Chapter 7
Revisiting Urban Economics for Understanding Urban Data

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Abstract The recent availability of data about cities and urban systems opens the exciting possibility of a 'new Science of Cities'. Urban morphogenesis, activity and residence location choice, mobility, urban sprawl and the evolution of urban networks are just a few of the important processes that can be discussed now from a quantitative point of view. Here, we will discuss how a data-informed approached can elaborate on urban economics models in order to get predictions in agreement with empirical observations. We will illustrate this approach on the polycentric organization of activity in cities and how it evolves when their population grow.

7.1 Introduction

With the availability of large amounts of urban data, we can now hope to bridge the gap between theoretical models and empirical observations. This will help us to provide a quantitative understanding of the phenomenon under study. In the case of a system as complex as a city, the hope is to construct solid, scientific foundations of urban systems (see Batty 2013; Barthelemy 2016). This effort is necessarily interdisciplinary (and this is not always easy, see O'Sullivan and Manson 2015): we have to build up on early studies in urbanism to discuss morphological patterns and their evolution, and on quantitative geography and spatial economics to describe the behavior of individuals, the impact of different transportation modes, and the effect of economic variables (such as the income, the renting market).

We will illustrate this type of approach on some aspect of mobility in cities. The strategy used is in general determined by the following main tasks: First, we extract robust empirical facts and useful information from large amounts of data; second, we identify the relevant mechanisms and parameters describing the behavior of the elementary constituents of the system (in most cases individuals, but also groups, companies or institutions). Then, using these mechanisms and parameters,

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we construct parsimonious models by combining tools and concepts from statistical physics with ingredients from urban economics and quantitative geography. Finally, we validate the model by data.

We will discuss empirical aspects in Sect. 7.2 with the study of the spatial structure of activities in cities. We will show how we can extract this information from mobile phone data in Sect. 7.2.1. This same dataset can also help us in describing mobility patterns and we will show in Sect. 7.2.2 how we can extract mesoscopic information from these large datasets. In Sect. 7.3, we will focus on theoretical approaches for explaining regularities observed empirically. We will start by discussing how to model the spatial distribution of activities, and then another problem which is the effect of income on commuting. In these theoretical approach we show that classical models fail to predict the empirical observations, therefore calling for the need for different frameworks. These examples highlight the importance of three essential ingredients in the modeling strategy that is used here: (i) extracting useful mesoscopic information from large datasets, (ii) describing complex quantities by random variables and (iii) complex actions by stochastic processes.

7.2 Empirical Studies

7.2.1 Extracting the Spatial Distribution of Activity

Cell phone networks, enable to capture large amounts of human behavioral data but also provide information about the structure of cities and their dynamical properties. We illustrate this point with mobile phone data recorded over two months and for 30 Spanish metropolitan areas (Louail et al. 2014, 2015). We can measure the density of users in certain areas of the city and applying filters such as the frequency of visit of a location and the duration of stay we can infer density of activity during the day in cities. The type of measures that we obtain is shown in Fig. 7.1 in the case of Vitoria and Bilbao.

We observe that there are essentially two types of cities. First we observe that usually smaller cities have a unique activity center (Fig. 7.1, left) and correspond to the classical image of the monocentric city organized around a central business district. Second for larger cities we observe a more complex pattern (Fig. 7.1, right) with more than one activity center.

In order to go further in the quantitative analysis of the spatial organization of activities in cities we have to determine the number of activity centers, or “hotspots”. We can see the density of users (or employment for example) as a two dimensional surface and the hotspots are the local maxima of this surface. In order to decide if a location can be considered as a local maximum, we introduced a parameter free method to introduce a threshold allowing to detect hotspots. This method, described in detail in Louail et al. (2014), relies on the Lorentz curve of the density at different points. It is based on the observation that the curvature of the Lorentz curve is

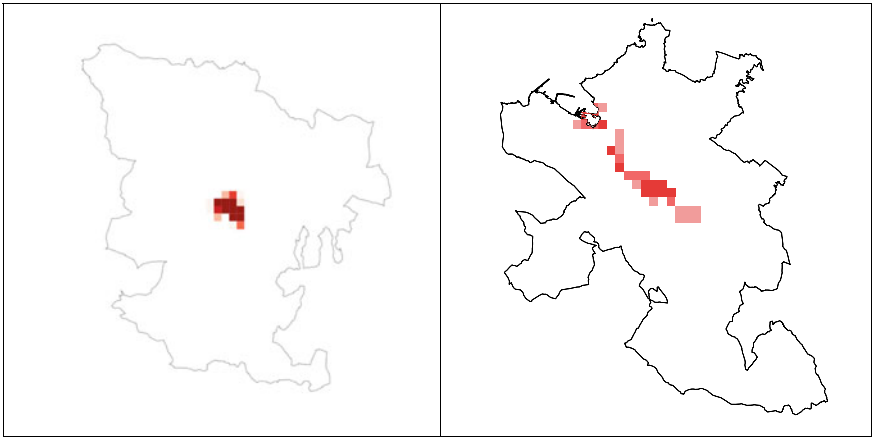


Fig. 7.1 Areas with large density of mobile phone users for (left) Vitoria and (right) Bilbao. The darker the area and the larger the density of users. Figure from Louail et al. (2014)

connected to the heterogeneity of the values of the density (the surface between the diagonal and the Lorentz curve is indeed directly related to the Gini coefficient). This curvature of the Lorentz curve $L(F)$ can be characterized by the slope of the curve at $F = 1$ and it is therefore natural to extract from this slope a threshold (see Fig. 7.2). This threshold (denoted by F_{LouBar} in Fig. 7.2), is naturally larger than the naive one given by the average. Indeed considering that every location with a value larger than the average is a very mild determination of a hotspot and F_{LouBar} would give the strongest constraint.

For a given value of the threshold we can then count the number H of hotspots and any result on this quantity should be robust with respect to small variations of the thresholding procedure. For example, we plot in Fig. 7.3 the number H of hotspots obtained by using the average for the threshold (i.e. each location with a density larger than the average is counted as a hotspot) and by using the quantity F_{LouBar}.

We observe on these results the existence of a robust behavior (confirmed by studies on employment data for 9000 US cities, Louf and Barthelemy 2013)

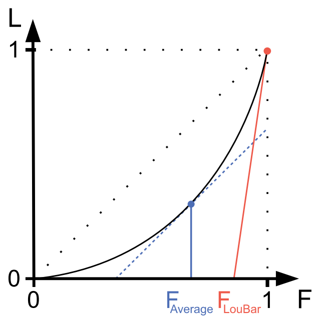
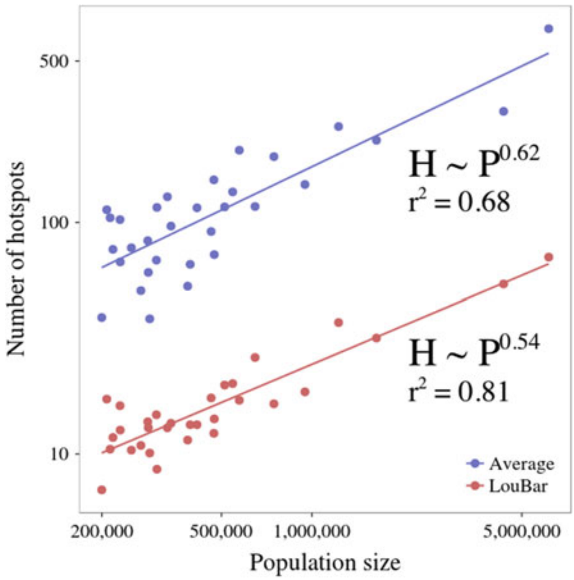


Fig. 7.2 Lorentz curve used for constructing a parameter-free method for determining hotspots. The intersection of the slope at $F = 1$ with the x-axis gives a threshold naturally related to the heterogeneity of the density distribution. Figure taken from Louail et al. (2014)

Fig. 7.3 Number of hotspots H versus the population of the city for two different values of the threshold (the average and the one obtained from F_{LouBar}). Both thresholds predict consistently a sublinear behavior. Figure taken from Louail et al. (2014)



$$H \sim P^{\beta}$$

where the exponent β is usually found to be around 0.5–0.6. The number of these hotspots thus scales sublinearly with the population size, a result that will serve as a guide for constructing theoretical models. The spatial structure of these hotspots is also of interest and allows us to distinguish different categories of cities, from monocentric and "segregated" where the spatial distribution is very dependent on land use (residential or activity), to polycentric where the spatial mixing between land uses is much more important. These results point towards the possibility of a new, quantitative classification of cities using high resolution spatio-temporal data.

7.2.2 A Typology of Mobility Patterns

The description of mobility patterns and their statistics is of great interest for understanding the functioning of a city at a large scale and the effect of infrastructures. Surprisingly enough, there were few studies of this problem and the main source of inspiration for many authors is the paper by Bertaud and Malpezzi (2003) who proposed the simple typology of journey-to-work trips shown in Fig. 7.4.

The first class of cities display the type (a) of mobility patterns which correspond to the classical idea of what happens in a monocentric city: all the flows (corresponding here to the journey-to-work commute) are converging to one unique center. In (b), Bertaud and Malpezzi proposed another type of organization where flows appear to be "random". In (c) we have the polycentric—or urban villages—organization with

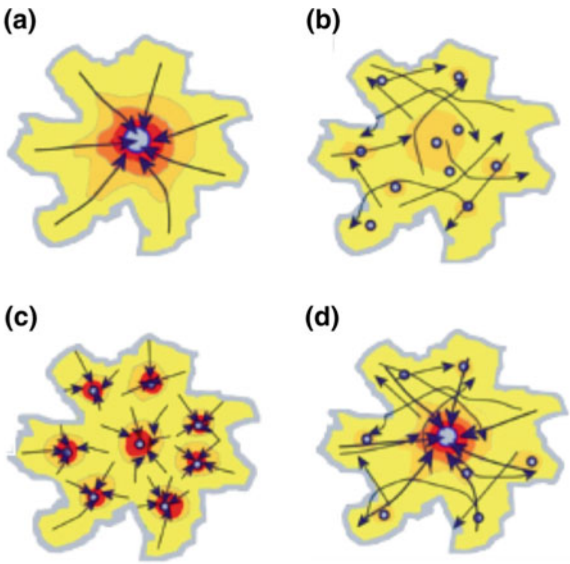


Fig. 7.4 A proposal for a typology of mobility patterns from home to work. The journey-to-work flows are indicated by arrows. Figure taken from Bertaud and Malpezzi (2003)

the existence of many activity centers with their own attraction basin. Finally, they conclude with the possibility of observing a pattern (d) resulting from the superimposition of patterns (a–c). Although this typology proposal is very reasonable it wasn't verified empirically.

The current mobile phone data however allows to test this typology. A more general problem with large datasets such as the one obtained with mobile phone data (or other sources such as GPS or RFIDs), is the extraction of a clear and simple footprint of the structure of large origin-destination matrices which contain the complete information on commuting flows, but are difficult to analyze and compare. We discuss here briefly a versatile method (Louail et al. 2015) which extracts a coarse-grained signature of mobility networks, under the form of a 2×2 matrix that separates the flows into four categories. The main idea is to separate working places in two different categories: either a location is an activity center (i.e. a hotspot) or not. We can also apply the method discussed above to residence places: we then obtain "residential hotspots" for which the population density is far above the average. The commuting flows from home to work can then be aggregated in the four different categories shown in Fig. 7.5.

We then obtain the four numbers I, C, D, R for each city that aggregate the flows. The quantity I represents the flow between residential hotspots and activity centers, while R describes the flows between non-hotspots (both residential and working). The two other quantities C and D represent flows between areas of different types.

We apply this method to origin-destination matrices extracted from mobile phone data for journey to work trips in the 30 largest Spanish urban areas (Louail et al. 2015), and lead to the result shown in Fig. 7.6. We observe that these cities essentially differ

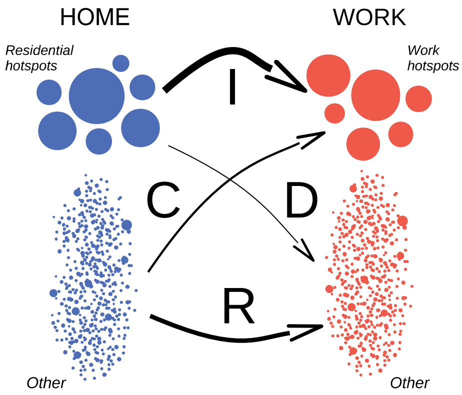


Fig. 7.5 Constructing the four different types of commuting flows: we separate both homes and working locations into hotspots and non-hotspots. Figure taken from Louail et al. (2015)

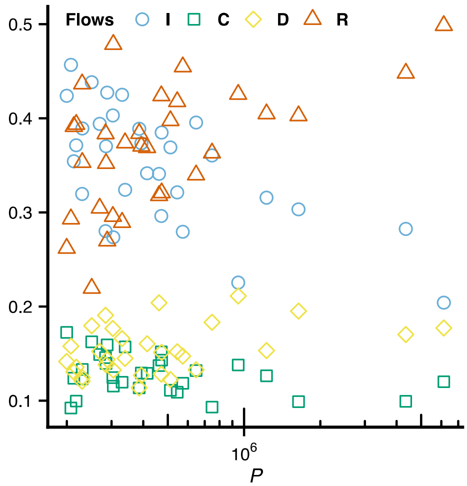


Fig. 7.6 The four different flows I, R, C, D for the 30 largest Spanish urban areas versus their population. We observe that large areas are mainly determined by R and I, the other quantities being negligible. Figure taken from Louail et al. (2015)

by their proportion of two types of flows: integrated (I) between residential and employment hotspots and random flows (R), whose importance increases with city size.

This result is in contrast with the naive expectation of a monocentric city where C flows dominate. For large cities “random” flows described by R are the most important. This might be due to the fact that it is easier to move around in large cities thanks to the various public transportation systems. We see here how the extraction of mesoscopic patterns from large datasets forces us to reconsider our view of cities and more particularly here the spatial organization of mobility patterns.

7.3 Theoretical Approaches: Modelling Strategies

The empirical study of activity centers discussed above shows that their number scales with the population of the city as

$$H \sim P^{\beta}$$

where $\beta \approx 0.5$–0.6. The theoretical question is therefore simple: how can we explain this behavior and can we predict the value of β? (or at least why do we have a sublinear behavior with $\beta < 1$?).

In order to understand the spatial structure of cities and in particular how the number of hotspots varies with population as discussed above, we have to model how an individual chooses her residence and workplace. In the following we will discuss the classical approaches to this problem and we will show that they are unable to predict and understand the scaling of H with P. We then show how we can integrate economical ingredient and make simplifying assumptions typical from statistical physics in order to get a model with predictions in agreement with the observation.

7.3.1 Classical Approaches: Krugman, Fujita and Ogawa

We first follow here Krugman (1996) for a simple model of activity clustering in cities. The distribution of companies is described by the density $\rho(x, t)$ and a fundamental quantity is the market potential given by

$$\Pi(x) = \int dz \rho(z) K(x - z)$$

where $K(x - z)$ is the kernel that describes the impact (or spillover effects) of a company located at z on the attractiveness of location x. We can compute the average market potential $\overline{\Pi}$ over the whole city and Krugman proposes then to describe the evolution of company density by the following equation

$$\frac{\partial \rho}{\partial t} = \gamma \left[\Pi(x, t) - \overline{\Pi} \right]$$

This nonlinear equation states that the density will increase at location where the market potential is larger than the average. Krugman then showed that a uniform distribution of companies $\rho(x, t) = const.$ is unstable where the most unstable mode is given by a quantity k^* that depends on the details of the system such as spillover effects (but is independent from the population size P). This means that the activities will indeed form clusters and this simple mechanism seems to explain the clustering of activities in cities. However, we would like to understand the scaling of the number of hotspots H and this model simply predicts

$$H \sim Ak^{*2}$$

where A is the surface area of the city. This model is therefore unable to predict the evolution of the spatial distribution of cities when the population grows (unless introducing an external assumption about how the area of the city varies with the population). In other words, this model predicts a constant value of H that is independent of the population P.

Another important approach for understand the spatial structure of cities was proposed by the economists Fujita and Ogawa (1982). In this model, agents optimize their utility and companies their profit. Focusing on agents, an agent will choose to live in location x and work in location y such that the quantity (which corresponds to the composite commodity) given by

$$Z(x, y) = w(y) - C_R(x) - C_T(x, y)$$

where $w(y)$ is the wage when working at y, $C_R(x)$ the rent at location x, and $C_T(x, y)$ the transportation to commute from home to work (for companies there is a similar equation for profits). In this model and for a monocentric organization of activities, Fujita and Ogawa choose to take $C_T(x, y) = t|x - y|$ independent of the traffic (i.e. without any congestion effect). They were able to show that this monocentric organization is actually unstable, in particular when transportation cost per unit distance (t) become too large. This formalism however does not allow to predict the number of activity centers when population grows.

We saw on these two examples models that are in agreement with the qualitative organization of the spatial structure of activity in cities, but are so far unable to provide a quantitative prediction. Even if these models are satisfying from an intellectual point of view, as long as their predictions are not in agreement with empirical measures, we can only place a low level of confidence in their ability to describe what is actually happening in cities. Ideally, we would like to have a model with a minimal number of parameters and which is able to predict a large number of unrelated empirical facts. The model described in the following didn't reach this level yet but at least is a proposal for an alternate modeling of cities that is sound from an economical point of view and in agreement with the scaling of the number of hotspots.

7.3.2 Complex Quantities as Random Variables

The problem is to compute the value of the exponent β. In this new way of modelling cities (Louf and Barthelemy 2013, 2014), we integrate ingredients of urban economics, and most importantly we replace an unknown, complex quantity by a random one, a concept introduced in the study of the spectra of heavy atoms (Wigner 1955). More precisely, we assumed that:

(1) At each time step, we add a new individual in the city.
(2) Each individual will optimize its own budget consisting of its wage minus residential and transportation costs.
(3) The wage is described as a random variable.
(4) Transportation costs through congestion integrate interactions between individuals.

With the assumption (1) dynamics is introduced in this system and we do not consider that cities are in equilibrium in contrast with many previous studies such as the Fujita-Ogawa model (1982). With (2), we integrate ingredients coming from urban economics that discussed for a long time the behavior of individuals. The assumption (3) is typical from statistical physics where replacing the wage $w(y)$—a complex quantity that results from a large number of interacting constituents—by a random variable $\eta(y)$ proved to be in some cases accurate (Wigner 1955). We note here that in the original model proposed by Fujita and Ogawa (1982), wages are endogenous variables and are an output of the model. This description leads to complications that however forbids to make clear testable predictions. In (4), the effect of congestion on the time spent to go from one point to another is described in transportation economics and describes effectively interactions between individuals. We use the generalized cost for transportation which is proportional to the time $\tau(x, y)$ needed to go from x to y and which is given by the Bureau of Public function (see for example Branston 1955)

$$\tau(x, y) = \frac{d(x, y)}{v}\left[1 + \left(\frac{T(x, y)}{C}\right)^{\mu}\right]$$

where $T(x, y)$ is the traffic between these two points, C is the capacity and v the average velocity of the road system between x and y, and μ is an exponent that characterizes the sensitivity of the system to the congestion and is generally of order $\mu \approx 2$–5 (see for example Branston 1976). This interaction is at the heart of the non-trivial collective behavior observed in this model and characterized by non-trivial exponent values.

Putting all these ingredients together, the model is now described by the maximization of the following quantity:

$$Z(x, y) = \eta(y) - td(x, y)\left[1 + \left(\frac{T(x, y)}{C}\right)^{\mu}\right]$$

(in this expression, we also note that the wage $\eta(y)$ can be understood as a number encoding the attractiveness of the location y). For this model, using mean-field arguments and numerical simulations, we were able to predict that the monocentric city is stable until a threshold P^* for the population above which another activity center becomes more interesting for individuals (Fig. 7.7).

This spatial splitting of the activity is driven in this model by the congestion: all individuals choose to go to the most attractive center (from the point of view of the

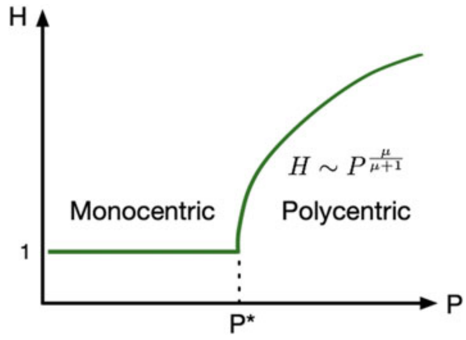


Fig. 7.7 Number of activity centers versus the population. For $P < P^*$, the system is monocentric and over this threshold the activity is dispersed over H different centers

wage), but this increases the transport cost (due to the congestion effect). Another center, less attractive but with a smaller traffic becomes then the most interesting working place. We can estimate this threshold and we can show that increasing the population leads to a larger number of activity centers. More precisely we show that

$$H \sim \left(\frac{P}{P^*}\right)^{\frac{\mu}{\mu+1}}$$

and therefore predicts $\beta = \mu/\mu+1$. This result demonstrates that independently from the value of μ, the behavior of H is sublinear with the population, in agreement with empirical observations. In this simple model, non-linear congestion effects imply that cities undergo a ‘dynamical’ transition from a monocentric to a polycentric structure as their population grow. Congestion is certainly not the only factor that favor the formation of different activity centers, but these results demonstrate that it plays at least a major role for understanding the spatial organization of cities.

Within this model we know the location of residence and work for all individuals and we can therefore estimate other quantities such as the delay spent in traffic jams or the CO_2 emitted by cars. The predictions for these quantities are in excellent agreement with measures for European or OCDE cities (Louf and Barthelemy 2013, 2014). On a more fundamental level, this model predicts that CO_2 emitted by cars or the total spent in cars is not a simple function of urban density. This is in sharp contrast with the celebrated result of Newman and Kenworthy (1989), showing that the gasoline consumption in a city is a decreasing function of the population density. Certainly more theoretical and empirical work is needed here for understanding this issue.

7.4 Discussion

The recent availability of large amounts of data enables us to reveal regularities in the spatial structure and mobility patterns in cities across countries. These regularities suggest that common mechanisms that encompass the differences of cities exist and govern the formation and evolution of these systems. Also, traditional assumptions can now be tested and in some cases a whole new modeling framework is needed in order to understand empirical observations.

Describing several (human) actions by stochastic processes and complex quantities resulting from the interactions of several agents by random variables are the key ideas that are exposed here and that allow to propose new models for understanding the evolution of cities. More generally, we tried to show in this paper, through examples about mobility in cities, how a combination of empirical results, economical ingredients, and statistical physics tools can lead to parsimonious models with predictions in agreement with observations. Obviously, many problems are left. In particular, we have now to integrate within this framework other transportation modes, and socio-economic factors such as the impact of the revenue on the spatial structure of cities. Our hope is that this interdisciplinary theoretical approach, informed by data, will lead to a solid understanding of systems as complex as cities.

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