Simplified Rotor Angular Velocity Estimation for a Permanent Magnets Synchronous Motor by Current and Voltage Measurements

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Abstract. This paper is devoted to the rotor angular velocity estimation of the permanent-magnet synchronous motor (PMSM). It is an actual problem, for example, in sensorless control. We consider a classical, two-phase model in the stator frame of the unsaturated, non-salient PMSM in the state-space representation. All parameters of the model except the stator windings resistance and rotor inertia are assumed to be known. On the first step, we find the relation between measured signals and angular velocity and excluding the unknown parameters of the motor. This relation is simplified using properties of the measured signals and represented as the first-order regression model, where the unknown parameter is the angular velocity. On the next step, we propose the estimation scheme, which is based on the gradient descent method. The efficiency is illustrated through a set of numerical simulations.

Keywords: Sensorless control · Permanent magnet synchronous motor · Parameter identification · Real-time

1 Introduction

The rotor angular velocity estimate can be used in the control loop instead of the measured value, for sensor fault detection or as a reserve system. It is actively studied as part of sensorless (self-sensing) algorithms, where mechanical variables, such position and speed, estimate by currents and voltages measurements [8].

Sensorless control has several benefits. Transducers mounting requires additional space for sensing element and wiring. High-resolution sensors are

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usually expensive. Estimators and observers can be used to decrease the cost of the drive system and increase the failure tolerance.

In this paper, the speed estimation for a permanent-magnet synchronous motor (PMSM) is considered. The overview of the main approaches is presented in [2]. We mention the following results. The observer-based position estimator is described in [4,5]. The main problem of such methods is performance degradation at low- and zero-speed. Methods presented in [9,10] use high-frequency signal injection, which improves performance for the low speeds. However, it requires additional hardware effort and cannot be used on the speeds near the maximum.

This work uses the relation between the rotor angular velocity and currents and voltages, which is described in [1]. In the mentioned paper the third order regression model was obtained, where parameters depend on the rotor angular velocity. In this paper, the order is decreased to one. All parameters of the mathematical model are assumed to be known except the stator windings resistance and rotor inertia. Although resistance can be measured, it depends on the temperature and changes over operating time.

2 Problem Statement

Consider a classical, two-phase $\alpha\beta$ model of the unsaturated, non-salient, PMSM given by [6] and [8]

$$\dot{\lambda}(t) = \upsilon(t) - Ri(t), \tag{1}$$
$$j\dot{\omega}(t) = -f\omega(t) + \tau_e(t) - \tau_l(t), \tag{2}$$
$$\dot{\theta}(t) = \omega(t), \tag{3}$$

where $\lambda(t) = [\lambda_1(t)\, \lambda_2(t)]^T \in \mathbb{R}^2$ is the stator flux, $i(t) = [i_1(t)\, i_2(t)]^T \in \mathbb{R}^2$ are the currents, $\upsilon(t) = [\upsilon_1(t)\, \upsilon_2(t)]^T \in \mathbb{R}^2$ are the voltages, R is the stator winding resistance, $j > 0$ is the rotor inertia, $\theta(t) \in \mathbb{S} = [0, 2\pi)$ is the rotor phase, $\omega(t) \in \mathbb{R}$ is the mechanical angular velocity, $f \geq 0$ is the viscous friction coefficient, $\tau_l(t) \in \mathbb{R}$ is the load torque, $\tau_e(t) \in \mathbb{R}$ is the torque of electrical origin.

The state-space representation of (1)–(3) has the following form [1]

$$L\frac{di(t)}{dt} = -Ri(t) - \lambda_m\omega(t)C'(\theta) + \upsilon(t), \tag{4}$$
$$j\dot{\omega}(t) = -f\omega(t) + \lambda_m i^T(t)C'(\theta) - \tau_l(t), \tag{5}$$
$$\dot{\theta}(t) = \omega(t), \tag{6}$$

where $L \in \mathbb{R}_+$ is the stator inductance, λ_m is the constant flux generated by permanent magnets,

$$C'(\theta) = \begin{bmatrix} -n_p \sin(n_p\theta) \\ n_p \cos(n_p\theta) \end{bmatrix} = n_p JC(\theta) = dC/d\theta, \tag{7}$$

$J \in \mathbb{R}^{2\times 2}$ is the rotation matrix

$$J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \quad C(\theta) = \begin{bmatrix} \cos(n_p\theta) \\ \sin(n_p\theta) \end{bmatrix}, \tag{8}$$

$n_p \in \mathbb{N}$ is the number of pole pairs.

The objective is to find the estimate $\hat{\omega}(t)$ of the constant angular velocity ω that provides exponential convergence of the error $\tilde{\omega}(t) = \omega - \hat{\omega}(t)$ to zero, *i.e.* there exist positive constants C and a such that

$$\|\tilde{\omega}(t)\| \leq Ce^{-at}, \tag{9}$$

$\|\cdot\|$ is some norm of the vector, under the following assumptions.

Assumption 1. *All model* (1)–(2) *parameters except the stator winding resistance R and the rotor inertia j are known.*

Assumption 2. *The currents $i(t)$ and voltages $v(t)$ are measured.*

Assumption 3. *The rotor angular velocity $\omega(t)$ is constant.*

The Assumption 2 is satisfied in the usual operation mode. In some cases $v(t)$ are not measured directly, but estimated with sufficiently high accuracy.

3 Main Result

Following [1] consider the equation based on (4)

$$Ri(t) + L\frac{di(t)}{dt} - v(t) = -\lambda_m \omega C'(\theta). \tag{10}$$

Applying the filter as proposed in [7]

$$(\cdot)_f = \frac{1}{Tp+1}(\cdot), \tag{11}$$

where $p = d/dt$ and $T \in \mathbb{R}_+$ is a design parameter, to (10) gives

$$R\frac{1}{Tp+1}i(t) + L\frac{p}{Tp+1}i(t) - \frac{1}{Tp+1}v(t) = -\lambda_m \omega \frac{1}{Tp+1}C'(\theta). \tag{12}$$

Substituting vectors components yields

$$R\zeta_1(t) + \xi_1(t) = \mu \sin(n_p\omega t + \alpha) + \varepsilon_1(t), \tag{13}$$
$$R\zeta_2(t) + \xi_2(t) = -\mu \cos(n_p\omega t + \alpha) + \varepsilon_2(t), \tag{14}$$

where $\varepsilon_1(t)$ and $\varepsilon_2(t)$ are exponentially decaying terms, because filter (11) is stable,

$$\zeta_1(t) = \frac{1}{Tp+1} i_1(t), \tag{15}$$

$$\xi_1(t) = L\frac{p}{Tp+1} i_1(t) - \frac{1}{Tp+1} \upsilon_1(t), \tag{16}$$

$$\zeta_2(t) = \frac{1}{Tp+1} i_2(t), \tag{17}$$

$$\xi_2(t) = L\frac{p}{Tp+1} i_2(t) - \frac{1}{Tp+1} \upsilon_2(t), \tag{18}$$

$$\mu = \lambda_m n_p \omega / \sqrt{1 + n_p^2 \omega^2 T^2}, \tag{19}$$

$$\alpha = \theta(0) - \arctan(n_p \omega T), \tag{20}$$

where $\mu \in \mathbb{R}$, $\alpha \in \mathbb{R}$ are transfer coefficient and phase shift respectively for (11) and sinusoidal signal with frequency $n_p\omega$.

Let us rewrite $\zeta_1(t)$, $\zeta_2(t)$ и $\xi_1(t)$, $\xi_2(t)$ explicitly

$$\begin{aligned}
\zeta_1(t) &= \frac{a_i}{\sqrt{T^2\omega^2 n_p^2 + 1}} \cos\left(n_p\omega t + \varphi_i - \tilde{\alpha}\right) \\
&= \frac{a_i}{\sqrt{T^2\omega^2 n_p^2 + 1}} \left(\cos\left(n_p\omega t + \varphi_i\right)\tilde{b} + \sin\left(n_p\omega t + \varphi_i\right)\tilde{a}\right) \\
&= \frac{1}{\sqrt{T^2\omega^2 n_p^2 + 1}} \left(i_1(t)\tilde{b} + i_2(t)\tilde{a}\right),
\end{aligned} \tag{21}$$

$$\zeta_2(t) = \frac{1}{\sqrt{T^2\omega^2 n_p^2 + 1}} \left(i_2(t)\tilde{b} - i_1(t)\tilde{a}\right), \tag{22}$$

$$\begin{aligned}
\xi_1(t) &= \frac{L a_i \omega n_p}{\sqrt{T^2\omega^2 n_p^2 + 1}} \cos\left(n_p\omega t + \varphi_i + \frac{\pi}{2} - \tilde{\alpha}\right) \\
&\quad - \frac{a_\upsilon}{\sqrt{T^2\omega^2 n_p^2 + 1}} \cos\left(n_p\omega t - \tilde{\alpha}\right) \\
&= \frac{L\omega n_p}{\sqrt{T^2\omega^2 n_p^2 + 1}} \left(i_1\tilde{a} - i_2\tilde{b}\right) - \frac{1}{\sqrt{T^2\omega^2 n_p^2 + 1}} \left(\upsilon_1\tilde{b} + \upsilon_2\tilde{a}\right),
\end{aligned} \tag{23}$$

$$\xi_2(t) = \frac{L\omega n_p}{\sqrt{T^2\omega^2 n_p^2 + 1}} \left(i_2\tilde{a} + i_1\tilde{b}\right) - \frac{1}{\sqrt{T^2\omega^2 n_p^2 + 1}} \left(\upsilon_2\tilde{b} - \upsilon_1\tilde{a}\right), \tag{24}$$

where $\tilde{a} = \sin\left(\arctan(n_p\omega T)\right)$, $\tilde{b} = \cos\left(\arctan(n_p\omega T)\right)$, $\tilde{\alpha} = \arctan(n_p\omega T)$.

Excluding R from (13)–(14) and neglecting the exponentially decaying terms we obtain

$$\xi_1(t)\zeta_2(t) - \xi_2(t)\zeta_1(t) = \mu\zeta_2(t)\sin(n_p\omega t + \alpha) + \mu\zeta_1(t)\cos(n_p\omega t + \alpha), \tag{25}$$

where $\xi_1(t)\zeta_2(t) - \xi_2(t)\zeta_1(t)$, $\zeta_1(t)$, and $\zeta_2(t)$ are measured signals, μ and $n_p\omega$ are unknown parameters.

Remark 1. The stator windings inductance can be excluded form (13)–(14) instead of R.

3.1 Angular Velocity Estimation

This section aims to find a linear regression model with constant parameters depending on the unknown angular velocity ω.

Substituting (21)–(24) into the left part of (25) yields

$$\begin{aligned}\xi_1(t)\zeta_2(t) - \xi_2(t)\zeta_1(t) &= \frac{L\omega n_p}{T^2\omega^2 n_p^2 + 1}\left(-i_1^2 - i_2^2\right)\\ &\quad + \frac{1}{T^2\omega^2 n_p^2 + 1}\left(\upsilon_2 i_1 - \upsilon_1 i_2\right)\\ &= -\left(\frac{a_i^2 L\omega n_p + a_i a_\upsilon \sin\phi_i}{n_p^2\omega^2 T^2 + 1}\right),\end{aligned} \tag{26}$$

where ϕ_i is the phase current shift, a_i and a_υ are currents and voltages amplitudes respectively

$$a_i = \sqrt{i_1^2 + i_2^2}, \tag{27}$$

$$a_\upsilon = \sqrt{\upsilon_1^2 + \upsilon_2^2}, \tag{28}$$

$$\sin\phi_i = \frac{i_2\upsilon_1 - i_1\upsilon_2}{a_i a_\upsilon}. \tag{29}$$

Substituting (21)–(24) into the right part of (25) gives

$$\begin{aligned}&\mu\zeta_2(t)\sin(n_p\omega t + \alpha) + \mu\zeta_1(t)\cos(n_p\omega t + \alpha)\\ &\quad = \frac{\mu}{\sqrt{T^2\omega^2 n_p^2 + 1}}\left[\left(i_2\tilde{b} - i_1\tilde{a}\right)\sin\left(n_p\omega t - \tilde{\alpha}\right)\right.\\ &\qquad \left. + \left(i_1\tilde{b} + i_2\tilde{a}\right)\cos\left(n_p\omega t - \tilde{\alpha}\right)\right]\\ &\quad = \frac{\mu}{\sqrt{T^2\omega^2 n_p^2 + 1}}\left[i_1\cos\left(n_p\omega t\right) + i_2\sin\left(n_p\omega t\right)\right]\\ &\quad = -\frac{\lambda_m n_p\omega a_i}{T^2\omega^2 n_p^2 + 1}\sin\phi_i.\end{aligned} \tag{30}$$

Combining (26) and (30) we obtain

$$-\frac{a_i^2 L\omega n_p + a_i a_\upsilon \sin\phi_i}{n_p^2\omega^2 T^2 + 1} = -\frac{\lambda_m n_p\omega a_i}{T^2\omega^2 n_p^2 + 1}\sin\phi_i, \tag{31}$$

$$a_\upsilon \sin\phi_i = \omega n_p\left(\lambda_m \sin\phi_i - a_i L\right). \tag{32}$$

The Eq. (31) can be represented in the linear regression form

$$\psi(t) = \theta\varphi(t), \tag{33}$$

where $\psi(t) = a_v \sin\phi_i$ is the regressand, $\theta = \omega$ is the unknown parameter, $\varphi(t) = n_p\left(\lambda_m \sin\phi_i - a_i L\right)$ is the regressor.

Various approaches can be used to estimate the unknown parameter θ. We propose the estimation algorithm, which is based on the standard gradient method [3]:

$$\dot{\hat{\theta}}(t) = k\varphi(t)\left(\psi(t) - \hat{\theta}(t)\varphi(t)\right), \tag{34}$$

where $\hat{\theta}(t)$ is the estimate of the parameter θ, $k \in \mathbb{R}_+$ is a constant gain.

The estimation converges to zero exponentially fast

$$\left\|\theta - \hat{\theta}(t)\right\| \leq C_1 e^{-\rho_1 t}, \tag{35}$$

where C_1 and ρ_1 are some positive constants, if the following conditions are satisfied [3]:

1. The regressor $\varphi(t)$ is bounded.
2. There exist the positive constant D, such that

$$\int_0^t \varphi^2(\tau)d\tau \geq Dt. \tag{36}$$

The regressor in (33) is constant and bounded. Inequality (36) holds for $\varphi(t) \not\equiv 0$. The objective (9) is achieved.

4 Numerical Examples

In this section, we present simulation results that illustrate the efficiency of the proposed estimation algorithm. All simulations have been performed in Mathworks MATLAB Simulink.

The model (4)–(6) parameters, which was used in the simulation, are shown in the Table 1.

Open-loop controller was used in the all experiments

$$\upsilon_1(t) = A(t)\cos(\xi(t)t), \tag{37}$$

$$\upsilon_2(t) = A(t)\sin(\xi(t)t), \tag{38}$$

where

$$A(t) = \frac{\lambda_1\lambda_2}{(p+\lambda_1)(p+\lambda_2)}A_0(t), \tag{39}$$

$$\xi(t) = \frac{\lambda_1\lambda_2}{(p+\lambda_1)(p+\lambda_2)}\xi_0(t), \tag{40}$$

Table 1. Parameters of the motor FAST1M6030 and external load.

Parameter (units)	Value
Inductance L (mH)	3.4
Resistance R (Ω)	0.47
Rotor inertia j (kg m^2)	$1.6 * 10^{-3}$
Pairs of poles n_p (–)	3
Magnetic flux λ_m (Wb)	0.4
Viscous friction coefficient f (N·m s/rad)	0.001
External load τ_l (N·m)	0.01

$A(t) = 60$, λ_1 and λ_2 are the tunable parameters; $A_0(t)$ and $\xi_0(t)$ are the desired amplitude and frequency of the voltage signals in steady state.

The experimental results for piecewise constant angular velocity (in the steady state), which have step change at time 200 s, are shown in Fig. 1. The following form of $\xi_0(t)$ was used

$$\xi_0(t) = \begin{cases} 60, 0 \leq t < 200s, \\ 66, 200s \leq t. \end{cases} \tag{41}$$

The estimation gain k was equal to 20. In this case, the estimation error $\tilde{\omega}(t)$ converges to zero in steady state.

In the second case, estimation of the time-varying rotor speed is investigated. The control signals were produced using

$$\xi_0(t) = 60 + \zeta_0(t), \tag{42}$$

$$\zeta_0(t) = \begin{cases} 0, 0 \leq t < 50s, \\ 0.06\sin(0.2t), 50s \leq t. \end{cases} \tag{43}$$

To increase performance of the estimator, the value of k was increased up to 50. The behaviours of the angular velocity signal $\omega(t)$, estimate $\hat{\omega}(t)$, and estimation error $\omega(t) - \hat{\omega}(t)$ are depicted in Fig. 2. There is a small estimation error, which depends on properties of the rotor speed and the estimator performance.

In Fig. 3 the behaviour of the estimate based on the corrupted by exponentially correlated noise $\delta(t)$ current signal $i(t)$ is illustrated. The noise signal $\delta(t)$ was modelled by a shaping filter $W(s) = 0.005/(0.00004s^2 + 0.0006s + 1)$ with frequency-bounded input white noise of power $N = 0.1$. Simulation parameters were the following

$$\xi_0(t) = 20, \quad k = 5. \tag{44}$$

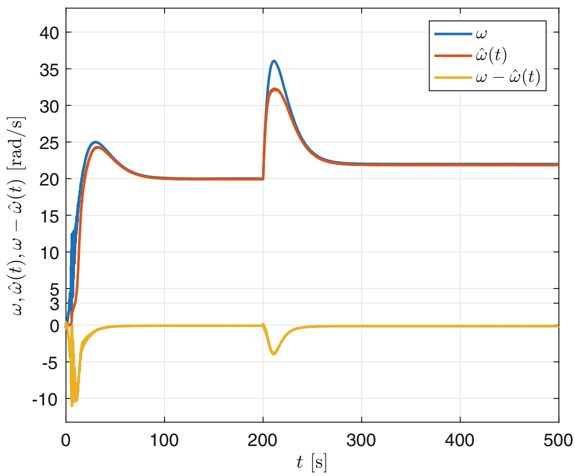

Fig. 1. The angular velocity, estimation, and estimation error

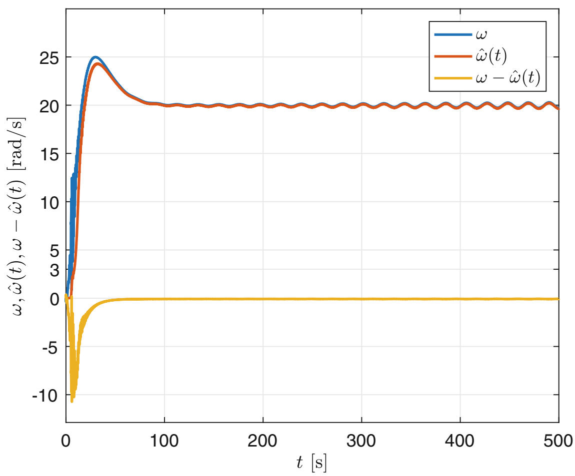

Fig. 2. The angular velocity, estimation, and estimation error for the time-varying rotor speed

In the case of noised measurements, the estimate $\hat{\omega}(t)$ does not converge to $\omega(t)$ and is also corrupted by noise. However, it is bounded.

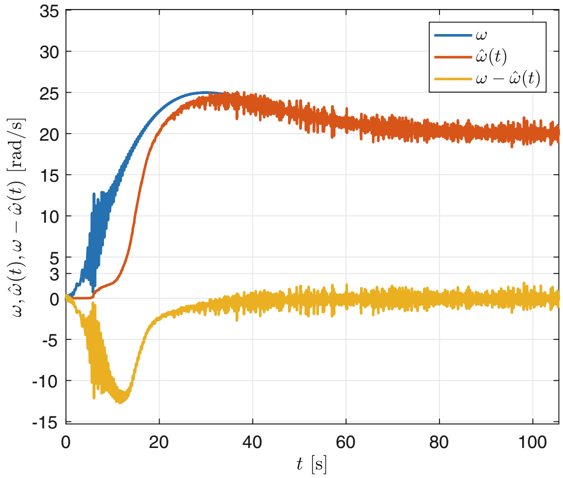

Fig. 3. The angular velocity, estimation, and estimation error for the case with additive noise in the measured signal $i(t)$

5 Conclusion

The simplified estimator for the PMSM rotor angular velocity based on currents and voltages measurements is described. All parameters of the drive are assumed to be known except the stator windings resistance and the rotor inertia.

For the rotation with constant angular velocity is proved that the velocity estimation error converges to zero exponentially fast. The estimator can handle cases with time-varying rotation frequency and noises in the measured signals. The estimation error in such cases is bounded, but don't converge to zero.

For the known stator windings resistance and unknown inductance the estimator can be accordingly modified. From Eqs. (13)–(14) one or another parameter can be excluded.

Future investigations will be devoted to the angular velocity estimation for the case with unknown permanent magnets flux constant λ_m and the stator windings inductance L. Also, time-varying rotation frequency will be considered.

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