Application of Fuzzy Sets in Reliability and in Optimal Condition Monitoring Technique Selection in Equipment Maintenance

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Abstract In order to improve the design of a system, we need to identify the least reliable component of the system. Unexpected failure of any component of the system may increase the maintenance and down time cost due to unavailability of the system. Though this is easy in simpler systems, it becomes a difficult task as the complexity of the system increases. A methodology using mathematical modelling facility of fuzzy set theory is presented here, which is effective in situations wherein the data available is mostly subjective and it is difficult to get precise quantitative data. After covering basic concepts of various uncertainty modelling theories and fuzzy sets, its application to reliability and fault tree is presented. In the second part of the chapter, multi-attribute decision making methods with application to ranking and optimal condition monitoring technique selection from maintenance engineering domain is presented. These include fuzzy set based Analytic Hierarchy Process (AHP), rating and ranking method, ranking by maximizing and minimizing sets, raking by cardinal utilities and suitability set method.

1 Introduction

In this competitive world, reliability and maintenance of components, sub-systems, systems etc. makes considerable impact on profitability of an enterprise. Reliability, i.e. survival probability of system, was researched world-wide at academic as well as industry level for last several decades. Consequences of inadequate maintenance of equipment was also studied for last many decades. Conventionally, reliability and maintenance modeling is carried out by probabilistic method. Probabilistic methods fared well as product life cycle was long and sufficient data gathering was possible. In present day scenario, product life cycles are short and as a result, there

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is acute need of newer ways for reliability and maintenance modeling. This chapter presents the application of a different approach i.e. fuzzy set theory approach in reliability and maintenance engineering.

2 Various Uncertainty Modeling Theories

There are two facets of looking at ‘uncertainty’. Uncertainty that arises from variability or stochasticity is random. Such uncertainty that is based on randomness is termed as ‘aleatory uncertainty’. The other kind of uncertainty is due to lack of information or data, ignorance, subjectivity in expressions, linguistics, ambiguity, etc. This uncertainty is a result of fuzziness and is known as ‘epistemic uncertainty’. Aleatory type uncertainty is dealt by Probability Theory that handles randomness. Epistemic uncertainty is dealt by possibility theory or fuzzy set theory.

Probability theory has its roots in the 17th century. Probability is an objective characteristic. It deals with the chance of occurrence of an event such as rolling a dice and getting numbers 1, 2, 3 4, 5 or 6. The conclusions of probability theory are tested by conducting an experiment or by experience. The basics of probability theory lie in set theory and the logic used is that either an element belongs to a set or not. Interval used in probability theory is [0, 1].

Fuzzy set theory was established and developed since the year 1965. Fuzzy set theory is presently a quite well established mathematical theory [1–4]. Fuzzy set theory and its applications developed very comprehensively over the last few decades and has attracted the attention of academicians, researchers, and practitioners world-wide. It considers subjective uncertainty in linguistics e.g. ‘beautiful child’, ‘cold day’, ‘very hot’ etc. The membership grade used here is subjective in nature. This theory focuses primarily on imprecision which is intrinsic in natural languages and is usually associated with the term ‘possibilistic’. The term ‘variable’ is used here in a linguistic sense. Possibility theory also uses [0, 1] interval.

Other prominent theories for uncertainty modeling are Bayesian theory, rough sets theory, vague sets theory, intuitionistic fuzzy sets theory, neutrosophic sets theory, soft sets theory, interval arithmetic theory, evidence theory etc.

Bayesian theory provides a mathematical framework wherein using probability concepts, inferences are drawn and reasoning is performed. It is based on Bayes’ theorem where posterior probability is calculated by updating the prior probability. It describes the probability of an event based on prior knowledge of conditions that might be related to the event. Analyst develops a set of initial beliefs and then adjusts them through experimentation and arrives at the Bayesian posterior probability.

Rough set theory proposed by Pawlak in 1982, introduced a new mathematical approach to model imperfect knowledge, i.e. imprecision or vagueness. In this theory, vagueness is expressed by a boundary region of a set. Rough set is defined by means of topological operations, interior and closure, called approximations. This theory is different than that of probability theory in statistics or fuzzy set theory that uses membership grades. This theory does not need any initial information or additional information about data. Interpretation of results is easier in rough set theory and is popular amongst people working in the field of computer engineering.

Intuitionistic fuzzy set and vague set are further extensions or generalizations of fuzzy sets. The definition of intuitionistic fuzzy sets was given by Atanassov in 1983 while vague sets were defined by Gau and Buehrer in 1993. Intuitionistic fuzzy sets are developed prior to vague sets. In Intuitionistic fuzzy sets, there is a membership grade and also a non-membership grade attached with each element. Moreover, there is a constraint that the sum of these two grades, i.e. membership grade and non-membership grade is less than or equal to one. In fuzzy sets, point-based membership values are used while in vague sets, membership values provided are interval based. In vague sets also, an object is characterized by two different membership functions, namely true membership function and false membership function. It is found that interval based membership functions used in vague sets capture data vagueness in a better way and are more expressive. In literature, researchers have considered both intuitionistic fuzzy sets and vague sets as equivalent, i.e. corresponding or similar in form and relations.

Florentin Smarandache in 1995 introduced a theory known as neutrosophic logic and sets. Neutrosophy refers to the knowledge of neutral thought and it represents the main distinction between sets, fuzzy sets and intuitionistic fuzzy sets. In this theory, a different viewpoint is proposed in which each proposition is estimated to have a degree of truth, a degree of indeterminacy and a degree of falsity. A neutrosophic set is a set where each element of the universe has a degree of truth, indeterminacy and falsity. In this type of sets, the indeterminacy is independent of truth and falsity values. All the three degrees i.e. of truth, indeterminacy and falsity, vary within [0, 1].

Another mathematical tool, i.e. theory of soft sets was proposed by Molodtsov in 1999. This theory also deals with uncertainties. Soft set is defined as a parameterized family of subsets of universe set where each element is considered as a set of approximate elements of it, i.e. of soft set. In short, the boundary of the set depends on the parameters. This theory is somewhat a generalization of fuzzy set theory.

Interval arithmetic (also popular as interval mathematics or interval analysis) was proposed by Ramon E. Moore in 1957. This theory characterises each value as a range of possibilities. It considers the computation of both the exact solution and the error term as a single entity i.e. an interval. It is an approach that puts bounds on rounding errors or measurement errors in mathematical computations. It develops numerical methods that yield consistent results. Apart from handling rounding errors and uncertainties, it also helps to get dependable results and definite solutions to equations and also for optimization problems. This concept is effectively used till now in numerous applications in mathematics, computer science and engineering.

Evidence theory (also referred as theory of belief functions) was first introduced by Dempster in 1967 and later extended by Shafer in 1976. This theory is popular as Dempster-Shafer theory. It provides a general framework for reasoning with uncertainty having influence of other theories like probability and possibility theories. Evidence theory is relatively closer to the classical probability theory. Evidence theory considers the evidence from different sources and computes degree of belief i.e. belief function which takes into account all available evidence. The major advantage of this theory is that this requires comparatively less information to describe the phenomenon than the classical probability theory. Evidence theory uses Dempster-Shafer combination rule instead of usual Bayes formula to update the belief function. Four important concepts in evidence theory are—frame of discernment, basic belief assignment, belief function and plausibility function.

3 Fuzzy Set Theory

Human communication and interpretation involves ambiguity and multiple meanings to its expressions. Fuzzy sets have helped researchers and academicians to model the amount of ambiguity and subjectivity. Variety of engineering and management problems involving ambiguous, imprecise, inexact or vague information are well solved using fuzzy sets in literature [1–4]. Fuzzy set theory is suitable to handle problems which lack of adequate data, inconsistency of data, experts differing in their opinions, ignorance, etc.

The originator of fuzzy set and fuzzy logic is Zadeh. In 1965, Zadeh while working in the field of control engineering, introduced fuzzy set concept. He developed the idea of partial belongingness of an object to the set. Traditionally in set theory or in probability theory, an object may belong or not belong to the set. The traditional set is a crisp set. The related logical scheme is like—it may be true or false. The logic of crisp set is extended to fuzzy set wherein the idea of partial truth or partial false is introduced. In practical world, the data values are vague in many of the applications. Fuzzy set theory handles the vagueness by generalizing the idea of membership in a set. An object may be a member of a set to some degree. Fuzzy set theory presents the concept of grades or values or function of membership. Therefore the membership of some object in the given set X may be no membership, full membership or partial membership. Each element of a fuzzy set is linked to a point-value from the unit interval of [0, 1] that is designated as the grade of membership in the fuzzy set.

4 Fuzzy Reliability

Conventional reliability is defined as the probability that a component or a system will perform its intended function adequately/satisfactorily for a specified period of time, understated operating conditions. Conventional reliability concept is based completely on the probability theory. The basic assumption in the conventional reliability is that there are only two states for a component or a system. These are operating or working state and failed state. With the growth in possibility theories in last few decades, there is a vital change in viewing the reliability concepts and assumptions. Reliability theories are modeled using fuzzy sets, mainly with the remarkable development of fuzzy set theory. Although conventional reliability theory cannot be out rightly excluded, the merits of fuzzy reliability must be used in parallel. There are many ideas proposed in the literature in the fuzzy reliability domain. This theory is based on both possibility assumption as well as probability assumption. It also considers binary state assumption and fuzzy-state assumption. Fuzzy reliability concepts use the combination of the above-said assumptions [5–8].

The conventional reliability is PROBIST reliability. It is based on the assumption of probability and the binary-states. PRO stands for probability and BIST stands for binary states. The states are deterministic i.e. crisp and the system will be in one of these two states, operating or failed, at any given point in time. The PROBIST reliability of the component or the system is computed using fuzzy sets which consider it as a fuzzy number. PROBIST reliability can be also in the form of linguistic values. Researchers have considered PROBIST reliability as a fuzzy number with membership functions of triangular, trapezoidal, normal, etc.

PROFUST reliability considers probabilistic assumption and the fuzzy state assumption. The failure time of the system is assumed to follow probability measures. But, in this case, operating and failed states are defined by fuzzy states. The system can be in one of the fuzzy states at any given point in time. As crisp state is one of the states of fuzzy set, PROFUST reliability is one of the cases of PROBIST reliability.

POSBIST reliability is based on possibility assumption and assumes the binary states, while POSFUST reliability is defined based on the concept of possibility theory and fuzzy states.

4.1 Example of PROBIST Fuzzy Reliability

PROBIST reliability is considered as Triangular Fuzzy Number (TFN) [9, 10] in this example. Figure 1 shows the reliability block diagram of a system consisting of six components. There are three subsystems configured in series. First subsystem connects two components A and B in parallel. Second subsystem has two components C and D linked in series and third subsystem again has parallel configuration with components E and F. Component reliabilities as symmetric TFN are as below:

$$
\begin{aligned}
\mathrm{R_A} &= [0.65, 0.75, 0.85] \\
\mathrm{R_B} &= [0.70, 0.75, 0.85] \\
\mathrm{R_C} &= [0.70, 0.80, 0.95] \\
\mathrm{R_B} &= [0.85, 0.90, 0.85] \\
\mathrm{R_F} &= [0.60, 0.70, 0.80]
\end{aligned}
$$

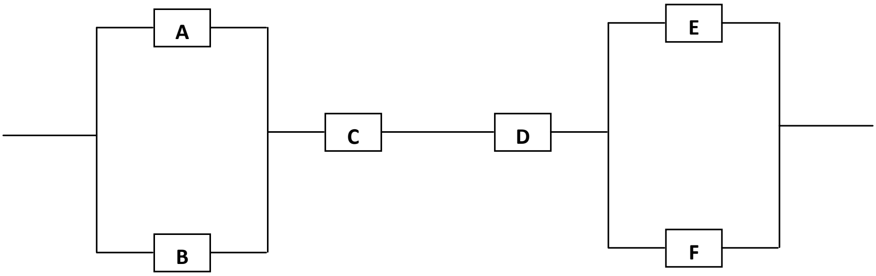

Fig. 1 Reliability block diagram

Series system reliability is given by,

$$[R_{SS}] = \prod_{i=1}^{n} R_i$$

Parallel system reliability is given by,

$$[R_{PS}] = 1 - \prod_{i=1}^{n} [(1 - R_i)]$$

Taking α-cut [9, 10] and solving for parallel subsystem reliability, we get

$$\begin{aligned}[R_{AB}] &= [1 - [\{1 - (0.65 + 0.1\alpha)\} \cdot \{1 - (0.70 + 0.05\alpha)\}], \\ &\quad 1 - [\{1 - (0.85 - 0.1\alpha)\} \cdot \{1 - (0.85 - 0.1\alpha)\}]] \\ &= [0.8950, 0.9375, 0.9775]\end{aligned}$$

$$\begin{aligned}[R_{EF}] &= [1 - [\{1 - (0.75 + 0.05\alpha)\} \cdot \{1 - (0.60 + 0.1\alpha)\}], \\ &\quad -[\{1 - (0.85 - 0.05\alpha)\} \cdot \{1 - (0.80 - 0.1\alpha)\}]] \\ &= [\,0.9000, \quad 0.9400, \quad 0.9700\,]\end{aligned}$$

$$\begin{aligned}[R_{AB}] &= [\,0.8950, \quad 0.9375, \quad 0.9775\,] \\ [R_C] &= [\,0.7000, \quad 0.8000, \quad 0.9000\,] \\ [R_D] &= [\,0.8500, \quad 0.9000, \quad 0.9500\,] \\ [R_{EF}] &= [\,0.9000, \quad 0.9400, \quad 0.9700\,]\end{aligned}$$

Table 1 PROBIST reliability values

A	R_{SS}(Lower)	R_{SS}(Upper)
0	0.479273	0.81069
0.1	0.493484	0.791743
0.2	0.507978	0.773099
0.3	0.522757	0.754756
0.4	0.537825	0.736709
0.5	0.553186	0.718957
0.6	0.568842	0.701495
0.7	0.584799	0.684322
0.8	0.601058	0.667433
0.9	0.617624	0.650827
1.0	0.6345	0.6345

Taking α- cut and solving for Series system reliability, we get

$$[R_S] = \{(0.8950 + 0.0425\alpha) \cdot (0.7000 + 0.1000\alpha) \cdot (0.8500 + 0.0500\alpha) \cdot (0.9000 + 0.0400\alpha), \\ (0.9775 - 0.04\alpha) \cdot (0.9000 - 0.1000\alpha) \cdot (0.9500 - 0.0500\alpha) \cdot (0.9700 - 0.0300\alpha)\}$$

For various values of α, PROBIST reliability bounds are presented in Table 1.

4.2 Conceptual Example of PROFUST Fuzzy Reliability

The illustrated example gives an indication of PROFUST reliability applied to a degrading system [11]. Consider a system consisting of five sub-systems that perform the operation simultaneously. If all five sub-systems work then we say that the system is functioning fully. However, if any of the sub-system fail then the system operates, but in degraded mode. As a result, system performance also degrades. For such a system there are more than two states which can be modeled using fuzzy sets. The solution methodology would involve developing a Markov model for degradable system. Markovian analysis considers the system as being in one of the several states, either in an operating state or failed state. Here the assumptions are that conditional probability of failure during any fixed interval of time is constant and so are the transition rates. Also, the probability that the system would undergo transition from one state to another state depends only on the current state of the system. The transition does not depend on any of the states the system has experienced earlier. Exponential distribution having memoryless property is used to model failure times of the sub-systems, as it satisfies Markovian property.

Let the universe of discourse be $U = \{S_0, S_1, S_2, S_3, S_4, S_5\}$; where fuzzy success states are given by $S = \{S_i,\ \mu_S(S_i)\,;\quad i = 1,\ 2,\ 3,\ 4,\ 5\}$; and corresponding fuzzy failure states are given by $F = \{S_i,\ \mu_F(S_i)\,;\quad i = 1,\ 2,\ 3,\ 4,\ 5\}$. μ is the membership grade. For Markovian analysis, consider the transition from fuzzy success state to fuzzy failure state as $T_{SF} = \{m_{ij}\,,\,\mu_{T_{SF}}(m_{ij})\,; \quad i = 1,\ 2,\ 3, 4,\ 5\}$ where T_{SF} represents and m_{ij} is the transition from state S_i to S_j. The next step is to build the membership functions and derive the equation for the PROFUST reliability using an appropriate method. Using the alpha-cut technique or any other method of fuzzy sets, values of PROFUST reliability can be computed.

4.3 Fuzzy Fault Tree

Fault tree analysis is one of the ways to compute probability of failure, and in turn obtain reliability of the system. Traditionally, the failure probabilities of components are obtained from past data. However, several complications such as continuously changing environment and availability of data about components make the calculation of system reliability difficult. If component failure probabilities are not accurate, then the top event probability, i.e. system failure probability will not be accurate as well. Therefore acute need was identified by the system designers to obtain the data from the field experts based on their subjective assessments. In literature, fuzzy sets and its arithmetic have dealt with such situations. The fuzzy sets are extensively used in fault tree analysis that provide a mathematical framework for imprecise and uncertain data situations.

In fuzzy fault tree analysis [7], triangular fuzzy numbers are utilized as possibility distribution of each of the primary event. In quantitative assessment of a fuzzy fault tree, the fuzzy data in the form of triangular fuzzy number is considered for each component at the bottom-most hierarchical level of the fault tree. Usual logic of the trees along with Boolean operations and fuzzy arithmetic is used to provide the failure possibility calculation of all the subsequent higher level events. Finally, possibility distribution for the top-most event i.e. possibility of failure of the system under analysis is determined.

In probabilistic fault tree analysis, exponential time to failure distribution with constant failure rate λ is considered. Based on available data, parameter λ of exponential failure time is a single point estimated value. In fuzzy fault tree analysis it is considered that the uncertainty exists in estimation of parameter λ. This uncertainty is modeled by defining λ as a triangular fuzzy number with triplets λ_1 as lowermost value, λ_2 is middle value and λ_3 as rightmost value i.e. $\lambda = (\lambda_1, \lambda_2, \lambda_3)$. An example is presented here for the fault tree depicted in Fig. 2.

The top event T can be expanded as:

$$\begin{aligned} T &= A_1 \cdot A_2 \\ &= (X_1 + X_2) \cdot (X_3 . A_3) \\ &= (X_1 + X_2) \cdot [X_3 \cdot (X_4 + X_5)] \end{aligned}$$

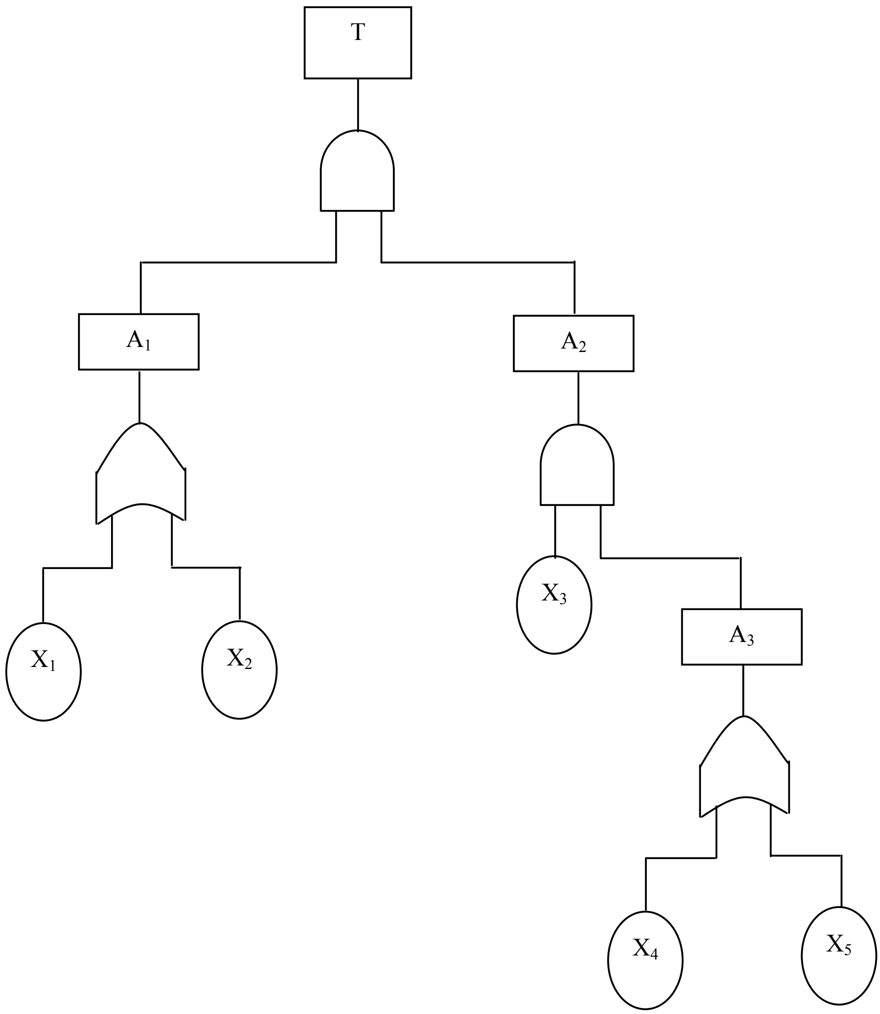

Fig. 2 Fault tree diagram

Given p_i, the failure probability of X_i, the failure probability of top event T is,

$$prob(T) = [1 - (1 - p_1)(1 - p_2)] \cdot p_3 \cdot [1 - (1 - p_4)(1 - p_5)]$$

The exponential time to failure distribution is assumed for all basic events with fuzzy failure rate λ_i i.e. in the form of TFN with triplets $\lambda_{i1}, \lambda_{i2}\, and\, \lambda_{i3}$. The intervals of confidence for the fuzzy failure probability p_i, for the level of presumption α, have been estimated using following equation:

Table 2 Fuzzy probability of the basic events

Basic events	Failure rate λ (crisp value)	Triangular fuzzy number					
		Failure rate			Probability		
X_1	0.0004	0.0001	0.0004	0.0006	0.09516	0.32968	0.45119
X_2	0.0007	0.0004	0.0007	0.00085	0.32968	0.50342	0.57259
X_3	0.0001	0.00005	0.0001	0.00025	0.04877	0.09516	0.22120
X_4	0.00045	0.0002	0.00045	0.0008	0.18127	0.36237	0.55067
X_5	0.00064	0.0005	0.00064	0.00075	0.39347	0.47271	0.52763

Table 3 Fuzzy Probability of the Top Event

Presumption level	Fuzzy probability of the top event	
	Lower bound	Upper bound
0.0	0.009660	0.133376
0.1	0.011807	0.122477
0.2	0.014195	0.111961
0.3	0.016830	0.101835
0.4	0.019711	0.092101
0.5	0.022840	0.082766
0.6	0.026216	0.073831
0.7	0.029837	0.065299
0.8	0.033701	0.057172
0.9	0.037804	0.049453
1.0	0.042141	0.042141

$$[F(t)]_\alpha = \left[1 - \frac{1}{e^{\lambda_{1t} + \alpha(\lambda_2 - \lambda_1)t}}, 1 - \frac{1}{e^{\lambda_{3t} - \alpha(\lambda_3 - \lambda_2)t}}\right]$$

$\alpha = 0$, gives p_{i1} as the lower bound and p_{i3} as the upper bound and $\alpha = 1$ gives p_{i2} which is the middle value. These are given in Table 2.

Table 3 shows the intervals of confidence at different levels of presumption for the top event probability with assumed $t = 1000$.

5 Fuzzy Multi Attribute Decision Making

Decision making in maintenance engineering is a real challenge for decision makers, i.e. engineer/manager as he/she faces a problem involving information and data uncertainty. Much of the decision-making in this field happens in an environment in which the objectives and the constraints are not accurately known. The major source of imprecision in maintenance decision making processes is fuzziness i.e. a kind of imprecision that is associated with fuzzy sets. Although many models have been developed over the years, its discussion is beyond the scope of this chapter. Here, we discuss Multi Attribute Decision Making (MADM) problems involving fuzziness that are solved using various fuzzy set based methods.

The basic MADM problem is to choose between or rank a set of alternatives, given some decision attributes (also known as criteria). Let $A = \{a_i\}; \quad i = 1,\ 2,\ 3, \ldots, n$ be the set of decision alternatives and $C = \{c_j\}; \quad j = 1,\ 2,\ 3, \ldots, m$ be the set of attributes according to which the desirability of an alternative is to be judged. The aim here is to obtain the optimal alternative with highest degree of desirability with respect to all relevant attributes [12–15].

5.1 Selection or Ranking of Condition Monitoring Techniques—Problem

The running example [16–21] explained here is related to the maintenance of a turbine. The purpose is to rank or select optimal alternative out of the three alternative techniques namely Vibration analysis (A_1), Lube oil/debris analysis (A_2), Endoscopic examination (A_3). Nine decision attributes are considered, namely, investment cost (C_1), operating cost (C_2), accuracy of the technique (C_3), repeatability of the instruments used (C_4), ease of use (C_5), environmental restrictions (C_6), technical expertise requirement (C_7), ease of maintenance (C_8), ease of mounting (C_9). This problem is solved using fuzzy AHP and other fuzzy set based methods.

5.2 Fuzzy Analytic Hierarchy Process (FAHP)

The Analytic Hierarchy Process (AHP), also known as priority theory, is a commanding and flexible decision making procedure that helps people set priorities and make the best decision when both qualitative and quantitative aspects of a decision are to be considered. The AHP engages decision makers in structuring a decision into smaller parts, proceeding from the goal to objectives to sub-objectives down to the alternative courses of action. Decision makers then make simple pair wise comparison judgments throughout the hierarchy to arrive at the overall priorities for the alternatives. The analytic hierarchy process allows users to assess the relative weight of multiple attributes in an intuitive manner. Saaty, in 1980, invented AHP methodology that established a consistent way of converting pair wise comparisons into a set of numbers representing the relative priority of each of the attributes. Many fuzzy set based AHP have been suggested by researchers [14, 15, 17].

The fuzzy set scale for intensity of importance in the form of TFN used in this problem is given in Table 4.

Table 4 Fuzzy Set scale for intensity of importance

Intensity of importance (TFN)	Definition	Explanation
[1, 1, 1]	Equal importance	Two activities contribute equally to the objective
[1, 2.5, 4]	Weak importance of one over another	Experience and judgment slightly favor one activity over another
[3, 4.5, 6]	Essential or strong importance	Experience and judgment strongly favor one activity over another
[5, 6.5, 8]	Demonstrated importance	An activity is strongly favored and its dominance demonstrated in practice
[7, 8.5, 10]	Absolute importance	The evidence favoring one activity over another is of highest possible order of affirmation

Table 5 shows a matrix of relative significance of each pair of attributes. Let r_{ij} be the numerical value assigned to the relative significance i.e. importance of attributes C_i and C_j. Fuzzy set scales for intensity of importance are available in the literature which may also be used. If both C_i and C_j, are equally important then $r_{ij} = 1$; if C_i is more important than C_j then $r_{ij} > 1$ and if C_i is less important than C_j then $r_{ij} < 1$. r_{ij} is a reciprocal matrix and it has positive entries throughout satisfying the reciprocal property i.e. $r_{ji} = \frac{1}{r_{ij}}$. Here, r_{ij} s are in TFN form and its reciprocal is obtained using inverse operation on TFN. Table 6 gives normalized average weights (priorities) matrix of the attributes. It is shown by researchers that normalized column and row weights are adequate as normalized Eigen vectors as used in the case of single value (crisp) intensity of importance, and as such the average of the row and column is taken as the final weight.

The three maintenance alternatives are then compared in pair wise manner under each criterion. These matrices are given in Tables 7, 8, 9, 10, 11, 12, 13, 14 and 15.

The final score of each alternative is then calculated by performing fuzzy multiplication of priority of attributes and priority of condition monitoring technique and then adding them up for each condition monitoring technique. This is given in Table 16. The final scores of techniques in TFN form are: Vibration analysis = [0.139, 0.294, 0.624], Lube oil/debris analysis = [0.104, 0.252, 0.659] and Endoscopic examination = [0.164, 0.371, 0.823]. Ranking of all three techniques is plotted in Fig. 3 in TFN form.

5.3 Linguistic Scales and Input from the Experts

A few more ranking methods are explained in subsequent sub- sections. For this, the ratings and weights have been expressed in linguistic terms. They are considered as linguistic variables, represented by the fuzzy sets. The grade membership for both the variables are considered as TFN's. Figures 4 and 5 show TFN representations of linguistic variables related to rating and weights on the scale [0, 1].

Table 5 Matrix of relative significance of decision attributes

Attributes	C_1	C_2	C_3	C_4	C_5
C_1	1	[0.13, 0.15, 0.2]	[0.1, 0.12, 0.14]	[0.1, 0.12, 0.14]	[0.13, 0.15, 0.2]
C_2	[5, 6.5, 8]	1	[0.17, 0.22, 0.33]	[0.17, 0.22, 0.33]	[0.25, 0.4, 1]
C_3	[7, 8.5, 10]	[3, 4.5, 6]	1	[0.17, 0.22, 0.33]	[5, 6.5, 8]
C_4	[7, 8.5, 10]	[3, 4.5, 6]	[3, 4.5, 6]	1	[5, 6.5, 8]
C_5	[5, 6.5, 8]	[1, 2.5, 4]	[0.13, 0.15, 0.2]	[0.13, 0.15, 0.2]	1
C_6	[5, 6.5, 8]	[1, 2.5, 4]	[0.17, 0.22, 0.33]	[0.17, 0.22, 0.33]	[0.17, 0.22, 0.33]
C_7	[7, 8.5, 10]	[3, 4.5, 6]	[3, 4.5, 6]	[3, 4.5, 6]	[1, 2.5, 4]
C_8	[5, 6.5, 8]	[0.25, 0.4, 1]	[0.17, 0.22, 0.33]	[0.13, 0.15, 0.2]	[0.17, 0.22, 0.33]
C_9	[0.17, 0.22, 0.33]	[0.25, 0.4, 1]	[0.13, 0.15, 0.2]	[0.1, 0.12, 0.14]	[0.13, 0.15, 0.2]
CS	[42.17, 52.72, 63.33]	[12.63, 20.45, 29.2]	[7.87, 11.08, 14.53]	[4.97, 6.7, 8.67]	[12.85, 17.64, 23.06]

Attributes	C_6	C_7	C_8	C_9	RS
C_1	[0.13, 0.15, 0.2]	[0.1, 0.12, 0.14]	[0.13, 0.15, 0.2]	[3, 4.5, 6]	[4.82, 6.46, 8.22]
C_2	[0.25, 0.4, 1]	[0.17, 0.22, 0.33]	[1, 2.5, 4]	[1, 2.5, 4]	[9.01, 13.96, 19.99]
C_3	[3, 4.5, 6]	[0.17, 0.22, 0.33]	[3, 4.5, 6]	[5, 6.5, 8]	[27.34, 36.44, 45.66]
C_4	[3, 4.5, 6]	[0.17, 0.22, 0.33]	[5, 6.5, 8]	[7, 8.5, 10]	[34.17, 44.72, 55.33]
C_5	[3, 4.5, 6]	[0.25, 0.4, 1]	[3, 4.5, 6]	[5, 6.5, 8]	[18.51, 26.2, 34.4]
C_6	1	[0.13, 0.15, 0.2]	[3, 4.5, 6]	[5, 6.5, 8]	[15.64, 21.81, 28.19]
C_7	[5, 6.5, 8]	1	[5, 6.5, 8]	[7, 8.5, 10]	[35, 47, 59]
C_8	[0.17, 0.22, 0.33]	[0.13, 0.15, 0.2]	1	[3, 4.5, 6]	[10.02, 13.36, 17.39]
C_9	[0.13, 0.15, 0.2]	[0.1, 0.12, 0.14]	[0.17, 0.22, 0.33]	1	[2.18, 2.53, 3.54]
CS	[15.68, 21.92, 28.73]	[2.22, 2.6, 3.67]	[21.3, 30.37, 39.53]	[37, 49, 61]	[156.69, 212.48, 271.72]#

RS Row Sum; *CS* Column Sum; # Total of Row Sum

Table 6 Average priority TFN's of attributes

Attributes	C_1	C_2	C_3	C_4	C_5
RS	[4.82, 6.46, 8.22]	[9.01, 13.96, 19.99]	[27.34, 36.44, 45.66]	[34.17, 44.72, 55.33]	[18.51, 26.2, 34.4]
CS	[42.17, 52.72, 63.33]	[12.63, 20.45, 29.2]	[7.87, 11.08, 14.53]	[4.97, 6.7, 8.67]	[12.85, 17.64, 23.06]
N	[0.017, 0.03, 0.052]	[0.033, 0.065, 0.13]	[0.10, 0.17, 0.29]	[0.12, 0.21, 0.35]	[0.068, 0.12, 0.22]
IN	[0.016, 0.019, 0.023]	[0.034, 0.049, 0.079]	[0.069, 0.090, 0.13]	[0.12, 0.15, 0.20]	[0.04, 0.06, 0.08]
Average priority of attributes	[0.017, 0.025, 0.075]	[0.034, 0.057, 0.105]	[0.085, 0.13, 0.21]	[0.12, 0.18, 0.275]	[0.054, 0.09, 0.15]

Attributes	C_6	C_7	C_8	C_9
RS	[15.64, 21.81, 28.19]	[35, 47, 59]	[10.02, 13.36, 17.39]	[2.18, 2.53, 3.54]
CS	[15.68, 21.92, 28.73]	[2.22, 2.6, 3.67]	[21.3, 30.37, 39.53]	[37, 49, 61]
N	[0.058, 0.103, 0.18]	[0.13, 0.22, 0.38]	[0.037, 0.063, 0.11]	[0.008, 0.012, 0.023]
IN	[0.035, 0.046, 0.064]	[0.27, 0.38, 0.45]	[0.025, 0.033, 0.047]	[0.016, 0.020, 0.027]
Average priority of attributes	[0.047, 0.075, 0.122]	[0.2, 0.3, 0.42]	[0.031, 0.048, 0.079]	[0.012, 0.016, 0.025]

N Normalized (Row); *IN* Inverted, Normalized (Column)

Usually, inputs are always taken from individual experts and are then combined as a part of group decision making. Means of combining expert's judgment include averaging method, averaging method with feedback, and combining method for symmetric triangular fuzzy sets. The combined resulting fuzzy set is then required to be interpreted in terms of linguistic variable. To get this, resultant fuzzy set is to be 'mapped' on to the nearest pre-defined fuzzy set. Some mapping methods are—distance measures such as Hamming distance, Euclidean distance, credibility score method, etc. The input data for ratings and weights are given in Tables 17 and 18 respectively.

5.4 *Rating and Ranking Method*

As per this method (developed by Kanhe), the rating of the alternative i with respect to decision attribute j is r_{ij}; $i = 1, 2, 3, \ldots, n$; $j = 1, 2, 3, \ldots, m$. The relative

Table 7 Investment cost (C_1)

Techniques	A_1	A_2	A_3	RS	N	Average priority
A_1	1	[0.17, 0.22, 0.33]	[1, 1, 1]	[2.17, 2.22, 2.33]	[0.12, 0.15, 0.21]	[0.125, 0.15, 0.21]
A_2	[3, 4.5, 6]	1	[3, 4.5, 6]	[7, 10, 13]	[0.4, 0.69, 1.15]	[0.5, 0.69, 0.95]
A_3	[1, 1, 1]	[0.17, 0.22, 0.33]	1	[2.17, 2.22, 2.33]	[0.12, 0.15, 0.21]	[0.125, 0.15, 0.21]
CS	[5, 6.5, 8]	[1.34, 1.44, 1.66]	[5, 6.5, 8]	[11.34, 14.44, 17.66]#		
IN	[0.13, 0.15, 0.2]	[0.6, 0.69, 0.75]	[0.13, 0.15, 0.2]			

Table 8 Operating costs (C_2)

Techniques	A_1	A_2	A_3	RS	N	Average priority
A_1	1	[3, 4.5, 6]	[5, 6.5, 8]	[9, 12, 15]	[0.35, 0.57, 0.91]	[0.5, 0.65, 0.84]
A_2	[0.17, 0.22, 0.33]	1	[5, 6.5, 8]	[6.17, 7.72, 9.33]	[0.15, 0.37, 0.57]	[0.15, 0.28, 0.41]
A_3	[0.13, 0.15, 0.2]	[0.13, 0.15, 0.2]	1	[1.26, 1.3, 1.4]	[0.05, 0.06, 0.09]	[0.06, 0.07, 0.09]
CS	[1.3, 1.37, 1.53]	[4.13, 5.65, 7.2]	[11, 14, 17]	[16.43, 21.02, 25.73]#		
IN	[0.65, 0.73, 0.77]	[0.14, 0.18, 0.24]	[0.06, 0.07, 0.09]			

Table 9 Accuracy of the technique (C_3)

Techniques	A_1	A_2	A_3	RS	N	Average priority
A_1	1	[0.17, 0.22, 0.33]	[0.13, 0.15, 0.2]	[1.3, 1.37, 1.53]	[0.05, 0.07, 0.11]	[0.59, 0.77, 0.11]
A_2	[3, 4.5, 6]	1	[0.17, 0.22, 0.33]	[4.17, 5.72, 7.33]	[0.17, 0.3, 0.51]	[0.16, 0.24, 0.38]
A_3	[5, 6.5, 8]	[3, 4.5, 6]	1	[9, 12, 15]	[0.38, 0.63, 1.04]	[0.52, 0.68, 0.91]
CS	[9, 12, 15]	[4.17, 5.72, 7.33]	[1.3, 1.37, 1.53]	[14.47, 19.09, 23.86]#		
IN	[0.067, 0.083, 0.111]	[0.14, 0.17, 0.24]	[0.65, 0.73, 0.77]			

importance of decision attribute j is called weight and is denoted as ω_j. The ranking of the alternatives is performed according to their rank:

$$R_i = \frac{\sum_{j=1}^{m} \omega_j \cdot r_{ij}}{\sum_j \omega_j} \quad ; i = 1, 2, \ldots, n$$

The optimal alternative is the one for which value of R_i is maximum. The method is further extended by Bass and Kwakernaak [22] who proposed following two phases for the method:

Phase 1: Determination of ratings of alternatives: $A = \{a_i\}; \quad i = 1,\ 2,\ 3, \ldots, n$ is the set of decision alternatives and $C = \{\tilde{c}_j\}; \quad j = 1, 2, 3, \ldots..m$ are the attributes that are fuzzy in nature. Let $\tilde{R}_{ij}$ be the fuzzy rating of alternative i with respect

Table 10 Repeatability of the instruments used (C_4)

Techniques	A_1	A_2	A_3	RS	N	Average priority
A_1	1	[0.17, 0.22, 0.33]	[0.13, 0.15, 0.2]	[1.3, 1.37, 1.53]	[0.06, 0.08, 0.12]	[0.064, 0.082, 0.12]
A_2	[3, 4.5, 6]	1	[0.25, 0.4, 1]	[4.25, 5.9, 8]	[0.19, 0.34, 0.64]	[0.19, 0.31, 0.55]
A_3	[5, 6.5, 8]	[1, 2.5, 4]	1	[7, 10, 13]	[0.31, 0.58, 1.04]	[0.38, 0.62, 0.88]
CS	[9, 12, 15]	[2.17, 3.72, 5.33]	[1.38, 1.55, 2.2]	[12.55, 17.27, 22.53]#		
IN	[0.067, 0.093, 0.111]	[0.19, 0.27, 0.46]	[0.45, 0.65, 0.72]			

Table 11 Ease of use (C_5)

Techniques	A_1	A_2	A_3	RS	N	Average priority
A_1	1	[0.17, 0.22, 0.33]	[0.13, 0.15, 0.2]	[1.3, 1.37, 1.53]	[0.06, 0.08, 0.12]	[0.064, 0.082, 0.12]
A_2	[3, 4.5, 6]	1	[0.25, 0.4, 1]	[4.25, 5.9, 8]	[0.19, 0.34, 0.64]	[0.19, 0.31, 0.55]
A_3	[5, 6.5, 8]	[1, 2.5, 4]	1	[7, 10, 13]	[0.31, 0.58, 1.04]	[0.38, 0.62, 0.88]
CS	[9, 12, 15]	[2.17, 3.72, 5.33]	[1.38, 1.55, 2.2]	[12.55, 17.27, 22.53]#		
IN	[0.067, 0.093, 0.111]	[0.19, 0.27, 0.46]	[0.45, 0.65, 0.72]			

to j and $\tilde{W}_j \in R$ be the weight or importance of attribute j. The rating of alternative i with respect to attributes j is fuzzy and is given by the grade membership function $\mu_{R_{ij}}(r_{ij})$. The relative importance (weight) of attributes j is given by a fuzzy set $\tilde{W}_j$ with grade membership function $\mu_{W_j}(\omega_j)$.

The evaluation of alternative a_i is a fuzzy set and is computed using $\tilde{R}_{ij}$ and $\tilde{W}_j$:

$$g(z) = g(\omega_1, \ldots \omega_n,\ r_{i1}, \ldots r_{in}) = \frac{\sum_{j=1}^{m} \omega_j \cdot r_{ij}}{\sum_{j=1}^{m} \omega_j}$$

Table 12 Environmental restrictions (C_6)

Techniques	A_1	A_2	A_3	RS	N	Average priority
A_1	1	[0.13, 0.15, 0.2]	[0.1, 0.12, 0.14]	[1.23, 1.27, 1.34]	[0.044, 0.055, 0.07]	[0.047, 0.058, 0.075]
A_2	[5, 6.5, 8]	1	[0.17, 0.22, 0.33]	[6.17, 7.72, 9.33]	[0.22, 0.34, 0.51]	[0.18, 0.26, 0.38]
A_3	[7, 8.5, 10]	[3, 4.5, 6]	1	[11, 14, 17]	[0.4, 0.61, 0.92]	[0.54, 0.69, 0.86]
CS	[13, 16, 19]	[4.13, 5.65, 7.2]	[1.27, 1.34, 1.47]	[18.4, 22.99, 27.67]#		
IN	[0.05, 0.06, 0.08]	[0.14, 0.18, 0.24]	[0.68, 0.77, 0.79]			

Table 13 Technical expertise requirement (C_7)

Techniques	A_1	A_2	A_3	RS	N	Average priority
A_1	1	[3, 4.5, 6]	[5, 6.5, 8]	[9, 12, 15]	[0.38, 0.63, 1.04]	[0.52, 0.68, 0.91]
A_2	[0.17, 0.22, 0.33]	1	[3, 4.5, 6]	[4.17, 5.72, 7.33]	[0.17, 0.30, 0.51]	[0.16, 0.24, 0.38]
A_3	[0.13, 0.15, 0.2]	[0.17, 0.22, 0.33]	1	[1.3, 1.37, 1.53]	[0.054, 0.07, 0.11]	[0.061, 0.077, 0.111]
CS	[1.3, 1.37, 1.53]	[4.17, 5.72, 7.33]	[9, 12, 15]	[14.47, 19.09, 23.86]#		
IN	[0.65, 0.73, 0.77]	[0.14, 0.17, 0.24]	[0.067, 0.083, 0.111]			

Membership function of μ_{zi} is defined as:

$$\mu_{z_i}(z) = min\ \{\min_{j=1}^{n} (\mu_{W_j}(\omega_j)),\ \ \min_{k=1}^{n} (\mu_{R_{ik}}(r_{ik}))\}$$

The final rating is therefore $\widetilde{R}_i = \{(r,\ \mu_{R_i})\}$ given by a membership function $\mu_{R_i}(r) = sup_r\ \ \mu_{z_i}(z)$.

Phase 2: Ranking: Once final ratings are obtained, ranking or rank ordering is carried out in this phase. A measure is established to distinguish the 'preferable alternatives' from each other and rank them. If ratings $\widetilde{R}_i$ are fuzzy, then preference set $\widetilde{P}_i = \{(p,\ \mu_{P_i}(p)\}$, is obtained using:

$$\mu_{P_i}(p) = \sup_r\ \mu_R(r_1, \ldots, r_m)$$

Table 14 Ease of maintenance (C_8)

Techniques	A_1	A_2	A_3	RS	N	Average priority
A_1	1	[3, 4.5, 6]	[0.25, 0.4, 1]	[4.25, 5.9, 8]	[0.22, 0.44, 0.92]	[0.21, 0.36, 0.69]
A_2	[0.17, 0.22, 0.33]	1	[0.25, 0.4, 1]	[1.42, 1.62, 2.33]	[0.073, 0.12, 0.27]	[0.082, 0.13, 0.24]
A_3	[1, 2.5, 4]	[1, 2.5, 4]	1	[3, 6, 9]	[0.16, 0.44, 1.04]	[0.25, 0.5, 0.85]
CS	[2.17, 3.72, 5.33]	[5, 8, 11]	[1.5, 1.8, 3]	[8.67, 13.52, 19.33]#		
IN	[0.19, 0.27, 0.46]	[0.09, 0.13, 0.2]	[0.33, 0.55, 0.66]			

Table 15 Ease of mounting (C_9)

Techniques	A_1	A_2	A_3	RS	N	Average Priority
A_1	1	[0.17, 0.22, 0.33]	[0.13, 0.15, 0.2]	[1.3, 1.37, 1.53]	[0.05, 0.07, 0.11]	[0.59, 0.77, 0.11]
A_2	[3, 4.5, 6]	1	[0.17, 0.22, 0.33]	[4.17, 5.72, 7.33]	[0.17, 0.3, 0.51]	[0.16, 0.24, 0.38]
A_3	[5, 6.5, 8]	[3, 4.5, 6]	1	[9, 12, 15]	[0.38, 0.63, 1.04]	[0.52, 0.68, 0.91]
CS	[9, 12, 15]	[4.17, 5.72, 7.33]	[1.3, 1.37, 1.53]	[14.47, 19.09, 23.86]#		
IN	[0.067, 0.083, 0.111]	[0.14, 0.17, 0.24]	[0.65, 0.73, 0.77]			

The above fuzzy set is effectively used to judge the degree of preferability of an alternative over all the other alternatives.

Arithmetic operations such as addition, multiplication and division of fuzzy sets are involved in the computation. There are two ways in which this can be performed: by considering only the left, middle and right side values of TFNs or with the help of interval of confidence at each presumption level α (or popularly called as α-cut). R_1 i.e. rating of Vibration Monitoring is [0.465217, 0.661905, 0.86], R_2 i.e. rating of Lube Oil Analysis is [0.473913, 0.646032, 0.8275] and R_3 i.e. rating of Endoscopic examination is [0.445652, 0.633333, 0.82125]. The membership functions of the final ratings are shown in Fig. 6.

It is evident from Fig. 6 that vibration analysis technique is slightly dominant over other two techniques. Nonetheless, one needs to investigate the case further to obtain the optimal technique. To do this, another decisive factor is used which is able to differentiate between 'preferable alternatives'. The preference set is obtained to investigate preferability of vibration monitoring alternative over the others. The preference set thus obtained is [−0.359158, 0.0222225, 0.4002175] and is plotted in Fig. 7. It is observed from Fig. 7 that vibration monitoring technique is the optimal technique under fuzzy decision attributes considered in the example.

Table 16 Computation of scores of condition monitoring techniques

Attributes		C_1	C_2	C_3	C_4	C_5
Average priority of attributes		[0.017, 0.025, 0.075]	[0.034, 0.057, 0.105]	[0.085, 0.13, 0.21]	[0.12, 0.18, 0.275]	[0.054, 0.09, 0.15]
Average priority of techniques	A_1	[0.125, 0.15, 0.21]	[0.5, 0.65, 0.84]	[0.59, 0.77, 0.11]	[0.064, 0.082, 0.12]	[0.064, 0.082, 0.12]
	A_2	[0.5, 0.69, 0.95]	[0.15, 0.28, 0.41]	[0.16, 0.24, 0.38]	[0.19, 0.31, 0.55]	[0.19, 0.31, 0.55]
	A_3	[0.125, 0.15, 0.21]	[0.06, 0.07, 0.09]	[0.52, 0.68, 0.91]	[0.38, 0.62, 0.88]	[0.38, 0.62, 0.88]
Scores of Techniques	A_1	[0.002, 0.003, 0.016]	[0.017, 0.037, 0.088]	[0.005, 0.01, 0.023]	[0.007, 0.015, 0.033]	[0, 0.007, 0.018]
	A_2	[0.008, 0.02, 0.07]	[0.005, 0.016, 0.043]	[0.014, 0.03, 0.08]	[0.023, 0.056, 0.151]	[0.01, 0.028, 0.083]
	A_3	[0.002, 0.003, 0.016]	[0.002, 0.004, 0.009]	[0.044, 0.088, 0.191]	[0.046, 0.112, 0.242]	[0.02, 0.056, 0.132]

Attributes		C_6	C_7	C_8	C_9
Average priority of attributes		[0.047, 0.075, 0.122]	[0.2, 0.3, 0.42]	[0.031, 0.048, 0.079]	[0.012, 0.016, 0.025]
Average priority of techniques	A_1	[0.047, 0.058, 0.075]	[0.52, 0.68, 0.91]	[0.21, 0.36, 0.69]	[0.59, 0.77, 0.11]
	A_2	[0.18, 0.26, 0.38]	[0.16, 0.24, 0.38]	[0.082, 0.13, 0.24]	[0.16, 0.24, 0.38]
	A_3	[0.54, 0.69, 0.86]	[0.061, 0.077, 0.111]	[0.25, 0.5, 0.85]	[0.52, 0.68, 0.91]
Scores of techniques	A_1	[0.002, 0.004, 0.008]	[0.1, 0.2, 0.38]	[0.006, 0.017, 0.055]	[0, 0.001, 0.003]
	A_2	[0.008, 0.02, 0.043]	[0.032, 0.072, 0.16]	[0.002, 0.006, 0.019]	[0.002, 0.004, 0.01]
	A_3	[0.025, 0.05, 0.096]	[0.012, 0.023, 0.047]	[0.007, 0.024, 0.067]	[0.006, 0.011, 0.023]

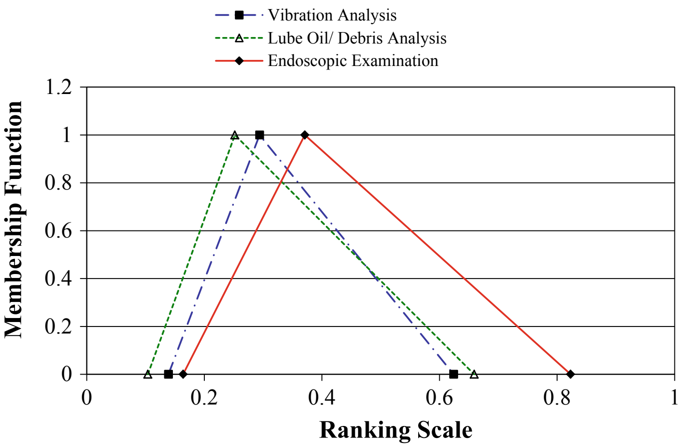

Fig. 3 Final ranking of techniques

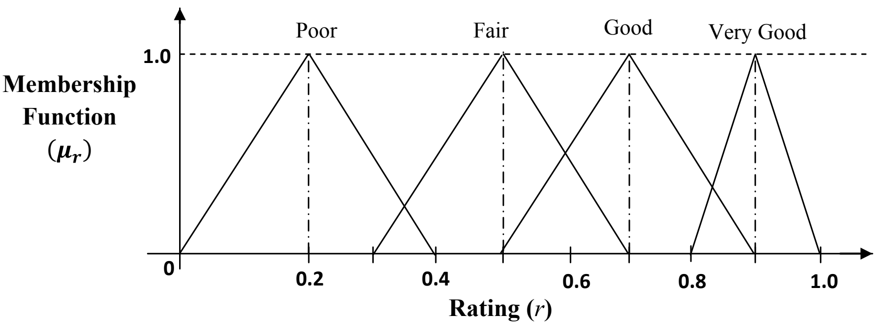

Fig. 4 Fuzzy sets representing ratings in linguistic terms/variables

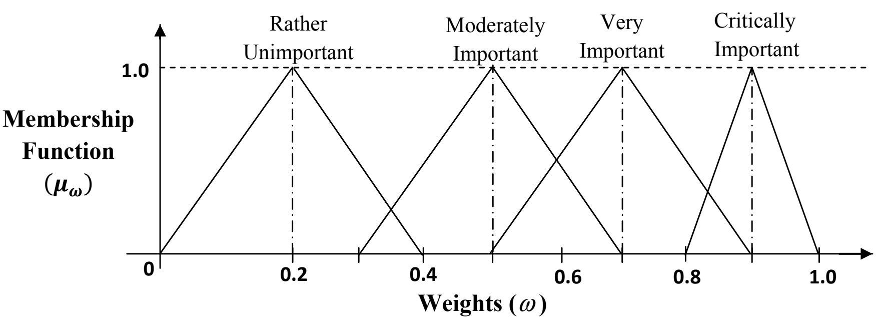

Fig. 5 Fuzzy sets representing weights in linguistic terms/variables

Table 17 Rating of the techniques with respect to decision attributes

Decision attributes	Alternatives/Techniques		
	Vibration analysis (A_1)	Lube oil/debris analysis (A_2)	Endoscopic examination (A_3)
Investment cost (C_1)	Fair	Fair	Good
Operating costs (C_2)	Good	Good	Good
Accuracy of the technique (C_3)	Good	Very Good	Good
Repeatability of the instruments used (C_4)	Good	Good	Very Good
Ease of use (C_5)	Good	Fair	Fair
Environmental restrictions (C_6)	Good	Fair	Fair
Technical expertise requirement (C_7)	Good	Good	Good
Ease of maintenance (C_8)	Good	Fair	Fair
Ease of mounting (C_9)	Fair	Good	Fair

Table 18 Weights of decision attributes

Decision attributes	Weights
Investment cost (C_1)	Moderately important
Operating costs (C_2)	Very important
Accuracy of the technique (C_3)	Critically important
Repeatability of the instruments used (C_4)	Very important
Ease of use (C_5)	Very important
Environmental restrictions (C_6)	Very important
Technical expertise requirement (C_7)	Very important
Ease of maintenance (C_8)	Very important
Ease of mounting (C_9)	Very important

5.5 *Ranking Fuzzy Sets Using* 'Cardinal Utilities'

Baldwin and Guild [23] proposed a relation $\widetilde{P_{ij}} = \{(r_i, r_j),\ \mu_{P_{ij}}(r_i, r_j)\};\ i \neq j$ with membership function as $\mu_{P_{ij}}(r_i, r_j) = f(r_i, r_j)$. This function expresses the 'difference' between the ratings of two fuzzy sets. Such a set is defined as $\tilde{O}(x_i) = \{x_i,\ \mu_O(x_i)\}$ with membership function $\mu_O(x_i) = sup_{r_i, r_j}\ \ min\ \{\mu_{R_i}(r_i),\ \mu_{R_j}(r_j),\ \mu_{P_{ij}}(r_i, r_j)\}$. The equation expresses the degree to which alternative x_i is preferable to its best rival alternative. $\tilde{O}(x_i)$ corresponds to *max-min composition* of $\widetilde{R}_i$, $\widetilde{R}_j$ and $\widetilde{P_{ij}}$.

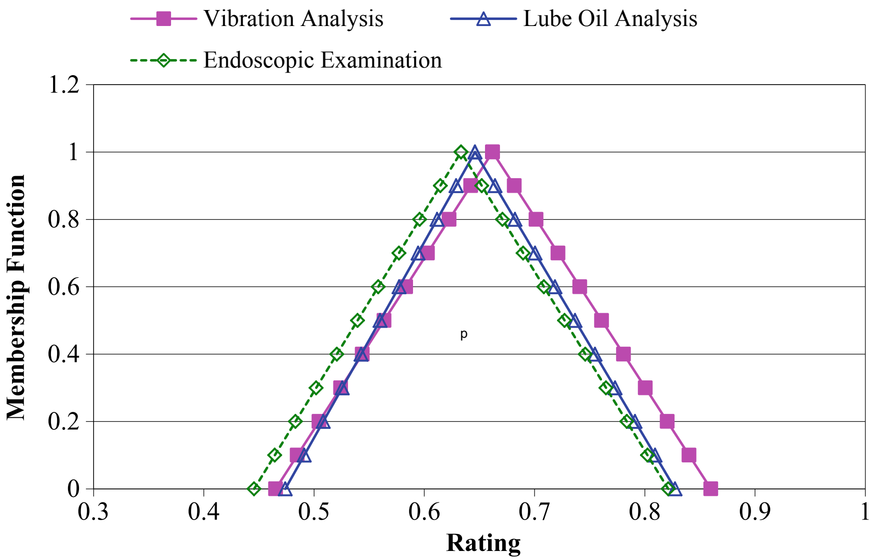

Fig. 6 Final ratings of techniques

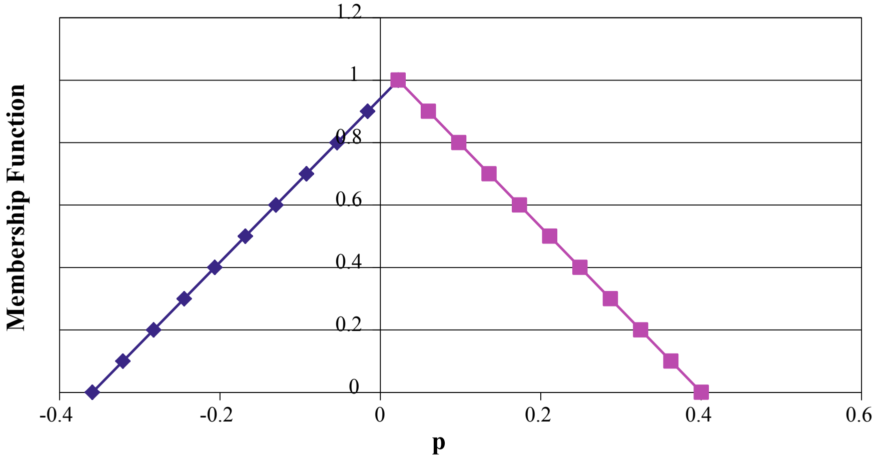

Fig. 7 Membership function of preferability of Technique 1 over Techniques 2 and 3

Without going into the specifics of mathematical analysis, the solution of the problem is presented as:

$$\mu_O(x_i) = \min_j \{\widetilde{\mu}_j\} = \min_j \left[\frac{\delta - \alpha}{1 + (\delta - \gamma) + (\beta - \alpha)}\right]$$

Fig. 8 Membership functions of ratings $\widetilde{R}_i$, $\widetilde{R}_j$

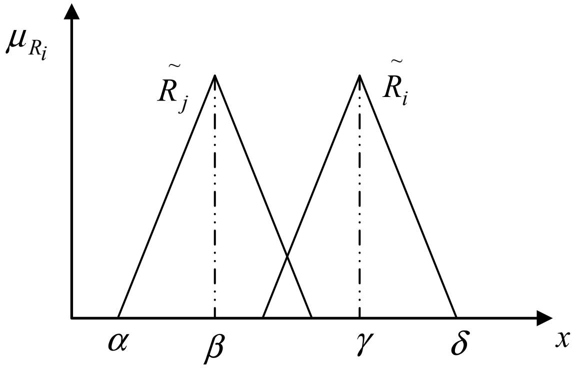

Table 19 Values of parameters obtained by cardinal utilities method

Parameters of the equation	R_1 and R_2		R_1 and R_3		R_2 and R_3	
	R_1	R_2	R_1	R_3	R_2	R_3
α	0.473913	0.465217	0.445652	0.465217	0.445652	0.473913
β	0.646032	0.661905	0.633333	0.661905	0.633333	0.646032
γ	0.661905	0.646032	0.661905	0.633333	0.646032	0.633333
δ	0.86	0.8275	0.86	0.82125	0.8275	0.82125
μ_j	0.28177	0.26288	0.29900	0.25714	0.27889	0.25539

The parameters of the above equations are depicted in the Fig. 8. Optimal maintenance technique is the one that has maximum $\widetilde{\mu}_j$ value obtained through above equation.

All the three maintenance techniques i.e. vibration analysis (with rating R_1), lube oil analysis (with rating R_2) and endoscopic examination (with rating R_3) are compared considering two techniques at a time. From Table 19 one can observe that $\widetilde{\mu}_j$ of technique 1 is more than $\widetilde{\mu}_j$ of technique 2 and 3, i.e. rating of R_1 is greater rating of R_2 and R_3. This means vibration analysis technique outperforms other two techniques as far as optimality is concerned.

5.6 *Ranking Fuzzy Sets by* 'Maximizing and Minimizing Sets'

This is a modification of ranking approach which is developed by Jain and is further modified by Chen [24] for better discrimination of the ratings. This method utilizes 'maximizing set $(\widetilde{M})$' and 'minimizing set $(\widetilde{N})$' having membership functions defined as:

$$\mu_M(r) = \left[\frac{r - r_{min}}{r_{max} - r_{min}}\right]^n$$

$$\mu_N(r) = \left[\frac{r_{\max} - r}{r_{\max} - r_{\min}}\right]^n$$

where $r \in [r_{min}, r_{\max}]$ is the real interval and n = 1 for linear, n = 2 for risk prone and n = 0.5 for risk averse membership functions.

To rank and get the fuzzy optimal alternative, $O(x_i)$ is obtained using

$$O(x_i) = \frac{R(x_i) + 1 - L(x_i)}{2}$$

where $R(x_i) = sup_r\ min\ \{\mu_{R_i}(r),\ \mu_M(r)\}$ and $L(x_i) = sup_r\ min\ \{\mu_{R_i}(r),\ \mu_N(r)\}$.

In this method the analysis is carried out by considering two techniques at a time. The membership functions of the three fuzzy sets (techniques) are obtained as below:

$$\begin{aligned}
\mu_{R1}(r) &= \frac{r - 0.465217}{0.196688} && ;0.465217 \le r \le 0.661905 \\
&= \frac{0.86 - r}{0.198095} && ;0.661905 < r \le 0.86 \\
&= 0 && ;otherwise
\end{aligned}$$

$$\begin{aligned}
\mu_{R_2}(r) &= \frac{r - 0.473913}{0.172119} && ;0.473913 \le r \le 0.646032 \\
&= \frac{0.8275 - r}{0.181468} && ;0.646032 < r \le 0.8275 \\
&= 0 && ;otherwise
\end{aligned}$$

$$\begin{aligned}
\mu_{R_3}(r) &= \frac{r - 0.445652}{0.187681} && ;0.445652 \le r \le 0.633333 \\
&= \frac{0.82125 - r}{0.187917} && ;0.633333 < r \le 0.82125 \\
&= 0 && ;otherwise
\end{aligned}$$

Computation of maximizing and minimizing sets for n = 1 is as below:

(1) Considering R_1 and R_2:

$$\begin{aligned}\mu_M(r) &= \frac{r - 0.465217}{0.394783} && ;0.465217 \le r \le 0.86\\ &= 0 && ;\mathit{otherwise}\end{aligned}$$

$$\begin{aligned}\mu_N(r) &= \frac{0.86 - r}{0.394783} && ;0.465217 \le r \le 0.86\\ &= 0 && ;\mathit{otherwise}\end{aligned}$$

(2) Considering R_1 and R_3:

$$\begin{aligned}\mu_M(r) &= \frac{r - 0.445652}{0.414348} && ;0.445652 \le r \le 0..86\\ &= 0 && ;\mathit{otherwise}\end{aligned}$$

$$\begin{aligned}\mu_N(r) &= \frac{0.86 - r}{0.414348} && ;0.445652 \le r \le 0.86\\ &= 0 && ;\mathit{otherwise}\end{aligned}$$

(3) Considering R_2 and R_3:

$$\begin{aligned}\mu_M(r) &= \frac{r - 0.445652}{0.381848} && ;0.445652 \le r \le 0.8275\\ &= 0 && ;\mathit{otherwise}\end{aligned}$$

$$\begin{aligned}\mu_N(r) &= \frac{0.8275 - r}{0.381848} && ;0.445652 \le r \le 0.8275\\ &= 0 && ;\mathit{otherwise}\end{aligned}$$

The ratings of the three techniques obtained by this method and maximizing and minimizing sets are plotted in Figs. 9, 10 and 11.

Ranking values for all three techniques are given in Table 20. When R_1 and R_2 combination is considered, optimal ranking value $(O(x_i))$ of R_1 is more than R_2 and in R_1 and R_3 combination also $(O(x_i))$ of R_1 is more than R_3. Optimal ranking value of R_2 is more than R_3 in R_2 and R_3 combination. Therefore, alternative 1 (vibration analysis) is an optimal technique as per this method.

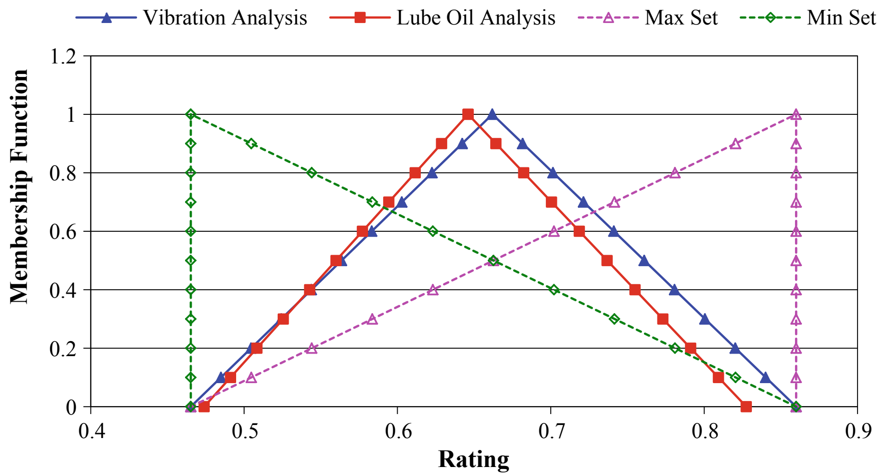

Fig. 9 Final ratings of Technique 1 and 2 (Chen Method)

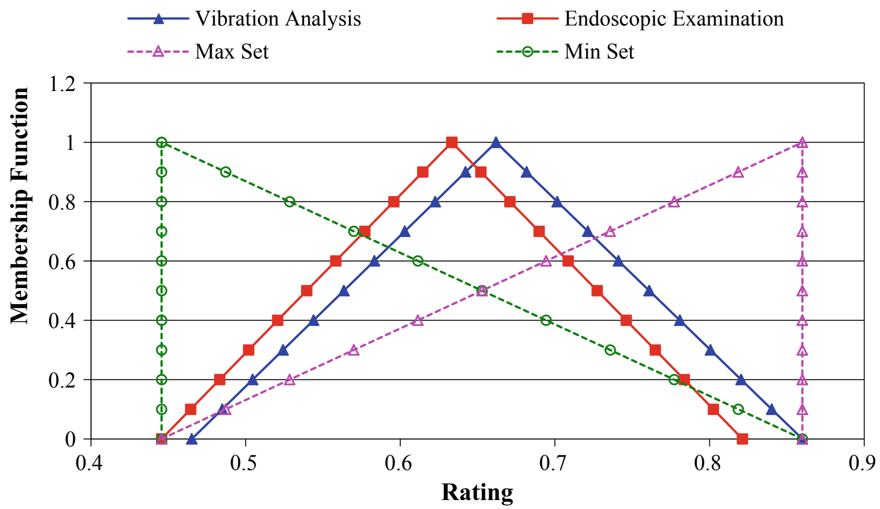

Fig. 10 Final ratings of Technique 1 and 3 (Chen Method)

5.7 Suitability Set and Dominance Relation

In this method, rating matrix and weights for attributes are determined followed by computation of suitability set for each alternative [21]. The suitability set for each alternative '*i*' is assumed as fuzzy weighted sum of ratings and is considered as an appropriate measure of suitability:

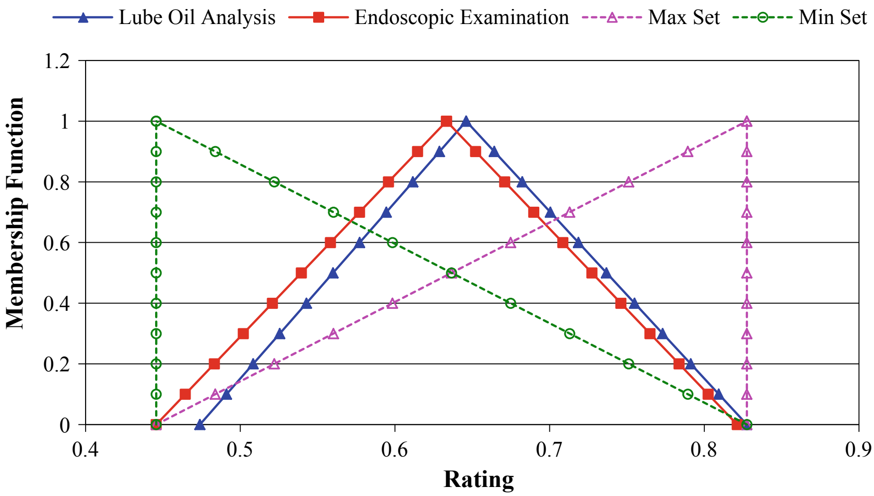

Fig. 11 Final ratings of Techniques 2 and 3 (Chen Method)

Table 20 Values of Parameters Obtained by Chen Method

Parameters of the equation	R_1 and R_2		R_1 and R_3		R_2 and R_3	
	R_1	R_2	R_1	R_3	R_2	R_3
$R(x_i)$	0.66635	0.63017	0.68149	0.62395	0.67919	0.6612
$L(x_i)$	0.670	0.68139	0.64967	0.68919	0.64031	0.67173
$O(x_i)$	0.498175	0.47439	0.51591	0.46738	0.51944	0.494735

$$S_i = \sum_j \omega_j \cdot r_{ij}$$

For choosing an alternative one need to know the concept of dominance. Dominance (δ) of the normal convex fuzzy set A over the normal convex fuzzy set B is defined by:

$\delta(A, B) = \vee_x(\mu_{\leq A}(x) \wedge \mu_B(x))$; where $\vee$ denotes maximum operation, $\wedge$ denotes the minimum operation and $\leq A$ is the fuzzy set '*less than or equal to A*' formed from A by setting:

$$\begin{aligned} \mu_{\leq A}(x) &= 1.0 \quad &; x < x^* \\ &= \mu_A(x) \quad &; x \geq x^* \end{aligned}$$

where x^* is the leftmost (lowest) value of x for which $\mu_A(x) = 1.0$.

Though one can get an alternative using dominance relation, it is not sufficient to make the choice as it does not take into account the shapes of suitability sets. To include the shapes of membership functions as well and gain confirmation about the optimal technique, 'difference function' is defined as:

$$g_k(S_1, S_{2,}, \ldots, S_n) = S_k - \frac{\sum\limits_{\substack{i=1 \\ i \neq k}}^{n} v(x_i) \cdot S_i}{\sum\limits_{\substack{i=1 \\ i \neq k}}^{n} v(x_i)}$$

where vector, $v(x_i) = min_j\{DR(i,j)\}$ and k corresponds to a position in $v(x_k) = 1$. The set obtained using the above difference function gives the degree of preference of the chosen alternative over the other alternatives. This procedure is particularly useful when two or more suitability sets, i.e. alternatives dominate the other sets/alternatives.

The suitability sets for three alternative techniques in the form of TFN are: $S_1 = [2.14 \quad 4.17 \quad 6.88], S_2 = [2.18 \quad 4.07 \quad 6.62]$ and $S_3 = [2.05 \quad 3.99 \; 6.57]$ and are shown in Fig. 12.

The next step is to get the dominance relation $\{DR(i,j)\}_\delta$, which is the dominance of S_i over S_j. $\{DR\}_\delta$ for this example is:

$$\{DR\}_\delta = \begin{bmatrix} 1 & 1 & 1 \\ 0.98 & 1 & 1 \\ 0.96 & 0.985 & 1 \end{bmatrix}$$

The first row of the $\{DR\}_\delta$ matrix has all entries as 1. This means vibration analysis (with suitability set S_1) dominates over other two alternatives, with weights

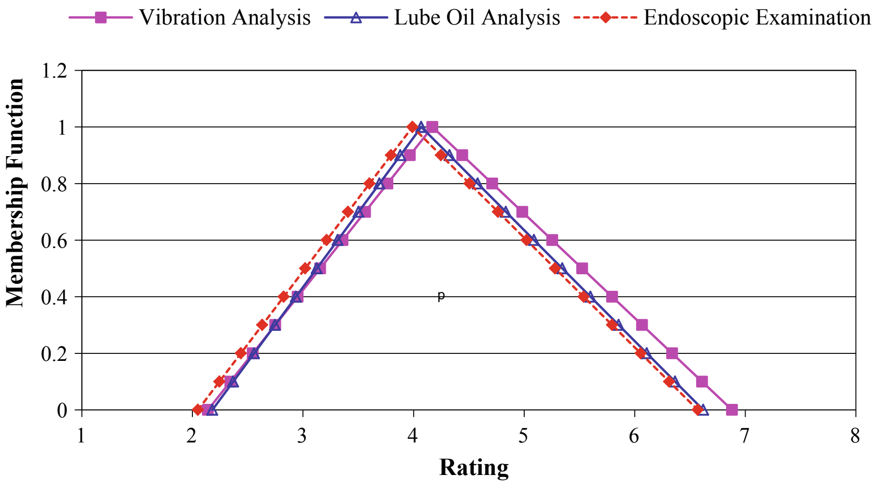

Fig. 12 Final ratings of techniques (suitability set method)

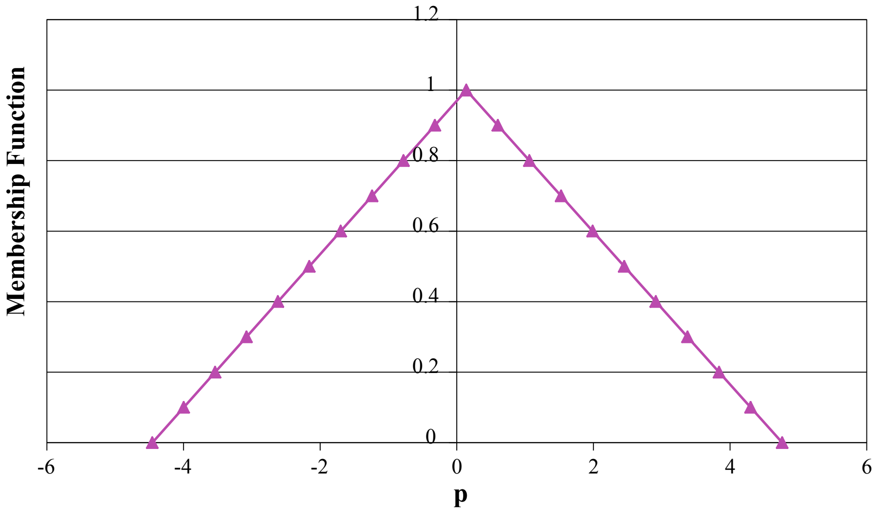

Fig. 13 Preferability of vibration analysis over lube oil analysis and endoscopic examination (suitability set method)

being [1, 0.98, 0.96]. Next, difference function is determined and degree of preference is obtained as [−4.455257732, 0.139587629, 4.764329897]. Plot of preferability of vibration analysis over lube oil analysis and endoscopic examination is shown in Fig. 13.

6 Scope for Hybrid Methods

Reliability, Availability Maintainability and Safety (RAMS) are closely interlinked areas and fuzzy sets application is attempted for several past decades. Readers may study the literature related to dynamic reliability models where fuzzy sets has been successfully applied by researchers. Fuzzy simulation of parameters of RAMS is yet another area that carries good potential for research investigations.

Hybrid approach which considers some uncertain parameters as probabilistic and some others as fuzzy numbers are attempted in the past. Methods for reliability or availability analysis consider all uncertain parameters to be either completely random or entirely fuzzy. However, if both uncertainties are present, as it often happens in real life scenario, there is a need to develop a right hybrid method. For those problems which involve some parameters that are justifiably represented by probability density function and other parameters which are considered to be more effectively represented by fuzzy numbers, the methods such as possibility-probability transformations, belief functions, etc. have been attempted previously. However this domain has lot of scope for further research investigations.

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