# How Mathematicians Learned to Stop Worrying and Love the Computer

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*How I learned to stop worrying and love the bomb,*
*subtitle to the 1964 movie Dr. Strangelove*

The modern, programmable, digital computer grew from theoretical mathematical results obtained in the 1930s and 40s by mathematicians such as Alan Turing (1912–54) in the United Kingdom, John von Neumann (1903–57), and Alonzo Church (1903–95) in the United States. Indeed, both Turing and von Neumann were involved in the design and construction of early digital computing devices for military purposes in their respective countries during the Second World War.[1]

Yet, when digital computers became available for scientific work, starting in the 1950s, hardly any (pure) mathematicians made use of them. Indeed, that state of affairs continued through the 1960s and the 1970s, and well into the 1980s, a period in which the computer grew to be a ubiquitous tool in the natural sciences, in engineering, and the worlds of business and finance.

[1]While the results of Turing, von Neumann, and Church gave a theoretical underpinning to the subsequent developments of computers, it is clear that the technology would have been developed anyway, and indeed such advances were already underway. For example, Konrad Zuse took out patents for computing devices in 1936 and 1941. And the ENIAC, 1943–46, was designed by engineers Eckert and Mauchly, before von Neumann became involved in the project. Moreover, theoretical and practical work on computing devices was done much earlier by Pascal (1642), Leibniz (1674), and Babbage (1822).

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While mathematicians' seeming lack of interest in computers might have seemed strange—indeed paradoxical—to anyone outside the field, to those on the inside it was not at all surprising. The computer had little to offer to the vast majority of research mathematicians.

There was no paradox here. Society's widespread layperson's assumption that mathematics is essentially "higher arithmetic" was always completely off base: (pure) mathematicians study abstract patterns and relationships.[2] For the most part, they use logical reasoning rather than numerical computation. Indeed, the early mathematical work on computing by Turing, von Neumann, Church, and others focused entirely on the theoretical concept of computation. Were it not for the demands of the war effort at the time, it is highly unlikely that Turing or von Neumann would ever have become involved in the design and construction of physical computing devices (Pilot ACE and the Manchester Mark I for Turing, ENIAC for von Neumann) and the execution of actual computations (codebreaking in Turing's case and the calculation of artillery range tables and the design of the atomic bomb for von Neumann[3]).

To be sure, many applied mathematicians were quick to make use of the new technology, and specialized areas of mathematics such as numerical analysis grew considerably with the availability of computers. But the vast majority of mathematicians spent most of their time in day-to-day research activities that had remained largely unchanged for over two millennia. Their work progressed under a widespread, though unstated, assumption that computers could not possibly play a role in the construction of proofs of theorems.

That assumption was given a significant jolt in 1976, with the announcement by two mathematicians in the United States, the American Kenneth Appel and the German Wolfgang Haken, that they had made essential use of a computer to solve a famous, long-standing open problem in mathematics: the Four Color Problem. Dating back to 1852, the problem had all the hallmarks of a theoretical problem for which a computer might be of no help. It asked for a proof that any map drawn on a plane can be colored using at most four colors so no two countries that share a stretch of border are colored the same. The answer will be yes or no; it is not about calculating a number.

If you try a few cases, say with maps of countries, states, or counties, you quickly start to believe the answer might be yes. But what about fictitious maps with thousands of regions, designed to require five or more colors? How do you deal with that possibility? Since there are infinitely many possible map configurations, it is not possible to prove the answer is yes by trying to color every possible one, even with a fast computer.

Or maybe the answer is no. A computer could perhaps be used to solve the problem in the negative; you could let the computer generate map after map and try to color them until it finds one that cannot be colored with four or fewer colors. Since the number of possible coloring configurations for a given map is finite, that could work.

[2]The term "higher arithmetic" has acquired a special meaning in the mathematical world. That is not what I am referring to here.

[3]Making my title for this article a bit more than an irresistible play on words.

But if the answer to the problem is yes, that approach would never end. To solve the problem affirmatively, which is what the majority of mathematicians believed was the case, a logical argument would be required.

Attempts to solve the puzzle by many mathematicians over the years ended in failure, until Appel and Haken eventually came up with an approach that worked. Their approach did indeed involve a logical argument, but on its own that argument did not solve the problem. Rather, they were able to show that if every map in a specific collection of 1,476 particular map configurations could be colored with at most four colors, then the same would be true for all maps. The two researchers then wrote a computer program to examine all possible four-color coloring schemes for those 1,476 maps in turn to see if, in each case, it could find one that worked for that map. That task would have taken too long for a human to complete, even a team of humans, but their computer completed the task in a few months. (Today's computers could do it much faster.) The computer search proved successful, and the Four Color *Problem* became the Four Color *Theorem*. Mathematics had entered a new era.

Initially, the Appel and Haken proof generated a considerable amount of controversy among mathematicians, many of whom regarded the use of a computer to prove a theorem in the same way sports fans object to the use of performance-enhancing drugs. But when, over the ensuing years, a number of other theorems were proved using arguments that likewise required use of a computer, the objections gradually died down. The writing was clearly on the wall—or rather, on the computer screen. As in many other walks of life, for mathematics, the computer was here to stay.

Even so, for the vast majority of mathematicians, things remained the same. Proving a new result still required the construction of a suitable logical argument. The only new twist was that it became accepted that the argument might, on occasion, depend on the successful execution of a computation (often an exhaustive search through a large but finite sets of possibilities), and such arguments were accepted as legitimate proofs. Referees of papers submitted for publication would check the logic in the traditional way and either take the computation on trust or, if feasible, arrange for an independently written computer program running on a different computer to check that the computational part did as the authors claimed.

Notable examples of computer-assisted proofs, as they became known, are

- Proof of Feigenbaum's universality conjecture in nonlinear dynamics (1982),
- Proof of the nonexistence of a finite projective plane of order 10 (1989),
- Proof of the Robbins conjecture (1996), and
- Proof of Kepler's sphere packing conjecture (1998).

There were also cases where computers were used to establish negative results; for example, Odlyzko and te Riele's disproof of the Mertens Conjecture (1985). But since such examples were rare, mathematicians, by and large, continued doing business as usual. The only time they used a computer was for email, after it was introduced in the 1980s, and for typing manuscripts. As things turned out, that latter use of computers for manuscript preparation provided the final impetus that resulted in the American mathematical community embracing the new technology for teaching and research.

In 1978, the Stanford mathematician and computer scientist Donald Knuth released the first version of his mathematical typesetting system TeX, which enabled mathematicians to type their own books and papers using a regular computer keyboard. Special commands were used to product Greek letters and mathematical symbols, and the program took care of organizing the layout on the page so that it was both mathematically correct and esthetically pleasing. There was a fairly steep learning curve as a new user mastered the typesetting language, which was made somewhat easier with the appearance in the early 1980s of LaTeX, a more user-friendly front-end package for TeX, developed by Leslie Lamport of SRI.

So great and so obvious were the benefits of using LaTeX, that some mathematicians quickly adopted it, but even with LaTeX there was still a significant learning curve, and many were put off. They could still see the advantages of typing their own manuscripts, however, and so they went with one of a number of what-you-see-is-what-you-get, mathematical word-processing systems that offered drop-down menus of alternative alphabets and mathematical symbols, an approach that was much easier to learn but did not produce the elegant page layout you got from TeX.

In 1987, Richard Palais of Brandeis University wrote a series of articles for the American Mathematical Society *Notices*, surveying for mathematicians the various mathematical word processing systems that were available at the time. The interest in those articles was sufficiently strong for mathematicians at the AMS to start talking about the society taking a proactive role in helping the community take advantage of the new working possibilities that computers were starting to offer, not only in preparing manuscripts but also in teaching and research. That led to a decision for the *Notices*, which was sent to all members ten times a year, to introduce a regular section "Computers and Mathematics", that would serve both to provide inspiration for mathematicians to make greater use of computers, and to act as an information exchange for the various possibilities computers offered in their work.

That same year, 1987, was also when I moved from the United Kingdom to the United States, to spend a year as a Visiting Professor at Stanford. My host, Jon Barwise, was the mathematician the AMS asked to edit the new *Notices* section, and the two of us talked about the upcoming new column on a number of occasions.

As mathematicians, we both had spectators' interest in the use of computers within traditional mathematics—indeed, Jon attended the lavish event launching Steve Wolfram's new mathematical software system *Mathematica* on June 23, 1988—but our main interests took different forms. Jon's interest was primarily that of a logician, and he soon began working with his Stanford colleague John Etchemendy to develop instructional software to teach formal logic (*Turing's World*, *Tarski's World*, and *Hyperproof*). My focus was more as part of my growing interest in what would become known as mathematical cognition, where the focus was on studying mathematics as a mental tool, looking at how it arose, and how it related to, fitted in with, and complemented other forms of thinking. From that standpoint, the use of computers to assist in doing mathematics was but one component of what I would end up calling "mathematical thinking".

The "Computers and Mathematics" section launched in the May/June 1988 issue of the *Notices*, with Barwise leading off with an essay in which he declared that the goal was to reflect, both practically and philosophically, on cases where computers were affecting mathematicians and how they might do so in the future; to act as an information exchange into what software products were available; and to publish mathematicians' reviews of new software.

Barwise edited the section through to February 1991, after which the AMS asked me to take it over. I held the reins from the March 1991 issue until the AMS and I decided to end the special section in December 1994. The reason? That six-and-a-half-year run of the special section had achieved the intended goal. The computer had become a staple tool for mathematicians, both in teaching and research.

The general format of each column was to start with some form of editorial comment, then, frequently, a feature article solicited by the editor, and then a number of reviews of new mathematical software. In all, we published 59 feature articles, 19 editorial essays, and 115 reviews of mathematical software packages (31 features, 11 editorials, and 41 reviews under Barwise, 28 features, 8 editorials, and 74 reviews under me).

At around the same time the "Computers and Mathematics" section was starting up, a number of mathematicians were developing a new subfield of mathematics called "Experimental Mathematics". In this field, one of the primary goals in using computers was to formulate conjectures that could subsequently be proved by conventional means—which cast the computer as an additional weapon in the pure mathematician's armory rather than a completely separate technological endeavor. In 1992, a new journal with that name as its title was established by the American mathematicians David Epstein, Silvio Levy, and the German-American mathematics publisher Klaus Peters. And in the fall of that year, the Canadian mathematicians Jonathan and Peter Borwein sent me their article "Some Observations on Computer Aided Analysis", written to introduce their new field to the mathematical community at large, which I published in the October issue of "Computers and Mathematics".

At the same time as the computer was starting to change mathematics research and applications, various instructors brought it into their classrooms. Computer Algebra Systems such as *Mathematica* and *Maple* were used to teach calculus in a new way, and a number of new textbooks to support such teaching came onto the market. Some of the articles and product reviews in "Computers and Mathematics" were devoted to the increasing use of computers in the world of university mathematics education. Things were starting to move very quickly.

When he introduced the last section he edited, Barwise had written:

> Whether we like it or not, computers are changing the face of mathematics in radical ways, from research, to teaching, to writing, personal communication, and publication. Over the past couple of years we have seen numerous articles about these developments.
>
> Computers are even forcing us to expand our idea about what constitutes doing mathematics, by making us take much more seriously the role of experimentation in mathematics. (I draw attention to a new journal devoted to experimental mathematics below.)
>
> One view of the future is that mathematics will come to have (or already has) two distinct sides: experimentation, which can exploit the speed and graphics abilities of programs like

> Maple and Mathematica, to allow us to spot regularities and make conjectures, and proof, very much in the style of today's mathematics. ...

> Whether we applaud or abhor all these changes in mathematics, there is no denying them by turning back the clock, anymore than there is in the rest of life. Computers are here to stay, just as writing is, and they are changing our subject.

It is surely obvious from those final remarks that the computer was seen as something of a threat by some mathematicians, and the "Computers and Mathematics" section was not without its detractors.

Taking over a month later, I began by saying that:

> This column is surely just a passing fad that will die away before long. Not because mathematics will cease to have much connection with computers, but rather, quite the reverse: the use of computers by mathematicians will become so commonplace that no one thinks to mention it any more.

When I wrapped up the section 4 years later, I wrote:

> With its midwifery role clearly coming to an end, the time was surely drawing near when "Computers and Mathematics" should come to an end. The change in the format of the *Notices*, which will take place at the end of this year, offered an obvious juncture to wind up the column. ...
>
> The disappearance of this column does not mean that the *Notices* will stop publishing articles on the use of computers in mathematics. Rather, recognizing that the use of computer technology is now just one more aspect of mathematics, the new *Notices* will no longer single out computer use for special attention. I will drink to that.
>
> The child has come of age.

And so mathematics moved on. In 2004, Jon Borwein and David H. Bailey published (together with coauthors in two cases) the first of what would be three major research monographs on experimental mathematics, and in 2008, Jon and I published our expository text *The Computer as Crucible: An Introduction to Experimental Mathematics* [1]. A year later, Wolfram released *Wolfram Alpha*, an online computational tool that, among other things, was able to execute practically any mathematical method or procedure—faster and more accurately than any human, and with effectively no restrictions on data size.

The computer had, by then, completely revolutionized all of procedural mathematics. Only the pure mathematicians, who focus on finding proofs of precisely worded theorems, remained almost entirely unscathed by the revolution.

In late 2016, after I learned of Jon's tragic early passing, I looked back on my own mathematical work in the 20 years after I edited my last *Notices* "Computers and Mathematics" section, some of it with Jon. My reflections prompted me to pen—more accurately type (on a computer)—an opinion piece for the *Huffington Post*, which was published on January 1, 2017, with the startling, but absolutely accurate title: "All the Mathematical Methods I Learned in My University Math Degree Became Obsolete in My Lifetime" [3]. For the fact is, that over a period of just under a quarter-century, during which time I moved from working in pure mathematics (i.e., focusing on proofs) to making use of mathematics to solve large-scale, real-world problems, my daily experience of doing mathematics changed from using methods and executing procedures to putting problems into a form where I could apply a powerful computational tool such as (in my case) *Wolfram Alpha* or *Mathematica*.[4]

True, by then I was no longer a pure mathematician, so my experience here is not typical of pure math. But it is typical of the way doing math has changed for the vast majority of mathematicians in the world. Besides, no one can look at the computer-intensive work of Jonathan Borwein and David H. Bailey in Number Theory, where they also use *Mathematica*, and pretend it is anything other than pure math. To be sure, some pure mathematicians make hardly any professional use of computers aside from email and an occasional Google search. But for a great many, the computer is now an integral part of how they carry out their work.

That then is the story of how mathematicians learned to stop worrying and love the computer. I could go on, and dig much deeper into the details. But, given the ease with which, given a few key issues (and associated key words), we can now all dig down on our own, I'll let you get a sense of the mathematical computer revolution by browsing the image gallery associated with this short article. The gallery can be viewed as a browser-playable slide presentation at

https://www.icloud.com/keynote/0C-SmMChXRYKuvGeWFTwz8l3g#BorweinPaperIMAGES and as a browser-viewable or downloadable PDF file at

https://web.stanford.edu/%7Ekdevlin/BorweinPaperIMAGES.pdf

**Further Reading**

For a complete index to everything published in the "Computers and Mathematics" section of the AMS Notices, see [4].

Accessible books the reader may find helpful are: [1, 2, 5, 6]

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[4]Full disclosure. I was a member of Wolfram's initial *Mathematica* Advisory Board in the products early years (we were all unpaid), so I naturally defaulted to using Wolfram products. But there were several CASs being developed around the same time, *Maple*, *Matlab*, *Magma*, *Sage*, etc.