow regional mathematics standards education were promulgated, which allowed Viktor Firsov, the head of the group that prepared them and one of the theoreticians behind their composition, to express his views one more time. For example, he deliberately pointed out the difference between standards and curricula. As he wrote: "a teaching curriculum expresses a concrete, methodologically conceived strategy and tactics for teaching" (Firsov 1998, p. 4). Consequently, there might be many different curricula. Standards, on the other hand, are introduced for the specific purpose of "establishing a manageable multiplicity of curricula." He went on: "Standards must become a normative foundation enabling a transition from schools that are still excessively uniform to schools of a new type" (p. 5).

Further, he answered those who came out against reducing the quantity of the content of education. Opponents of such a reduction were divided by Firsov into two groups, the first of which allegedly equated a large quantity of requirements with a large amount of assimilated knowledge—such opponents were invited to present genuine, rather than falsified, facts about the knowledge of Russia's schoolchildren and were additionally informed “about the quantity of content that is considered sufficient for mandatory education by educators in most developed countries” (p. 6). (Note that Firsov thus took the unanimity of these educators for granted.) The second group, according to Firsov, was made up of those who were concerned only with “the elite,” with those who in their opinion “determine the prospects for the country’s development” (here, Firsov caustically pointed out that the supporters of these views fail to specify which country they have in mind, thus hinting that these “elite” schoolchildren can later move to other countries, likely the very ones whose educators he had cited just a little while earlier). This group was invited to recognize that reducing the scope of standards did not rule out preserving the scope and size of specific curricula.

In general, the article allowed for various alternative ways of following and not following standards; for example, Firsov noted that it was possible to allow for following standards with delays if the parents assumed responsibility in the event of any difficulties (for example, in the event of a transfer to a different school). Moreover, he allowed for not following standards at all but then without government financing and without a state certificate (no one else, however, was ready to grant such rights).

The very term “standard” at that time was understood quite broadly, nor was it entirely clear what new regulatory measures might appear. In St. Petersburg, for example, it was deemed sufficient to publish examination materials under the title “Standards” (Standarty 1993) and leave it at that (these “Standards,” which will be discussed below, did in fact establish what was required of the students to a certain extent).

It may be said that this period of the development of standards ended with a competition, which was won predictably by the draft prepared by the Russian Academy of Education, the materials of which were published in 1998 (Lednev et al. 1998). This document, however, had little impact on schools at the time. Schools continued to function as they had done before, using curricula prepared and published by the ministry or, to put it perhaps even more precisely, using textbooks that in one way or another corresponded to these curricula.

Kuznetsova’s collection (Kuznetsova 1998) contains some curricula and some other documents, including a “Mandatory Minimum Content in Mathematics for Basic Schools,” written, as the text explains, “on the basis of existing curricula that follow the temporary standards and the methodological letters of the Ministry Directorate of General Secondary Education…which were introduced into teaching practice between 1992 and 1997” (p. 60). This document lists various topics and concepts that students are required to learn (it is not indicated how fully or how deeply) and is quite traditional. The curricula include curricula for classes with an advanced course in mathematics and, conversely, curricula for classes with insufficient preparation (sometimes called “correctional”). But, to repeat, teachers were guided first and foremost by textbooks, which were supposed to correspond to these curricula.

4 Textbooks and Certain Basic Tendencies in the Development of Mathematics Education

The main textbooks at that time were those prepared in the mid-1980s, which had gone through a nationwide competition (Abramov 2010). It should be noted that some of them continue to play a dominant role to this day (for example, the geometry textbooks edited by L.S. Atanasyan), while others are currently being used in somewhat revised versions prepared by somewhat altered lists of authors. The changes that took place, including the increase in the number of publishers that put out literature for schools, led to the appearance of new textbooks. Without attempting to list all or even many of the books that came out during those years (see, for example, Karp and Vogeli 2011), we will point out some important trends.

One of them—which pertained to the organizational side of things—must be mentioned at once: even if monitoring over which literature was being used by teachers and their classes did exist, it was incomparable to what had been in place during the classic Soviet period. Consequently, a teacher could require parents to acquire one or another textbook (relatively cheap ones) and use this textbook in class, pretty much without asking anyone. Naturally, such a system was not officially encouraged, nor were parents always pleased about having to spend additional money, but this was quite possible and even widespread. Specific organizations specialized in approving textbooks and labeling them with various certifying designations were formed; for example, textbooks were approved at the federal level by the Ministry on the recommendation of the so-called Federal Council of Experts. Such a certifying designation made it possible to purchase textbooks using state resources, which, of course, radically influenced their distribution (such resources, however, might still be lacking, for various reasons, in various regions). But, to repeat, there were other ways of doing it as well.

Below, we will discuss three trends that seem to us the most significant. The first of them was the rapid increase in the number of so-called classes with an advanced course in mathematics.

4.1 On Classes with an Advanced Course in Mathematics

Schools with an advanced course in mathematics, which appeared in the late 1950s and early 1960s, quickly and deservedly won a very high reputation. They selected gifted students who were interested in mathematics; their curricula were designed by wonderful research mathematicians along with highly qualified teachers; these curricula were supplemented by extracurricular activities; and indeed, the teaching of even nonmathematical subjects and the general atmosphere in these schools were noticeably different from what was found in ordinary schools (Karp 2011). The authorities had a contradictory attitude toward these schools—their special atmosphere, and their selectivity in general, which was based on abilities and results, irritated the authorities; but on the other hand, in the era of the scientific-technological revolution, trained professionals were indispensable. What was evident, in any event, was that the graduates of these schools were splendidly prepared for college entrance examinations, and admission to colleges was based specifically on the results of such examinations, which were conducted by each college independently, which made the preparation for such examinations a goal of paramount importance for students and parents, giving rise to an enormous market of preparatory courses, tutors, special textbooks, and the like.

The new liberalization now allowed many schools, as has already been said, to increase the number of hours allocated to mathematics in certain upper grades and to call them "mathematical" (while possibly certain other grades with a small number of hours for mathematics were given a "humanities" label). Not everything was identical everywhere, of course, but on the whole, it can be said that in a very large number of cases, schools that did so did not set particularly difficult educational goals for themselves. If schools that had appeared under the supervision of outstanding mathematicians like Kolmogorov, Gelfand, Smirnov, and others decades earlier had aimed at educating future research mathematicians, deliberately developing difficult courses to this end (which at the very least always included calculus with proofs), the new classes now being formed usually did not aim past college entrance examinations, and these examinations focused on materials from general education schools—which, of course, did not rule out the inclusion of difficult problems.

Colleges were also interested in the new schools. The prestige of engineering professions was falling at this time—engineers were losing their jobs en masse and retraining for new occupations. Every engineering college was concerned with securing a sufficiently large number of matriculating students. Nor must it be forgotten that the matriculating students' level of preparation usually also did not make the colleges very happy. The way out in large cities was found in the creation of the "school-college" system, in which certain classes were designated as being connected with certain colleges, which in some way influenced the curricula of these classes, and most importantly offered certain benefits to the graduates of these classes when they applied to college.

While the "old" schools with an advanced course in mathematics usually allocated at least 8–9 h, per week to mathematical subjects, the "new" schools could as easily allocate 6 or 7 h, and most importantly even when mathematics was granted 8 or 9 h, often it turned out to be preferable to use "ordinary" textbooks from general education schools, designed for a smaller number of hours (say, five for all mathematical subjects). Additional time was allocated for solving problems, which teachers could, for example, draw from manuals for those applying to college.

Such classes with an advanced course in mathematics gradually started to appear not only in high school (grades 10–11) but also earlier—beginning with the eighth grade. There were isolated attempts to start even earlier, but they met with no success. Curricula began to take shape that were recommended for such classes (even if they did not possess a strictly mandatory character), see, for example, Karp and Nekrasov (2000, pp. 32–39). Special textbooks for such classes also began to appear—especially for grades 8–9, since books for students who were applying to colleges were not well suited for use at these grade levels. These were, first and foremost, problem books. We should mention, for example, such texts as Galitsky et al. 1992 or Karp 1993, whose tables of contents correspond—or correspond with certain minor caveats—to standard textbooks but which contain problems whose level and difficulty are substantially higher than those of ordinary problems.

Later, so-called supplemental chapters to textbooks began to appear. As Atanasyan et al. (1996) explain, for example:

> The manual contains additional chapters for the course in geometry for eighth grade. For each chapter in the basic textbook, there is an additional chapter in the manual. The additional chapters, as a rule, do not repeat the material presented in the basic textbook, but this material is broadly used; therefore, before beginning to study any chapter, the students must study the [corresponding] chapter from the basic textbook. (p. 3)

In reality, it was probably better for students not to study first one chapter from one book and then one chapter from the other but to examine parts of the additional chapters as they studied the basic textbook. But in any event, the manual contained additional problems and additional theorems.

We do not possess (and it is unlikely that anyone possesses) information about how many of the students in the country used these "advanced" curricula in their studies during those years. We would argue (based on the figures we know from St. Petersburg) that certainly this number was not more than 10%. Nonetheless, it is evident that the appearance of classes with an advanced course in mathematics and the possibilities they opened up raised the mathematical preparedness of a considerable proportion of teachers and likely benefited many students as well. At the same time, it is also clear that the confusion and befuddlement about what was meant in each particular case by the term "an advanced course" could only cause harm.

4.2 Textbooks for Humanities-Oriented Classes

While studying an advanced course in mathematics had a big history by the 1990s, the goal of special instruction in mathematics for future humanities students was only now formulated for the first time. The reformist principles cited above included humanityzation of education; moreover, the reformers explained that the need for this was connected:

> *First,* with the rejection of technocratic and scientistic traditions, which over the last 200 years have evolved within the global system of education under the influence of a rationalistic view of the world as a kind of inanimate mechanism that may be disassembled under analysis— whether it be a human being, society, culture, etc. And *second*, with the ambition to overcome the clearly observable schism in the culture of education between a humanities and a technical component, to overcome their growing separation. (Dneprov 1998, p. 50)

Even Dneprov himself, however, wrote specifically about the humanities and corresponding school subjects, arguing that their role had to be expanded and their teaching fundamentally changed. Now it appeared, however, that it was desirable to "humanitize" mathematics as well. Expanding the role of the humanities was often equated with reducing the role of the mathematical subjects and even with changing the very style of mathematical instruction. Practically speaking, however, the problem consisted of writing a textbook that could be used for teaching mathematics in grades 10–11 where only 3 h per week were allocated for mathematics.

There arose a confusion (or, if one wishes, a discussion) about what exactly ought to be done (Sarantsev 2003). Probably the first textbook for humanities-oriented students that appeared, Butuzov et al. (1995), began with the following announcement:

> We see the purpose of our book as consisting first and foremost in conveying an idea of the most fundamental mathematical concepts, whose knowledge, in our view, must be a part of the general cultural literacy of a person of any profession. We have attempted as far as possible to tell about the application of mathematics in various spheres of human activity, to acquaint you with certain pages of history and the creators of this remarkable science. This book examines a number of questions that do not belong to traditional school programs in mathematics, but which are important for certain professions related to the humanities. Among them is elementary probability theory, the basic concepts of statistics, and certain others. (p. 4)

Clearly, it is not easy to implement such a program. Moreover, such a program itself can give rise to questions. How does one determine which mathematical concepts are parts of general cultural literacy? In what level of detail should those who know almost no mathematics be told about its applications? Can an account of the life of, say, Galois, which was indeed quite dramatic, replace knowledge of mathematics (even at a much more modest level than is required for understanding Galois theory)? How seriously should one take the goal of teaching schoolchildren those sections of mathematics which might be useful to humanities students?

In practice, however, such questions were never raised nor indeed were special textbooks typically used—in very many so-called humanities-oriented classes, ordinary textbooks for general education schools were used, whose content was impossible to cover in the allotted time. But no one especially insisted on mastering the material either.

The textbook written by the author of this chapter and A.L. Werner (Karp and Werner 2000, 2001), which came out at the end of the period that we are discussing and which reflected the experience of working with humanities-oriented classes and their graduates, which will be discussed below, was written partly in order to return to the traditional order of things—that is, to achieving certain clear outcomes in teaching. At the same time, the orientation of traditional Russian textbooks toward developing computational and other algorithmic skills (necessary, at least at that time, to future engineers) was seen as unnecessary to future professionals in the humanities. The authors' view was that future professionals in the humanities had to be acquainted with the mathematical way of thinking and in general with the activity involved in mathematics, not by being told about the way in which mathematicians think but by involving students in mathematical work (even if such work obviously could be carried out only on such very modest materials as were accessible to such students). Technically, the course was substantially lightened; many proofs disappeared; but at the same time, the authors attempted to offer students many opportunities for mathematical analysis and reasoning, for constructing examples, for comparing various situations, as well as histories of the development of various concepts, and so on. Consequently, the textbook addressed not so much applications of mathematics, as the very concept of modeling, that is, the translation of what was observable in the ordinary world around us into mathematical language.

Other textbooks also appeared (for example, Bashmakov 2004). It should be noted that in the late 1990s and early 2000s, the problem of writing textbooks for humanities-oriented schools was recognized as one of paramount importance, and the ministry held several competitions, with the support of the World Bank, to produce sets of textbooks for such schools (which were won by the aforementioned textbook by Karp and Werner and the problems books and other materials that supported it). In itself, the methodological problem posed then remains difficult and needs further study, although some outcomes were achieved. Unfortunately, during the next phase, in connection with new transformations, this work was interrupted and halted (Karp 2011).

4.3 New Textbooks for General Education Schools

Mass-scale general education schools themselves, however, also needed changing. One area in which changes were necessary has already been mentioned: the teaching of probability theory and statistics was in fact absent from Russian general education schools. During the Kolmogorov reform period (that is, in the late 1960s and 1970s), attempts were made to change this situation and to introduce elementary discrete mathematics into the schools. They did not meet with success, however, and the cause of this likely did not lie only in teachers' lack of preparedness. It is impossible to avoid the thought that the uniform determinate system of thinking, which was supported by the state, was poorly suited for fundamentally different, "probabilistic" thinking. We know of no official decisions not to cover probability theory, but no such decisions were necessary. On the contrary, what was needed was the will to begin to teach such theory, and until a certain point such a will was lacking. With the beginning of the 1990s, gradually and not without difficulties, discrete mathematics began arriving in the schools (Bunimovich 2011).

Another demand of the age was connected with the development of electronic technologies. We can point to various attempts at making use of the new possibilities, but we can detect no notable changes in the textbooks or workbooks of that time. Assignments that required the use of any kind of technology were practically nonexistent.

New textbooks, however, did continue to appear. Kuznetsova et al. (2011) specifically compare the new textbooks by Dorofeev et al. with textbooks that had appeared earlier. Below, we will likewise confine ourselves to discussing this textbook, without looking into others (about them, see Karp and Vogeli 2011), including textbooks in other subjects (geometry or algebra and elementary calculus).

Looking at the textbooks by Dorofeev et al., one can note differences with previous textbooks of various types. First, there are differences that are purely methodological and pertain to content. Such differences are many (not to mention the appearance of a systematic presentation of stochastics, which begins in the textbook for the fifth grade): for example, as Kuznetsova et al. (2011) point out, there is a change in the balance between the arithmetical and algebraic components of the beginning of the course in favor of the former. More attention is devoted to arithmetic and to working with numbers (if one wishes, one can detect here the influence of the technological revolution—we now work more directly with numbers than with algebraic symbols in real life, too). Ideas that effectively belong to analytic geometry, that is, to the connection between graphs and equations, appear quite early on. In general, various graphs appear in the textbooks earlier and in greater numbers than in previous textbooks (it may be argued that here, too, there is a certain influence of technology—although one that is not typically explicitly discussed—since children now encounter graphs more often than they did before). On the other hand, the study of certain topics is organized more gradually and sometimes introduced later than it had been previously. Here we see what is arguably the second fundamental distinctive feature of these textbooks, which is organizational in character: these textbooks, more than previous ones, take the individual characteristics of the schoolchildren into account. Note, for example, the appearance of the sections entitled "For those who are interested" (which subsequently, in one way or another, appeared in other textbooks as well). These sections offered additional "theoretical" material and additional problems at the same time. And the main sections, too, contained two groups of problems: more simple problems (A) and more difficult problems (B). The possibility for differentiation was already embedded in the textbook. The textbook also began paying more attention to independent work, and above all to self-checking—there appeared special assignments aimed at this.

And yet, there were no revolutionary changes either in these or in other textbooks of the time. They continued the existing tradition as they had done before, quite noticeably differing from American textbooks, for example, both in terms of their outward appearance (more modest, smaller in dimensions, cheaper) and in terms of content (more systematic and more proof-oriented).

5 Innovations in the System of Conducting Examinations

The system of conducting examinations was changed repeatedly under Soviet rule (Karp 2007a). By the end of Soviet rule, the system consisted of three rounds of exams: the first, upon completion of the basic school—eighth grade, in the system of numeration that existed at the time (later it began to be called ninth grade); the second, a graduation exam, upon completion of secondary school—grade 10 (later 11); and finally, those who wished to enroll in institutions of higher education also took entrance exams there. Naturally, there were no examinations in certain subjects, but in mathematics there were often even two examinations—for example, upon completion of school, a written examination in algebra and elementary calculus, and an oral examination in geometry (by the 1980s and 1990s, this last examination had often become what was called an elective examination, that is, it was taken by those who wanted to take it, while others could choose to take an examination in a different subject, say, history).

The graduation examination in algebra and elementary calculus, at this time, lasted 4 or 5 h but contained only five (later six) problems, which not only had to be correctly solved, however, but whose solutions also had to be written out in a correct and well-argued manner. Meanwhile, the grading criteria were mainly based on the notion that if five problems had been fully solved, a student was given the highest grade of 5; if four problems had been fully solved, a student was given the grade of 4, and so on. Naturally, things did not always go as smoothly as this—a problem could be solved partially or with deficiencies; there were special criteria for how many deficiencies were acceptable for each grade; it was specified which deficiencies were considered simply omissions and which were considered mistakes, etc. (Chudovsky et al. 1986). Note that the appearance of a sixth problem on the examinations constituted an important development: students acquired a choice, even if only a small one. From the 1970s on, graduation examinations were conducted at two levels—a basic level and an advanced level (for general education school and for schools with an advanced course in mathematics, respectively).

Understandably, such a system came under a great deal of criticism, which became harsher as the role of examinations in the new post-reform country grew: it was said that schools had acquired more freedom and had been liberated from trifling minute-by-minute control, yet in the end—on the exams—they had to show what they had taught their students. The following considerations were among those expressed at this time:

- Why are graduation and entrance exams conducted separately? Shouldn't they be unified, as is done in many countries? Why, in addition, are entrance exams conducted separately by each college? Are there really two separate mathematics for two quite similar engineering colleges? (We should recall the appearance of the "School-College" system, which on the whole supported at least a partial and limited unification of graduation and entrance exams.)
- Along with classes with an advanced course in mathematics and general education classes, humanities-oriented classes also began to appear: was a special exam necessary for such students?
- Since schools and students were acquiring greater rights than they had previously with regard to structuring the material being studied, and could pay somewhat more attention to certain topics and consequently less attention to others, the exams had to reflect this as well. The students must have greater choice about what to solve.
- In general, the role of each isolated skill is shrinking compared to what it had been before. Naturally, when studying trigonometry, it is important to get the students to be able to solve trigonometric equations of the required types, but upon graduation from school, and even more so later, what becomes important is

not the specific ability to solve these types of problems but a broader ability to work with mathematical texts. Therefore, it would be desirable to structure the problems in some new way.

- Discussions about partial solutions went on for years, as did discussions about the strictness with which solutions should be presented (sometimes the demands for strictness were completely exorbitant). Proposals were made to switch to multiple-choice tests or to problems with short answers that could not be argued about and would also be far easier to check.

The issues and questions posed above were discussed both in the period examined here (before 1999) and later, so we will return to them below. Here, we would merely note that answers to these questions did appear, in one way or another, and they appeared not only in a centralized manner. At a certain time, the ministry introduced examinations for humanities-oriented classes (Zvavich and Shlyapochnik 1994), but at this time, it was also possible to organize examinations at a regional level. Thus, Moscow and St. Petersburg began to organize their own graduation examinations (Karp 2003).

St. Petersburg's examinations, for example, were conducted at four levels: there were two advanced levels—in addition to the usual examination for classes with an advanced course in mathematics, a so called *elite-specialized* examination was offered, which was indeed very difficult but which from a certain time on began to be counted for all those who passed it as an entrance examination to a leading college, St. Petersburg University. The main difference between these examinations and others was their structure: students were offered not isolated problems, but blocks of interconnected questions, and they were furthermore given a relatively broad choice. Thus, the version of the examination for classes with an advanced course in mathematics offered two so-called mandatory blocks of four problems and three more blocks, of which students had to choose one (such a system could accommodate for existing differences between curricula). In the view of those who wrote these examinations, systems of problems composed into blocks taught students to look for connections between problems, by comparing and checking their solutions, learning to generalize what they had noticed, and so on (Karp 2002, 2003).

Local approaches were also employed in other places. And the ministry itself prepared new offerings, publishing a collection by Dorofeev et al. (1999), which became the basis for examinations. Every student and every school owned this book, and problems from it were systematically solved both in class and at home. On examinations, students would be asked to solve several problems from this text (the numbers were given out only before the beginning of the exam). This practice was referred to as "open exams." Exams based on this text were meant to be conducted at two levels—level A for students who had 3 h of mathematics per week and level B for those who had 4.5–5 h. (The idea of "open exams" had already appeared earlier for exams for eighth (ninth) grade; see below.)

Graduation examinations in schools of the "School-College" system were also counted as entrance examinations for schools connected with a college, which was hardly fair with respect to other schools, but at the same time could serve as a step toward the formation of some kind of more general system of uniting graduation and entrance examinations, but at later stages, these experiments were abandoned.

From today's vantage point, the practice of conducting examinations in ninth grade during this time also appears as something transitional. At the beginning of this period, examinations were based on the manual MP RSFSR (1985) (the latest edition by Chudovsky and Somova 1995), which introduced the idea of "open exams," which, it was believed, helped to reduce stress, eliminate unpredictability, prepare for the examination systematically, and so on. Later, a new collection by Zvavich et al. (1994) appeared, which contained somewhat more varied and difficult problems. But this collection was not used for long, and it was replaced by the problem book by Kuznetsova et al. (2002), while the number of problems given on the examination increased somewhat (from 6 to 10)—although their difficulty probably decreased. Behind these changes was the question (possibly not always acknowledged): what exactly should be tested—specific skills or the ability to operate with their more complex combinations? As an example of a problem from the collection by Zvavich et al. (1994), consider the following (I.599) from the first part, which was considered the easier part:

> Determine whether the following inequality is true: $\cos(149^0 + x)\cos x + \sin(149^0 + x)\sin x < 0$.

This somewhat cumbersome problem nonetheless requires the execution of several steps, the most substantive of which is specifically the determination of what needs to be done, that is, the development of a plan. Undoubtedly, students need to know the formula for the cosine of the difference of two angles; then they can see at once that the question concerns whether it is true that $\cos 149^0 < 0$. And yet the very formulation of the problem is somewhat more open than, say, "simplify the expression and check whether such-and-such an inequality holds," which at once steers students down the path of following learned algorithms.

In fact, the manual by Kuznetsova et al. (2002) also contained not a few interesting new methodological approaches. But it, too, was destined for a relatively brief life.

6 Public Opinion About the Teaching of Mathematics

Before moving on to an examination of the new political period that began with the coming to power of Vladimir Putin, we should say a little more about public opinion concerning the changes that occurred (to a certain extent, we have already said something about it when discussing standards and the content of mathematics education). It is hardly possible to represent all existing points of view here, but we should still like to describe those opinions which were widespread among research mathematicians. Again, without claiming to give an account of all opinions, we will confine ourselves to opinions expressed in a book published already at the beginning of the Putin period, *Education We Can Lose* (below we will quote from its second edition, Sadovnichy 2003). This volume, which was edited by Sadovnichy, academician and rector of Moscow University, contains articles by the academicians Anosov, Arnold, Kudryavtsev, Nikolsky, and by Igor Sharygin, a prominent figure in mathematics education (it contains other materials as well, for example, an article by Professor Melnikov, a prominent figure in the Communist Party and a mathematician by training, and even an interview with the writer Alexander Solzhenitsyn). It should be noted that although the collection contains a speech given by Putin in 2001, and although the authors sometimes refer to documents that were fresh at the time, their main pathos is directed against the changes that took place during the previous period and the plans for their continuation (even if only in rhetoric).

The common trait shared by all the authors, which is also expressed in the book's title, is their confidence in the merits and even world leadership of Soviet mathematics education. "Programs outlining what should be taught when in secondary school took shape in Russia over the last two centuries. The fact that the choices made were quite good is attested to by the fact that, with respect to the fundamentality of education in the natural sciences and mathematics, the Russian school has undoubtedly occupied the first place in the world," writes L.D. Kudryavtsev (p. 125). Admittedly, I.F. Sharygin ironically comments that people will say that the theorem about the supremacy of the Russian school has not been proven, but neither does he himself offer any proofs of this nor does he allow for any doubts.

Consequently, the authors take it for granted that what was done before, on the whole, is what must continue being done. "Why abandon that which has so recommended itself?" asks the same L.D. Kudryavtsev, and he goes on:

> An analogous situation exists in schools with algebraic and trigonometric transformations of quite complex expressions: in the opinion of certain critics, too much time is spent on this in schools, while later on students rarely have to do with such matters. This objection is again beside the point, since the main purpose of solving exercises with algebraic and trigonometric transformations is to cultivate the skills necessary for carrying out goal-oriented analytic transformations. (p. 130)

More generally, the narrowly utilitarian approach—"we will teach that which will be useful in life"—is decisively rejected by practically all of the authors, who emphasize the developmental value of mathematics, including even its value for developing morality (Sharygin); furthermore, the authors emphasize the ignorance of the reformers, who do not understand and do not know the applications of mathematics, and therefore, for example, wish to eliminate logarithms from the schools as something useless (Arnold).

The need for reforms is sometimes acknowledged, but with very sizable reservations. D.V. Anosov notes that in the 1930s, approximately one fourth of all children graduated from school, while later the proportion of those graduating from school increased—"Under such circumstances, schools inevitably must change" (p. 95). But then he goes on to remark that the goals of the reforms have not been sufficiently thought through and articulated and that their implementation has not been successful. L.D. Kudryavtsev praises Kolmogorov, who reformed the content of education, for including vectors and elementary calculus in the school program, but at the same time laments the disappearance of Newton's binomial theorem and complex numbers. S.M. Nikolsky welcomes the appearance of probability theory and statistics in schools but notes that this concerns only 2% of school mathematics education.

It appears the destruction began quite a long time ago, when the share of materials involving proofs began to diminish, but the changes of recent years are especially irritating to the authors because of their explicitly formulated aim of fighting "scientification," "the scientistic approach," "subject-centeredness," and the like, which they interpret as hostility to science, which is responsible for the reduction in the number of hours allocated for mathematics and its role in education in general.

Moreover, the authors by no means confine themselves exclusively to the teaching of selected students. L.D. Kudryavtsev writes:

> I regard the tendency to concentrate serious education in mathematics and the natural sciences in specialized higher grades as deeply misguided—on the one hand, this lowers the overall level of education for those who will not study in such specialized classes, and on the other hand, it makes learning more difficult for those who will study in such classes because they have not acquired sufficient necessary knowledge at an appropriate age. (p. 127)

The author has specifically mass-scale schools in mind. On a different note, the authors reject the idea that "the school program must be abridged so as to be made accessible to all." They are concerned with "normal children," as S.M. Nikolsky puts it: "These children are quite healthy and quite capable of handling the difficulties of elementary mathematics: fractions, equations, sines, logarithms. Such children are many, and progress depends on them—they are the ones I am speaking about" (p. 160). What to do with the rest, however many they might be (and S.M. Nikolsky himself notes that they not only exist but will continue to exist), is not discussed—Nikolsky merely remarks that a class composed of such children should be called not a general education class but something else.

Without entering into a debate about what seems justifiable in the cited pronouncements of these Russian research mathematicians, we should merely note that the change in the perception of what was happening was itself important: the conviction that whatever else might be wrong, the teaching of mathematics in Russia was excellent and the finest in the world, was clearly giving way to the view that things were not going well. It is noteworthy that the collection from which these pronouncements are drawn is quite anti-American in tone (naturally, there are differences among the authors in this respect): the United States turns out to be the source of the foolishness taking place in Russia, while simultaneously, we are told that American educators want to change and improve their own mathematics education (certain documents to this effect are included in the collection), and, additionally, some of the authors (I.F. Sharygin) voice the thought that, by forcing such a flawed approach onto Russia, the United States wishes to turn Russians into its slaves. All of these notions are not very easy to reconcile, but it is important to note that methodological conservatism has at least sometimes gone hand in hand with political conservatism.

7 New Times

On December 31, 1999, Vladimir Putin became the acting president of Russia, and several months later, he was elected president. New times began, which quite quickly began to be contrasted with the previous times, despite the fact that the whole career of the new leader had taken place precisely during the period that was now being denigrated. People began saying that Russia had to rise from its knees, that it was necessary to construct a "vertical of power"; there was talk of "sovereign democracy" and much else. Looking at this period as far as possible as a whole, without attempting to reconstruct the changes that took place year by year, we should repeat that there is no reason to think that each change in politics gave rise to a change in mathematics education. Nonetheless, the connection between the two is evident.

Official rhetoric about mathematics education and mathematics has been very positive. We might cite, for example, the "Conception for the Development of Mathematics Education in the Russian Federation," which was endorsed by the Russian government on December 24, 2013. It opens with a discussion of the special role of mathematics and its indispensability both for the development of society and for individual development. It is noted that the Russian system of education is the heir of the Soviet system, which, however, needs to be improved, even as its virtues need to be preserved. Problems are listed: motivational problems, connected with the underestimation of mathematics education; content-related problems, for example, that the needs of future experts in mathematics are not sufficiently taken into account; and personnel problems, including the shortage of teachers and their poor preparedness. Aims are formulated: to elevate Russian mathematics education to a leading position in the world. How this is to be achieved, however, is discussed in terms that are rather broad: it is merely stated that education must offer "each student an opportunity to achieve the level of mathematical knowledge necessary for further successful life in society," "provide each student with developmental intellectual activity at an accessible level," and so forth.

At the same time, the sense of a crisis in mathematics education has probably been even stronger than it was in previous years. In support of this claim, we will confine ourselves to listing several recent articles in Russia's main journal of mathematics education, *Matematika v shkole* ("Mathematics in the School"): "Diagnosis: Mathematically Illiterate" (Bogomolova 2014), "The Decay of Mandatory Knowledge" (Novikov 2018), "The Crisis in Secondary Mathematics Education Through a Teacher's Eyes" (Ryzhik 2013, 2014), "Signs of a Crisis in the Domestic Methodology of Mathematics Instruction" (Savvina 2017), and "The Mathematics Curriculum of 2015, or the Triumph of Unprofessionalism" (Shevkin 2015a, 2015b).

On the other hand, explanations of this crisis and recommendations about finding a way out of it vary widely. The cited article by Savvina (2017), for example, contains the following appeal:

> to make sense of the worldview that is the cause of the crisis. To understand that borrowing a conception of education as a commercial service from Protestant Western civilization with its market ideology goes against Russia's tradition and national interests. "The main function of schools is to reproduce the civilizational code, transmit traditions, fortify the country"

> [Kaiumov 2014]. In connection with which, it is necessary to appreciate the danger of introducing the latest innovations (competencies, universal learning activities, and other consequences of the market ideology in education) into pedagogical discourse." (p. 7)

Instead, the author proposes focusing on Russia's own historical experience, which she had previously associated with the idea of the comprehensive development of the personality, whose origin she attributes to Byzantine influences. It may be said that the very direction and character of the "discourse" proposed by the author reflect the changes that have taken place in the country.

8 Once More About Standards

Whereas during the previous phase, standards functioned more as an educational idea, which acquired practical significance only to a very limited extent, during the new phase, they have acquired much greater force. The standards are updated—changes in them are authorized by ministerial decree—and even different generations of standards are spoken of. Without going into all the distinctions between them, we can confine ourselves to an analysis of one of the existing documents—the Federal State Educational Standard for Basic General Education (FGOS 2018). There are also standards for elementary education and high schools.

As the very first paragraph of the document states, the Standard "represents the totality of the requirements that must be fulfilled in the implementation of a basic education program of basic general education" (p. 6). The same general part lets it be known that the Standard "was developed with due regard for regional, national, and ethnic distinctions among the peoples of the Russian Federation" (p. 7). The Standard is oriented toward the formation of a graduate who "loves his region and his Homeland, knows the Russian language and his native language, respects his people, its culture and spiritual traditions" and is also ready for self-development and continuous education; "who actively and with interest learns about the world, recognizes the value of work, science, and creativity" and is also "socially active, respects law and order, and holds his actions up to moral values" (p. 8).

In order to achieve all this (and much more), the Standard puts forward requirements for demonstrating the assimilation of the program. These requirements are assembled into three groups: requirements for personality-oriented results, meta-subject results, and subject results.

The first group of requirements includes the following:

1. The formation of a Russian civic identity: patriotism, respect for the Homeland, for the past and present of the multinational people of Russia;...
2. The formation of a responsible attitude toward learning...
3. The formation of a holistic worldview corresponding to the contemporary level of science...

......

7. The development of communicative competence...

......

11. The development of an aesthetic consciousness (pp. 10–11).

The meta-subject results, which the Standard stipulates, must demonstrate the following:

1. An ability independently to determine the goals of one's education ...
2. An ability independently to plan the means for achieving the goals, including alternative means ...
3. An ability to correlate one's actions with the planned outcomes (p. 12).

And much else—including, for example, reading comprehension; the ability to create, employ, and transform signs and symbols, models, and schemas for solving academic and cognitive problems; and the development of ecological thinking.

Subject requirements are formulated separately for each subject area. The subject area of "Mathematics and Informatics" contains a comparatively large number of requirements, of different types. These include the following:

The formation of conceptions of mathematics as a method for comprehending reality, enabling the description and study of real processes and phenomena;
The recognition of the role of mathematics in the development of Russia and the world;
The possibility of giving examples from Russian and world history of mathematical discoveries and their authors (p. 25).

But there are also requirements such as the following:

Mastery of the system of functional concepts, the development of the ability to use functional-graphic representations for solving various mathematical problems, for describing and analyzing real-world relations:

defining the position of a point based on its coordinates and the coordinates of a point based on its position in a plane;
determining the values of a function based on its graph: its domain, range, zeroes, intervals of sign-constancy, increasing and decreasing intervals, maximum and minimum;
constructing graphs of linear and quadratic functions;
operating at a basic level with the concepts of sequence, arithmetic progression, geometric progression;
using the properties of linear and quadratic functions and their graphs to solve problems from other academic subjects (p. 26).

Note that the strictly mathematical requirements differ from what was required during the 1990s or even the 1980s for the most part only due to a certain imprecision and lack of concreteness—everything pertaining to functions listed above was also required previously, except now the Standard does not specify at what level these knowledge and skills are required. Of course, one might ask whether a basic school graduate must be able to find the domain of a function *not* based on its graph. This is not required explicitly, although the Standard does require proficiency with "techniques for solving inequalities" (p. 26). The requirements pertaining to discrete mathematics are clearly (relatively) new—as has already been said, discrete mathematics was not studied in school in the USSR. Also new, of course, is the inclusion of informatics (something close to computer science) in the same subject field as mathematics and the formulation of corresponding requirements for computer scientific and algorithmic literacy or for the formation of skills and aptitudes enabling safe and effective conduct in working with computer programs.

The Standard goes beyond the two sections described above—it contains two other sections, “Requirements for the Structure of the Basic Educational Program” and “Requirements for the Conditions of Implementation of the Basic Education Program.” The latter deals mainly with financial-economic, informational, personnel-related, psychological-pedagogical, and other requirements. As for the section on structure, it not only contains formal requirements (for example, what a program must include and how the system for assessing results must be described) but also underscores certain substantive requirements, including the need to develop so-called universal learning activities, that is, activities not specific to the subject—the focus here is not on the ability to solve inequalities but, for example, on developing the students’ capacity for self-development and self-improvement and on other results prescribed by the requirements in the first two groups (which are not subject-oriented).

The authors of the textbooks used in schools must follow the requirements of the Standard. The subject requirements are arguably less rigid than the requirements from the other groups. In 2014, a great deal was written in the press about the fact that Ludmila Peterson’s mathematics textbook was not included in the so-called Federal List of Textbooks (that is, textbooks that are approved by the state and may be purchased using state funds), since in the expert opinion of Lyubov Ulyakhina, “The content of the textbook fails to facilitate the formation of patriotism. The protagonists of the works of Gianni Rodari, Charles Perrault, the Brothers Grimm, A.A. Milne, Astrid Lindgren, Erich Raspe, dwarves, elves, fakirs with snakes, and the three little pigs are hardly suited to cultivate a feeling of patriotism and pride in one’s country and one’s people.” Responding to reporters’ questions, the expert stated:

> I conducted an expert evaluation in accordance with the state educational standard for all textbooks in all subjects. One of the first questions that I had to answer was whether a textbook developed [students’] personal qualities. And another clause contained the word “patriotism”—and it was necessary to give an answer. “Does the textbook cultivate patriotism, love and respect for the family, the homeland, one’s people, one’s region?”—this was the complete formulation [of the question]. It was impossible to shun this clause, and the question had to be approached honestly. (see Znak 2014)

Whether or not we share the expert’s opinion about the fact that the mention of Snow White—a representative of a foreign culture—in a fourth grade textbook is not conducive to the formation of patriotism, we must acknowledge that normatively requiring each textbook systematically to cultivate patriotism and conducting expert evaluations of such cultivation can only lead at the very least to contentious debates—a single, universally accepted hierarchy of values is hardly conceivable in such matters. A certain bias is therefore inevitable, which is unlikely conducive to improving the quality of teaching (note that even during the years of Stalin’s anti-cosmopolitan campaign, excessive demands for patriotism in mathematics classes—as opposed to literature or history classes—were not welcome: for the cultivation of patriotism, extracurricular time was set apart; Karp 2007b, Karp 2010).

Without entering into a discussion of all the opinions voiced about these issues, we should only like to note that sharply negative assessments of the Standard were also expressed. Consider, for example, an article entitled “Mathematics Education

Under the Chariot of the FGOS [Federal State Educational Standard]" (Malyshev 2016). Its authors believe that, by concentrating on teaching universal learning activities, the writers of the Standard and those who follow them have undermined the teaching specifically of mathematics, offering students unsubstantive and artificial assignments in its place. But even in this article, the author rather quickly begins to talk not so much about the Standards, as about the assessments carried out in the schools, and first and foremost about the so-called EGE (Edinyi Gosudarstvennyi ekzamen – Uniform State Exam).

9 Final Assessment: The Uniform State Exam (EGE) and the State Final Assessment (GIA)

Probably the most frequently used word in discussions about education in Russia during the first and second decades of the twenty-first century has been "EGE." The idea of combining entrance and graduation examinations began to be discussed, as we have already mentioned, back in the 1990s, but only in 2001 did the ministry begin conducting experiments in a number of regions. At the same time, other local experiments in the field of examinations were discontinued: the examinations had to come from Moscow. The number of regions encompassed by the experiments grew, and from 2008 on, the EGE began to be conducted across the whole territory of Russia and no longer as an experiment. The essence of the procedure, as it took shape in 2009, is that students take examinations in a centralized manner, based on universal monitoring and measuring materials developed by a central agency for the whole country (although this agency may employ people from different regions, not only from Moscow), which are then checked in a centralized manner—by special local commissions, with possible subsequent emendations and changes by a national commission—with a view to giving an assessment (number of points) based on developed common criteria. A student who has received a certain number of points in mathematics—usually even a very low one—acquires the right to obtain a secondary school diploma (if there are no problems with other subjects, of course), and this same number of points is sent to the colleges to which the student has applied.

The practice and the very idea of the EGE has not only political and organizational but also methodological aspects. We should say at once that one of the main virtues of the EGE was proclaimed to be the reduction of corruption. One of the theoreticians behind the introduction of the EGE, Moscow Higher School of Economics rector Yaroslav Kuz'minov, said in an interview in 2009:

> The first consequence of the introduction of the EGE is a quite serious decline in corruption in admissions to colleges. In the mid-2000s, we estimated admissions corruption, i.e. bribes given for admission to colleges, in the amount of $300-$400 million. At present, this flow of money has diminished approximately fourfold... We observe no growth of corruption in the schools in recent years. And this is understandable. Schools have no direct relation to the administration of the EGE. (Kuz'minov 2009)

College teachers, whose incomes had declined substantially since the early 1990s, were accused (and probably not unjustifiably) of making the materials of their college entrance examinations available for purchase, in one form or another. Naturally, once these materials became inaccessible to these teachers, they could no longer sell them.

The second achievement of the EGE was seen to consist in the fact that it gave talented children from the provinces an opportunity to enter the leading colleges in the country, to which they had simply been unable to travel to take their examinations previously. The same Yaroslav Kuz'minov (2012) even said that objections against the EGE stemmed from the fact that Moscow and St. Petersburg residents had lost their privileged positions:

> A deep-seated cause of public antagonism toward the EGE consists not in methodology, but in its social effect. The interests of considerable groups—specifically, the populations of the country's largest cities—have been encroached upon. The residents of Moscow, St. Petersburg, Yekaterinburg, Novosibirsk, Nizhny Novgorod, in the 15 years following the fall of the USSR, had the opportunity almost monopolistically to enjoy an important social good: free higher education for their children in the best colleges. They were simply nearer to them and had average incomes that allowed them to pay for preparation courses for college entrance examinations. Residents of other regions, small cities and towns, on the other hand, found themselves outside the system of preparation for the best colleges—both in terms of their places of residence, and in terms of their incomes, which were two or three times lower than those of people residing in the megalopolises.

Of course, much was also said about the need for an honest tool for measuring the outcomes of education, which the EGE is supposed to become. The director of the Tsaritsyno Center of Education in Moscow, for example, expressed himself in the following cautious terms:

> We have never had a single objective tool for measuring the quality of children's education—neither during the Soviet, nor during the post-Soviet period. The EGE can be such a tool, provided its content is sound and the procedure for administering it absolutely clear, objective, open, effective, and transparent. (Materialy 2004, p. 274)

We should note at once that altogether different views about these questions were expressed as well. We will confine ourselves to just one quote (albeit a long one) from an article by Alexander Abramov (2009), "Three Myths That We Have Lost":

> I believe that the collapse of three myths has been documented this year. Actually, the myth of the EGE's democratic character never needed debunking. There are few student dormitories left in Moscow and St. Petersburg. Do-gooders who would finance the education of talents from the boondocks for five-six years are also nowhere to be seen…
>
> Few people believe the myth of the EGE's anti-corruption influence. College corruptionists have more than compensated for their losses on entrance exams by creating a far more profitable system of proceeds from 10-12 exam sessions. Meanwhile, the market in diplomas, tutoring, and "services" related to taking the EGE has been flourishing.
>
> The novelty of the season is the collapse of the myth about the EGE's objectivity... The fact is that in the heat of the struggle for objectivity and quality of education, the following decision was made: schoolchildren who received two failing grades—in mathematics and in the Russian language—would be given no school diplomas. Clearly, however, blackballing multitudes of angry young men in times of crisis is a risky proposition... The problem

found a three-stage solution. At the first stage—during the administration of the exams—a blind eye was turned to "shenanigans." Judging by reports on the internet, the means of acquiring decent grades are quite varied. Nor is this due only to the vast upsurge in mobile communication technologies available to exam-takers. At the second stage, an extremely low threshold for a passing grade was set by force—below ground level, as they say. At the third stage—when students re-took exams they had failed—two thirds of them overcame their illiteracy in the Russian language and mathematics in a couple of weeks. As a result, the projected exam figures were achieved: only 2–3% of graduates did not get a diploma.

Methodologically, the examination has changed several times and quite probably will yet change more than once. The earlier versions contained multiple-choice assignments (called "assignments A"). Here is an example of one such assignment (Nekrasov et al. 2007, p. 14):

Find the value of the following expression: $\sqrt[6]{3^7 \cdot 4^5} \cdot \sqrt[6]{3^5 \cdot 4}$

1. 24
2. 6
3. 36
4. $4\sqrt{3}$

These assignments were denounced as going against the Russian tradition, which precisely required students to demonstrate their reasoning, rather than to choose an answer; they were met with a wave of criticism and were gradually removed from the materials.

A second group (for a long time called "assignments B") consisted of assignments in which short answers had to be given. Here are a few examples:

- Find the maximum of the function $y = \frac{x^3}{3} + \frac{x^2}{2} - 2x - 2\frac{1}{3}$ (Nekrasov et al. 2007, p.18).
- Triangle ABC is inscribed in a circle with a center at O. Angle BAC is 32°. Find angle BOC. Give the answer in degrees. (Demonstration version 2019 https://www.examen.ru/add/ege/demonstracionnye-varianty-ege/).

Finally, there are also assignments (group "C") that require detailed answers (these, by contrast with the preceding ones, are checked by expert commissions). Here are two examples of such assignments (https://www.examen.ru/add/ege/demonstracionnye-varianty-ege/):

- a) Solve the equation $2\sin\left(x + \frac{\pi}{3}\right) + \cos 2x = \sqrt{3}\cos x + 1$. b) Indicate the roots of this equation that belong to the segment $\left[-3\pi; -\frac{3\pi}{2}\right]$.

- Find all positive values of a, for each of which the system $\begin{cases} (|x|-5)^2 + (y-4)^2 = 9 \\ (x+2)^2 + y^2 = a^2 \end{cases}$ has a single solution.

Here, we should note that after the EGE had been given for several years, it was recognized that it was somewhat strange to offer the same version of the examination to all students. Those who had covered the minimal program had no chance of even understanding many if not all of the assignments in which a detailed answer had to be given. The gap between “easy” and “difficult” problems was very great. The mandatory nature of the EGE destroyed “humanities-oriented” classes, since their graduates did not prepare for problems that were comparatively technically difficult. But even classes that used ordinary textbooks, which contained such problems, were not particularly helpful when only 3 or even 4 h were allocated per week for the entire course in mathematics (both algebra and geometry). On the other hand, it was necessary to include in exam problems above the minimum level in order to meet the requirements of colleges with a technological or economic orientation. In 2015, the proposal was made to conduct the examination in mathematics on two levels—a basic level and a specialized level—with only the results of the specialized exam being suitable for college applications. The specialized version approximately followed the versions that had been offered previously, while the basic version was easier—it consists (for 2018–2019) of 20 short-answer problems at the so-called basic level of difficulty. The specialized version contains eight short-answer problems at a basic level of difficulty, four short-answer problems at an advanced level of difficulty, and seven problems requiring detailed answers at a high level of difficulty. The time allowed for the exam is 240 min or during some years 235 min.

Naturally, over decades of conducting the EGE, an enormous quantity of data has been collected about its results. We might confine ourselves, for example, to statistics pertaining to Tomsk region in 2014 (Sokolov 2014). During this year, the lowest score demonstrating assimilation of the school curriculum in mathematics was 20 (in 2013, it was 24)—this referred not to the initial, “raw” score (the greatest possible raw score was 33), but to the score after a recount according to some rules (the greatest possible being 100). In other words, the lowest “passing” score was one fifth of the total. In Tomsk region, 120 graduates (2.26%) got less than this lowest score, and of them, 13 graduates did not get a single point. One graduate got the maximum score (100) in 2014, while eight had gotten the maximum in 2013, zero in 2012, and eight in 2011. The average score in 2014 in the region was 48.11 (in the Russian Federation as a whole, it was 39.6); in 2013, it was 48.04 (in the RF, 49.6) (p. 51). Also, Table 5.4 seems highly indicative (p. 53).

Also worthy of note are statistics about the distribution of high scores (80 or above) in the schools. Altogether, there were 109 students who scored at this level, of whom 46 attended the same Tomsk lyceum, 7 others the same gymnasium, 6 others two other gymnasia (3 students in each); 13 other educational institutions had two such graduates each, and 24 other educational institutions had one such graduate each (pp. 53–54). It is not difficult to compute that in dozens of Tomsk region schools, there was not a single student who scored 80 or higher. On the other hand, we also have statistics about schools with the best average score (pp. 53–54): only in 1 school it is higher than 80, in 10 schools it is between 60 and 69, and in 42 schools it is between 50 and 60. Once again, it is not difficult to compute that over half of this year’s graduates in Tomsk region attended schools in which the average score was below 50.

Without reproducing the other statistics, we will confine ourselves to noting that problem C1, which required a detailed answer, was solved by 22.4%, problem C2 by

Table 5.4 Distribution of graduates over ranges of test scores

Range of scores	Number of graduates with scores within the range	Percentage of graduates with scores within the range
0–10	46	0.86%
11–20	231	4.33%
21–30	606	11.37%
31–40	1225	23.0%
41–50	805	15.1%
51–60	960	18.01%
61–70	769	14.42%
71–80	616	11.54%
81–90	67	1.26%
91–100	6	0.12%

3.4%, problem C3 by 7.7%, problem C4 by 1.6%, problem C5 by 0.8%, and problem C6 by 3.9% (p. 59). The report cited here gives no figures about the students who failed to solve a single problem from this section, but they constituted at the very least 60%.

Naturally, these figures should not be extrapolated onto all of Russia and across all years, but nonetheless in our view they convey a certain sense of the results.

Comparing the results from different regions by year, and noting sometimes significant improvements or unexpected differences in results between regions (oblasts), it is impossible not to wonder whether the examinations are always conducted fairly, whether the hope of obtaining an honest tool for assessing achievements, which was mentioned above, has been fulfilled. "Anomalously high results" on the EGE have been repeatedly mentioned in the press; there have already been reports (https://echo.msk.ru/news/2277336-echo.html) about lawsuits against officials who, in one way or another, sold exam materials; there have also been accounts of versions of examinations appearing on the Internet before the examination was given. Of note, for example, is the assertion made by St. Petersburg teacher, Dmitry Gushchin, in 2018 (for example, https://sibmama.ru/EGE-2018-2.htm), who stated that, prior to the examination, problems appeared online which were then reproduced with only slight differences on the distributed copies of the examination. For example, the materials published beforehand contained the problem: solve the inequality $2\log_3(1-2x)-\log_3(1/x-2)\le\log_3(4x^2+6x-1)$; while the examination itself contained the problem: solve the inequality: $2\log_2(1-2x)-\log_2(1/x-2)\le\log_2(4x^2+6x-1)$, in which 3 in the base of the logarithm was changed to 2. The ministry, however, denied any leak, claiming that the problems were different, and that the resemblance between them was only natural and due to a certain standardization.[4]

[4] At the ministry's behest, legal proceedings were begun, in the course of which Gushchin's charges were found to be groundless, since the court found nothing unlawful in the coincidences indicated by him. As the book is going into production, the case is being reviewed a second time.

Standardization does indeed exist—long before the examinations, students know (and completely legally) not only that, for example, some problem will be devoted to plane geometry, but also that this problem will involve, say, two alternative arrangements of the configuration of figures being discussed. There are entire collections devoted to solving specific problems, for example (Yashchenko and Zakharov 2014), "EGE 2014. Mathematics. Problem B8. The Geometric Meaning of the Derivative: Workbook." Mathematics instruction is thus inevitably structured by the examination: we study not mathematics in general, but specifically how to solve problem B8. Let us add that when entrance examinations to colleges existed, in one way or another fundamentally new problems appeared each year—a high-level college considered it necessary to invent them; now, by contrast, this erstwhile variety has vanished. The uniformity that has been introduced in a certain sense surpasses that of Stalin's times.

Writing about the EGE, Bashmakov (2010) rightly compares the educational system with a physical system whose state changes when it is measured. It is evident that the monopoly of the EGE could not but change the system. Note that both the supporters of the EGE and its opponents, in comparing the examination with the systems that exist outside of Russia, usually fail to notice that in these systems there is usually no monopoly with regard to conducting the examinations: in Britain, there exist various examination commissions; in France and Germany, examinations are conducted at the regional level; in the United States, there could not even be a uniform examination, since there is no uniform curriculum. But the monopoly—or, to use another expression, the unified vertical of power—was a political decision.

Criticism of the EGE has been varied. It has been noted that first-year college students admitted based on their EGE scores could not subsequently solve the same kinds of problems. Such an assertion was made, for example, by Sadovnichy, academician and rector of Moscow State University, who stated in 2009: "We have conducted two tests using materials from the EGE in two departments—the mechanics-mathematics departments and the computational mathematics department. About 40% passed the test; 60% failed" (Sadovnichy 2009). Goldina and Gil'derman (2010) tested students at a far less prestigious college (Moscow Automobile and Road Construction State Technical University) and also reached the conclusion that EGE scores reflecting the knowledge of first-year students had been greatly inflated. Not without irony, they reported that an elective course had to be offered in which first-year students were taught to complete the square and the like. There have been many such articles (see also, for example, Deminsky 2010).

The decline in requirements pertaining to the culture of reasoning has also been mentioned. Nesterenko (2009) noted that the increase in the number of problems brings with it a decrease in their difficulty and continued as follows:

> The sharp decline of requirements on the exams leads to a hollowing-out of the content of education in the schools. Teachers, contrary to their calling, are forced to drill schoolchildren to solve standard sample exercises, distributed in advance. There is no need to teach that which is not very important for obtaining a positive grade on the exam. (p. 68)

In the same article, however, he also noted that the examination as a whole is somewhat difficult—and he remarked that the difference in difficulty between problems of various groups has only limited relation to how they are scored.

It has also been said that the EGE produces crammers instead of creative human beings, that it pushes students away from an understanding of mathematics, and so on (for example, Smolin et al. 2009). Despite this criticism, however, the EGE remains in use.

Upon graduating from the basic school (nine grades), students must take the so-called State Final Assessment (Gosudarstvennaya Itogovaya Attestatsiya—GIA), which in its new form is also called the Basic State Exam (Osnovnoy Gosudarstbennyi Ekzamen—OGE). Without discussing the examination in detail, we should say that, for example, in 2019 it consists of two modules: "Algebra" and "Geometry." The examination contains 26 problems in all. The "Algebra" module contains 17 problems: 14 problems in part 1 and 3 problems in part 2. The "Geometry" module contains nine problems: six problems in part 1 and three problems in part 2. The problems include multiple-choice problems, short-answer problems, and problems that require a detailed solution. The following exercises from a sample examination that appears on the GIA official site (http://gia.edu.ru/ru/) convey an idea of the level and types of problems found on the examination:

Algebra, part 1

- The following table shows how ninth-graders are graded for running 30 m:

	Boys			Girls		
Grade	«5»	«4»	«3»	«5»	«4»	«3»
Time, seconds	4.6	4.9	5.3	5.0	5.5	5.9

- What grade will a girl receive if she runs this distance in 5.62 s?
  (1) grade "5" (2) grade "4" (3) grade "3" (4) a failing grade.
- There are buns on a plate. They are identical in appearance. Four have meat inside, eight have cabbage, and three have apples. Petya takes one of them at random. Find the probability that it will have apples inside.
- Solve the equation $x^2 + x - 12 = 0$. If the equation has more than one root, write the greater of the roots as the answer.

Part 2

- A fisherman sets out in a motorboat from a dock at 5 a.m., traveling upstream. After some time, he drops anchor, fishes for 2 h, and comes back at 10 a.m. on the same day. What distance from the dock does he travel, if the speed of the river current is 2 km/h, and the speed of the boat is 6 km/h?
- Construct the graph of the function $y = \dfrac{x^4 - 13x^2 + 36}{(x-2)(x-3)}$ and find the values c for which the line $y = c$ has exactly one point in common with the graph.

Geometry, part 1

- In an isosceles triangle ABC with base $\overline{AC}$, the exterior angle at vertex C is 123°. Find the angle BAC. Give the answer in degrees.
- Which of the following assertions is true?

  1. Through a point external to a given straight line, a straight line can be drawn parallel to the given line.
  2. A triangle with sides 1, 2, 4 exists.
  3. Any parallelogram has two equal angles.

In your answer, write down the numbers of the assertions you have chosen without spaces, commas, or other symbols.

Part 2

- In a parallelogram ABCD, point E is the midpoint of side $\overline{AB}$. It is known that EC = ED. Prove that the given parallelogram is a rectangle.

As can be seen even from these examples, the gap between the first and the second parts is very substantial. At the same time, the overall knowledge required of the students is not very great (in the past, far more was covered in the Soviet 9-year (8-year) school, including, for example, trigonometry). A certain amount of space is allocated for what may be called real-world problems.

10 Specialization

An analysis of the examination problems reproduced above shows that some students are expected to learn only the most basic facts (the first part is for them), while others are expected receive a relatively full education in mathematics—something like what all students were claimed to have received in the past—and often even something at a much higher level (the last problems in section C or the last of the algebraic problems from the OGE reproduced above would have been considered very difficult even in Soviet times, of course). Connected with these expectations is a key idea that has become widespread in recent decades: the division of education into a basic and a specialized level.

As Lukicheva and Mushtavinskaya (2005) explain:

> Specialization in education is a means of differentiating and individualizing education by changing the structure, content, and organization of the educational process, in which students' interests, inclinations, and abilities are more fully taken into account. (p. 6)

Technically, in the opinion of the same authors, specialized education can include, for example, education in physics-mathematics, physics-chemistry, biology-geography, social sciences-humanities, industry-technology, art-aesthetics, defense-athletics, and so on, and so forth (pp. 10–11). At the same time, one can distinguish between basic courses in mathematics (in general education classes for 4 h per week, and in specialized classes for 6 h per week) and the elective courses that actually make the class a specialized one. Among the possible elective courses in mathematics, the authors list the following: “Mathematical Models in the Natural Sciences,” “Methods of Mathematical Modeling in the Humanities,” “Theory and Practice of Consumer Behavior,” “Ecology in Numbers,” “Applications of Trigonometry,” “Functions. Graphs,” “Remarkable Theorems and Facts of Algebra,” “Optimization Problems,” “Problems with Parameters,” “Absolute Value,” and many others (pp. 23–27). In other words, elective courses can vary greatly: they can be devoted to further study of the questions of school mathematics; they can present certain sections of mathematics non-usually studied in school; or they can be devoted to certain general questions or to the applications of mathematics in one or another specific field.

The book cited above represents only one possible view of specialized education. The difference between the teaching of basic courses in schools and classes with different specializations, for example, has also been discussed. As Bashmakov (2010) writes:

> The term specialization is typically interpreted as follows. Any high-school student must select one specialized trajectory of education. Though there is a list of primary specializations, the number of specializations may be infinite. Any subject may be studied at two levels: basic and specialized (advanced); i.e. specialization has become equated with the level of studies. To make a specialized school (class), it is sufficient to decide which of the main subjects will be studied at the basic level and which at the advanced level. (pp. 157–158)

And he goes on:

> This approach still maintains the idea that mathematical knowledge and skills should be arranged along a straight line and this line may be chopped up as necessary. Anything lying outside the straight line will be billed as “elective courses” and used in the manner of an optional condiment. (p. 158)

As far as can be judged, universal specialization has not yet arrived: a system in which students must, in effect, choose a profession at the end of ninth grade cannot be inculcated very easily, let alone a system of elective courses that assumes that there are people in schools who are capable of teaching such courses. Bashmakov’s remark seems justified—it is easy to pour out more tea for some, less for others, stronger for some, more watery for others, but it is far more difficult to do so with mathematics.

What is important, however, is that the idea of specialization, which had once carved a path for itself with difficulty for a very small group of students especially interested in mathematics, now, on the contrary, is offered to all students, including those who are completely unprepared for such a choice. What is really at stake here is determining early on those students who will not receive a serious course in mathematics. In this way, individual wishes—first and foremost, the wish to study a little less mathematics—are indeed satisfied to a certain degree (at least, they may be considered satisfied), but how enduring such wishes are, and what should be done if they change, is not entirely clear (these very questions, as we have said above, were also being discussed decades ago).

11 Gifted Students

Identifying and supporting the gifted has come to be considered an objective of paramount importance. President Putin personally meets with gifted students at the specially formed Sirius Center; indeed, the government site kremlin.ru explains that:

> The Sirius Educational Center was formed on the basis of olympic infrastructure on the initiative of the head of state in December 2014. The goal of the center's work is the early identification, development, and further professional support of gifted children from all regions of Russia, who have manifested outstanding abilities in the spheres of the arts, sports, natural scientific disciplines, and also those who have achieved success with technological creativity.

And it continues:

> Each month, 600 children of ages 10–17, who are accompanied by over 100 teachers and coaches, receive free education at the center. The educational program, which lasts 24 days, includes both specialization-oriented classes and developmental recreation, master classes, creative talks with professionals recognized in their fields, a complex of health-promoting procedures, and during the school year—general education classes. (news from July 21, 2017)

Naturally, far from all of these children have manifested their giftedness specifically in the field of mathematics, but there are mathematically gifted children among them also. No expense is spared for these programs. The president has long talks with Sirius students, which are ubiquitously reported on in the press. Not only that, but it is reported that the very name "Sirius" was chosen by the president himself, in honor of the brightest star.

The times when specialized mathematics schools were almost officially referred to as a cancerous growth on the healthy body of Soviet education (Karp 2011) have gone. The government of Moscow, for example, now yearly gives out grants to the best schools, a large number of which specialize in mathematics (including specialized schools affiliated with major colleges). Nor is this surprising, since the metrics on which the selection is based include the results of Olympiads, the EGE, and the OGE (see the official site of the mayor of Moscow: www.mos.ru).[5]

The position of schools specializing in mathematics, however, is not all that sunny: all of the problems mentioned above remain; to some extent, the concept of the specialized mathematics school has been blurred; many specialized schools exist, and the differences between them are not always clear to everyone; the ties to the mathematics community, although they remain relatively strong in some places, nonetheless are clearly weaker than during the most flourishing years; the orientation of schoolchildren has changed, and a future as a mathematician is often no longer seen as anything enviable and desirable; and finally and most importantly, the weakening of mathematics instruction in the classes that precede specialized education is taking its toll, as we will see below (Karp 2011).

[5] Nor should we forget that the winners of Olympiads enjoy special privileges when enrolling in college: a successful Olympiad performance is taken to be the equivalent of top results on the EGE.

We should also note the appearance of a large number of various new forms of extracurricular work in mathematics, including mathematical competitions, and consequently the growth of the corresponding literature (Marushina 2016): this process began already during the preceding period, in the 1990s, but it continues successfully to this day. We might recall such books as Bratus' et al. (2003), Yashchenko (2005), Blinkov et al. (2007), Chulkov (2009), and Tikhonov and Sharich (2012). It is also important to point out the noticeable growth in the number of publications for children of elementary-school and middle-school age who are interested in mathematics (for example, Katz 2013, Kozlova 2008).

Special organizations and institutions for facilitating the development of mathematics education have appeared—the first of these that must be mentioned is the Moscow Center for Continuous Mathematics Education, MTsNMO (Sossinsky 2010). Initially organized as a kind of front to provide legal cover for the Independent University of Moscow, a scientific and academic mathematical center, MTsNMO has become a leading and influential institution working with hundreds and thousands of schoolchildren, supported and recognized by the Moscow and Federal governments, possessing its own publishing house, and exerting a noticeable influence on mathematics education across the country.

Another important development has been that, along with traditional competitions and problems, more attention in working with schoolchildren has started being devoted to problems that require prolonged thinking—research problems. Sgibnev (2013), for example, is a book devoted to such problems. As an example of a research problem, let us cite a question discussed in this book: how many ways are there of representing a fraction as a sum of fractions whose numerators are 1 (this question may be said to go back to ancient Egypt). This problem—with restrictions and in particular situations—was worked on by a sixth grader over quite a long period of time and with a certain degree of success. Such research activity is certainly no less useful than traditional "sports-like" Olympiad mathematics.

Successes in this "sports-like" mathematics, however, have started declining, despite the care and attention it receives on all sides. From 2010 until 2017, at the top International Mathematical Olympiad, Russia proved incapable of placing among the three prizewinners; indeed, in 2017, the country's national team took only the 11th place; and twice before that, the eighth. Of course, in each competition, a team's results reflect not only its efforts but also the efforts of the other participants; specialized preparation, based in no small measure on Russian methodology, is now widespread in other countries as well. Nonetheless, the yearslong lack of success also tells of existing problems. In 2018, Russia was able to return to one of the top three spots, being beaten only by the national team of the United States. This was preceded by consultations at the highest educational level—with the minister of education and the presidential assistant for education (Kak Rossiya vernulas'... 2018)—which reveals what kind of importance was assigned to the success and failure of the national team.

In the words of MTsNMO head, Ivan Yashchenko (Kak Rossiya vernulas'... 2018), a fundamental role was played by the precise and formalized selection process, whose rules were published in advance and required good performances on five Olympiads, including training Olympiads, the Russian National Olympiad, and the Romanian Master of Mathematics:

> This is also very important, because everyone understood that the system would be absolutely transparent. Both the children and the regional teachers knew the formula based on which the team would be selected. This was not a situation in which one had to "become friendly" with the coach in some way. There was no subjective element.

These changes were accompanied by changes in the way in which the preparation of the Olympiad was organized: "We sat down and agreed that our goal was to raise the Russian national team to a higher level, brushing aside the ambitions of particular regions, the ambitions of 'personal students,'" Yashchenko writes. "After all, if the best teacher starts preparing everyone, it may well turn out that his pupils will not make it onto the national team, while a stronger schoolchild from a different region, on the contrary, will make it, and precisely thanks to the higher-quality preparation, which this child would not have been able to obtain at home."

Kirill Sukhov, a teacher at the Presidential Physics and Mathematics Lyceum No. 239, and a representative of the St. Petersburg (Leningrad) school, was chosen as head coach for the team, and this choice paid off. Whether the existing problems, which may be surmised from Yashchenko's explanations, have been lastingly put to rest, and whether these were the only problems to blame, the future will show.[6]

It has been suggested that the causes of the difficulties lie deeper and cannot be reduced to various administrative and organizational oversights. Sergey Rukshin, who has spent his whole life working on Olympiads and preparing students for them, and who is the head of the office affiliated with school No. 239 from which the top coach for the national team had to be recruited, explains in a large interview (Rukshin 2018) that schools are not meeting the goal of preparing students in a way that would make it possible subsequently to identify them for mathematical work. Noting that the math circles of the Soviet period included few children from working-class families, he says that this was not due to a lack of ability on the part of such children:

> The families of such children simply didn't have a cultural tradition aimed at identifying a child's abilities in different fields. This was precisely the task of general education schools. We may not be able to teach everyone equally well. We don't need a cashier in a supermarket who knows how to take the integral. We have no need whatsoever for a worker in a supermarket, who arranges sunflower oil on a shelf, who remembers biology well…. But in order to identify who is capable of what, we need, at least at the initial stages, to teach everyone using the same curricula, to give them equal opportunities, and to select an elite from among people who had more or less equal starting positions….

Thus, even to him, a person who has spent his whole life working on Olympiads, the selection of this elite appears to be no simple matter: "Please show me the little gadget that, if you point it at a person, will show his giftedness or far-off successes," he exclaims. Further, he comes to the conclusion that conditions everywhere must therefore be identical, which means also identical textbooks and curricula; he

[6] In 2019, Russia took sixth place in the International Olympiad in Mathematics.

remarks that problems with education lead to problems with national security, and so on, employing a rhetoric that is quite consonant with official rhetoric. What is important, however, is that the actual official policy, for all its official concern with the gifted and with the elite, seems to him dangerous precisely for the education of the gifted—no Olympiads and no math circles will be able to help, if children have had no initial acculturation to mathematics and if no motivation to study mathematics has been instilled in them. It will not be possible either properly to designate certain children as gifted or even to select some deliberately narrow group among them. Ineluctably, very many capable individuals will be overlooked.

12 Mathematics Teacher Preparation

The Soviet system of teacher preparation included, first and foremost, education at a pedagogical college (4 or 5 years). By contrast with the usual practice in many other countries, in the USSR, the decision to prepare to become a teacher was made upon entering college; thus, all students in the mathematics department of a pedagogical institute prepared to become teachers of mathematics, and, say, courses in calculus were offered not to everyone wishing to study this subject in general but specifically to future teachers. The course of study included the study of mathematical disciplines—mathematical analysis, geometry, algebra, number theory, probability theory, and others; disciplines of the ideological type—the history of the Communist Party of the Soviet Union, philosophy, political economy, scientific atheism, and scientific communism; disciplines of the pedagogical-methodological type—general didactics and general psychology, age-specific psychology and pedagogy, methodology of mathematics instruction (of the *general* methodology, which covered general principles of instruction, and of a *specific* methodology, which covered the teaching of the basic topics of the school course; the course of methodology also included a course in school-level problem solving); and finally, additional disciplines—foreign languages, physical education, and the like. In addition, students conducted student teaching in schools (general information about mathematics teacher preparation and certain bibliographic references may be found in Stefanova 2010).

It should be noted that the courses in mathematics were quite broad, for example, the course in geometry using the textbook by Bakel'man (1967) included the axiomatic construction of Euclidean and non-Euclidean geometries, affine and projective transformations, differential geometry of curves and surfaces, and elementary topology. The course in geometry was taught over several semesters, and hundreds of hours were allocated to it, which included both lecture hours and practical problem-solving sessions. The course in mathematical analysis was very different from traditional calculus courses in the United States, if only because all theorems were strictly proved. In the courses on methodology, possible ways of structuring lessons were analyzed in detail, and possible mistakes that students could make and possible difficulties that they could encounter in each topic were examined—which could be done because the textbooks were stable, that is, they did not change for decades. The ideological courses, and frequently also the psychological-pedagogical ones, usually involved attending and studying lectures and analyzing various works (for example, by Lenin) in practical study sessions.

In general, the system was quite rigid, in the sense that it offered students very little free choice: the elective courses were very few in number—literally two or three over the whole time of study. It is natural to wonder to what degree the covered material was assimilated: this was tested by means of several tests in each semester-long course and usually by means of an oral examination in which students had to reproduce the proofs of randomly chosen theorems. It is no less difficult to say to what extent the study, say, of the topology of closed surfaces facilitated a deeper understanding of the standard school course, which graduates subsequently had to teach.

From the beginning of the 1990s, noticeable changes began to take place in the pedagogical colleges. First, they began to be renamed into universities—prior to this, a university (or as people now say, a classical university) was intended in its basic design to prepare research scientists (which does not mean, of course, that university graduates did not end up becoming school teachers). The universities were few in number; for example, Leningrad (St. Petersburg) had only one. Consequently, the renaming added prestige to the former institutes. Second, ideological courses disappeared—they were replaced (with certain organizational changes) by general courses in the humanities and social-economic courses, for example, the history of Russia (note that for quite a long time, these courses continued being taught by the same people who had worked in ideological or humanities departments previously). Third, in a number of pedagogical colleges, the numbers of students admitted to mathematics departments declined noticeably—other disciplines became more popular. Gradually, other changes also took place, which will be discussed below—briefly and without examining the entire process of change.

An important organizational step occurred when Russia joined the so-called Bologna Process, whose official goal consists in the harmonization of higher education across Europe and the formation of a common European educational space. Russia signed the requisite documents in 2003. Consequently, it became necessary to reconstruct teaching plans and to organize the teaching process in a new way, dividing the previously uniform preparation of specialist-teachers into the preparation of holders of bachelor's degrees and master's degrees.

These reforms were by no means always welcomed. Rukshin (n.d.), who has already been cited above, expressed himself in quite decisive terms, characterizing the Bologna Process as one of the more odious developments:

> The so-called "Bologna Process" and the transition to a two-tier system of "bachelors and masters" destroyed carefully designed and balanced teaching plans for the preparation of specialists and absolutely needlessly increased the duration of the courses of study at the universities.

There were discussions about the deep disparity between the system of bachelor's and master's degrees in the European countries and the United States, on the one hand, and in Russia, on the other. "In Russia, a first-year bachelor's student immediately begins acquiring knowledge in a narrow specialized field. In the physics faculty, he studies physics and higher mathematics, while in the philological faculty, he studies general linguistics and literary theory" (Vakhitov 2013). According to Vakhitov (2013), in this way, the idea of making bachelor's degree students far more free in choosing their own educational trajectory still fails to be realized, and what we have instead is simply a truncated version of the same specialized preparation that had existed before.

Already at the very beginning of the restructuring of higher education, Professor Testov (2005) of Vologda expressed doubts about its viability and worthwhileness, writing about the importance of a fundamental mathematics education and reasonable ways of combining it with professional education, in which Russia was alleged to have experience that the West was only beginning to acquire, and clearly perceiving a certain futility in the preparation of bachelor's degree holders, who, even after obtaining a fundamental education, would be completely unable to apply it in practice.

Ten years later, Dalinger (2015) from Omsk, offering a kind of overview of various (negative) views of the reforms, quoted from the proceedings of a conference on mathematics education that had taken place in 2007: "it can be confirmed that what the Bologna Process has brought to Russia so far has been mainly destruction; the illusions and unfounded hopes have been dispelled" (p. 397). However, he quotes V.P. Odinets:

> This is the fault not of the process itself, but of those persons who have overseen and are overseeing its implementation in Russia without thinking about the consequences or without understanding them. (p. 397)

The experience of the implementation of the new system (which, despite the objections, was implemented) is described in an article by Stefanova (2010). Bachelor's degree holders at the Herzen State Pedagogical University in St. Petersburg are prepared over 4 years, and upon completing which, those who have received a bachelor's degree acquire the right to work in basic schools (9-year school). In order to work in high schools (grades 10–11), they must obtain a master's degree. The course is divided into three stages: two 2-year stages for obtaining a bachelor's degree and one 2-year stage for obtaining a master's. Furthermore,

> The first stage may be characterized as the stage of general preparation. At this stage, students study subjects that represent all fields within a given specialization and select one field for subsequent study.
>
> The second stage is devoted to preparing students in the field which they have selected, and also to providing them with professional teacher preparation (in the case of future teachers of mathematics, this includes preparation for the basic school, i.e. grades 5–9).
>
> At the third stage, preparation in a specific field continues at a higher level, now with a certain degree of professional specialization. (Stefanova 2010, pp. 296–297)

Each of these stages includes the teaching of disciplines in the same, previously mentioned spheres—mathematical, pedagogical-methodological, and others. Federal state standards for higher professional education (FGOS 2019) require future teachers with bachelor's degrees to develop various competencies of a general cultural nature (for example, the holder of a bachelor's degree "possesses high-level reasoning ability, is capable of generalizing, analyzing, receiving information, setting an aim and choosing a means to achieve it"), of a general professional nature (for example, "is able to take responsibility for the results of his professional activity"), and of a professional pedagogical nature (for example, "is able to use the opportunities of the educational environment, including the information environment, to provide for a high-quality teaching-learning process").

Specialized knowledge is also not forgotten, of course, although the already cited Dalinger writes as follows (Dalinger 2017):

> One of the main criticisms of contemporary educational standards is the obvious disparity between the number of hours allocated to the study of a discipline, in this case, mathematics, and the amount of material necessary for the education of a future teacher of mathematics.

And to prove his point, he cites the following figures:

> In 1963, at the mathematics faculty of the Gorky Omsk State Pedagogical Institute, the teaching plan for the preparation of a specialist teacher of mathematics (with a course of study lasting four years) allocated 1000 h for the study of mathematical analysis and 192 for the study of additional chapters in mathematical analysis, while in 2016 the teaching plan for a bachelor's degree in the sphere of "Pedagogical Education," with the specialization "Mathematics Education" (with a course of study lasting five years) 540 h are allocated to the study of mathematical analysis (this is the workload,[7] of which 234 h are hours conducted at the college), and 108 h are allocated for additional chapters in mathematical analysis (this is the workload, of which 26 h are hours conducted at the college).

And he concludes:

> As experience shows, the sharp reduction in the number of hours for a bachelor's degree in the mathematical disciplines leads to students not developing either the celebrated subject knowledge, abilities, and skills, nor the competencies proclaimed by contemporary standards.

Without looking into how these figures were obtained, and without discussing how much they owe to the general standards (which merely require that the total number of lecture hours must not exceed 27 per week, without counting physical education classes) and how much to the specific college in question—which, to be sure, makes decisions under the pressure of existing circumstances—we will say that the widespread and not ungrounded plea for greater independence for the students can be naturally understood precisely as a plea for a reduction in the number of lecture hours and hours conducted at the college.

In keeping with the standards, programs of instruction and teaching plans are developed by each college separately (and subsequently approved by the ministry), for which reason a general analysis of the content of courses in mathematics and mathematics education in all pedagogical universities is hardly feasible. We should merely mention the attempts by Alexey Semenov, rector of Moscow Pedagogical University, to reduce specialized preparation in mathematics—virtually minimizing it, as critics claimed, to what is directly connected with

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[7] As far as we can understand, the author means that the first number ("workload") is the total number of hours allocated for the study of the course, including independent work.

schools—while increasing the time for pedagogical practical training and psychological-pedagogical courses. These attempts were discussed in the press (for example, Privalov 2016), and Semenov himself (Semenov 2016) expressed his views as follows:

> Preparation in college must touch on the content of the school program; our graduates must get A-pluses in what they will teach in schools. Traditionally, pedagogical colleges are seen as inferior versions of ordinary universities—professors take ordinary courses, simplify them, and teach them to students, believing that the students will become smarter as a result and will begin to understand the school program. That's not so simple—I know that a large number of courses in mathematics have no relation to the school program, and as a result, graduates can solve school problems only at a grade C-level—because their professors have not taught them.

However, no discussion ensued. Semenov was fired from the post of rector, and it is not entirely clear what the real reason for this was. Questions about how teaching at a pedagogical college should be structured, in light of the new situation, undoubtedly remain.

In conclusion, we must also say a word about the system of professional development for mathematics teachers. In the USSR, it was usually implemented at special centers, for example, in Leningrad (St. Petersburg), such a center was first called the Institute for the Improvement of Teachers, then the University of Pedagogical Mastery, and now it is called the Academy of Postgraduate Pedagogical Education, which reflects not only a love for pretty names but also certain changes in the institution's activity. In the past, teachers regularly took courses here (usually, once every 5 years) in which difficult topics from school mathematics were analyzed, problems were solved, and so on, In addition, there were specialized courses, devoted, for example, to teaching certain curricula, or using certain textbooks, or teaching certain age groups (Karp 2004).

As far as can be judged, and without claiming to know everything taking place in all regions of the country, the system continues to be preserved to a certain degree, and among the aspects preserved are the courses offered every 5 years. At the same time, the overall number of hours of classes accessible to teachers has diminished somewhat, and such classes are also becoming less centralized and can be organized in different places and on different foundations (while remaining in most cases free for teachers).

13 Discussion and Conclusion

This chapter does not claim and cannot claim to give a complete description of what has happened in mathematics education in Russia over the last 30 years. We have no information about many regions of the country, about the real experience of change in each of them, about how the change has been perceived by all participants in the process, and much else, and moreover it appears that such information is not only not in this author's possession but to a large extent not in anyone else's possession either—it has not been collected.

And yet from the information we do possess, it is clear that the process has been a very contradictory one and that hoping to conclude its description with a flat verdict—that a catastrophe has occurred or, on the contrary, that conditions have been created for the harmonious development of every personality—is altogether impossible. It should not be forgotten that the development of mathematics instruction in the whole world has also gone through a certain crisis (Karp 2017), and probably one of the inevitably surprising aspects of what has happened in Russia is that the development of computer technologies, which has exerted an enormous influence on the style and problems of mathematics instruction in the West, has clearly had less of an impact on Russian education.

This does not mean that in Russia no thought has been given to using computers and that computers have not been used in practice. We can, without hesitating, name numerous books and articles about using computers and computer programs in mathematics classes (Dubrovsky and Bulychev 2017; Ovsyyannikova 2017), and even in what was discussed above, there are examples in which computers were used in one way or another—for instance, the research problems (Sgibnev 2013) that we have mentioned were, of course, solved with their help. Nonetheless, one can state affirmatively that the role and place of computer technologies in the teaching of mathematics have differed from what it is in many other countries.[8] Although the Russian school course in mathematics is changing, it continues to retain very many old features—this seems wonderful when it concerns such features as the orientation toward proofs and substantiation, but it raises doubts when it comes to certain parts of the content of the course. Ryzhik (2013) writes that requirements for college entrance examinations have led to:

> the appearance of a “mathematics for prospective college students,” or more precisely a “pseudo-mathematics,” which has been responsible for the appearance of specific topics in the school course. Its adoption was facilitated through the use of methodological literature of fleeting significance. Examples: absolute value problems; problems on the position of the roots of a quadratic trinomial; problems with parameters—in frightening quantity and frightening in quality; the height of absurdity—logarithmic equations (inequalities) with an unknown base, and in the form of a trigonometric function to boot. The deformation of the school course in mathematics, which came about inevitably from having an eye on college, has led to the disappearance of other useful sections of the course (for example, straightedge and compass construction problems have practically disappeared). (p. 7)

It is not difficult to notice that all this had already happened long before 1991, or that some of the listed types of problems (even listed above as the absurdities) might not be useless in certain circumstances, or that straightedge and compass construction problems also cannot be considered a novelty and they have disappeared from the programs of most countries in the world, even if for different reasons. Nonetheless, it is impossible not to agree that the Russian course—at least, the

[8] As an anecdote, we might recall that Minister of Education Dmitry Livanov once demanded that each textbook in the country have an electronic supplement (Ministerstvo 2014), which in practice led to the development of rather simplistic problem sets with a selection of answers in the style of the 1960s.

course oriented toward the stronger students—is overloaded with technical details and moreover that the new times have only brought several new types of problems, which have, however, entered the schools in a far more aggressive fashion than their predecessors, since their source, the EGE, plays a far greater role than separate college exams.

The discarding of old problems is seen by many, and possibly not without reason, as a betrayal of the classical traditions of thinking about difficult problems—and indeed, artificial and cumbersome logarithmic equations, with trigonometry thrown in as well, are at a minimum no less useful than ten one-step exercises of the same type, in which students must, say, calculate the base 2 logarithm of 4 or 8, such as may be easily found on school tests in certain Western countries. As for problems that are substantive without being cumbersome—problems that seem natural from today's point of view—not very many of those have appeared in recent decades in the textbooks.

One innovation in mathematics education over recent decades has been an increased attention in the upper grades to problems with real-world content; as has been shown above, such problems are now represented both on the EGE and the OGE and hence are also discussed in classes and in textbooks. These problems, however, are usually very simple (while the mathematics problems that actually arise in real life are usually very difficult—so difficult that it is usually impossible to give them in full versions in school). The result has been a kind of division: it is expected that weak students will solve these simple practical problems, while those who are stronger will be given classical Russian "theoretical" problems ("theoretical" in the sense that it is usually impossible or very difficult to invent a practical application for them, and they are given to students for their intellectual development). But here we come to the social problematics of the development of mathematics education.

As has already been repeatedly noted, it is difficult to talk about this topic, if only because of its politicized character. The changes that have taken place in the country—which have been called "democratization," even though they hardly merit such a name—nonetheless have undoubtedly given the country's citizens certain previously absent rights. Consequently, we can conceive of the present situation as one in which we are faced with the question: does democratization lead to the improvement of mathematics education or not? Consequently, criticism of the condition of mathematics education becomes part of the political discussion. It is no accident that in discussions of mathematics instruction, readers can often observe an unrestrained anti-Americanism and, more broadly, an anti-Western stance, often with the paradoxical conclusion that Americans are forcing American-style education onto Russia in order to enslave Russia more securely (although in that case, it would probably be more logical for them to restructure their own education along Russian lines at the same time).

In reality, everything is far more complicated. To begin with, the development of Soviet school mathematics itself does not fit into the anti-Western canon. It is not difficult to show that Russian prerevolutionary mathematics education, on whose traditions Soviet education was based after 1931, the year of the crackdown against revolutionary pedagogy (Karp 2010), developed under the strong influence of French and German mathematics education. Subsequently, Western education—for various reasons, which cannot be discussed in detail here—began to get restructured in one direction, with a decrease in attention to what was substantive-mathematical, proof-oriented, and generally "scientific" in schools, which became increasingly mass-scale; while Soviet education—in part under the pressure of competition with the West, in part due to the privileged position of the non-ideologized sciences of physics and mathematics, and in part by becoming hostage to its own ideological dogma of equality—extended the existing traditions practically to the level of universal education, which in turn led to methodological improvements.

Historians of education can point to other situations in which, as it were, the outskirts of the civilized world acquired knowledge from more advanced regions, only to surprise their former teacher-countries with this very knowledge decades later, when this knowledge had for various reasons waned in these latter countries (for example, we might recall how Charlemagne invited Alcuin from distant York; it is hardly possible to deduce from this English superiority over "Eastern"—meaning Italian—education.)

The artificial conditions created by the Soviet Union brought considerable successes to mathematics education. The same conditions prevented mathematics education from developing in a natural way, for example, by restricting the writing of new textbooks. When the Soviet Union collapsed and the situation began to change, mathematics education inevitably found itself worse off, if only because it ceased to be virtually the only kind of education that was truly unrestricted by the state. Many problems (for example, the same relatively archaic character of the course, or mass-scale grade inflation, or the leaks of copies of exams) existed in the USSR, but they by no means always became a subject for public discussion. Over recent decades, much has become more open, which cannot be equated with a worsening of education.

On the other hand, in the new situation, perhaps the main virtue of the Soviet system has been abandoned in a practically open manner: the orientation (even if it was often only a demagogic one) toward achieving equality of opportunities for all students, that is, toward providing all students with a relatively deep, proof-oriented course. In the new times, despite the discussions about democratization and democracy (which also took place during the Soviet period, however, even if with somewhat different overtones), the formation of an elite began to be openly talked about, with the further implication (not necessarily mentioned out loud) that membership in the elite was a hereditary quality, even if its doors were not closed to certain other potential members—"the especially gifted." The stratification observable at the very beginning of the period examined above became increasingly more noticeable, thereby also causing considerable damage to the selection of these very "gifted" children, as has already been discussed. In addition, the number of those sufficiently prepared to become successful teachers turns out to be insufficient (and the individuals who become teachers are usually by no means the most exceptional and talented from a mathematical point of view, as is quite understandable). This insufficient preparation of "ordinary" children is subsequently difficult to make up for in college classrooms.

There is no need to repeat that from an economic perspective, Russia has gone through a difficult period—naturally, when teachers, including mathematics teachers, went for months without receiving their salaries, as happened during certain years of the Yeltsin period, this could not but impact the state of education, including the prestige of the teaching profession, exerting a long-term influence on the development of teaching. But one should not attribute all problems in this field to these economic difficulties—in the “fat” Putin years, when high oil prices raised the standard of living, many problems in the teaching of mathematics remained.

At the same time, recent decades have also seen many successes and achievements. Education is based on traditions, and the traditions are still alive—there are still millions of people who cannot imagine textbooks almost without proofs and many thousands if not millions of people who assume that their children will attend math circles. Under abnormal Soviet conditions, people became teachers who probably would not have done so under other conditions, choosing other fields for the expression of their talents and knowledge. The coming of remarkable teachers into the schools helped to create special standards of teaching—there are still many people who had wonderful teachers or who became teachers under their influence.

As we have seen, in the Russian press (including the professional press), the ideal is usually conservative: let’s go back to what we have lost, that is, to the Soviet model (as though it were possible to go back to it now, even if the Soviet political system were reconstructed). Reformers are seen as those who reproduce in Russia a system from other countries, that is, reject that which was created previously. Whether it will be possible to carry out genuine reforms while preserving the unique achievements and traditions of Russian mathematics education—and while enabling its free transformation and development, that is, its existence under the conditions of a genuinely democratic society—only the future will tell.

References

Abramov, Alexander M. 2009. Tri mifa kotorye my poteryali [Three Myths That We Have Lost]. *Nezavisimaya gazeta* (September 1).

———. 2010. Toward a history of mathematics education reform in Soviet schools (1960s–1980s). In *Russian mathematics education. History and world significance*, ed. Alexander Karp and Bruce Vogeli, 87–140. Hackensack, NJ: World Scientific.

Atanasyan, Levon S., Valentin F. Butuzov, Sergey B. Kadomtsev, Sergey A. Shestakov, and Irina I. Yudina. 1996. *Geometriya. Dopolnitel’nye glavy k shkol’nomu uchebniku 8 klassa* [Geometry. Supplementary chapters to the school textbook for eighth grade]. Moscow: Prosveschenie.

Bakel’man, Ilya Ya. 1967. *Vysshaya geometriya* [Advanced geometry]. Moscow: Prosveschenie.

Bashmakov, Mark I. 2010. Challenges and issues in post-Soviet mathematics education. In *Russian mathematics education. History and world significance*, ed. Alexander Karp and Bruce Vogeli, 141–186. Hackensack, NJ: World Scientific.

———. 2004. *Matematika. Uchebnoe posobie dlya 10–11 klassov gumanitarnogo profilya* [Mathematics. Textbook for grades 10–11 with a humanities profile]. Moscow: Prosveschenie.

Blinkov, A.D., E.S. Gorskaya, and V.M. Gurovits. 2007. *Moskovskie matematicheskie regaty* [Moscow *Mathematics regattas*]. Moscow: MTsNMO.

Bogomolova, E.P. 2014. Diagnoz: Matematicheski malogramotnyi [Diagnosis: Mathematically illiterate]. *Matematika v shkole* 4: 3–9.

Bratus', Tatiana A., Natalya A. Zharkovskaya, Alexander I. Plotkin, Tatiana E. Savelova, and Elena A. Riss, 2003. *Kenguru-2003. Zadachi, resheniya, itogi* [Kangoroo-2003. Problems, solutions, results]. St. Petersburg: IPO.

Bunimovich, Evgeny A. 2011. Combinatorics, probability and statistics in the Russian school curriculum. In *Russian mathematics education: Programs and practices*, ed. Alexander Karp and Bruce Vogeli, 231–264. Hackensack, NJ: World Scientific.

Butuzov, Valentin F., Yury M. Kolyagin, Gennady L. Lukankin, Eduard G. Poznyak, Yury V. Sidorov, Mariya V. Tkacheva, Nadezhda E. Fedorova, and Mikhail I. Shabunin. 1995. *Matematika. Uchebnoe posobie dlya uchaschikhsya 10 klassov obscheobrazovatel'nykh uchrezhdenii* [Mathematics. Textbook for tenth-grade students of general education institutions]. Moscow: Prosveschenie.

Chudovsky, Alexander N., Lidia A. Somova. 1995. *Sbornik zadanii dlya provedeniya pis'mennogo ekzamena po matematike v 9 klasse obscheobrazovatel'nykh ucherezhdenii* [Problem book for written exams in mathematics for grade 9 in general education schools]. Moscow: Mnemozina.

Chudovsky, Alexander N., Lidia A. Somova, and Vladimir I. Zhokhov 1986. *Kak gotovit'sya k pis'mennomu ekzamenu po matematike* [How to prepare for the written exam in mathematics]. Moscow: Prosveschenie.

Chulkov, Pavel V. Ed. 2009. *Vesennii turnir Archimeda* [Spring tournament of Archimedes]. Moscow: MTsNMO.

Committee on Education of St. Petersburg. 1994. *Sbornik normativnykh i metodicheskikh materialov* [Collection of Normative and Methodological Materials]. St. Petersburg.

Dadayan, A.A., E.P. Kuznetsova, R.I. Isaeva, A.V. Ivanel, A.A. Mazanik, and A.A. Stolyar. 1988. Oshibki na ekrane [Mistakes on the TV screen]. *Matematika v shkole* 4: 43–46.

Dalinger, Viktor A. 2015. Bolonsky protsess i rossiiskoe matematicheskoe obrazovanie v pedagogicheskikh VUZakh [The Bologna process and Russian mathematics education in pedagogical colleges]. *International. Journal of Experimental Education* 2: 396–400.

———. 2017. Podgotovka uchiteley matematiki v usloviyakh novykh gosudarstvennykh standartov po napravleniiu "Pedagogicheskoe obrazovanie", profil' "Matematicheskoe obrazovanie" [Mathematics teacher preparation under new state standards in the sphere of "pedagogical education," specialization: "Mathemetics education"]. *Sovremennye problem nauki i obrazovaniya*: 1.

Deminsky, V.A. 2010. EGE-2009 i uroven' matematicheskoy podgotovki studentov-pervokursnikov [The EGE-2009 and the level of mathematical preparation of first-year students]. *Matematika v shkole* 2: 62–65.

Dneprov, Eduard D.1998. *Sovremennaya shkol'naya reforma v Rossii* [Contemporary school reforms in Russia]. Moscow: Nauka.

———.1999. *Tri istochnika i tri sostavnye chasti nyneshnego shkol'nogo krisiza* [Three sources and three components of today's school crisis]. Moscow: Yabloko.

Dorofeev, Georgy V, Georgy K. Muravin, Elena A. Sedova. 1999. *Matematika. Sbornik zadanii dlya podgotovki i provedeniya pis'mennogo ekzamena za kurs sredney shkoly* [Mathematics. Problem book for preparing and conducting a written exam for the high school course]. Moscow: Drofa.

Dubov, E.L. 1994. Nuzhen ne standard, a urovnevaya differentsiatsiya [What we need is not a standard, but level differentiation]. *Matematika v shkole* 2: 12.

Dubrovsky, Vladimir N., and Vladimir A. Bulychev. 2017. MathKit and math practicum. In *Current issues in mathematics education*, ed. Alexander Karp, 13–27. Bedford, MA: COMAP.

Eidel'man, Tatiana N. 2007. God realizovannykh utopii: Shkoly, uchitelya i reformatory obrazovaniya v Rossii 1990 goda [The year of realized utopias: Schools, teachers, and education reformers in Russia in 1990]. *Novoe literaturnoe obozrenie* 1 (83): 350–367.

Firsov, Viktor V. 1998. K kontseptsii proekta standarta [Toward a conception of a blueprint for a standard]. *Matematika v shkole* 3: 2–9.

FGOS 2018. *(Federal'nyi gosudarstvennyi obrazovatel'nyi standard) osnovnogo obschego obrazovaniya. Izdanie 7* [Federal State educational standard for basic general education. Edition 7]. Moscow: Prosveschenie.

______. 2019. *(Federal'nyi gosudarstvennyi obrazovatel'nyi standard) vyshego professional'nogo obrazovaniya po napravleniu obrazovanie i pedagogika* [Federal State educational standard for higher professional education in Education and Pedagogy]. http://fgosvo.ru/fgosvpo/7/6/1/5. Accessed 16 Jan 2019.

Galitsky, Mikhail L., Alexander M. Goldman, and Leonid I. Zvavich. 1992. *Sbornik zadach po algebre dlya 8–9 klassov. Uchebnoe posobie dlya uchaschikhsya shkol i klassov s uglublennym izucheniem matematiki* [Collection of problems in algebra for grades 8–9. Textbook for students of schools and classes with an advanced course in mathematics]. Moscow: Prosveschenie.

Gladky, Alexey V. 1994. Kakim ne dolzhen byt' standard [What a standard must not be like]. *Matematika v shkole* 2: 4–7.

Gnedenko, Boris V., and Dmitry B. Gnedenko. 1994. Standart obrazovaniya – Vzglyad v buduschee [A standard for education – A view of the future]. *Matematika v shkole* 3: 2–3.

Goldina, V.N., and S.A. Gil'derman. 2010. Sravnenie itogov testirovaniya pervokursnikov Moskovskogo avtomobil'no-dorozhnogo instituta s rezul'tatami EGE [Comparison between test results and EGE results of first-year students at the Moscow automobile and road construction state technical university]. *Matematika v shkole* 2: 38–42.

Institut obscheobrazovatel'noy shkoly. 1993. Standard srednego matematicheskogo obrazovaniya [Standard for secondary education in mathematics]. *Matematika v shkole* 4: 10–23.

Kaiumov, Oleg R. 2014. O problemakh, svyazannykh s mezhtsivilizatsionnymi zaimstvovaniyami v pedagogike [On problems associated with Intercivilizational borrowings in pedagogy]. *Vestnik Eletskogo gosudarstvennogo universiteta im. I.A.Bunina* 34: 7–12. Elets: EGU im. I.A.Bunina.

Kak Rossiya vernulas' v lidery Mezhdunarodnoy matematicheskoy olimpiady [How Russia Returned to Being a Leader in the International Mathematical Olympiad]. 2018. https://indicator.ru/article/2018/07/17/rossiya-mezhdunarodnaya-matematicheskaya-olimpiada-imo-2018/. Accessed 16 Jan 2019.

Karp, Alexander. 1993. *Sbornik zadach dlia 8–9 klassov s uglublennym isucheniem matematiki* [Mathematics problems for 8–9 grades of the schools with an advanced study of mathematics]. St. Petersburg: Obrazovanie.

———. 2002. Math problems in blocks: How to write them and why. *Primus* 12 (4): 289–304.

———. 2003. Mathematics examinations: Russian experiments. *Mathematics Teacher* 96 (5): 336–342.

———. 2004. Conducting research and solving problems: The Russian experience of inservice training. In *The work of mathematics teacher educators: Exchanging ideas for effective practice*, ed. T. Watanabe and D. Thompson, 35–48. Association of Mathematics Teacher Educators (AMTE).

———. 2007a. Exams in algebra in Russia: Toward a history of high-stakes testing. *International Journal for the History of Mathematics Education* 2 (1): 39–57.

———. 2007b. The Cold War in the Soviet school: A case study of mathematics. *European Education* 38 (4): 23–43.

———. 2010. Reforms and counter-reforms: Schools between 1917 and the 1950s. In *Russian mathematics education. History and world significance*, ed. Alexander Karp and Bruce Vogeli, 43–85. Hackensack, NJ: World Scientific.

———. 2011. Schools with an advanced course in mathematics and schools with an advanced course in the humanities. In *Russian mathematics education: Programs and practices*, ed. Alexander Karp and Bruce Vogeli, 265–318. Hackensack, NJ: World Scientific.

———. 2017. Reflecting on the current issues in mathematics education. In *Current issues in mathematics education*, ed. Alexander Karp, 1–12. Bedford, MA: COMAP.

Karp, Alexander and Vladimir Nekrasov. 2000. *Matematika v Peterburgskoy schkole* [Mathematics in St. Petersburg school]. Resource guide. St. Petersburg: Spetsial'naya literatura.

Karp, Alexander, and Bruce Vogeli, eds. 2011. *Russian mathematics education: Programs and practices*. Hackensack, NJ: World Scientific.

Karp, Alexander and Alexey Werner, 2000. *Matematika 10* [Mathematics for grade 10]: Textbook, Moscow: Prosveschenie.

———, 2001. *Matematika 11* [Mathematics for grade 11]: Textbook, Moscow: Prosveschenie.

Karp, Alexander, and Alexey Werner. 2011. On the teaching of geometry in Russia. In *Russian mathematics education: Programs and practices*, ed. Alexander Karp and Bruce Vogeli, 81–128. Hackensack, NJ: World Scientific.

Katz, Zhenya. 2013. *Pirog s matematikoy* [Pie with mathematics]. Moscow: MTsNMO.

Kozlova, Elena G. 2008. *Skazki i podskazki. Zadamiya dlya matematicheskogo kruzhka* [Tales and Hints. Tasks for the mathematical circle]. Moscow: MTsNMO.

Kuz'minov, Yaroslav. 2009. *Rektor VShE: Blagodarya EGE snizilas' korruptsiya v VUZakh* [Higher School of Economics Rector: Thanks to the EGE, Corruption in Colleges Has Declined]. https://www.yuga.ru/news/175140/. Accessed 16 Jan 2019.

———. 2012. *My khuzhe kitaitsev?* [Are We Worse Than the Chinese?]. *Argumenty i fakty* (July 4). https://www.hse.ru/news/1163611/55983879.html. Accessed 16 Jan 2019.

Kuznetsova, Galina M. 1998. *Matematika. 5–11 klassy. Sbornik normativnykh dokumentov* [Mathematics. Grades 5–11. Collection of normative documents]. Moscow: Drofa.

Kuznetsova, Liudmila V., Evgeny Bunimovich, Boris P. Pigarev, and Svetlana B. Suvorova, 2002. *Algebra. Sbornik zadanii dlya provedeniya pis'mennogo ekzamena po algebre za kurs osnovnoy shkoly. 9 klass* [Algebra. Problem book for conducting a written exam in algebra for the basic school course. Grade 9]. Moscow: Drofa.

Kuznetsova, Liudmila, Elena Sedova, Svetlana Suvorova, and Saule Troitskaya. 2011. On algebra education in Russian schools. In *Russian mathematics education: Programs and practices*, ed. Alexander Karp and Bruce Vogeli, 129–190. Hackensack, NJ: World Scientific.

Lednev, Vadim S., Nikolay D. Nikandrov, and Mariya N. Lazutova. 1998. *Uchebnye standarty shkol Rossii. Kniga 2. Matematika. Estestvenno-nauchnye distsipliny* [Leaning standard in Russian schools. Volume 2. Mathematics. Natural sciences]. Moscow: Sfera. Prometey.

Lukicheva, Elena Yu. and Irina V. Mushtavinskaya 2005. *Matematika v profil'noy shkole* [Mathematics in specialized schools]. St. Petersburg: Prosveschenie.

Malyshev, I.G. 2016. Matematicheskoe obrazovanie pod kolesnitsey FGOS [Mathematics education under the chariot of the FGOS]. *Matematika v shkole* 7: 3–6.

Marushina, Albina A. 2016, *Mathematical extracurricular activities in Russia. Unpublished doctoral dissertation.* Columbia University.

Materialy discussii ob itogakh i perspektivakh EGE [Discussion materials about the outcomes and prospects of the EGE]. 2004. *Voprosy obrazovaniya* 3: 273–287.

Ministerstvo obrazovaniya. 2014. Do 1 yanvaria sleduiuschego goda u kazhdogo uchebnika dolzhna poyavit'sia elektronnay versiya [Every Textbook Must Have an Electronic Version Before January 1 of Next Year]. https://xn%2D%2D80abucjiibhv9a.xn%2D%2Dp1ai/%D0%BD%D0%BE%D0%B2%D0%BE%D1%81%D1%82%D0%B8/4298. Accessed 19 Jan 2019.

MP (Ministerstvo Prosvescheniya) RSFSR 1985. *Sbornik zadanii dlya provedeniya pis'mennogo ekzamena po matematike v vos'mykh klassakh obscheobrazovatel'nykh shkol RSFSR* [Problem book for conducting a written exam in mathematics for grade 8 in general education schools of the RSFSR]. Moscow: Prosveschenie.

Nekrasov, Vladimir B., Dmitry D. Guschin, and Leonid A. Zhigulev. 2007. *Matematika. Uchebno-spravochnoe posobie (Gotovimsya k EGE)* [Mathematics. Learning-reference manual. (Let's prepare for the EGE)]. St. Petersburg: Prosveschenie.

Nesterenko, Yuri V. 2009. Nekotorye zamechaniya v sviazi s demonstratsionnoy versiey zadaniya dlya EGE v 2010 godu [Some remarks in connection with the sample copy of the EGE assignment for 2010]. *Matematika v shkole* 10: 67–76.

Novikov, Sergey P. 2018. Proizoshel raspad obyazatel'nogo znaniya [The decay of mandatory knowledge]. *Matematika v shkole* 1: 65–71.

Ot Glavnogo uchebno-metodicheskogo upravleniya obschego srednego obrazovaniya Gosobrazovaniya SSSR. 1991. O trebovaniyakh k matematicheskoy podgotovke uchaschikhsya srednikh shkol [On the requirements for the mathematical preparation of secondary school students]. *Matematika v shkole* 3: 3–4.

Ovsyyannikova, Irina. 2017. Using technology in mathematics education. In *Current issues in mathematics education*, ed. Alexander Karp, 39–43. Bedford, MA: COMAP.

Privalov, Alexander. 2016. O zavershenii odnoy reformy [About the conclusion of one reform]. *Expert* 43 (1005): 14.

Rukshin, Sergey E. 2018. *Interview*. https://www.youtube.com/watch?v=8LUX0drb1NM&t=3078s. Accessed 16 Jan 2019.

———. n.d. *Neprodumannye reformy obrazovaniya i nauki kak ugroza natsional'noy bezopasnosti strany* [Ill-Conceived Reforms in Education and Science as a Threat to the National Security of the Country]. http://www.zavuch.ru/news/news_main/511/. Accessed 16 Jan 2019.

Ryzhik, Valery I. 2013. Krizis srednego matematicheskogo obrazovaniya glazami uchitelya [The crisis in secondary mathematics education through a teacher's eyes]. *Matematika v shkole* 10: 3–10.

———. 2014. Krizis srednego matematicheskogo obrazovaniya glazami Uchitelya [The crisis in secondary mathematics education through a teacher's eyes]. *Matematika v shkole* 1: 3–9.

Sadovnichy, Viktor A. Ed. 2003. *Obrazovanie, kotoroe my mozhem poteryat'* [Education we can lose]. Moscow: Moscow State University.

Sadovnichy: Okolo 60% pervokursnikov MGU, proshedshie po EGE, "provalili" pervuiu zhe kontrol'nuiu [Sadovnichy: About 60% of Moscow State University First-Year Students Who Passed the EGE "Failed" the Very First Test]. 2009. https://www.newsru.com/russia/28sep2009/60nemgu.html. Accessed 16 Jan 2019.

Sarantsev, Gennady I. 2003. Gumanitarizatsiya matematicheskogo obrazovaniya: Fantazii i real'nost' [The humanityzation of mathematics education: Fantasies and reality]. In *Formirovanie matematicheskikh ponyatii v kontekste gumanitarizatsii obrazovaniya*, ed. Gennady I. Saransev et al. Saransk: MGPI im. M.E.Evsev'eva.

Savvina, Olga. 2017. Priznaki krizisa otechestvennoy metodiki prepodavaniya matematiki [Signs of a crisis in the domestic methodology of mathematics instruction]. *Matematika v shkole* 2: 3–8.

Semenov, Alexey L. 2016. Semenov nepravil'no rukovodit MPGU, nado eto delat' inache [Semenov is steering Moscow State Pedagogical University the wrong way, things should be done differently]. Interview. *Meduza*. https://meduza.io/feature/2016/10/18/semenov-nepravilno-rukovodit-mpgu-nado-delat-eto-inacheon. Accessed 16 Jan 2019.

Sgibnev, Alexey I. 2013. *Issledovatel'skie zadachi dlya nachinaiuschikh* [Research problems for beginners]. Moscow: MTsNMO.

Shatalov, Viktor F. 1979. *Kuda i kak ischezli troiki* [Where and how did "threes" disappear?]. Moscow: Pedagogika.

———. 1980. *Pedagogicheskaya proza* [Pedagogical prose]. Moscow: Pedagogika.

———. 1987. *Tochka opory* [Support point]. Moscow: Pedagogika.

Shevkin, Alexander V. 2015a. Programma po matematike 2015 goda ili Torzhestvo neprofessionalizma [The mathematics curriculum of 2015, or the triumph of unprofessionalism]. *Matematika v shkole* 8: 3–11.

———. 2015b. Programma po matematike 2015 goda ili Torzhestvo neprofessionalizma [The mathematics curriculum of 2015, or the triumph of unprofessionalism]. *Matematika v shkole* 9: 3–17.

Smolin, Oleg, Sergey Kazarnovsky, Anatoly Kasprzhak, Svetlana Nikulina. 2009. Krizis otmeniaet EGE? [Does crisis cancel EGE?] *Echo Moskvy*, https://echo.msk.ru/programs/assembly/578535-echo/. Accessed 16 Jan 2019.

Sokolov, B.V. 2014. Analiz rezul'tatov EGE-2014 po matematike v Tomskoy oblasti [Analysis of EGE-2014 results in mathematics in Tomsk region]. In *Analiz rezul'tatov gosudarstvennoy itogovoy attestatsii vypusknikov 2014 goda obscheobrazovatel'nykh uchrezhdenii Tomskoy oblasti v forme edinogo gosudarstvennogo ekzamena. Informatsionno-analiticheskii otchet i metodicheskie rekomendatsii,* ed. P.I. Gorlov, 50–61 Tomsk: TOIPKRO.

Sossinsky, Alexey B. 2010. Mathematicians and mathematics educators: A traditions of involvement. In *Russian mathematics education. History and world significance*, ed. Alexander Karp and Bruce Vogeli, 187–222. Hackensack, NJ: World Scientific.

Standarty matematicheskogo obrazovaniya. Opyt provedeniya raznourovnevykh vypusknykh ekzamenov po matematike v Sankt-Peterburge [Mathematics education standards. Conducting multi-level graduation exams in mathematics in St. Petersburg]. 1993. St. Petersburg.

Stefanova, Natalya L. 2010. The preparation of mathematics teachers in Russia: Past and present. In *Russian mathematics education. History and world significance*, ed. Alexander Karp and Bruce Vogeli, 279–324. Hackensack, NJ: World Scientific.

Stefanova, Natalya, Natalya Podkhodova, and Viktoriya Snegurova. (Eds.). 2009. *Sovremennaya metodicheskaya sistema matematicheskogo obrazovaniya* [The contemporary methodological system of mathematics education]. St. Petersburg: RGPU.

Testov, Vladimir A. 2005. Matematika i Bolonskii protsess [Mathematics and the Bologna process]. *Vyshee obrazovanie v Rossii* 12: 40–42.

Tikhonov, Yuliy V. and Vladimir Z. Sharich 2012. *Komandno-lichnyi turnir shkol'nikov "Matematicheskoe mnogobor'e" 2008–2010* [Team and individual students tournament: Mathematical all-round]. Moscow: MTsNMO.

Timoschuk, M.E., and L.M. Nozdracheva. 1994. Nedostatki proekta standarta [The shortcomings of the blueprint for a standard]. *Matematika v shkole* 2: 11–12.

Vakhitov, Rustem. 2013. Bolonskii protsess v Rossii [The Bologna process in Russia]. *Otechestvennye zapiski* 4 (55).

Yashchenko, Ivan V. 2005. *Priglashenie na matematicheskii prazdnik* [Invitation to a mathematics festival]. Moscow: MTsNMO.

Yashchenko, Ivan V., Petr I. Zakharov 2014. *Matematika. Zadacha B8. Geometricheskii smysl proizvodnoy: rabochaya tetrad'* [Mathematics. Problem B8. The geometric meaning of the derivative: Workbook]. Moscow: MTsNMO.

Znak 2014. Patrioticheskoe vychitanie [Patriotic Subtraction]. https://www.znak.com/2014-04-08/pochemu_odin_iz_samyh_populyarnyh_uchebnikov_po_matematike_ne_proshel_gosudarstvennuyu_ekspertizu. Accessed 16 Jan 2019.

Zvavich, Leonid I., Dmitry I. Averyanov, Boris P. Pigarev, and Tatiana N. Trushanina. 1994. *Zadaniya dlya provedeniya pis'mennogo ekzamena po matematike v 9 klasse* [Problems for conducting a written exam in mathematics in grade 9]. Moscow: Prosveschenie.

Zvavich, Leonid I. and Leonid Ya. Shlyapochnik. 1994. *Zadachi pis'mennogo ekzamena po matematike za kurs sredney shkoly* [Problems of the written exam in mathematics for the secondary school course]. Moscow: Shkola-Press.