Chapter 1
Development of Mathematics Education in the Czech Republic (1989–2018): From a Search for Structure to Mathematical Literacy

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Abstract Education has always played an important role in the history of the region of the current Czech Republic. Five phases of development after 1989 can be differentiated: (a) deconstruction (1990–1991)—characterized by opposition against the current state; (b) partial stabilization (1991–2000)—decision-making based on what needed to be addressed and analysis of the state of education; (c) system reconstruction (2001–2004)—the beginning of the National Program for the Development of Education (2001) and subsequent Educational Framework Programs; (d) implementation (2005–2015)—elaboration of school education programs, and (e) the current period. This chapter describes the characteristics of these phases, legislative documents, and the development of approaches to the concept of mathematics education, mathematics teacher education, textbooks, and forms of assessment. This chapter concludes with a reflection on the problems with mathematical education in the Czech Republic and the outlook for the future.

Keywords Development of mathematics education in the Czech Republic · History · Mathematics education · Mathematics teacher education · International comparison · Curricular reform after 2001

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J. A. Colmenius: "… Our first wish is for the full power of development to reach all of humanity, not of one particular person or a few or even many, but of every single individual, young and old, rich and poor, noble and ignoble, men and women – in a word, of every human being born of Earth, with the ultimate aim of providing education to the entire human race regardless of age, class, sex and nationality.

Secondly, our wish is that every human being should be rightly developed and perfectly educated, not in any limited sense, but in every respect that makes for the perfection of human nature …" (Pampaedia or Universal Education)

1 Introduction

The main goal of this chapter is to describe the development that mathematics education has undergone in the territory of the Czech Republic since the so-called Velvet Revolution. The Velvet Revolution is a term used to describe the changes the country underwent from November 17 until December 29, 1989, that had the result of overthrowing the communist regime and installing a pluralistic democratic system. This chapter also summarizes the current state of (mathematics) education in the Czech Republic.

The former Czechoslovakia split into two independent countries in 1993, the Czech and Slovak Republics. The focus of this chapter, when describing the years after 1993, is on the Czech Republic. A lot of attention is paid to the positive changes in that period. However, the authors also pinpoint some of the prevailing problems in the area of mathematics education.

This chapter is divided into several parts. It presents an educational background on the topic of Czech mathematics education, including a brief account of the historical development of the territory of the current Czech Republic. The reader will become familiar with the milestones of Czech history and see the roots of contemporary mathematical education in the Czech Republic. The following section focuses on legislation in the area of education and changes it underwent after 1989 and 1993. It also presents information on various concepts in mathematics education, the content of school mathematics, mathematics textbooks, the use of ICT in mathematics education, and assessment. Without doubt, the role of teachers in, and their impact on, mathematics education is immense, which is why much attention is paid to mathematics teacher training. This chapter also presents information on the relationship of the general public to education and on the opportunities it has to influence what is happening, for example, in the form of public discussions in various forums. The conclusion of this chapter presents possible perspectives on the field of mathematics education as implied by the present state as they are perceived by the authors of this chapter.

2 Background

2.1 History

The history of the region known as the Czech lands is very rich. This territory has changed hands many times and has been known by a variety of different names. With respect to this chapter, let us state here only that the Lands of the Bohemian Crown were incorporated into the Austrian (Habsburg) Empire in 1526 and later became part of the Austro-Hungarian Monarchy. In 1918, after World War I, the Czechoslovak Republic was established. It was a multinational state of the Czechs (the majority population), Germans, Slovaks, and minorities (Hungarians, Russians, Poles).

The territory of Czechoslovakia was reduced considerably in 1938 when the country lost Sudetenland to Nazi Germany, the eastern part of Cieszyn Silesia and a part of Northern Slovakia to Poland, and a part of Carpathian Ruthenia and Eastern and Southern Slovakia to Hungary. Slovakia proclaimed independence on March 14, 1939. The rest of Czechoslovakia was occupied by Nazi troops on March 15, 1939, and on March 16 the creation of the Protectorate of Bohemia and Moravia as a part of the German Reich was proclaimed.

World War II ended with the defeat of Germany, and thus Czechoslovakia was re-established, although Carpathian Ruthenia became part of the Soviet Union. Three million German inhabitants of Czechoslovakia were expelled from the country and sent to Germany and Austria on the basis of presidential decrees (the Beneš decrees) with the consent of the allies. The postwar Czechoslovak Republic was dependent on the Soviet Union. Reforms were implemented that led to a gradual elimination of private ownership and ensured a monopoly of power. The country was taken over by the communists in a coup d'état in February 1948, and the country became part of the Soviet bloc.

The 1960s witnessed a time of liberalization that culminated in 1968 in the so-called Prague Spring. However, the reform movement was suppressed by the invasion of the Warsaw Pact armies on August 21, 1968. Russian occupying troops stayed in the country until 1991. The Prague Spring and invasion in 1968 were followed by 20 years of the so-called normalization. The communist regime became less and less popular in the 1980s. The Velvet Revolution overthrew the communist regime. The first free elections in 1990 brought a victory for democratic parties. The country started the process of liberalization and oriented itself toward western democratic countries. Since then its economy has been privatized and new political parties have been established.

One of the quickly developing problems of the new democratic country was its multinational structure. The problem resulted in a splitting of the country. Czechoslovakia ceased to exist at the end of 1992 without a plebiscite after negotiations failed between the top political representatives of both parts of the country. The Czech and Slovak Republics came into existence on January 1, 1993. The countries were quickly internationally recognized as successor states of the former

Czechoslovakia. The Czech Republic entered NATO on March 12, 1999. The 2003 plebiscite confirmed that the country would enter the EU, which happened on May 1, 2004. In 2007, the Czech Republic became a part of the Schengen Area. The Czech Republic is a member of the so-called Visegrád group that came to existence in the time of the former Czechoslovakia (February 15, 1991), which now defends the common interests of the Czech Republic, Slovakia, Poland, and Hungary in the European Union.

2.2 *Educational Tradition in the Region of Today's Czech Republic*

Education has always played an important role in the history of the region of the Czech Republic. Gergelová Šteigrová (2011) highlighted as historical milestones the foundation of Charles University in Prague and the life and work of Jan Amos Comenius.

The University in Prague, the affiliation of one of the authors, was founded in 1348 by Charles IV, the King of Bohemia and Roman Emperor. At that time, it was the first *Studium generale* north of the Alps and east of Paris. It was modeled on universities in Bologna and Paris, and within a very short time it achieved international recognition. Mathematics education was, according to testimony, on a higher level than mathematics education at the University of Paris (Hankel 1874 in Mikulčák 2010). At the bachelor's level, it included the processes of practical addition, subtraction, multiplication, and division. On the master's level, students focused on the fundamentals of Euclid. The quality of the university was important not only for the development of mathematical thinking, but it also affected the level of education in the whole region. The rector of the university published school rules with curricula and appointed some teachers. It is not surprising that population groups in the region that would not have received any education in other countries were often able to read and write (e.g., Hussite women in the fifteenth century) (Mikulčák 2010).

This tradition was part of the background of the personality of Jan Amos Comenius (1592–1670), one of the greatest Czech thinkers. He gained a reputation not only as a theorist in pedagogy but also wrote several textbooks for different school levels and formulated a number of recommendations for mathematics education from pre-school to secondary school levels. He emphasized that pupils should learn arithmetic and geometry comprehensively (Mikulčák 2010). For example, in his treatise *Schola Ludus* ("Playful School" 1654) he showed how pupils should get to know and interpret basic mathematical and geometrical concepts through a form of dramatic improvisation. General pedagogical principles were developed in *Didactica magna* ("The Great Didactic" 1657) and *Orbis Pictus* ("The World in Pictures" 1658).

2.3 Mathematics Education in the Newly Established Czechoslovakia

If we want to understand the situation of the system of education after 1989, several facts from earlier in the twentieth century must be mentioned. The newly established Czechoslovak Republic (1918) inherited the educational system of Austrian-Hungarian Empire. Although the Minor Education Act (1922) was passed, the system of education did not transform much. Lower secondary schools remained a part of the educational system and served the function of a social filter: a consequence of school fees and demands on pupils' cultural level was that only about 40% of pupils were able to finish their studies (Walterová 2011). Any attempts to innovate the teaching of mathematics were in the hands of outstanding primary school teachers. In secondary schools, the process was supported by the Union of Czechoslovak Mathematicians and Physicists. The Union is one of the oldest (and still existing) learned societies in our territory. It was founded in 1862. Its original mission was to improve the teaching of physics and mathematics at all levels and types of schools and to support and develop these disciplines. The members of the union were mostly teachers from upper secondary schools and institutions of higher education. The minority were university teachers and scientists.

However, pedagogical reform inspired by American sources was gradually gaining importance in the interwar Czechoslovakia, which resulted in the establishment of several reformed schools in the 1930s. The development of reformist ideas was stopped during the period of World War II, because this was the period of German pressure on Czech schools.

After World War II, a renaissance of reformist ideas had no time to develop. The communist coup d'état in 1948 brought a gradual increase of influence from the Soviet Union on the system of education. However, this influence was not only negative. The positive effects were publications of translations of methodological (mathematical) literature from Russian and the availability of a large number of academic works in Russian. Also, Russian translations of French, English, and German works were very valuable as they were not available in Czech or Slovak.

However, Soviet influence also brought the new conception of undifferentiated (uniform) schools, which in practical terms meant the replacement of several streams of education (secondary grammar schools, *stadt schule*, *real schule*) by one type of school with the same demands on pupils of the same age. The intervention was a political move and was interpreted by the communist leaders as providing equity in education for all pupils. Compulsory school attendance lasted 9 years. The first 5 years were called the national school and the following 4 years were called the middle school. This was followed by a third level, which was not compulsory. This level included 3-year-long vocational schools, 4–5-year-long professional schools, and 4-year-long gymnasiums (grammar schools). Students at professional schools and gymnasiums finished by passing the Maturita (school leaving examination). The view of the status of the Maturita examination was gradually changing. Although it was still the necessary condition for being accepted to university, it was

not the only condition for being accepted. Its content and form were affected by the contemporary social and political changes. The communist government tried to build a regime based on the Soviet model. The school was to be connected to the everyday life of the working class. The Maturita exam was no longer to be an elite exam. The goal was to change it to make it accessible and passable to a greater number of working-class students and politically reliable supporters of the regime so that they could start their studies at university. In consequence, the prestige and level of the exam dropped significantly. It was also possible to pass the exam by studying only part time while working and simultaneously studying at a so-called evening school or in the 1-year "working class prep" course (Morkes 2003). Mathematics was an elective subject of the oral part of the Maturita exam.

However, it became obvious that the uniform requirements did not reflect the needs of everyone—weaker pupils could not cope with them and talented pupils were not developed sufficiently. The political decision made in reaction to the economic crisis of the 1950s was to have students graduate quickly and then have them participate in "building communism" as soon as possible. This resulted in the establishment of the 11-year school system, which followed the Soviet model. Compulsory school attendance was shortened to 8 years. The following 3 years were elective, offering upper secondary general education. Before this concept was passed into law, it was subject to a wider discussion. Teachers who disagreed with its concepts were forced to leave their jobs. Shortening the length of education by 2 years required a change in curriculum, from a cyclical model to one organized in a linear fashion. The systematic teaching of disciplines started in the 7th grade and finished in the 11th grade. There was no review before the school leaving exam. This meant that pupils who left school after 8 years of compulsory education did not have a complex, comprehensive education and lacked a lot of knowledge that was taught from the 9th to 11th grades (e.g., analytical geometry, congruence, solid geometry).

The 11-year school system focused on providing education in the sciences (mathematics, physics, and chemistry). Language education (with the exception of Russian) remained compulsory but was not strong. Also, the status of biology declined. The 11 years of school were concluded by the Maturita exam; the written part of it was done in Czech and Russian and the oral part included compulsory mathematics. The Maturita exam in mathematics had a time limit of 15 min.

The system changed in the 1960s. Compulsory school attendance was restored to 9 years. The system diverted from the Russian model. Compulsory education was provided at what was known as 9-year basic schools, which had 5-year-long primary and 4-year-long lower secondary levels. Upper secondary studies lasted from 2 to 5 years. The 3-year-long upper secondary comprehensive school was primarily a preparation for university studies. However, its curriculum also included work practice in an industry. This work practice was later omitted from the curriculum, and the schools began to specialize in arts or sciences. Studies concluded with the Maturita exam. The compulsory part was Czech (or Slovak), Russian, or elected subjects including mathematics.

Gymnasiums (grammar schools) were re-established after 1968. Their curricula came out of a historical tradition but were also inspired by international trends. In the beginning this resulted in the improved quality of upper secondary education. However, *normalization* (the period of political oppression after the unsuccessful reform movement in 1968) resulted in a decline in the prestige of gymnasiums when the so-called dorm schools for the working class were established in 1974. Their purpose was to prepare young, talented, politically conscious and loyal members of the working class for university studies. The acceptance criteria were political-engagement based. The schools were 1-year long and concluded by the Maturita exam. Their graduates were accepted to universities without entrance exams. These schools ceased to exist as late as 1984.

The 1978 Act of Education introduced a 10-year compulsory school attendance. Practically, it meant that the lower secondary school level was shortened by 1 year, which was followed by two compulsory years of upper secondary education. All students were obliged to study at some kind of upper secondary school—gymnasiums, professional, or vocational schools. Vocational school students could sit for the Maturita exam and then be accepted to universities. Members of the working class could also sit for the Maturita exam in one of the shorter evening schools or for distant studies. These measures were in line with the vision of the Communist party that the majority of the young should graduate from an upper secondary school. The content of the Maturita exam changed. Czech language study was still compulsory. At certain types of schools (technical schools), Maturita in mathematics became compulsory at certain periods.

The tradition of high-quality education did not vanish completely even during the hard times. There were a number of excellent teachers at schools, thanks to whom the level of education remained good (Novotná et al. 2019). This holds true especially for mathematics education, in which a great interest in quality was retained. It was crucially important that school mathematics education was in the spotlight of several scientists in the field of mathematics. Let us name here at least B. Bydžovský and E. Čech. Čech was the author of a number of demanding textbooks for secondary schools, which emphasized the structure of mathematics. He also formulated the demand that schools should develop pupils' activity, develop their interest in mathematics, their language, and their ability to think independently. The textbooks presented mathematics as a system (Boček and Kuřina 2013).

An important turning point came in the 1960s. Innovation in mathematics education was gradually breaking through thanks to the influence of similar efforts in Europe and the United States as well as by the Kolmogorov reform in the Soviet Union. Czechoslovakia, like many other countries, had its period of New Math—another terminology was used of course. (For more details, see Jelínek and Šedivý 1982a, b).

A specialized department was established in the Institute of Mathematics, Czechoslovak Academy of Sciences, *Section for Didactics of Mathematics.* Its work was characterized by the effort to connect research in teaching practices with curricular research, both on theoretical and on practical levels. Emphasis was put on connections between school mathematics and everyday reality, geometrization, and

learner-centered approaches where the teacher's role was to support discovery (i.e., experimenting, problem-based teaching, genetic approach). The aim of the Section for Didactics of Mathematics was to reduce fact-based teaching, to develop a psychological-genetic approach, to put emphasis on working methods in mathematics, on laboratory and problem-based approaches, and to reinforce algorithmic components (Vondrová et al. 2015a, b). The collaboration with university teachers and researchers in the field of mathematics by the staff team of the Section for Didactics of Mathematics was supported as a fundamental direction of research. The team cooperated with experimental schools too.

The partial results and experience from this experiment were used in applied research conducted by the Research Institute of Education. The role of the institute was to modify the didactical system of school mathematics from the primary to upper secondary school levels with respect to the current state of mathematical disciplines. Its role was also to introduce the basic elements of logic and to get rid of isolation in various parts of mathematics by linking curricular content on the basis of knowledge of sets and by preparing pupils to understand mathematical structures (Kabele 1968). The system was tested in select primary and lower secondary schools. Then in 1976 a set of textbooks was designed and "set" mathematics started to be taught in all schools (Kabele 1968), first at the secondary level (described in more detail in e.g., Mikulčák 1967), then in grades 1–9. Given the scope and radical nature of the changes, mathematics became the flagship of the innovation project with the publication of Ministerstvo školství (1976) which gave it its informal name of "sets."

This was also the time when more attention turned to mathematically gifted pupils. There were more than 100 classes for extended mathematics education at basic schools. Special learning materials were produced for them. Mathematically gifted pupils were also taken care of within the framework of the Mathematical Olympiads. The stimulus for this was very weak results from Czechoslovak participants at the International Mathematical Olympiad in the 1960s and the beginning of the 1970s. The cause of these failures was not worse knowledge or abilities of the Czechoslovak participants in comparison to their competitors from countries with traditionally good results, such as the Soviet Union or Hungary; the cause was inadequate preparation and lack of experience at competitions. That is why a correspondence seminar was established. It resulted in an improvement especially in the years 1979–1983 (Mikulčák 2007).

Although it is often said that pedagogy was strongly subordinate to ideology, the modernization movement was inspired not only by the Soviet Union but also by different sources. Tichá (2013) discussed how, for example, Bruner, Papy, Freudenthal, and the results of the International Congress on Mathematical Education (ICME) in 1988 inspired the work of the Section for Didactics of Mathematics.

3 Legislation Before and After 1989

3.1 Selected Legal Anchoring Before 1989

Compulsory 6-year school attendance was introduced for boys and girls in Bohemia and Moravia in 1774. It was extended to compulsory 8-year school attendance in 1869 in the Austro-Hungarian Empire, which also included today's territory of the Czech Republic. The national school consisted of institutionally separated levels: a 5-year general school (*Volkschule* in German) and a 3-year municipal school (*Stadtschule* in German); boys, not girls, were allowed to study at different types of secondary schools (grammar schools, gymnasium, real gymnasium) instead of municipal schools. The only public secondary school girls could attend were pedagogical schools. Other secondary schools for girls were private. Up until 1918, education for girls was not supported by the state. Curricula for *Volksschule* were based upon basic subjects (reading, writing, counting, measuring, physical education, religious education), to which biology, geography, and history were added in *Stadtschule.*

When Czechoslovakia was established (1918), it adopted the laws of the Austro-Hungarian state. Only partial changes were made. Predominant in the countryside were 1–2-year general schools, which allowed the establishment of schools in even the smallest villages. The presence of rural schools helped the development of literacy. The school system in larger towns was composed of 5-year general schools and then followed up by 3-year municipal schools.

The end of World War II was followed by a very short period of partial changes and was terminated by the Act of Education in April 1948. The 5-year national school was followed by a 4-year middle school. A higher level of general education was provided by 4-year secondary grammar schools. Technical education was provided by various types of 4-year technical schools (concluded by the *Maturita* [graduation] examination). Skilled workers were educated in vocational schools. Mathematical education officially proclaimed to follow Soviet models. For example, a conference of the Union of Czechoslovak Mathematicians and Physicists on a new conception of teaching mathematics was held in August 1949. There E. Čech introduced the participants to the principles of scientific quality in teaching mathematics. His vision was for school mathematics to be built on the structure of mathematics as science.

3.2 Development of Education and Legislation After 1989

After 1989, ideological dependence, unilateralism, dogmatism, pressure, emphasis on discipline, and formalism in education were strongly criticized. Kotásek (2004) differentiated four phases of the initial period:

1. Deconstruction (1990–1991)—characterized by resisting against the current state
2. Partial stabilization (1991–2000)—decision-making on what needs to be addressed and analyzing the state of education

3. System Reconstruction (2001–2004)—White paper (National Program for the Development of Education) (Kotásek et al. 2001) and subsequent Educational Framework Programs
4. Implementation (2005) —adoption of the Education Act.

3.2.1 Deconstruction of the Socialist System of Education (1990–1991)

At the beginning of this period, tight ideological control ended and the state monopoly of education was broken down. The management of the education system was consecutively decentralized mainly by delegating a number of decision-making powers to municipalities, schools, and their headmasters. Schools had more freedom in the allocation of lessons to each subject. Several private and denominational schools were established. Most mathematics teachers rejected uniform textbooks based on set mathematics, looked for other ways of interpreting the topics, and asked for new textbooks. The challenge was taken up by several teams of authors. New series of textbooks were developed quickly and quickly became used in practice at schools.

The recognition of the rights of pupils (or their parents) to choose their educational path according to their abilities and interests as an integral part of the overall liberalization process started to be taken into account.

Many aspects of teachers' work changed. On the one hand, teachers gained significant space for their independent creative work in preparation of the content of individual subjects, in the choice of teaching methods, and the choice of textbooks. On the other hand, it meant greater burdens on teachers; many teachers had to deal with a temporary loss of their professional competence (pedagogical as well as technical), which necessitated partial, and sometimes even full, requalification. For example, Krajčová and Münich (2018) report the insufficient development of ICT literacy among older teachers at the time.

Regarding the organization of the system, a note should be made that compulsory 10-year school attendance (discussed earlier) was abolished after the year 1989, after the Velvet Revolution, when the country returned to the model of 9 years of compulsory school attendance: 5 years of primary and 4 years of lower secondary levels at basic schools.

Upper secondary school education builds on basic school education. There are gymnasiums (grammar schools), technical and professional secondary schools, and vocational schools. Generally, the length of study is 4 years (sometimes, but very rarely, longer), if the school ends with Maturita. There are a wide range of apprentice schools preparing students mostly for various crafts that do not end with Maturita.

The 1990s saw the restoration of 8- and 6-year grammar schools, which previously existed between World War I and II. These schools prepare their students for university studies. This means very early differentiation of pupils. Pupils who are

interested in studying at these schools are admitted if they pass the entrance exams after the fifth and seventh grades of school attendance.

3.2.2 Partial Stabilization (1991–2000)

This period focused on the adjustment of the existing curriculum and the creation of space for the implementation of new curricular projects. The Ministry of Education, Youth and Sport[1] initiated a process in which the existing curriculum, called Basic School, was modified by a group of authors in 1996 (Jeřábek 1998). What was added in this document was an explicit definition of fundamental curricular content (referred to as "What the pupil should have mastered"). The minimum number of lessons for each educational area was specified. The rest of the lessons, the so-called disposable lessons, allowed schools to specialize. This change did not have much impact on the content of mathematics education.

Apart from the project Basic School, there were other projects ratified by the Ministry too. Each school could make a decision as to which of the projects to follow.

The Association of Teachers of Primary Education created a project called National School (MŠMT 1997), alternative school curricula for compulsory school education (i.e., first to ninth grade). The ideas that came out of it were that it was essential to differentiate education according to the decisions of a particular school to the maximum degree possible and to adapt education to the needs of individual pupils. In contrast to the above described Curricula for Basic School, less time was given to mathematics (about 20% less). As it was known, the common teaching content (compulsory minimum content for all schools at the given level) was specified.

Similar projects were the programs called Educational program Municipal School (MŠMT 1996) (grades one through five) and the follow-up Civic School (grades six through nine). These projects did not only define what a pupil should and can master but also present model problems.

In Table 1.1, we present how the introduction to the concept of numbers in the first grade is treated in the different curricular projects.

Besides those projects, other traditional alternative programs (Waldorf pedagogy, Montessori, Dalton) were introduced in the Czech Republic. Montessori mathematics especially became very attractive to a number of parents.

There is another fact that must be mentioned here. The mid-1990s witnessed the establishment of the first lower secondary grammar schools, which represented a more academic strand of education. The tradition of these schools goes back to the interwar years and already at those times they were criticized for their elitist nature (Mikulčák 2010).

Nationwide discussion on the concept of education in the Czech Republic, description of problems, and the definition of future development was based on

[1] In the following text, it is just called the Ministry.

Table 1.1 The concept of numbers in different curricula

National school	Basic school	Municipal school
Knowledge of numbers 0–20 Position of numbers on a number line Orientation on number line Writing numbers to 20 Comparing numbers and signs of inequality	**Content** Numerical series Relations greater than, smaller than, and equals Numbers Symbols >, <, =, +, - ***** Counting objects in a given set. Choosing various specific sets with a given number of elements Reading and recording numbers Comparing numbers Solving and posing word problems involving the comparison of numbers Solving and posing word problems and addition and subtraction **What the pupils should have mastered** Count the elements of a given set to 20 (including 20). Create a specific set (beads, marbles, etc.) with a given number of elements up to 20 (including 20). Compare numbers and sets of elements up to 20. Read and record numbers from 0 to 20 Solve word problems requiring the comparison of numbers from 0 to 20 **Examples of extending contents** Measuring lengths using segments (e.g., centimeters)	**Pupils should master (level Z)** Figures and numbers. Reading and recording numbers. Numbers as amounts and positions comparing numbers Guessing the amount The principle of the decimal system Mathematical notation and expression. The needed mathematical symbols. **Activities, tools, inspiration** Marking and comparing numbers on a number line and in the context of measuring. Arrow diagrams. Estimating the number of objects and numerical figures. In the number 3724, the number 37 also expresses the number of hundreds. **Pupils can master (level R)** Negative numbers in real life (thermometer, lift, bank account). Roman numerals. **Increasing accent** Flexibility of the use of the decimal system (decomposition and models). Notion of magnitude of numbers. Use of natural language and its gradual precision. **Decreasing accent** Decimal positional notation system Formal isolation of domains of numbers

analyses conducted in the Czech Republic (MŠMT 1998) and abroad (OECD 1997; Čerych 1999b). It came out of the study "České vzdělání a Evropa: strategie rozvoje lidských zdrojů při vstupu ČR do EU" (Czech Education and Europe: Strategy for the Development of Human Resources on Entering the EU, Čerych 1999a). The process ended with the creation of the White Paper (Kotásek et al. 2001).

3.2.3 The Period of System Reconstruction (2001–2004)

The period started with the issuing of the *National program for the development of education in the Czech Republic: White Paper* (Kotásek et al. 2001).

White Paper (2001) and Act of Education (2004)

The White Paper represented a proposal for a system of education that defined the ideological background, general goals, and actionable programs by which the development of the education system in the intermediate-term should be directed. It was a binding foundation on which all specific action plans of the Ministry were to be planned. The direction of Czech educational policy was expressed by the main strategic lines that are presented schematically in Fig. 1.1.

The White Paper did not focus on individual subjects; but it also defined the basic principles of education. From the viewpoint of mathematics education, most important was the focus on the needs of life in a knowledge-based society. This allowed the financing of a number of projects focusing on the development of mathematical literacy and inquiry-based mathematics (and science) education. The White Paper was planned to be the fundamental document for the medium-term horizon (until about 2010).

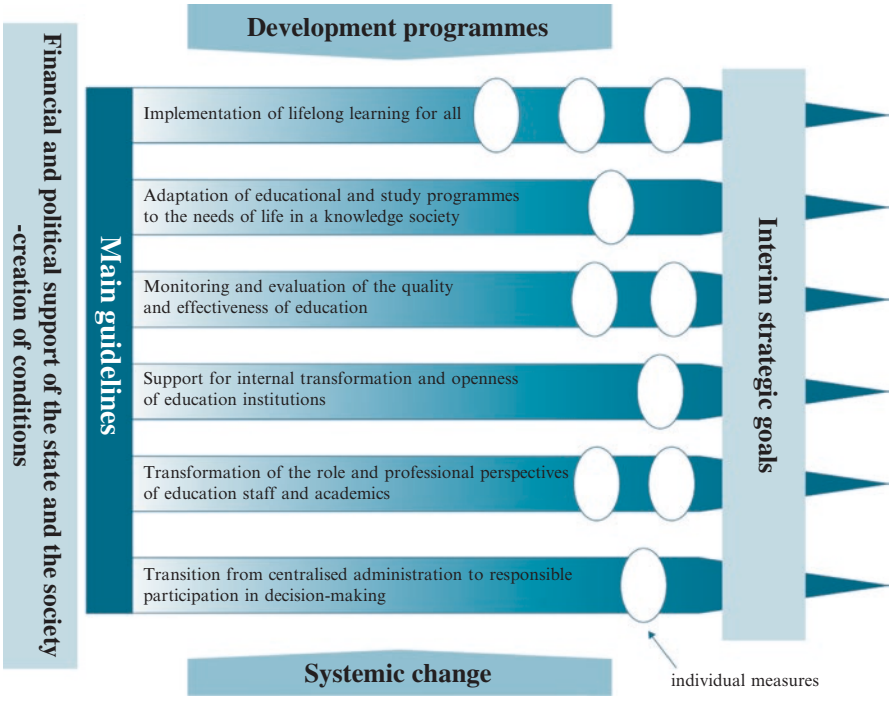

Fig. 1.1 Schema of strategic changes (Kotásek et al. 2001, p. 92)

The White Paper was followed by the new Act of Education (Parliament 2004) which came into force in 2005. It legitimized the scheme of the system of education presented (in Fig. 1.2). The Act of Education stated that the Ministry would issue *Framework Education Programs* (FEP) for each field of education in preschool, primary, secondary, art, and language education. The Act also stated that each school (its headmaster and teachers) was responsible for the creation of their own School Education Programs (SEP). Thus, a two-level curriculum was enacted (the schema is presented in Fig. 1.3).

Framework for Education Programs (FEP) and School Education Programs (SEPs)

FEPs define the compulsory content, range, and conditions of education. They are necessary for the development of School Education Programs (SEPs), the assessment of educational outcomes, and for the development and certification of textbooks and other teaching texts. SEPs are developed based on school levels while reflecting their particular regional needs. They allow schools to make maximum use of local materials and personal conditions.

Janík et al. (2010) summarize that FEPs create the following innovations:

- At the level of organization and management of an education system, the development and implementation of curricula becomes decentralized.
- At the level of conception and goals of school education, emphasis is put on the development of key competencies.
- The concept of inclusive education is promoted.
- At the level of school educational content, they aim to surpass encyclopedic approaches of education through the enrichment of curricula by adding new educational content, structuring educational areas and fields, and creating cross-curricular links.

If we compare FEP BE with previous curricular projects, we see that the main difference is in the formulation of educational outcomes as competencies. This approach is used in all FEPs and puts emphasis on the process of life-long learning. The schematic in Fig. 1.4 shows the structure of FEP for basic education (pupils aged 6–15) (FEP BE English translation from 2007).

The FEP BE demands the development of:

- learning competencies
- problem-solving competencies
- communication competencies
- social and personal competencies
- civil competencies
- working competencies

Development of competencies is the general goal, which is not linked explicitly to any of the subjects. Disciplinal development is expected to be supported in the solution of various types of problems across subjects.

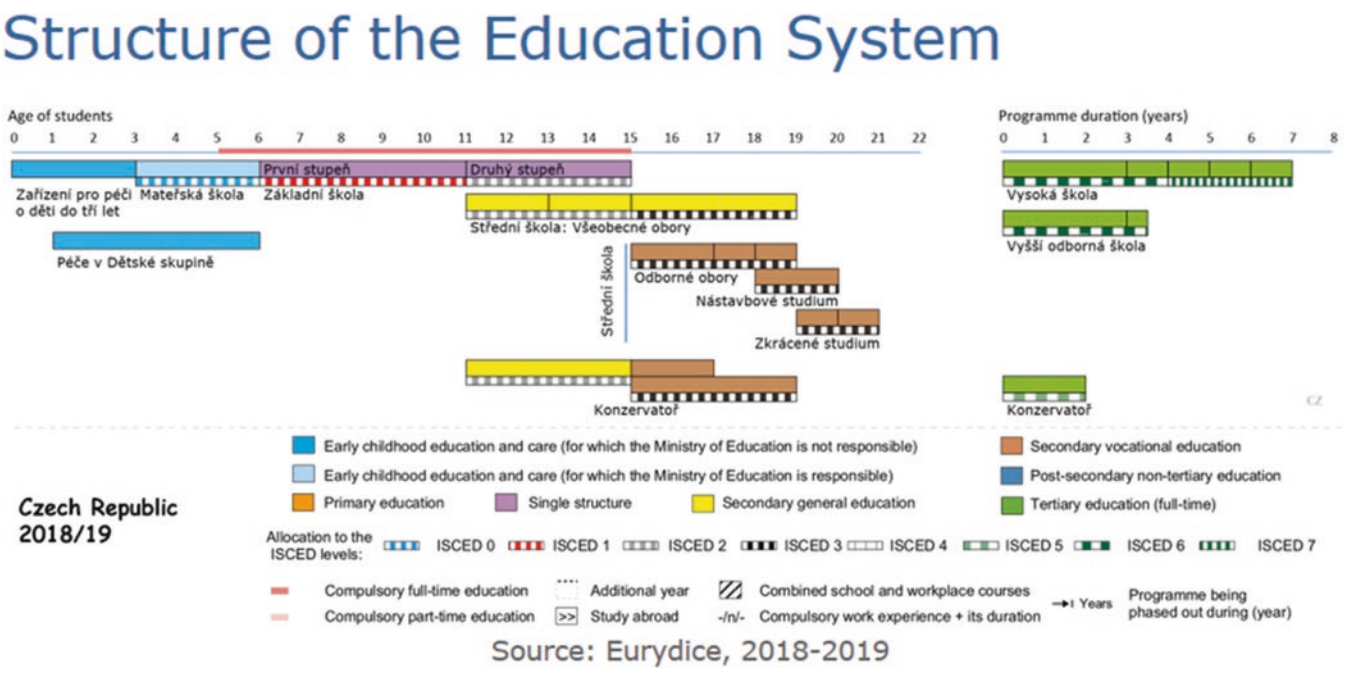

Fig. 1.2 Structure of the Czech educational system, (Source: Euridice, 2018–2019. https://eacea.ec.europa.eu/national-policies/eurydice/content/czech-republic_en)

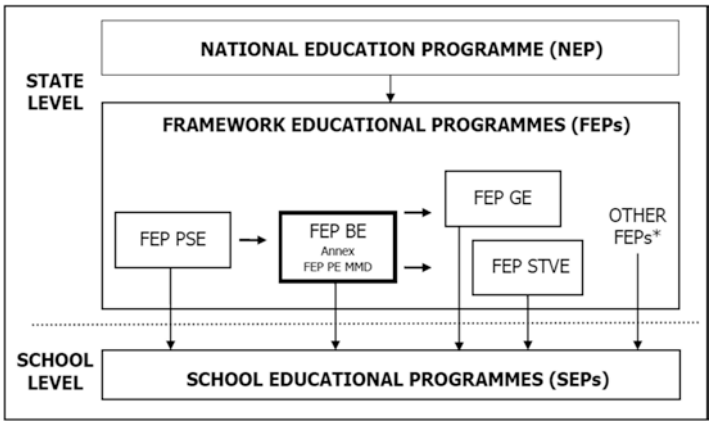

Fig. 1.3 Structure of Czech curricular materials (Source: FEP BE 2007, p. 6)

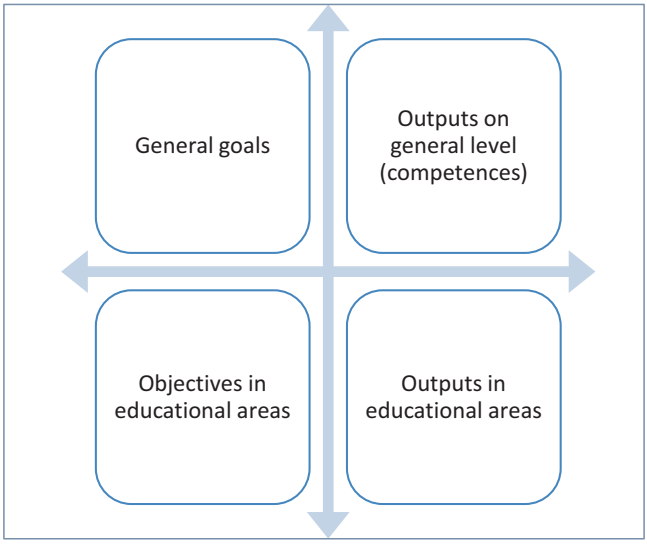

Fig. 1.4 Structure of FEP BE

How are the demands of these general outcomes formulated? Let us illustrate this using the problem-solving competency that is defined in the FEP for preschool education, the FEP BE, and the FEP for upper secondary education (grammar schools). The components of the problem-solving competence copy to a certain degree the stages of solving a problem (Polya 1945). The various content of areas of school education means that different subjects offer different opportunities for the development of problem-solving competency. In general, topics in the area of mathematics and science provide ample opportunities for solutions to rational-logic problems and deterministic problems.

One of the problems is that the competences are formulated very vaguely (Straková et al. 2009). Also, its gradation (for 6-, 15-, and 19-year-old pupils) is in our opinion not convincing. Table 1.2 presents a comparison of requirements for problem-solving competency.

How to form, shape, and develop pupils' key competencies? Setting key competencies as target categories of basic education is problematic. For one thing, their formulation is rather vague (see the examples in Table 1.2). Also, student achievement is difficult to assess (Janík et al. 2010, p. 24). Mathematics teachers are used to focusing on educational outcomes with achievements that are verifiable and quantifiable, i.e., based on knowledge of curricular content and the ability to work with this content. This stems from the very essence of mathematics, which has a solid knowledge structure (Müller and Steinbring 2004).

Another question is how a teacher should work with these competencies when planning, conducting, and evaluating a lesson. It is recommended that teachers should first work with the educational content part of the FEP BE related to educational areas (EA). Here they find characteristics of the EA, objectives of the EA, and educational content for each EA.

The questions still open are how well teachers have managed and continue to manage to fulfill the goals defined by the FEP and what the impact of the introduction of FEP is on the performance of Czech pupils. We think an answer to these questions can be sought in the results of Czech pupils in international comparative studies. These are presented in more detail in Sect. 7.

3.2.4 Implementation (2005–2015)

This period was characterized by the elaboration of school education programs (SEPs). SEPs were obligatory documents. Preparation of the SEP was presented as:

> … manifestation of the school's pedagogical autonomy as well as its responsibility for teaching methods and outcomes. For this reason, the individual parts of the SEP are prepared with the participation of all of the teachers of the school in question, who are also co-responsible for implementing the SEP in the conditions of their school." (FEP BE 2007, p. 212).

Groups of teachers handled the newly gained autonomy on different levels (Janík and Knecht 2007). Some schools invited support from the outside and worked on the SEP in teams. In other schools, the responsibility was delegated to one teacher for each subject. The SEP for each subject was developed in isolation from other subjects, often very formally. Pedagogical research focusing on this period agrees that the reform was not well understood by teachers who had very contradictory attitudes toward it (Janík and Knecht 2007).

The first stage of this period, when FEPs were prepared, transitioned to the stage when FEPs were implemented by schools (2007–2015). Even this period is marked by contradictory attitudes. Some teachers speak of the reform as a very unpleasant matter that bore a lot of extra, unpaid work and unnecessary formalities and had no

Table 1.2 Comparison of components of problem-solving competence for FEP Preschool Education, FEP BE, and FEP in secondary grammar schools

FEP Preschool Education (6 years old) 2018, p. 11 and 12	FEP BE (15 years old) 2007, p. 11	FEP Upper Secondary (19 years old) 2007, p. 9
A child/student finishing the pre-school/basic/upper secondary education		
… takes notice of events and problems in their environment. …has a positive echo to their active interest that is a good motivation for solving further problems and situations	… notices the most various problem situations at school and outside of school; recognizes and understands a problem; reflects on discrepancies and their causes; considers and plans ways to address problems while employing his/her judgment and experience	… recognizes a problem, elucidates its nature, divides it into parts
… decides problems up to their ability; tries to decide known and repeated situations independently (through imitation or repetition) and the more exigent ones with support and aid of an adult	… is able to find information useful for solving problems; identifies identical, similar, and different features; applies the knowledge acquired to discover various solutions; is not discouraged by a failure should there be one and persistently seeks conclusive solutions to a problem	…forms hypotheses, proposes gradual steps, considers the application of various methods when solving problems or verifying a hypothesis. … applies appropriate methods and prior knowledge and skills when solving problems; apart from analytical and critical thinking, the pupil uses also creative thinking while employing imagination and intuition. … is open to using various methods when solving problems, considers a problem from various sides
… decides the problems according to their immediate experience, proceeds via trial and error, proves, experiments, spontaneously brings out new solutions to problems, searches for different possibilities and variants, has their own original ideas and applies previous experiences	… solves problems independently; selects suitable ways to solve problems; uses logical, mathematical and empirical methods when solving problems	
… can decide between virtual and real problems, uses logic, mathematic and empiric procedures, grasps simple algorithms for solving various tasks and situations, and uses them for further ones. … states precise ideas about numbers, expresses sizes, uses numerical and mathematic concepts and perceives elementary mathematic relations	… tests in practice the correctness of problem-solving methods and applies sound practices when addressing similar or new problem situations; monitors progress when overcoming problems	… interprets critically the acquired knowledge and findings and verifies them; finds arguments and evidence for claims, formulates, and defends well-founded conclusions

(continued)

Table 1.2 (continued)

FEP Preschool Education (6 years old) 2018, p. 11 and 12	FEP BE (15 years old) 2007, p. 11	FEP Upper Secondary (19 years old) 2007, p. 9
… differentiates between function solutions leading to an aim and non-functional solutions and is able to make a choice between them. …understands to avoid a problem solution that does not lead to a target and that timely and prudent solutions are an advantage; realizes that they are able to affect situations due to their actions. … is not afraid to fail when not only their success is appreciated but also their endeavors	… thinks critically; makes prudent decisions and is able to defend them; realizes the responsibility for decisions; is able to evaluate the results of their decisions	… considers the possible advantages and disadvantages of the individual solution variants, including the assessment of their risks and consequences

positive impact on education. Teachers did not oppose the idea of educational reform, but their interpretation of the objectives of the reform was different from the expected aims (Straková 2010). Questionnaire surveys (Janík et al. 2011) confirm that teachers failed to understand that the aim of the reform was to reinforce autonomy of schools, to support decentralization, to allow schools to take part in the creation of curricula, and innovation at the level of educational outcomes and content. On the other hand, teachers appreciated the opportunity to choose methods of work more freely.

In this period, the Analysis of Fulfilment of the Objectives of the National Program for the Development of Education (Straková et al. 2009) was expanded. A group of experts invited by the Ministry stated in this document that the objectives formulated in the White Paper were not achieved. Some problems were caused by a too general formulation of goals. Some goals were not ordered hierarchically from the point of view of importance, and causal links predominated. Only in some cases was operationalization (i.e., expression by measurable signs) of the goals of individual measures done. Indicators of achievement for the goals were largely absent. Inequity in education was described as one of the persistent problems of the Czech system of education because the performance and results of pupils in the system were closely related to the economic and social status of their families.

Early diversification of pupils into a more academic stream of education (the so-called lower secondary grammar schools) also received negative public assessment as hindering equity in the Czech education system. The OECD found that:

> A majority of Czech parents from a low socio-economic background send their children to local schools, while only a small number of parents from higher socio-economic background choose to do so. School choice can further increase inequities if mechanisms are not in place to lessen the negative effects. Czech pupils who struggle in school are often streamed into special schools with reduced curricula. Streaming pupils based on academic

> ability and early tracking can negatively impact their educational outcomes if education pathways do not provide high curricular standards and opportunities to transfer between tracks. Early tracking occurs at age 11 (compared to the OECD average of 14), and differentiation of educational pathways has increased. Research shows a strong relation between tracks chosen and socio-economic background. At age 11, 13% of the pupil cohort, mostly from high socio-economic backgrounds, enters lower secondary grammar schools. (OECD 2013, p. 6)

The above-presented conclusions were confirmed by the results of Czech pupils in the international comparative PISA (Programme for International Student Assessment) study of 2015 (Blažek and Příhodová 2016). Analysis of results achieved in tests of mathematical literacy shows that Czech schools replicate social inequality. Secondary analysis of PISA 2015 results (ČŠI 2017, p. 20) shows that if a pupil with a lower socioeconomic background (SEB) attends a school with pupils of above-average SEB, i.e., the average SEB of the school is high, the pupil achieves better results than if they attend a school with a below-average SEB. This relation is very strong among the population of 15-year olds. In other words, if a significant group of pupils leave elementary school and go to lower secondary grammar schools at a relatively early stage, the results of the whole population are negatively affected because SEB in the elementary school declines.

3.2.5 System Reconstruction (2015 and Further)

The phases formulated by Kotásek (2004) can be amended with the current phase, which is referred to as the period of *system reconstruction*. In 2014, the White Paper was replaced by new strategic material, Strategy of Educational Policy of the Czech Republic until 2020 (MŠMT 2014). In contrast to the many priorities declared in the White Paper, this material focuses only on a few strategic goals:

– reduce inequality in education.
– promote quality teaching and teachers as a key condition of education.
– institute responsible and effective management of the education system.

The integration of children with special needs started; however, a lack of funds, teaching assistants, and training of teachers in this area made the process very painstaking. The problems of children from socially and culturally disadvantaged backgrounds persist because support for their education fails and social inequality is replicated. Also, the number of pupils with a limited command of Czech is increasing. This problem has not yet been solved. Simultaneously to all these changes, the FEP BE is currently being modified.

More information about the situation in Czech schools during this period can be found in (Dvořák et al. 2010, 2015).

4 Conception of Mathematics Education

The current conception of mathematics education is influenced by the ideas of constructivism, which have been adapted to the conditions of teaching mathematics in Czech schools by Hejný and Kuřina (2015). They developed the conception of didactic constructivism (Hejný and Kuřina 2015) which comes out of the idea of mathematics as a human activity that, through solving tasks and problems, leads to looking for connections, creating concepts, generalizing statements, and proving them. Part of this activity is the development of mathematical models of reality. This approach was further developed by other authors (Stehlíková 2004; Stehlíková and Cachová 2006; Molnár et al. 2008; FEP BE 2007). The teacher's role is to encourage pupils' activity. In places where pupils can work with their existing knowledge and experience, they are encouraged to use them. The teacher steps in at places where recapitulation and the summary of knowledge, sample solutions, and deduced algorithms are necessary. It is the teacher who knows the pupils best and can make decisions about methods and procedures for the given lesson (Hejný et al. 2004). The aim is to guide the teachers to use such teaching methods and forms of work that, according to their long-term experience, raise pupils' interest in new knowledge, encourage dialog between the teacher and their pupils, enable the individualization of teaching and introduction of a variety of modern technologies, and stimulate pupils' teamwork. Success is not measured by the amount of mastered encyclopedic knowledge, but by the ability to solve problems and to respond adequately to unexpected situations. The FEP states that "Owing to its activity-driven and practical nature using appropriate methods, education motivates pupils to continue learning and leads them to a learning activity and to finding that it is possible to seek, discover, create and find suitable ways of solving problems" (FEP BE 2007, p. 9).

The diagnostic criterion is not the speed and reliability with which pupils are able to imitate the teacher's mathematical activity but the level of understanding they have achieved on a scale of cognitive mechanism. Individualization is a two-way process. One aspect is taking into account different cognitive types of pupils (e.g., girls' approaches are often different from boys' approaches), and the other one is connected to the speed of progress in their appropriation and grasping of concepts and procedures. Individualization with respect to different cognitive styles is enabled by a variety of model spectra and the accepting of individual algorithms.

4.1 Characteristics of the Educational Area of Mathematics and Its Application in FEP BE

Curricular documents express explicitly the demands on pupils' activity in mathematics lessons:

> In basic education, the educational area of Mathematics and its application is based primarily on activities typical for working with mathematical concepts and for using mathematics

> in real-life situations. It provides the knowledge and skills necessary for practical life and facilitates the acquisition of mathematical literacy. (FEP BE 2007, p. 27)

This clearly shows that the understanding of the sense of pursuing mathematics was gradually transforming and was modified to indicate the development of a mathematical literacy. Because of this indispensable role, mathematics permeates all basic education and creates the preconditions for further successful learning.

Education for mathematical literacy places an emphasis on a thorough understanding of basic ways of thinking in regard to mathematical concepts and their interaction. Pupils gradually learn various mathematical concepts, algorithms, terminology, and symbols, as well as methods for their application.

The educational content of this area is divided into four thematic areas in (FEP BE 2007):

- Numbers and numerical operations in stage one, which is followed up and expanded upon in stage two with the thematic area of numbers and variables
- Dependencies, relations, and working with data
- Two- and three-dimensional geometry
- Non-standard application problems and tasks.

Pupils learn to use technological devices (calculators, computer software, and various types of educational software) and to use some other aids, which enable pupils with difficulties in performing numerical calculations or in geometrical techniques to succeed. Students also improve in critical work through their use of informational sources (FEP BE 2007).

Instruction in this educational area focuses on the formation and development of key competencies by guiding pupils toward the application of mathematical knowledge and skills in practical activities, the development of memory, and the cultivating of combinatory and logical thinking. It also guides them to apply mathematical knowledge toward abstract and precise thinking, acquire a repository of mathematical tools, effectively use acquired mathematical skills, gain experience in the use of mathematical modeling, use precisely and succinctly the language of mathematics, include mathematical symbols, perform analyses, co-operate, and trust in their own problem-solving skills and abilities.

4.2 Problems with Teaching Mathematics

Representatives of the Union of Czech Mathematicians and Physicists stated that the overall state of mathematics teaching at our schools is not good. Mavrou and Meletiou-Mavrotheris (2014, p. 507) claim that:

> The methods of teaching mathematics in schools have been identified as contributing to the falling interest in mathematics. Empirical classroom research over several decades shows that, with some notable exceptions, mathematics instruction has been characterized by traditional, abstract formulation … Ideas are presented in an overly theoretical and abstract manner without sufficient opportunities for students to engage in problem-solving and experimentation.

In recent years, mathematics has begun to be viewed as a meaning-making activity. Some educational leaders and professional organizations in mathematics have been advocating for the adoption of more active learning environments that motivate learners and encourage them through authentic inquiry to establish the relevance and meaning of mathematical and scientific concepts (Samková et al. 2016). They have been stressing the fact that the core of school mathematics and science should no longer be the teaching of techniques and calculations that computers can do much faster and more reliably. Instead, the core should be the development of problem-solving skills that students will need to effectively live and function in a highly complex society. This shift is being reflected in educational policies and official curricula which advocate pedagogical approaches that support inquiry-based learning of mathematics.

Despite the extensive calls for learner-centered, inquiry-based pedagogical models, changing teaching practices is proving to be quite difficult. There is strong evidence that the implementation of inquiry-based teaching and learning of mathematics is not fully accepted. The authors' observations from Czech classrooms confirm that pupils and teachers often prefer problems in which the use of an adequate solving algorithm is evident. They prefer problems where they can determine the algorithm without any doubt. Thus, the burden of the tricky and difficult search for understanding of the problem is elevated, and the teacher's role is simplified to the mere discovery of the places where pupils make mistakes and to evaluation the "correctness" of their solutions (Novotná 2010).

In recent years, a lot of attention in research has been paid to critical areas of mathematics education from the point of view of teachers (Rendl et al. 2013) and of learners (Vondrová et al. 2015b). In the case of teachers, the authors tried to pinpoint what areas teachers consider critical, which pupil problems they consider as critical, and how they try to prevent them. What turned out to be difficult was to distinguish clearly whether teachers see pupils' problems as stemming from their conceptual understanding or rather their mastery of procedures. Teachers did not speak about more cognitively demanding skills, such as finding a solving strategy, argumentation, and reasoning. Also, they did not mention higher levels of conceptual understanding, such as explicit connections of the concept with the procedures in problem-solving, using different means of representations, and application in a variety of contexts.

In the case of the pupils, among what was discovered were deficiencies in the mental representation of a continuum of rational numbers, conceptual understanding of an algebraic expression as an object to be manipulated, breach of relationship between theoretical and spatial graphic spaces when interpreting and using a picture in geometry, and the conceptual understanding of measure in geometry. A strong tendency to use formulas in geometry and give preference to calculations over reasoning was identified (Rendl et al. 2013). Word problems manifested as one of the most critical sites of difficulty. Pupils also struggled with grasping a text, mathematization of situations when passing from specific to abstract representations, and making records or sketches. Pupils often perceived a graphical representation as a tool for explanation or clarification, but not for solving a problem.

5 Content of Mathematics Education and Textbooks

The textbook is often considered an important source of a pupil's (but also a teacher's) knowledge because:

- It is one of the program projects of education.
- It represents an important stage in didactical transformation of cultural contents into school education (Moraová 2018a, b).
- It is an inseparable part of a teacher's and pupil's everyday activity in lessons and outside of lessons too, as it is usually easily available to pupils (in comparison to other items of the multimedia system of didactic aids and tools).

The traditional conception of a textbook as a tool for handing over curricular content to pupils is gradually being overcome. Textbooks are now conceived of in a more complex way, as the mediator or facilitator of pupils' learning based on their own activities (Moraová 2018a, b).

At present, Czech teachers, or more often headmasters (Moraová 2014a, b), can choose between more than ten sets of textbooks for basic education, all of which have the official certificate of the Ministry. In contrast, the period before 1989 during the communist regime was a time when schools were allowed only one prescribed set of textbooks (no other had the official certification of the Ministry). Now there is a wide range of textbooks both with and without certification of the Ministry. The list of certified textbooks is available at the Ministry website. The selection of textbooks used by a school is up to the decision of the school (its headmaster). The Act of Education allows schools to use textbooks without official certification on the condition that they meet the demands of Framework Education Program and School Education Program. The headmaster must be able to explain why they have chosen an uncertified textbook, including why and how it better suits the school.

We will limit ourselves to mentioning only one set of textbooks—the set of mathematics textbooks of Hejný and his team on elementary education. This set of textbooks has sparked discussion both in professional and in parental communities. Hejný's concept of teaching mathematics is based on scheme-building (its theoretical background is presented in Hejný 2014). The basis for building schemes of mathematical knowledge is the creation of collections of isolated and generic models.[2] The process of solving a problem is connected to the activation of some of these schemes, their intertwining, and restructuration. Mathematics education is based on a set of tasks and problems that together form learning environments (LE). Basic properties of learning environments were formulated by Wittmann (2001) who stressed that they enable us to formulate a series of problems which help a pupil to understand deep ideas of mathematics. Hejný added three more requirements: "connection to a pupil's life experience, long-term nature (usable for pupils

[2] An isolated model is used in the case of future knowledge. A generic model is created from the process of generalization from a collection of isolated models (Hejný 2014).

of different ages, at best from grade 1 to grade 12), and differentiated nature (catering to needs of individual pupils)" (Hejný 2012, p. 46). Hejný and his coauthors created about 20 learning environments, for example:

> "– area starts with generic models of the area of square, rectangle, triangle, … within the environments 'tessellation,' 'paper folding,' 'grid paper,' 'geoboard,' and 'stick shapes';
>
> – small natural numbers start with generic models of address, status, operator of change and operator of comparison within the environments 'stepping,' 'money,' 'pebbles,' 'rhymes,' 'ladder,' …;
>
> – fractions start with generic models of 'one half,' 'one quarter,' 'divide into halves,' 'equal sharing,' … within the environments 'pizza', 'stick,' and 'chocolate'." (Hejný 2012, p. 47)

What the authors of this series of textbooks stress is an increase in pupils' interest in mathematics. A comparison of the results of pupils educated using the Hejný method and other methods show no significant differences (ČŠI 2017).

Mathematics textbooks deserve the attention they are due. As research studies show, planning and conducting a mathematics lesson often depends largely on the mathematics textbook. This means pupils spend a great deal of time working with a textbook and inhabiting its world. Most research focusing on mathematics textbooks analyzes their mathematical content and didactical treatment, including the proportion of textual and non-textual elements, language difficulty, and correspondence to Framework Education Programs. However, there are many more aspects that can be analyzed and studied. As Moraová (2018a, b) states, a textbook is not just a pedagogical document; it is a cultural artifact created in and for a specific society with specific cultural norms. This is crucially important with respect to whether mathematics is perceived by pupils as important not only at school but also in everyday life. It also affects whether the particular textbook will motivate the learners to gain new mathematical knowledge and skills.

6 ICT in Mathematics Education

A relatively new issue in mathematics education is the use of information technology in mathematics classrooms and its impact on teaching, methods, techniques, and approaches to mathematical content. Recent studies in mathematics education show that despite many national and international incentives whose goal is the integration of ICT into mathematics classrooms, such integration in schools is underdeveloped (Cox and Marshall 2007; García-Campos and Rojano 2008; Černochová and Novotná submitted for publication). Furthermore, the speed of the introduction of new technologies into mathematics classrooms is significantly slower than the speed of their development.

There are many reasons why the introduction of ICT into classrooms is a slow and painstaking process (Jančařík and Novotná 2011a). First, ICT offers a wide range of resources, which in consequence makes teachers unsure which of them to

use as well as when and how to use them. Also, there is a lack of information on the potential advantages and dangers of the use of ICT activities in lessons. Another reason is that teachers do not have any prior experience with the use of ICT in mathematics lessons from their own school years and pre-service training, which causes them to feel incompetent in using the devices. Unlike the new generation of pupils, who are digital natives, most teachers do not feel at home and comfortable in the area and have no hands-on experience with mathematical software (Černochová, Novotná submitted for publication).

ICT has a huge potential impact on teaching mathematics (Artigue 2002; Ruthven 2007; Vaníček 2010), but it is not always used for the benefit of the learners. Examples from practice show that in many cases teachers use technology "for show" (Jančařík and Novotná 2011a, b). These theatrical examples contribute very little to the development of mathematical knowledge and may be even counterproductive. Use of information technology in education should always be governed by the principle of efficiency. It is appropriate to use information technologies only in situations in which they really bring benefit, open new perspectives, or significantly decrease the amount of time needed for technical calculations.

As digital technology enters various areas of our lives and a wide range of activities, the extent of what we must be able to do with it increases. In schools, work with information technology is very often limited to one subject (named differently in different schools—information and communication technology, computing, informatics). However, it is difficult to develop pupils' skills in this area separately from other schoolwork. The concept of ICT in the Framework Education Program for Elementary Education is now being transformed according to the Strategy of Digital Education 2020 to make it reflect the needs of society and the labor market more. One of the priorities is to "improve pupils' competence in the area of work with information and digital technologies." The new conception of digital literacy penetrates all educational areas. New educational resources are presently being developed for children and pupils and piloted in selected schools. The new approach to the development of digital technology plans the integration of work with digital technologies in all subjects across the curricula. The expected outcome for the competence of pupils is to allow them to work with technology confidently, safely, critically, and creatively. This competency should be developed by all teachers in schools (MŠMT 2018).

7 Assessment

When discussing assessment, we have to distinguish between two areas. One of them is assessment in the classroom and the other is the high-stakes testing that provides the basis for evaluation of efficiency in the system of education.

7.1 Assessment in a Classroom

The predominant form of assessment in the Czech educational environment is summative (assessment of learning) (Žlábková and Rokos 2013). Formative assessment (assessment for learning) is less common. It still remains more or less on a theoretical level. For example, there are publications focusing on formative assessment whose authors present various examples of how to use methods of formative assessment, but the majority of them focus on the arts and humanities (Starý and Laufková et al. 2016). Mathematics teachers do not have sufficient support. The available examples of good practice are random and do not provide a systematic basis for the development of teachers' assessment skills and competence.

7.2 High-Stakes Testing

The testing of the whole population in selected cohorts has little tradition in the Czech Republic. Czech School Inspection started performing this kind of testing several years ago, and there have been several rounds of selective testing of different cohorts and different educational outcomes and competences (ČŠI 2017).

Apart from these, there are two exams in which large group of pupils participate: entrance exams to upper secondary schools and the state school exit (Maturita) exam.

7.2.1 Centralized Entrance Exams to Upper Secondary Schools with the Maturita Exam

On entering an upper secondary school whose graduates pass the state Maturita exam, the applicants must pass an entrance exam. The exam consists of a written test in mathematics and Czech language. Since 2017 the entrance exams have been centralized, organized by the Centrum pro zjišťování výsledků vzdělávání (The Center for Assessment of Educational Outcomes).

Although the tests are centralized, their results may not be used for assessing the quality of education or for any other form of comparison between schools. Before these entrance exams, many pupils take part in massive preparatory courses organized by private companies. Thus, pupils do not have equal opportunities under the exam.

7.2.2 School Exit Examination (Maturita)

The school exit examination after secondary school—the so called Maturita (from the word mature—the exam on the threshold of adulthood) has had a long tradition in our country. The basis for the Maturita exam was the standardized state exams opening the door to universities to those candidates who passed successfully. This

type of exam came to the Czech territory in 1849. The Maturita has always taken the form of an oral (and in some subjects written) exam sat in several subjects. Czech language and literature became compulsory exam subjects when Czechoslovakia was established (1918). Mathematics has never been compulsory for all types of upper secondary schools.

The changes in 1989 initiated a discussion on introducing a state Maturita exam (centralized didactical tests). One of the incentives was a recommendation from the OECD. It was expected that the results of a state Maturita exam could replace university entrance exams. There were several failed attempts to start a centralized exam. This ended when the state Maturita exam was put into practice in the spring of 2011. The Maturita has two levels of difficulty: basic and advanced. The Act of Education defines the Maturita exam as having two parts—the profile part organized by each school and the centralized part identical for all schools in the country. The centralized part consists of two compulsory exams: one in Czech language and literature and another either in a foreign language (taught at the school) or mathematics.

The issue of mathematics as a part of the Maturita exam and pupils' performance on it has attracted much attention. The Maturita in mathematics was planned to become compulsory at secondary grammar schools in 2020/2021 and the following year in all upper secondary schools. The only exception were art schools and schools for nurses and social workers. Up till now, a majority of school leavers choose a foreign language rather than mathematics. In 2018, mathematics was selected by 23.4% of graduates of which 22.3% failed the didactical test. The policymakers and public are currently discussing whether the compulsory Maturita in mathematics should or should not be postponed or not introduced.

7.2.3 International Comparative Studies

The Czech Republic had already joined the TIMSS (Trends in International Mathematics and Science Study) in 1995 and later got involved in PISA and other surveys and studies. In the beginning, the performance of Czech pupils was well above the average of the OECD countries in both tested populations (11- and 14-year-old pupils). The Czech Republic was by far the best performing post-communist country. However, the following testing brought disappointment. The test results of 8th graders on the TIMSS grew worse after 1999 (Tomášek et al. 2008). In the research study PISA, the performance drop of the Czech Republic between 2003 and 2009 was the greatest among all the countries that took part in both of the cycles (Palečková et al. 2010). In mathematics, the fourth graders dropped to the below-average zone in the 2007 TIMSS and their decline in comparison to 1995 was the greatest compared to all European and OECD countries involved in both surveys (Fig. 1.5). Czech's fourth graders struggled the most with problems on fractions and decimals, which was a consequence of including this curricular content in higher grades than in other countries (Potužníková et al. 2014). This does not mean that Czech pupils were significantly better in mastering other topics. Even

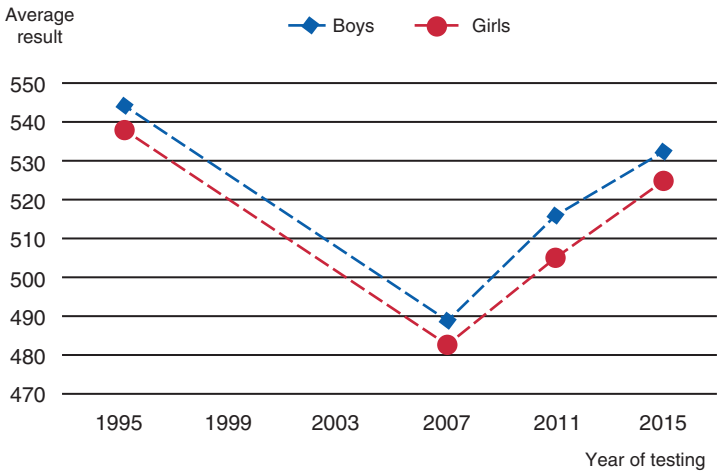

Fig. 1.5 Comparison of results of Czech 11-year-old boys and girls in mathematics TIMSS over the past 20 years. (Taken from Tomášek et al. 2016, p. 11.) NB: Only 14-year-old pupils were tested in 1999. The Czech Republic did not take part in the testing in 2003

if all problems involving fractions and decimals were to be excluded from analysis, the performance of Czech pupils would remain only average (Dvořák 2010).

It is difficult to determine the reasons for this drop in Czech pupils' performance. It is very likely a combination of several factors. For example, one of them may be pupils' lack of interest in mathematics or unwillingness to engage with it (Basl 2009), a result of insufficient development of mathematical reasoning that is an obstacle to mastering more difficult topics in higher grades (Hejný and Jirotková et al. 2013), or changes in society and in the place of mathematics within curricula (Dvořák 2010). The performance of pupils in mathematics improved in 2011 and improved again in 2015. However, the achieved results were still worse than in 1995.

One of the possible reasons for the decline may also be the addition of one extra year to the primary school level (from 4 to 5 years); this was the case for fraction problems. Another reason may be the introduction of FEP; the result of which was that much more attention at schools had to be paid to administrative demands than to the quality of teaching. Also, the FEP defines the required minimum of knowledge and skills.

These comparative studies provided some information that help to characterize the system of education. PISA 2000 implies that the Czech Republic is a country where pupils' performance is very strongly dependent on the socioeconomic status of their parents (Straková 2009).

A survey of fifth and ninth grade pupils' results in Czech, mathematics, foreign languages, and areas from other subjects was conducted in 2016–2017. The least successful subject was mathematics in the ninth grade, where every third pupil was able to solve only two-fifths of the test problems correctly. The mathematics results of the fifth graders corresponded to the expectations. The results differed regionally. The most successful group were pupils from Prague. As expected, pupils from

lower secondary grammar schools performed on average better than elementary school pupils. The results of fifth graders in mathematics were not affected by the extent to which teachers used the techniques of the so-called Hejný method (it is not used by 57% of the surveyed teachers, and it is more widespread in non-state schools). The results are more closely connected to what the teacher regards as the main goal of teaching mathematics (helping pupils find positive attitudes to mathematics or teaching the entirety of the FEP) (ČŠI 2017).

8 Mathematics Teacher Education

8.1 Pre-service Mathematics Teacher Education

The first educational institutions for teachers were established in 1774 (Mikulčák 2010). The Austro-Hungarian Education Act of 1869 introduced teacher-training institutes. Students of teacher-training institutes graduated after taking the Maturita, but the graduates were not fully qualified for the teaching profession. Having passed the examination, they next had to undergo a prescribed practical training. Then they passed an examination in pedagogical competence and acquired professional qualification. Since the nineteenth century, teachers in primary and lower secondary schools have tried to elevate their training to the university level. Charles University, and later other universities, initiated teacher training courses (Novotná 2019).

After the establishment of Czechoslovakia in 1918, the Ministry as well as the public started to oppose the concept of university education for primary school teachers. In 1946, the government set down provisions for the establishment of faculties of education, mostly within universities. In 1953, faculties of education were changed to higher schools of education. Three of these (located in Prague, Olomouc, and Bratislava) were later renamed pedagogical colleges. In 1959, 18 institutes of education were set up, bearing the title regional universities. At the time, pre-service primary teacher education was 3 years long and 4 years for lower secondary school. Upper secondary school teachers had to get their training at universities (e.g., at faculties of natural sciences, arts, or the Faculty of Mathematics and Physics of Charles University). Pedagogical institutes existed until 1964 when faculties of education were brought into existence again (Mareš 2007).

Until 1989, teacher training was the same at all faculties in the country which trained future teachers. All the institutions had the same study plans, curricula, textbooks, number of lessons, and student duties. Primary school teachers were generalists, and secondary school teachers studied to teach two subjects. The possible combinations of the two subjects were fixed; students could not choose a combination of their personal preferences or interests. Each education faculty had a list of allowed subject combinations. The most common combinations of mathematics were with physics, chemistry, biology, geography, and physical education. For example, a combination of mathematics and Czech language was not offered; it was

regarded as too time demanding. The number of lessons and duties was divided into three more or less equal parts: one-third was the common core subjects (especially pedagogy and pedagogical psychology), and the other two-thirds were the two studied subjects. Teaching practice was part of the common core subjects.

The basic change in university teacher education, which took place after 1989, opened the way for faculties to define the content and methodology of teacher training independently. Much discussion was generated by questions on the issues of teaching practice and the relationship between teaching a specific subject and the common core (pedagogical and psychological components) of teacher training. A lot of attention was paid to the teacher's work with pupils (diagnostics, communication, evaluation, etc.) (Beneš and Rambousek 2007).

The traditional way of training prospective teachers of mathematics emphasized scientific knowledge. Nowadays, facing the necessity of preparing teachers for a new flexible school system, the focus is much more on the didactic aspects of teacher training. The starting point is to state which parts of mathematics are necessary for future mathematics teachers.

Not all faculties in the Czech Republic have moved toward this new system of teacher training. Some of them stress only on pure mathematical content and tend to ignore the didactic aspects. In the study model for lower and upper secondary school teacher training, there is a great variety in the organization of the program. For example, noticeable differences can be found in the content and extent of knowledge that pre-service teachers have to master in subjects at the master's level, concerning how much attention is paid to the school teaching practice as well as how much emphasis is put on the common core studies (e.g., pedagogy, psychology, Czech language, foreign language, human biology, philosophy of education, educational technology, and introduction to logic). The attention paid to common core subjects has, in general, been reduced, and the focus has been shifted to studied disciplines, both the subject content knowledge and the pedagogical content knowledge (in the sense of Shulman 1987)

The basic legislative framework for the training of teachers and other education staff is the Higher Education Act of 1998. Based on that act, the responsibility to define the content and organization of teacher training programs was handed over to each relevant higher education institution. Internal regulations of individual teacher training colleges thus set out curriculum content and its organization for both standard and alternative ways of obtaining the teacher qualification. However, each study program, even the programs for life-long teacher education, had to be accredited. Starting in 1990 the accreditation was done by the Ministry, whose advising body was the Accreditation Commission. The Accreditation Commission [Akreditační komise][3] was established in 1990 by the law and was closed in 2016. It took care of the quality of higher education and carried out comprehensive evaluations of teaching, scientific research, development and innovative work, and artistic

[3] Statute for the Accreditation commission available from http://www.msmt.cz/areas-of-work/tertiary-education/statute-of-the-accreditation-commission?highlightWords=soubor

and other creative activities of higher education institutions. The chief means of achieving these objectives were by:

– evaluating the activities of higher education institutions and the quality of accredited activities and publishing the results of such evaluations
– reviewing other issues affecting the system of higher education, when asked to do so by the Minister, and expressing its standpoint on these issues.

As per the government's decision in September 2016, the Accreditation Commission was replaced by the National Accreditation Authority for Higher Education [Národní akreditační úřad pro vysoké školství][4]. This is an independent body with the power to make decisions without any approval from the Ministry.

The qualification needed for the teaching profession is given by Act No. 561/2004 (2004). The original demand was that all teachers would have to be graduates of pre-service teacher education programs. However, a lack of qualified teachers in some regions resulted in the modification of this original condition in its amendments. There are now more ways of gaining the needed qualification. However, the condition that all teachers must be graduates of a master's degree program remains. Several faculties offer programs oriented at the obtaining, extension, and deepening of teacher qualification (not offering an academic degree but guaranteeing the pedagogical qualification). Teacher education which is not provided in higher education institutions, but at lower levels of the education system, is regulated by the Education Act and related decrees (Eurydice 2016).

8.2 University Programs Preparing Teachers for Their Profession

In 2017, the Ministry accepted the Framework of Requirements on Study Programs providing qualifications for regulated teaching professions (MŠMT 2017). It is a methodological document that helps to evaluate university programs which prepare teachers for their profession. The material is relevant for study programs preparing pre-primary, primary, lower secondary, and upper secondary schools as well as programs training special pedagogues, psychologists, and pedagogical staff for after-school clubs and other childcare institutions. The material formulates general grounds on whose basis the Ministry assesses study programs.

Fulfilling the framework requirements guarantees that graduates of the study program will be adequately equipped for regulated teaching professions. The main goal for the division of the individual components of the study program is to define a well-balanced proportion between each of them. The bottom limit in each of the

[4] Government regulation No. 274/2016 coll. on standards for accreditation in higher education available from http://www.msmt.cz/vzdelavani/vysoke-skolstvi/preklad-zakona-o-vysokych-skolach?highlightWords=accreditation

Table 1.3 Proportion of components of professional teacher training (MŠMT 2017)

Component	Pre-primary	Primary	Lower secondary	Upper secondary
	in % of study plan			
Teaching propaedeutic	23–30	26–32	20–25	20–25
Subject component with the subject didactics (for lower secondary education divided into first and second subject and both subject didactics; for upper secondary divided into first and second subject)	45–50	50–55	25–30 25–30 10–15	25–33 25–33
Teaching practice	10–15	10–15	8–10	8–10
Work on bachelor/diploma thesis	5–10	5–10	5–10	5–10

components determines the minimum and is binding. The top limit can be exceeded; it is only a recommendation. The proportion of the individual components of the studies is given in percent, credits, and hours according to the European Transfer and Accumulation System. The teaching propaedeutic component involves pedagogical-psychological training, special pedagogical training, general didactic, developmental psychology, and other disciplines (e.g., ICT in education, foreign language). Teaching practice is perceived as supervised and reflected, both in the form of lesson observations, and ongoing, continuous teaching practice.

Table 1.3 presents the distribution of individual components for pre-primary, primary, lower secondary and upper secondary teacher training.

Examples of the differences between the designs of mathematics teacher training programs are presented in Novotná (2019).

8.3 Teacher Professional Development

The in-service training of education staff includes study programs for unqualified teachers which lead to formal teacher qualification and programs leading to other qualifications (such as the school advisor qualification) or to professional development courses (for continuing education). This in-service training is delivered within the lifelong learning system at higher education institutions, in establishments for the in-service training of education staff, or other institutions accredited by the Ministry. Additionally, this type of education can be gained through self-study.

In-service teacher education is not compulsory in the Czech Republic. In the mandatory[5] pedagogical documentation of schools, there is a 1-year school plan for in-service teacher education. At the end of the school year, this information must be published in an annual school report. Schools have, in their budgets, finances for

[5] Although in-service teacher education is not compulsory in the Czech Republic, the headmaster is expected to encourage and keep a record of participation of the teachers who attend in-service training seminars and courses.

covering the costs of the training. At this point, there is nothing that would force or motivate in-service teachers to participate in in-service teacher education if they are already fully qualified teachers. The in-service teacher education plan is the headmasters' responsibility, and it is up to the headmasters to motivate their teachers to participate. However, if the teachers are not motivated, their attendance at the training will be purely formal. There is a lack of quality control regarding in-service teacher training courses. Most of them are run by private bodies despite the need to have the seminars accredited by the Ministry.

9 Discussion and Public Opinion

Discussions on educational issues take place at many different levels. They are not just direct "face-to-face" discussions. They are in the media, in the press, in special journals, magazines, websites, and also in the non-expert media. The discussions focus on all aspects of education, such as educational content, forms, methods, and financing of schools. The Czech population has traditionally shown a lot of interest in education. Voices of the people are, however, far from unified. On the contrary, many contradictory opinions are often expressed.

The Ministry, which prepares strategic documents in cooperation with other departments and educational experts, permits a public debate on the drafts of proposed documents. Discussions are organized on drafts that are presented to the professional public. All inhabitants of the Czech Republic have the right to express their opinion on the issue and its solution. However, the Ministry or other government institutions may not consider these opinions as relevant and may not take them into account. The web pages of the Ministry provide contacts where inquiries from the press and media may be addressed. The website also offers Ministry press releases, both for the media and the general public. Various professional and interest-based organizations that focus on education use this as an opportunity to express their views on educational issues. The website also provides an archive of Ministry press releases since 2009.

Illustration 1: The Ministry organizes discussions on possible changes to the FEP, on state Maturita exams, on entrance exams for upper secondary schools, on the possibility of the introduction of the cut-off score (i.e., the minimum score needed to be accepted to a study program concluded by the state Maturita exam), or on support of novice teachers, etc.

Discussions, round tables, and public hearings on educational issues with education experts and others are also organized by the Committee on Science, Education, Culture, Youth, and Sports of the Parliament of the Czech Republic. Members of professional organizations are also invited to participate in these discussions. In the case of mathematics education, the corresponding organization is the Society of Mathematics Teachers of the Union of Czech Mathematicians and Physicists. Conclusions and unresolved issues from these discussions are usually published.

Illustration 2: On March 14, 2018, the Committee on Science, Education, Culture, Youth, and Sports of the Parliament of the Czech Republic held a meeting. One of the discussed issues was teaching mathematics at the primary and secondary school levels. Representatives of the Union of Czech Mathematicians and Physicists were invited to the meeting. Most of the discussion focused on the use of the Hejny method (https://www.h-mat.cz/en) in teaching mathematics. The discussion continued (and in fact still continues) even after the meeting finished, both in the press and on other platforms. The society of teachers of mathematics of the Czech Union of Mathematicians and Physicists published its opinion on its webpage (the official opinion is available in Czech from https://suma.jcmf.cz/_files/200000120-64c5b65c62/stanisko%20SUMA%20k%20metod%C4%9B%20Hejn%C3%A9ho.pdf).

Discussions on current issues in Czech education also are held at another level—at the country's regions and regional governments. Regions are responsible for education on the regional level; they establish the majority of upper secondary schools. Also, meetings of regional departments of education with the press and the public are organized at this level. At these meetings, it is quite common to hear the opinions of companies and businesses because they are the potential employers in the region and they often take part in the training of future graduates. In addition, MPs or representatives of the Ministry are invited to these meetings. If this level is compared to the level of the national government, we can state that regional authorities have a clearer idea of the situation in the terrain of their region than national government officials. However, their role is not to prepare strategic documents. Regions also need to know the views not only of professional groups but also of the public. The public is therefore invited to contribute to these discussions, although not much attention is paid to their opinion. The situation always changes in a pre-election period, when all the parties try to get votes by promising to improve education in their regions. However, many of these promises remain only promises, and their implementation after the election never happens.

Illustration 3: Discussions at the regional level usually do not focus on individual subjects, i.e., they do not focus exclusively on mathematics. They address issues of high relevance for the particular region. For example, the Vysočina region initiated a discussion on upper secondary school education. The Liberec region initiated a discussion on projects in education, on school and kindergarten capacity, and on improving the quality of schools. If many regions regard the same issue as highly relevant, the regional representatives may propose to the Ministry a discussion of the issue on the national level.

The third important level is the local level. Local authorities are responsible for elementary and preschool education. These are the levels closest to the average citizen. The public is invited to join in the discussions. The stimulus for the discussion might be a local issue or issues coming from above. Although there is no relevant research focusing on this aspect, we can assume that there is less interest for making strategic decisions about the system than in making decisions on local issues.

Citizens have the right to attend the meetings of local councils. However, they are allowed to enter discussions only with the consent of the local council. The extent to which citizens are active in these meetings varies greatly, from intense involvement to a lack of interest.

A burning issue at all levels is who should be making decisions about education. The answer to this question is tied to the question of who bears the responsibility for education. Responsibilities of two types are to be considered: individual and social responsibilities. Individual responsibility is born by pupils/students themselves and their parents. However, neither pupils nor parents have much freedom to make decisions in the current system. The question is how to adjust the decision-making process to get pupils and parents more involved.

Social responsibility is connected especially (but not exclusively) with the role of the state in education. The state should, for example, set conditions under which parents and their children can make responsible, sensible decisions, and it should guarantee equity. However, it is not easy to answer how this should be achieved (Pol 2007; Štefflová and Švancar 2004).

The platform through which the public can influence what is happening at schools is the school board. The public can also join open discussion forums to express its opinions on what is happening in the education system. An example of such a forum is the methodological portal www.rvp.cz, where there are for example:

- a discussion module that contains news, such as what is new in forums and in the community, online meetings, and forums focusing on a selected subject.
- archives of discussions on the methodological portal.
- a guide through the updated FEP.
- links to other websites of interest for teachers and educators (see e.g., https://www.suma.jcmf.cz/).

Documents connected to education and the educational system, such as archives of press releases before 2010 and articles from various areas of education (topics include teachers, the state Maturita exam, Inclusive Education, the Comprehensive National Entrance Exam, Education Reform, and Amendment to the Act of Education), are available at the portal www.eduin.cz. The portal also publishes articles on current issues in education, opinions, and discussions.

There are also several professional organizations and societies. They may organize discussions on education from their own initiatives or may be invited to participate in discussions on educational issues. Some of these organizations, e.g., the Union of Czech Mathematicians and Physicists, are very active. The Union not only takes part in discussions but also frequently initiates them.

10 Conclusions

Let us conclude this chapter by drawing attention to several general issues that are not linked exclusively to mathematics education, although they have an impact on it. Since 1989, the Czech educational system has evolved significantly from a

centralized, unified system to an open and pluralist system. The responsibility for education is shared by the Ministry and the regional and local authorities. Private persons and institutions are able to enter the domain of educational debate and policy.

One of the events that generated discussion on education and was at the beginning of the ongoing changes was the significant drop in Czech pupils' performance both in the TIMSS and the PISA from 1995 to 2007 (see Fig. 1.5). In order to stop this decline and bring about improvement in pupils' performance on an international scale, it was recommended that pupils actively participate in constructing their own knowledge. Also, the search for new teaching approaches was encouraged.

As stated in the text above, interest in educational issues is quite intensive at all levels of the Czech Republic. The website www.eduin.cz (see l. 1223) hosts a discussion that was begun by the initiative "Education first" that comes out of the idea that "education is a basic value of our society." This initiative not only names problems but also proposes possible solutions. The following are considered the fundamental problems of our system (and are not listed according to their priority):

– The world changes very fast and the Czech system of education lags behind.
– People see education as an urgent issue.
– Czech pupils have the greatest dislike of schooling out of all the OECD countries.
– The role of the teacher is changing, but neither teachers not the public are ready for this change.
– The Czech Republic invests nearly the smallest portion of its GNP into primary and secondary education compared to other OECD countries.
– Teachers are growing old but only about 40% of graduates from faculties of education go on to teach.
– Salaries for headmasters and teachers are below the national average.

The initiative recommends that the good parts of the Czech system of education should be preserved. This is the extensive network of public universities, high level of literacy among the population, the relatively good level of autonomy among schools and headmasters, a low early school leaving rate, and the activity of parents whose aim is to establish new and innovative schools. Teachers and headmasters should be supported by a change in the system of benefits and a better career system. Some of the issues that the initiative perceives as problematic are uniform state entrance exams (they were introduced into the system in 2016–2017), the current organization of the state Maturita exam (upper secondary school leaving exam), and the Educational Staff Law.

Let us stress here that no positive change can happen unless due attention is paid to teacher training. Until 1989, the system of life-long teacher education was very formal and in terms of the professional development of teachers not very functional. This system of in-service training was abolished in 1991 and has so far not been replaced by a well-structured system. In-service teacher education is not compulsory in the Czech Republic. The in-service teacher education plan is the headmasters' responsibility, and it is up to the headmasters to motivate their teachers to participate. If the teachers are not motivated, their attendance is largely a waste of time (Novotná 2019).

It also seems that there is a lack of control over the quality of offered in-service teacher training courses. Most of them are run by private bodies, and although the seminars must be accredited by the Ministry, many of them seem to be run for profit rather than for improving teaching at schools (Novotná 2019).

The last regulation that defined in-service teacher education is from September 2005. A new legislative framework is still in the preparation phase, and it is not clear when it will be approved. However, it promises a career system for teachers that would incorporate in-service teacher training as a prerequisite to career growth and promotion. The prepared career system is designed to help increase the prestige of the teaching profession in society, stimulate interest in the teaching profession among young people, and to give new and experienced teachers a perspective on professional growth throughout their professional lives. The career system will be based on the principle of interdependence in professional development, the career system, remuneration, the rights and obligations of teachers to develop their professional skills, and the right to choose possible career paths.

The authors of this chapter are aware of the fact that in this chapter they could present only a small part of what could be said about the development and trends of the Czech educational system. Interesting results could be received by a deeper analysis of doctoral studies in the theory of education, as well as by analysis of educational organizations, their requirements, students' motives for enrolling in courses, etc.

However, our study of the organization of the educational system and teacher education and its changes since 1989 points toward some important issues for further research: on the one hand, the influence of stereotypes, on the other hand, the effort to prepare teachers open to new educational approaches and teaching methods flexible when including new approaches into their own teaching, and shifting toward a pupil-centered type of teaching.

The new challenges do not come from the educational system only, but from the outside world. For example, the multicultural dimension of modern society constitutes one of the most significant changes to have influenced schools in many European countries, especially at the primary and middle school level. The teaching profession is all the more difficult because the teacher is usually not sufficiently prepared to deal with a new classroom context as well as the added challenge of reaching pupils of a migrant background, coming from countries with different cultures and languages. The teacher is seldom aware of the need to rethink and, if necessary, modify his/her methodological and pedagogical approach. Additionally, the teachers do not know how to use the multicultural dimension in a constructive way when teaching. This attitude is even more evident among mathematics teachers, who often consider their subject universal and culture-free.

An important discussion on the future of the Czech system of education took place on February 29, 2019. About 200 major actors in the system of education in the Czech Republic participated. This was the official, formal opening of the Preparation for the Strategy of Educational Policy of the Czech Republic until 2030. The aim of the conference was to open to the public and experts a debate on key challenges and visions of education in the Czech Republic as well as to look for

ways of achieving it. Part of the conference focused on revisions to the current framework of education programs. The new strategy should answer all the key problems of today's system of education. It will define the vision, priorities, goals, and measures in the area of educational policy in the Czech Republic for the period extending to the horizon of 2030. The process of making the draft of Strategy 2030 should be open and transparent. The aim is to take into account the results of consultations with the greatest possible number of actors in the educational system of the Czech Republic.

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