







Mathematical Modelling of Residual Lifetime of Pumping Units of Electric Power Stations

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Abstract. With long-term operation of pumping and other equipment, there is often a need to extend the life and assess the residual life. Modern life-time assessment methods are based on three-dimensional modelling and the finite element method. Within the framework of these methods, geometric models are constructed that take into account the thinning of the walls of the pumps as a result of operation. By constructing geometric models, finite element models are constructed. These models are made taking into account the conditions of loading of the structure, tightening of bolted connections, technological and temperature loads. The constructed models allow to assess the strength of the pump casing and the main bolted connections during normal operation and gyro testing. The paper assesses the cyclic strength of these structural elements. The results of the work confirmed a sufficient residual life-time for safety pump operation in the next 15 years.

Keywords: Life-time prediction · Long-term operation · Finite element model · Pumps

1 Introduction

The problem of the exhaustion of the design life-time of the equipment of nuclear and thermal power plants is relevant for Ukraine. The limited financial resources do not allow building new power units. A rational solution in this case is to conduct a set of measures to assess the residual life-time and extend the design life-time [1, 2].

For pumping units of thermal power plants and nuclear power plants during operation, thinning of the walls of the housing and the cover is typical. The cause of thinning is corrosion and erosion wear. As a result of wall thinning, the redistribution of mechanical stresses in the walls and pump cover occurs. The change in stresses has a negative effect on the static and cyclic strength of the pump unit. Thus, to assess the residual life-time, it is necessary to calculate a stress state. Stress data should be used to evaluate static and cyclic strength.

The depletion of the life-time is significantly affected by the operation of the pump in various modes. Pumps operate in the following modes. There are normal operating conditions (NOC), hydrotest mode (HM).

To assess the life-time various methods are using [3–14]. These methods based on different ideas. Some works use the technique of assessing the reliability and probability of failure [3, 4, 7, 9, 10]. Others based on deterministic models [5, 6, 8]. All these works use the principle of determining the stress state, on the basis of which the life-time is estimated.

2 Objectives

The objectives of the paper are to estimate the residual static strength and residual life-time of the X45/90a centrifugal pump. This pump operates in the NOC and HM. For the assessment of wall thickness due operation the results of ultrasonic test is used.

The investigation of a residual life-time has made on example of the centrifugal pump of type X45/90a. This pump was made in 1985 by the Kataysky pumping plant. Design service life is 30 years. Technical characteristics and parameters of the pump are given in Table 1.

Table 1. Technical characteristics and parameters of the X45/90a pump

Characteristics	Values
Weight	218 kg
Working environment	Boron Water
Supply of working fluid	45 m ³ /h
Pressure of pumped liquid on an input	–
Pressure of pumped liquid on an input	from –40 to +90 °C
Temperature of pumped liquid on an input	0,9 MPa
Pressure of pumped liquid at the exit	0,675 MPa
Temperature of hydro tests	10 °C

The planned life-time extension of the X45/90a pump is 15 years (until 2032). For details of pump case is selected walls thickness for assessment of residual strength. According to the thickness measuring of the pump the minimum sizes of walls are determined (the procedure of measurements is carried out in 2017). It is accepted that wall thinning evenly in all body parts by 3.7% per 100 working hours (the average operating time of this pump makes 135 h a year). These data correspond to the average speed of erosion-corrosive wear of pumps of similar types. The accepted values of uniform wall thickness of case shaped parts for the X45/90a pump taking into account the planned term of extension of 15 years are given in Table 2. Values have selected as rounded to the next smaller value.

Table 2. The accepted values of thickness of walls case of the x45/90a pump

Case shaped part	Actual wall thickness, mm	The expected operating time taking into account the planned extension term, h	The expected thinning of walls by the end of the planned extension term, %	Wall thickness for assessment of residual strength, mm
Case wall	14.2	2025	7.5	13.1
Cover	16.4	2025	7.5	15.1
Inlet pipe	15.8	2025	7.5	14.6
Outlet pipe	20.5	2025	7.5	18.9

3 Development of the Pump Model

For carrying out pump strength calculations the technology of three-dimensional finite-element modeling has been used. ANSYS software for engineering calculations is used.

The three-dimensional geometrical model for carrying out calculations is given in Fig. 1 in a general view and in Fig. 2 in a section. Walls thicknesses of case parts are set according to data of Table 3.

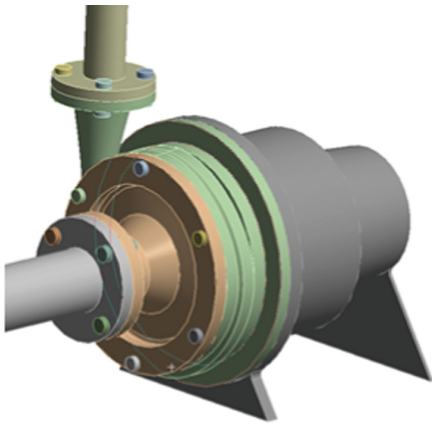


Fig. 1. Three-dimensional geometric model of the pump

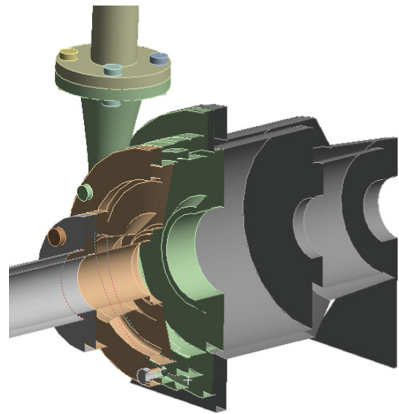


Fig. 2. Three-dimensional geometric model of the pump by section of the vertical plane

For calculation of stress the finite element (FE) mesh has applied on geometrical model. A linear finite element of hexagonal and tetrahedral shape has been used. Cover and the entering branch pipes are constructed with regular mesh of hexagonal FE, and the pump casing is meshed by free tetrahedral mesh. After splitting into a mesh of finite elements all details of the pump have no degenerate FE. The basic structural elements have at least two elements on thickness and the ratio of the parties is sustained. The mesh used for calculations is presented in Fig. 3.

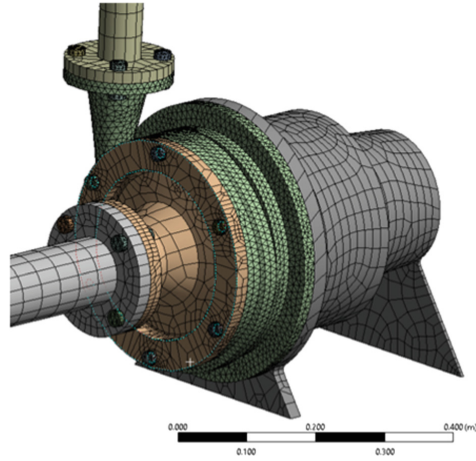


Fig. 3. The finite element model mesh (general view).

Table 3. Mechanical characteristics of the material 12Cr18H9Ti

Temperature, T, °C	Elastic modulus, E, GPa	Poisson ratio, ν	Density, ρ , kg/m ³	Yield strength, σ_y (R _{0.2}), MPa	Ultimate tensile strength, σ_B (R _m), MPa	Allowable stresses ^a , $[\sigma]$, MPa	Coef. of thermal conductivity, λ , W/(m °C)	Coef. of linear expansion, α , mkK ⁻¹
20	205	0.3	7900	196	441	131	15	16.4
50	202	0.3	7900	196	422	131	–	16.4
100	200	0.3	7900	196	412	131	16	16.6
150	195	0.3	7900	177	402	118	–	16.8
200	190	0.3	7900	167	383	111	18	17

Table 4. Mechanical characteristics of steel 1020

Temperature, T, °C	Elastic modulus, E, GPa	Poisson ratio, ν	Density, ρ , kg/m ³	Yield strength, σ_y (R _{0.2}), MPa	Ultimate tensile strength, σ_B (R _m), MPa	Allowable stresses ^b , $[\sigma]$, MPa	Coef. of thermal conductivity, λ , W/(m °C)	Coef. of linear expansion, α , mkK ⁻¹
25	200	0.3	7859	216	402	144	52	11.5
50	197	0.3	–	206	392	137	–	11.5
100	195	0.3	7834	206	392	137	50.6	11.9
150	192	0.3	–	206	392	137	–	12.2
200	185	0.3	7803	196	373	131	48.6	12.5
250	180	0.3	–	196	373	131	–	12.8
300	175	0.3	7700	177	363	118	46.2	13.1

The following materials as steel grade 12Cr18H9Ti (for the case) and steel 1020 (for bolted connections) are selected for calculations. Their physical and mechanical characteristics of materials are respectively presented in Tables 3 and 4. Rated

allowable stress for the elements of the equipment and pipelines loaded with internal pressure is accepted minimum of the following values calculated by (1). Where R_m is the tensile strength; and $R_{0.2}$ is the yield strength.

$$[\sigma] = \min\{R_m/2.6; R_{0.2}/1.5\} \quad (1)$$

For bolted and stud connections pressure calculated by (2).

$$[\sigma]_w = R_{0.2}/2 \quad (2)$$

4 Mathematical Models

To calculate the stresses, displacements and temperature fields finite element method is used. In the framework of this method, the volume of the investigated part is meshed into finite elements. For each finite element, its resolving equations are written. The solution of the equations is stitched in the mesh nodes. This leads to the solution of a system of linear equations of order equal to the number of degrees of freedom of all finite elements.

The work consistently solves the problem of stationary heat conduction (3) and the problem of static stress distribution (4).

$$[M]\{\dot{T}\} + [K]\{T\} = \{f_T\} \quad (3)$$

$$[K]\{q\} = \{f_N\} + \{f_{Temp}(\{T\})\} \quad (4)$$

where $[M]$ – global thermal capacitance matrix;

$[K]$ – global stiffness matrix;

$\{T\}$ – nodal temperature vector;

$\{q\}$ – nodal displacement vector;

$\{f_T\}$ – temperature loading vector;

$\{f_T\}$ – mechanical loading vector;

$\{f_{Temp}(\{T\})\}$ – mechanical loading due thermal expansion vector.

5 Analysis of Static Strength

For assessment of residual strength of the pump taking into account the possible extension of service life, calculations on static and cyclic strength at two options of loading are carried out:

1. to the normal operation conditions (NOC);
2. to hydrotest mode (HM).

For calculation on static strength under normal conditions and during hydro tests the group of strength characteristics should be used. They are applied under different

conditions of loading and/or different actual or conditional types of the stress-strain state arising in construction.

5.1 The Analysis of Static Strength of the Case at NOC

The following calculation parameters are set:

- rigid sealing on the supporting surfaces of pump racks;
- rigid fastening in the axial direction for the pipe supplying the working fluid and its outlet pipe. In the plane perpendicular to the axis of these pipes, elastic supports with a stiffness of 0,1 MN/m have placed;
- volume force is gravity;
- the bolt tightening torque according to Table 5;
- on internal surfaces convective heat exchange “water – steel” is set (coefficient of heat emission, 500 W/ m² °C);
- on outer surfaces convective heat exchange “steel – air” is set (coefficient of heat emission, 5 W/ m² °C);
- ambient temperature is accepted by 25 °C;
- external pressure is specified as normal.
- internal pressure to equally calculated pressure in each step and pump outlet pressures.

Table 5. Standard forces and torques of tightening

Standard size	Preload force, kN	Tightening torque, N·m
M12	9.0	41
M16	17.0	102

At the first stage the problem of the prestressed state caused only by tightening of studs has solved. In Fig. 4 distribution of strain intensity (equivalent strain by Mises’s criterion) which is formed in the studs under this loading mode.

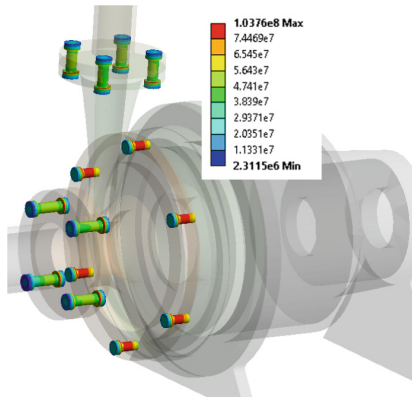


Fig. 4. von Mises stress in the bolts and studs after its tightening, Pa.

The analysis of the pre-stressed state of the studs showed the presence of compressive stresses in the studs equal 103,7 MPa, which is less than the maximum allowable value of 137 MPa. The high level of stress has formed in the pump casing (Fig. 5), at the same time the received stress (98,5 MPa) is also less then limit one (131 MPa).

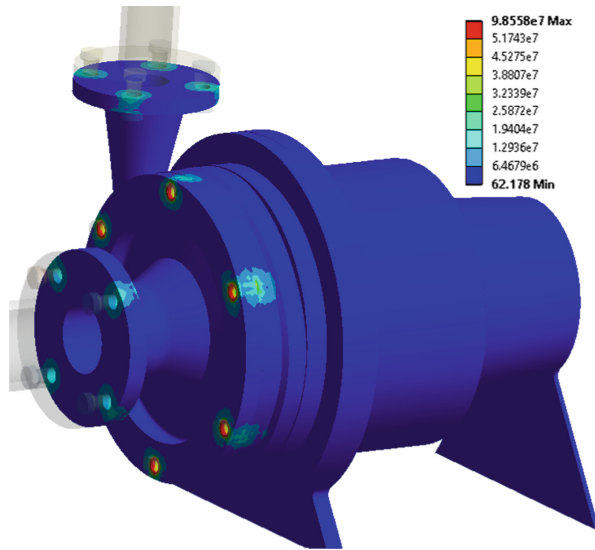


Fig. 5. von Mises stress in the pump casing and covers after bolts and studs tightening, Pa.

In Fig. 6 shows the temperature distribution across the pump casing.

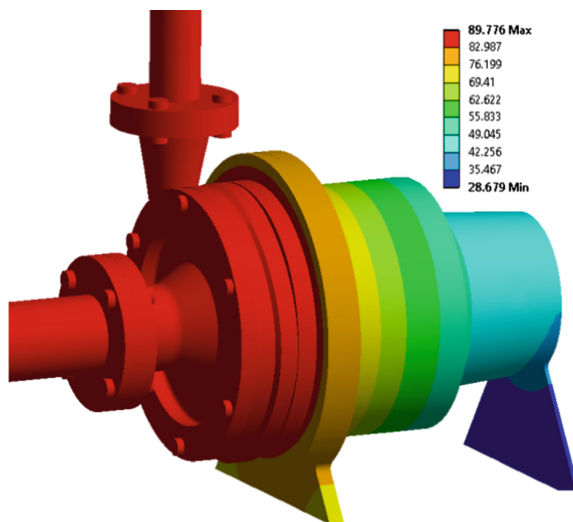


Fig. 6. Temperature distribution over the pump casing at NOC, degrees of Celsius.

The pressure distribution scheme is shown in Fig. 7.

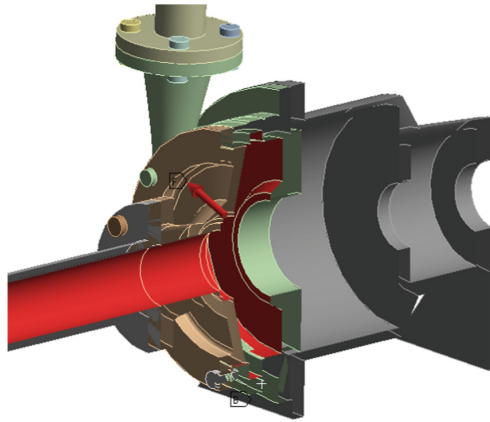


Fig. 7. Loading the pump with internal pressure at the normal operation conditions.

The results of calculations with this loading scheme are shown in Figs. 8 and 9. Maximum stresses formed in the elements of the body.

According to the results of the calculation (Figs. 8 and 9), the strength condition is satisfied (the maximum stress value is $(\sigma) = 98.9$ MPa, which is less than the limit value $[\sigma] = 131$ MPa).

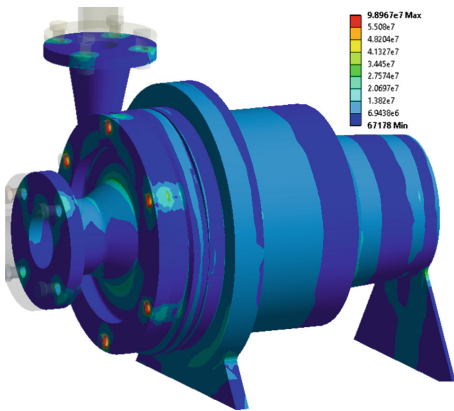


Fig. 8. von Mises stress in the pump case under normal conditions operation, a general view, Pa.

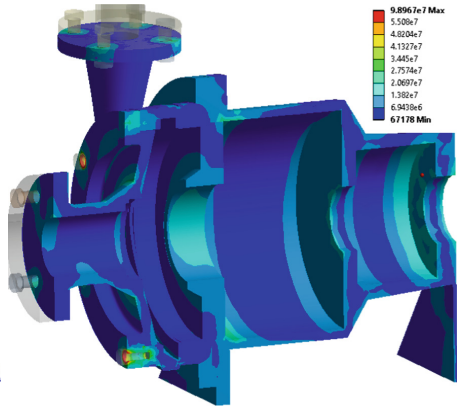


Fig. 9. von Mises stress in the pump case under NOC, a sectional view, Pa.

5.2 Analysis of the Strength of Bolts

Figure 10 shows the von Mises equivalent stress distribution in bolts and studs under NOC.

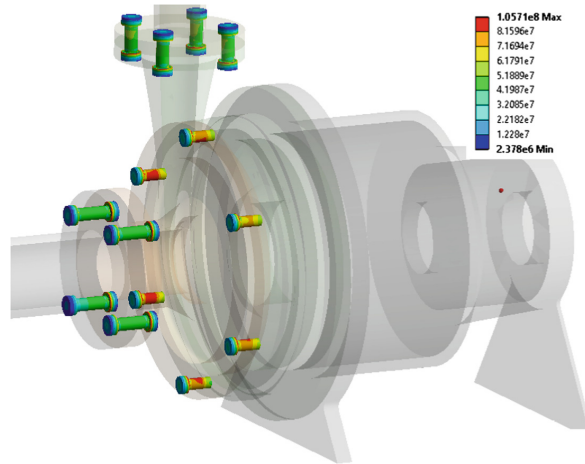


Fig. 10. von Mises stress in bolts and studs at NOC, Pa.

The maximum stress in the bolted joint is 105.7 MPa, which is less than the limit value $[\sigma] = 137.0$ MPa. Thus, the strength condition is satisfied.

5.3 Static Strength Analysis with Hydrotests

To check the system integrity, it is tested by increased pressure. It is also calling hydrotesting. To assess the strength in pump hydrotesting, such boundary conditions are used that corresponded to normal operating conditions, with the exception of internal pressure. Internal pressure is set constant throughout all sections of the pump.

The results of calculations of the stress-strain state under such a loading scheme are shown in Fig. 11.

Maximum stresses are formed in the back of the pump. According to the results of the calculation (Fig. 11), the strength condition is executed (the maximum stress value is $\sigma = 100.9$ MPa, which is less than the limiting value of 177 MPa).

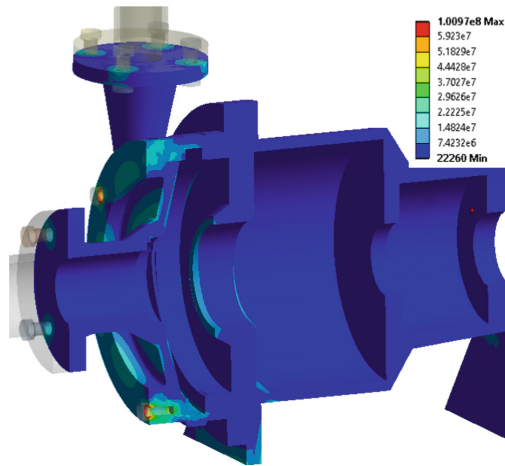


Fig. 11. von Mises stress in the pump case under during hydrotesting, Pa.

5.4 Analysis of the Strength of Bolted Joints with Hydrotests

The stress distribution in bolted joints during hydrotesting is shown in Fig. 12.

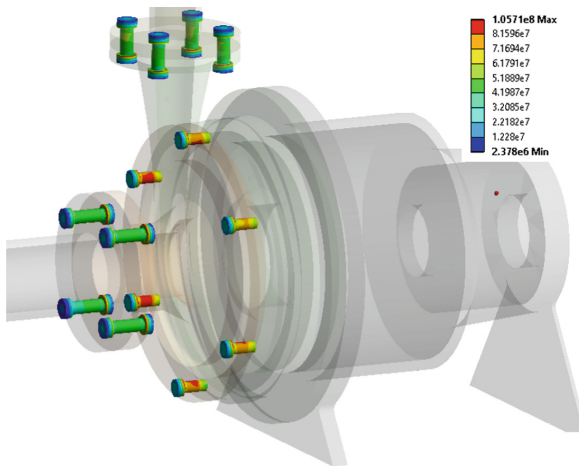


Fig. 12. von Mises stress in bolts and studs during hydrotesting, Pa.

The maximum stress in the bolted connection is 105.7 MPa, which is less than the limit value 144 MPa. Thus, the strength condition is satisfied.

6 Analysis of Cyclic Strength

When calculating on cyclic durability the following operation modes presented in Table 6 are considered.

Table 6. Pump operation modes

Mode	Approbation of the pump, with inclusion in work on operating modes	Hydrotesting
	<i>6 launches per year</i>	<i>1 test per 8 years</i>
Quantity of cycles in the mode for 30 years	6·30 = 180	4
Quantity of cycles in the mode for the planned extension term in 15 years	6·15·1.15 = 104	2
Total	284	6

For calculation the allowed tension amplitude $[\sigma_{aF}]$ at the set allowed number of cycles $[N_0] \leq 10^{12}$ is defined on cyclic strength at least from two received values according to (3) and (4):

$$[\sigma_{aF}] = \frac{E \cdot e_c}{n_\sigma (4 \cdot [N_0])^m} + \frac{R_c}{n_\sigma [(4 \cdot [N_0])^{m_e} + \frac{1+r}{1-r}]} \tag{5}$$

$$[\sigma_{aF}] = \frac{E \cdot e_c}{(4 \cdot n_N \cdot [N_0])^m} + \frac{R_c}{(4 \cdot n_N \cdot [N_0])^{m_e} + \frac{1+r}{1-r}} \tag{6}$$

And at $[N_0] \leq 10^6$ it is allowed to use the minimum value received from dependences for finding of stress amplitude $[\sigma_{aF}]$ according to (5) and (6):

$$[\sigma_{aF}] = \frac{E \cdot e_c}{n_\sigma (4 \cdot [N_0])^m} + \frac{R_{-1}}{n_\sigma \left[1 + \frac{R_{-1}}{R_m} \cdot \frac{1+r}{1-r} \right]} \tag{7}$$

$$[\sigma_{aF}] = \frac{E \cdot e_c}{(4 \cdot n_N \cdot [N_0])^m} + \frac{R_{-1}}{1 + \frac{R_{-1}}{R_m} \cdot \frac{1+r}{1-r}} \tag{8}$$

Where n_σ is safety factor of strength on tension;

n_N is safety factor of strength on number of cycles;

m and m_e (10) are also characteristics of material;

r is a coefficient of asymmetry of cycle of tension;

R_c is a characteristic of strength (7);

e_c is a characteristic of plasticity (8);

R_{-1} is limit of endurance at symmetric cycle (9);
 Z_c is relative throat of lateral section of sample at static destruction ($Z_c = Z$ at $Z < 50\%$ and $Z_c = 50\%$ at $Z > 50\%$).

$$R_c = R_m(1 + 1.4 \cdot 10^{-2}Z) \tag{9}$$

$$e_c = 1.15 \lg \frac{100}{100 - Z_c} \tag{10}$$

$$R_{-1} = 0.4 \cdot R_m \tag{11}$$

$$m_e = 0.132 \lg \left[\frac{R_m}{R_{-1}} (1 + 1.4 \cdot 10^{-2}Z) \right] \tag{12}$$

Use of (7–10) and information about physical characteristics of material (Tables 3 and 4) allows defining design characteristics of fatigue parameters for the studied pump (Table 7).

Table 7. Characteristics of fatigue at temperature 100 °C

Steel	R_m , MPa	$R_{p0.2}$, MPa	e_c	m	m_e	n_σ	n_N	$Z, \%$
12CR18H9TI	461	189	0.002	0.5	0.0528	2.0	10	40
1020	392	206	0.0019	0.5	0.0528	1.5	5	38

Settlement curves of fatigue are defined for the cyclic deformation set like asymmetry. The coefficient of asymmetry is defined by (13):

$$r = \frac{(\sigma)_{\max} - 2(\sigma_a)}{(\sigma)_{\max}} \tag{13}$$

Where is $(\sigma)_{\max}$ is the maximum specified stress in the studied cycle; (σ_a) is amplitudes of the specified stress in this cycle.

Settlement probes by (5–8), allow, having chosen minimum of couples (5)–(6) and (7)–(8) values to construct curves Weller (Fig. 13). The analysis of the given curves on graphics allows choosing as curve fatigue minimum of given (means to $[N_0] < 10^7$) curve calculated by (5), and $10^7 < [N_0]$ by (7).

The strength condition in the presence of various cyclic loads is checked by (14)

$$\sum_{i=1}^k \frac{N_i}{[N_0]_i} = a \leq [a_N] \tag{14}$$

Where N_i is a number of cycles for i type during operation;

k is total number of types of cycles,

$[N_0]_i$ is the allowed number of cycles for i type,

a is the accumulated fatigue damage, which extreme value $[a_N] = 1$.

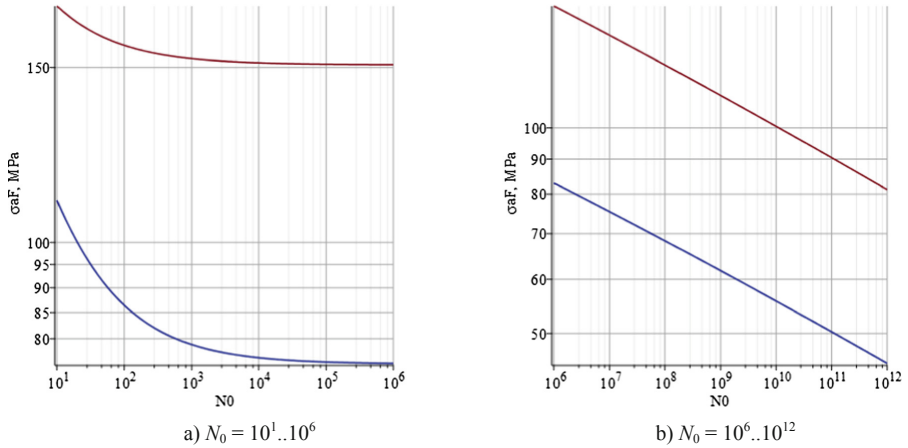


Fig. 13. Settlement curves of fatigue (Weller's Curves) for steel 12Cr18H9Ti at temperature of 100 °C.

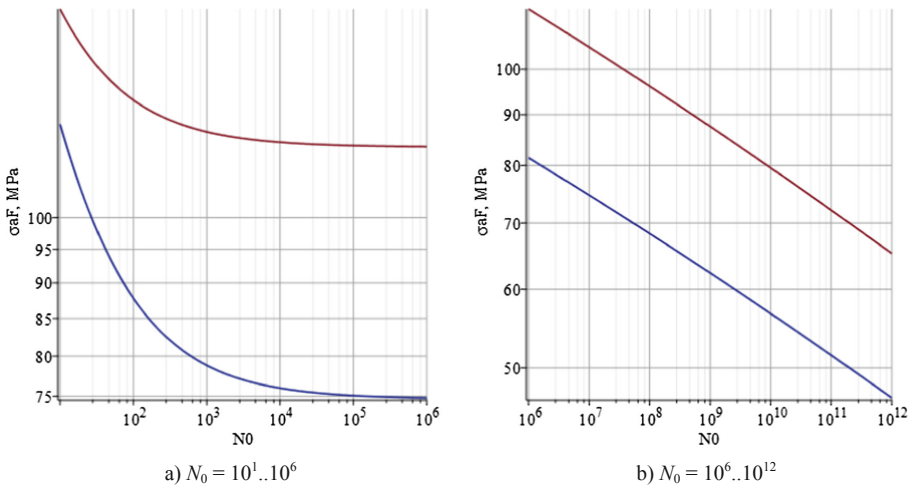


Fig. 14. Settlement curves of fatigue (Weller's Curves) for steel 1020 at temperature of 100 °C.

Amplitude of tension of cycle is defined as (15):

$$[\sigma_{aF}] = (\sigma^{\max} - \sigma^{\min})/2 \tag{15}$$

For the case under normal operation conditions $\sigma_{aF} = 20.7$ MPa, the number of cycles exceeds 10^{12} therefore have accept $N_0 = 10^{12}$. On the other hand for situation of hydrotests $\sigma_{aF} = 32.7$ MPa, the number of cycles exceeds 10^{12} therefore have accept $N_0 = 10^{12}$. Thus, for the considered pump casing structural elements taking into account the planned extension term at quantity of cycles of inclusions (284 cycles) and

accounting of hydrotests (6 cycles), value close to zero ($a \approx 0$) that is less than permissible value $[a_N] = 1$. The condition of strength is satisfied (Fig. 14).

For the bolted joints admissible $\sigma_{aF} = 3.5$ MPa, the number of cycles exceeds 10^{12} therefore have accept $N_0 = 10^{12}$. On the other hand for situation of hydrotests $\sigma_{aF} = 3.8$ MPa, the number of cycles exceeds 10^{12} therefore have accept $N_0 = 10^{12}$. Thus, for the considered pump casing structural elements taking into account the planned extension term at quantity of cycles of inclusions (284 cycles) and accounting of hydrotests (6 cycles), value close to zero ($a \approx 0$) that is less than permissible value $[a_N] = 1$. The condition of strength is satisfied.

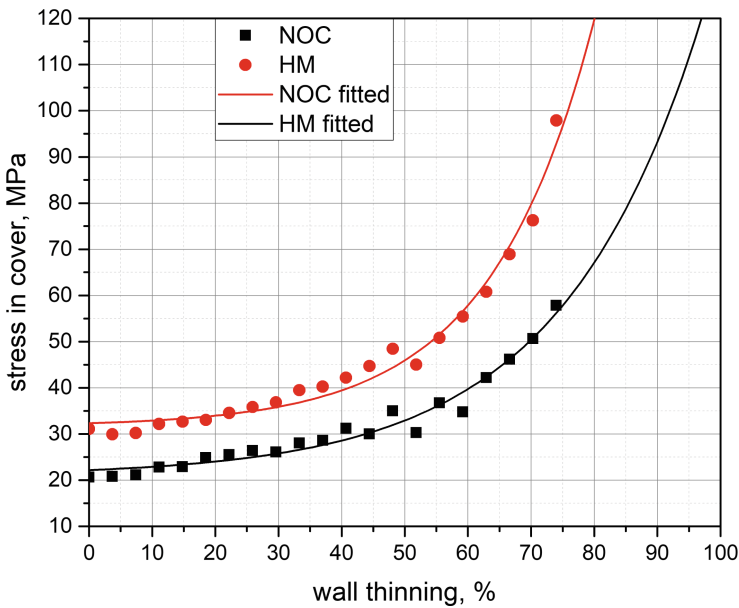


Fig. 15. Dependence of stresses on wall thinning

7 Lifetime Estimation

One of the high-loaded elements of the pump is the cover. It is also subject to great wear during operation. To determine the effect of wear of the cover on its stressed state, a series of calculations have been performed. These calculations have been performed for the two operation modes NOC and HM. To determine the dependence, the cover wall thickness has been varied, after which the maximum stresses in it has been determined. The result of the calculations is shown in Fig. 15. To summarize the obtained data, the data was approximated by a function of the form (16). The obtained parameters of this function are presented in Table 8.

$$\sigma_{aF} = a_1 \exp(x/a_2) + a_3 \tag{16}$$

To determine the number of cycles to failure, dependence (16) has been substituted into Eqs. (5)–(13). As a result of the calculations, the dependence presented in Fig. 16 has been obtained. As can be seen from the figure, with an increase in wear, the number of cycles to failure decreases. The NOC operation mode has a large life-time, because the lower pressure is applied to the cover. For HM mode, higher pressures lead to smaller life-time.

Table 8. Parameter of approximation

	a ₁ , MPa	a ₂	a ₃ , MPa
NOC	1.2707	22.264	20.917
HM	0.6912	16.504	31.642

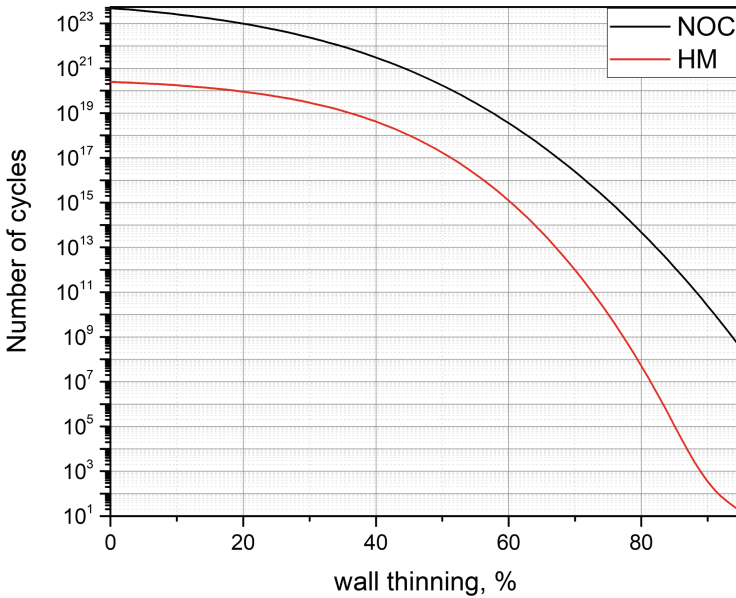


Fig. 16. Dependence of number of loading cycles on wall thinning

8 Seismic Strength Analysis

An important element in ensuring the safety standards of nuclear and thermal power plants is the determination of the seismic stability of the structure. For this purpose, the stresses that occur during typical earthquakes are estimated. Figure 17 shows typical

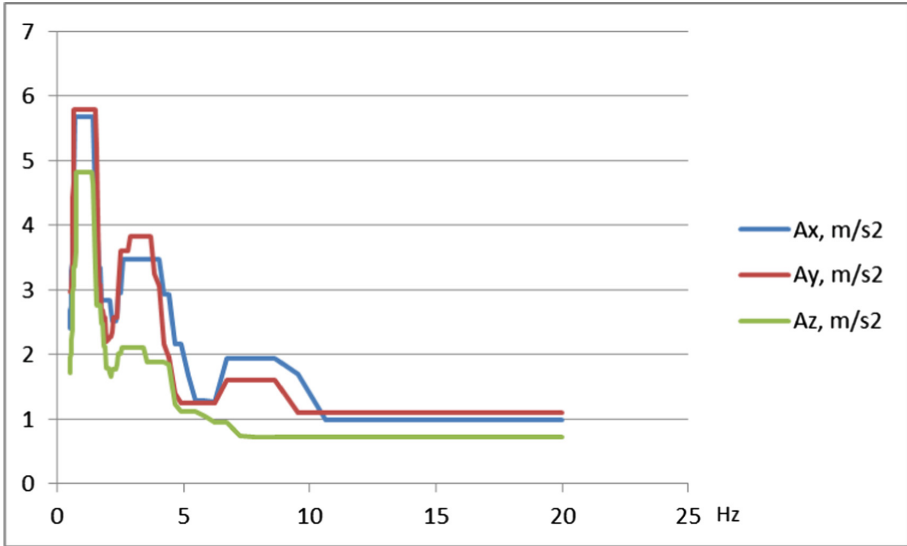


Fig. 17. Typical spectrum of room vibration in accelerations by directions, m/s^2

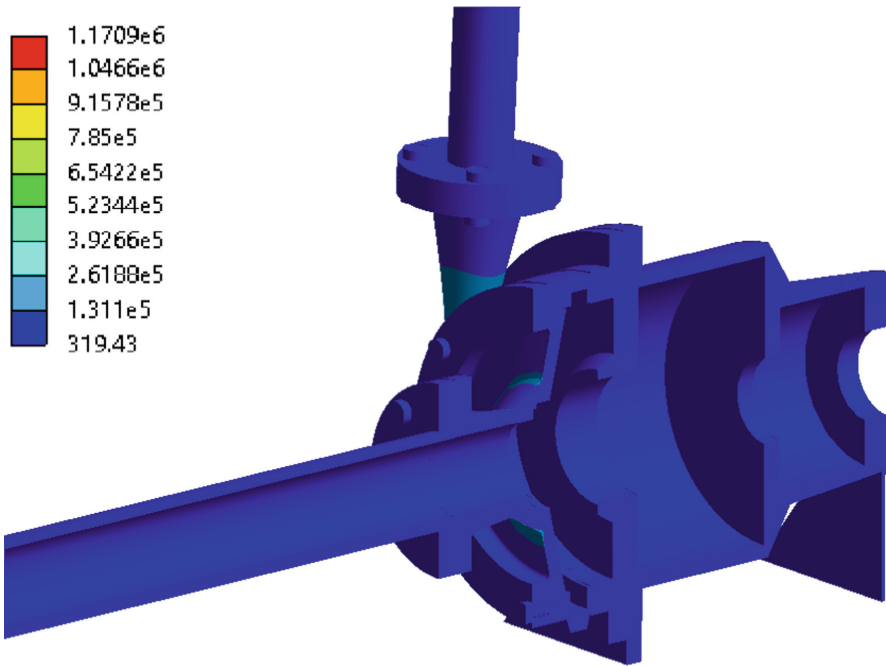


Fig. 18. von Mises stress in pump caused by seismic loading, Pa

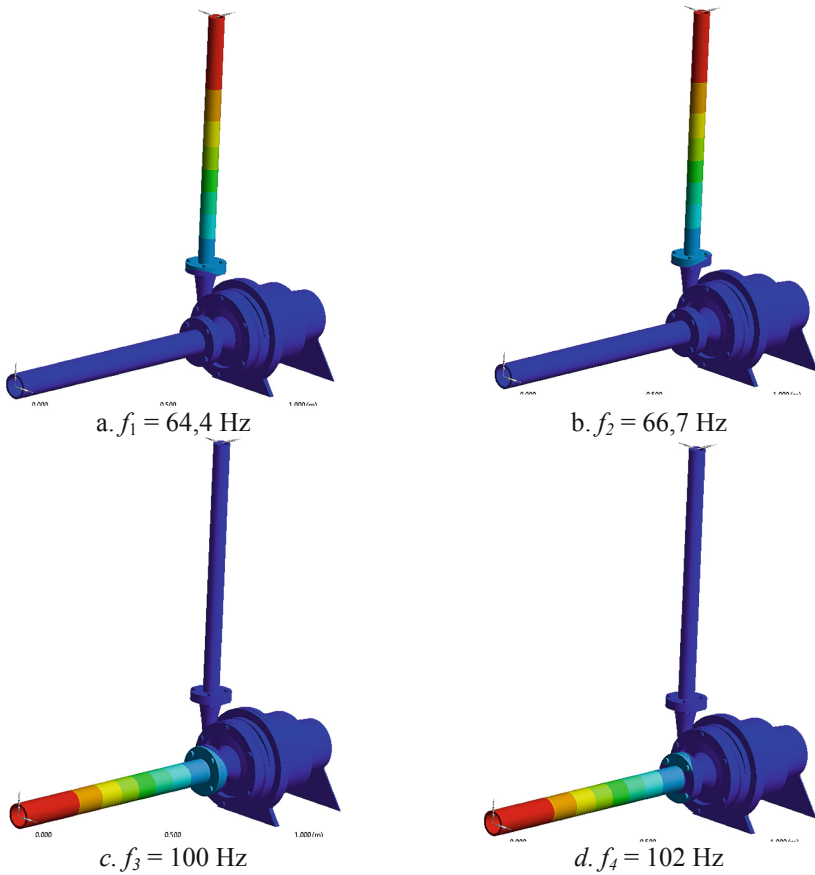


Fig. 19. Eigenfrequencies and forms of pump

acceleration spectra for equipment room. The first four eigen forms and frequencies of the structure are shown in Fig. 19.

Figure 19 shows that the first eigenfrequencies are higher than the frequencies of the earthquake spectrum (Fig. 17). This means that during earthquake the resonance phenomenon will not be observed. Therefore, the expected stresses will be low.

From Fig. 18 it can be seen that the stress due to earthquakes throughout the structure is less than 1.1 MPa. Therefore, their contribution to the total stress state is less than 1% and they do not affect the strength of the structure.

9 Conclusions

The paper discusses steps to determine the residual life-time of a centrifugal pump. Distributions of equivalent stresses in the pump construction elements under normal operating conditions and hydrotesting mode are obtained. The amplitudes of

stress cycles has been calculated, on the basis of which the life-time has been determined, as well as the possibility of extending the design life-time. According to the results of calculations, the design life-time of pumps with a corresponding thinning of the walls can be extended for 15 years.

The paper also estimates the number of cycles for rupture at various levels of wear on the pump cover. The dependences of stresses on wall thickness are obtained.

The results of seismic strength calculations showed that the stresses resulting from earthquakes contribute about 1% to the total stress state and may not be taken into account in further modelling.

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