Crossing Numbers of Beyond-Planar Graphs

Markus Chimani[1], Philipp Kindermann[2(✉)], Fabrizio Montecchiani[3],
and Pavel Valtr[4]

[1] Osnabrück University, Osnabrück, Germany
`markus.chimani@uos.de`
[2] University of Würzburg, Würzburg, Germany
`philipp.kindermann@uni-wuerzburg.de`
[3] University of Perugia, Perugia, Italy
`fabrizio.montecchiani@unipg.it`
[4] Charles University in Prague, Prague, Czech Republic
`valtr@kam.mff.cuni.cz`

Abstract. We study the 1-planar, quasi-planar, and fan-planar crossing number in comparison to the (unrestricted) crossing number of graphs. We prove that there are n-vertex 1-planar (quasi-planar, fan-planar) graphs such that any 1-planar (quasi-planar, fan-planar) drawing has $\Omega(n)$ crossings, while $\mathcal{O}(1)$ crossings suffice in a crossing-minimal drawing without restrictions on local edge crossing patterns.

1 Introduction

The *crossing number* of a graph G, denoted by $\mathrm{cr}(G)$, is the smallest number of pairwise edge crossings over all possible drawings of G. Many papers are devoted to the study of this parameter, refer to [22,25] for surveys. In particular, minimizing the number of crossings is one of the seminal problems in graph drawing (see, e.g., [2,3,23]), whose importance has been further witnessed by user studies showing how edge crossings may deteriorate the readability of a diagram [20,21,26]. On the other hand, determining the crossing number of a graph is NP-hard [5] and can be solved exactly only on small/medium instances [7]. On the positive side, the crossing number is fixed-parameter tractable in the number of crossings [15] and can be approximated by a constant factor for graphs of bounded degree and genus [10].

A recent research stream studies graph drawings where, rather than minimizing the number of crossings, some edge crossing patters are forbidden; refer to [4,9,11,12] for surveys and reports. A key motivation for the study of so-called *beyond-planar graphs* are recent cognitive experiments showing that already the

---

Research in this work started at the Bertinoro Workshop on Graph Drawing 2019. MC was supported by DFG under grant CH 897/2-2. FM was supported in part by MIUR under grant 20174LF3T8 AHeAD: efficient Algorithms for HArnessing networked Data. PV was supported by the Czech Science Foundation grant 18-19158S.

D. Archambault and C. D. Tóth (Eds.): GD 2019, LNCS 11904, pp. 78–86, 2019.
https://doi.org/10.1007/978-3-030-35802-0_6

Table 1. Lower and upper bounds the crossing ratio of beyond-planar graphs.

Graph class	Lower bound	Upper bound
1-planar	$n/2 - 1$	$n/2 - 1$
quasi-planar	$\Omega(n)$	$O(n^2)$
k-quasi-planar	$\Omega(n/k^3)$	$f(k) \cdot n^2 \log^2 n$
fan-planar	$\Omega(n)$	$O(n^2)$

absence of specific kinds of edge crossing configurations has a positive impact on the human understanding of a graph drawing [13,18]. Of particular interest for us are three families of beyond-planar graphs that have been extensively studied, namely the k-planar, fan-planar, and k-quasi-planar graphs; refer to [9] for additional families. A *k-planar drawing* is such that each edge is crossed at most $k \geq 1$ times [19] (see also [16] for a survey on 1-planarity). A *k-quasi planar* drawing does not have $k \geq 3$ mutually crossing edges [1]. A *fan-planar drawing* does not contain two independent edges that cross a third one or two adjacent edges that cross another edge from different "sides" [14]. A graph is *k-planar* (*k-quasi-planar*, *fan-planar*) if it admits a k-planar (k-quasi-planar, fan-planar) drawing; a 3-quasi-planar graph is simply called *quasi-planar*.

In this context, an intriguing question is to what extent edge crossings can be minimized while forbidding such local crossing patterns. In particular, we ask whether avoiding local crossing patterns in a drawing of a graph may enforce an overall large number of crossings, whereas only a few crossings would suffice in a crossing-minimal drawing of the graph. We answer this question in the affirmative for the above-mentioned three families of beyond-planar graphs. Our contribution are summarized in Table 1.

1. In Sect. 2, we prove that there exist n-vertex 1-planar graphs such that the ratio between the minimum number of crossings in a 1-planar drawing of one such graph and its crossing number is $n/2 - 1$. This result can be easily extended to k-planar graphs if we allow parallel edges.
2. In Sect. 3, we prove that there exist n-vertex quasi-planar graphs such that the ratio between the minimum number of crossings in a quasi-planar drawing of one such graph and its crossing number is $\Omega(n)$. Similarly, a $\Omega(n/k^3)$ bound can be proved for k-quasi-planar graphs.
3. In Sect. 4, we prove that there exist n-vertex fan-planar graphs such that the ratio between the minimum number of crossings in a fan-planar drawing of one such graph and its crossing number is $\Omega(n)$.

The lower bound in Result 1 is tight. Since fan-planar and quasi-planar graphs have $\mathcal{O}(n)$ edges, the lower bounds in Results 2 and 3 are a linear factor from the trivial upper bound $\mathcal{O}(n^2)$, and it remains open whether such an upper bound can be achieved (see Sect. 5). All results are based on nontrivial constructions that exhibit interesting structural properties of the investigated graphs.

Notation and Definitions. We assume familiarity with standard definitions about graph drawings and embeddings of planar and nonplanar graphs (see, e.g., [8,9]). In a drawing of a graph, we assume that an edge does not contain a vertex other than its endpoints, no two edges meet tangentially, and no three edges share a crossing. It suffices to only consider *simple* drawings where any two edges intersect in at most one point, which is either a common endpoint or an interior point where the two edges properly cross. Thus, in a simple drawing, any two adjacent edges do not cross and any two non-adjacent edges cross at most once.

We define the *k-planar crossing number* of a k-planar graph G, denoted by $\mathrm{cr}_{k\text{-pl}}(G)$, as the minimum number of crossings over all k-planar drawings of G. The *k-planar crossing ratio* $\varrho_{k\text{-pl}}$ is the supremum of $\mathrm{cr}_{k\text{-pl}}(G)/\mathrm{cr}(G)$ over all k-planar graphs G. Analogously, we define the *quasi-planar* and the *fan-planar crossing number* of a graph G, denoted by $\mathrm{cr}_{\mathrm{quasi}}(G)$ and $\mathrm{cr}_{\mathrm{fan}}(G)$, as well as the *quasi-planar* and the *fan-planar crossing ratio*, denoted by ϱ_{quasi} and ϱ_{fan}.

2 The 1-planar Crossing Ratio

An n-vertex 1-planar graph has at most $4n-8$ edges and a 1-planar drawing has at most $n-2$ crossings, that is $\mathrm{cr}_{1\text{-pl}}(G) \leq n-2$ [16]. Observe that for $\mathrm{cr}(G) < \mathrm{cr}_{1\text{-pl}}(G)$ it has to hold that $\mathrm{cr}(G) \geq 2$. It follows that the 1-planar crossing ratio is $\varrho_{1\text{-pl}} \leq n/2-1$. We show that this bound can be achieved.

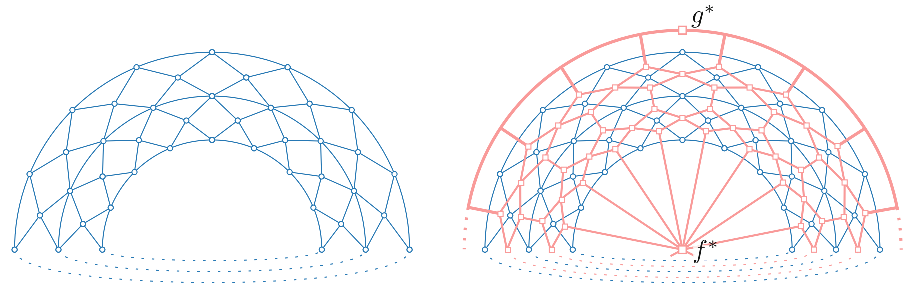
Theorem 1. *For every $\ell \geq 7$, there exists a 1-planar graph G_ℓ with $n = 11\ell+2$ vertices such that $\mathrm{cr}_{1\text{-}pl}(G_\ell) = n-2$ and $\mathrm{cr}(G_\ell) = 2$, which yields the largest possible 1-planar crossing ratio.*

The construction of G_ℓ consists of three parts: a rigid graph P that has to be drawn planar in any 1-planar drawing; its dual P^*; a set of *binding* edges and one *special* edge that force P and P^* to be intertwined in any 1-planar drawing.

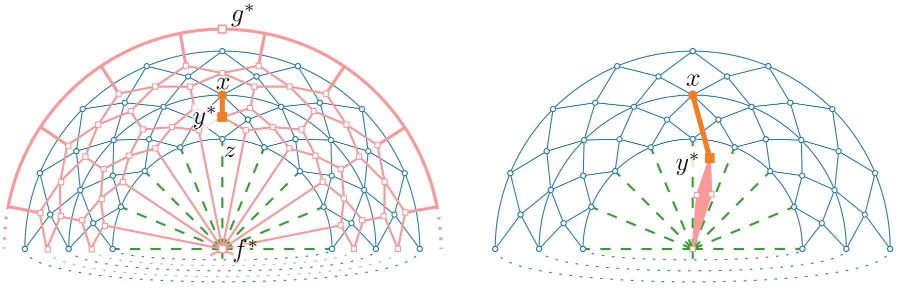
To obtain P, we utilize a construction introduced by Korzhik and Mohar [17]. They construct graphs H_ℓ that are the medial extension of the Cartesian product of the path of length 2 and the cycle of length ℓ; see Fig. 1a. They prove that H_ℓ has exactly one 1-planar embedding on the sphere, and that embedding is crossing-free. We choose $P = H_\ell$ as our rigid graph and fix its (1−) planar embedding (when we will refer to P, we will usually mean this embedding).

Let P^* be the dual of P, obtained by placing a dual vertex h^* into each face h of P and connecting two dual vertices if their corresponding faces share an edge; see Fig. 1b. Since P has 5ℓ vertices and 11ℓ edges, by Euler's polyhedra formula it has $6\ell+2$ faces; thus, P^* has $6\ell+2$ vertices and 11ℓ edges.

Obviously, $P \cup P^*$ can be drawn planar, as both P and P^* are planar and disjoint. All faces of P have size 3 or 4, except two large (called *polar*) faces f and g of size ℓ. We create a graph G' by adding ℓ *binding* edges to $P \cup P^*$ between f^* (the vertex of P^* corresponding to face f) and the vertices of P that are incident to f. This forces f^* to be drawn in face f in any 1-planar drawing. In the full version [6] we prove the following lemma, cf. Fig. 1c and d.



(a) The graph P (b) The graph $P \cup P^*$



(c) $\mathrm{cr}_{1\text{-pl}}(G_\ell) \leq n - 2$ (d) $\mathrm{cr}(G_\ell) \leq 2$

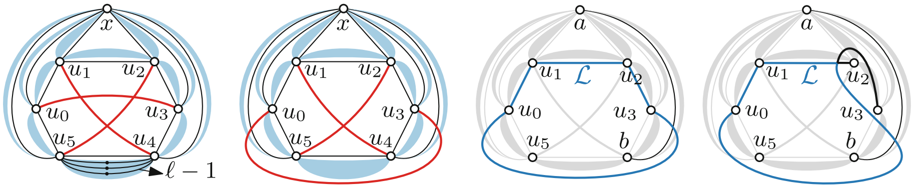
Fig. 1. Construction of the graph G_ℓ in the proof of Theorem 1. Blue circles and edges are P; red squares and bold edges are P^*; green dashed edges are the binding edges; and the orange very bold edge is the special edge. (Color figure online)

Lemma 2. *G' has only two types of 1-planar embeddings (up to the choice of the outer face): a planar one where P^* lies completely inside face f of P; and a 1-planar embedding where f^* lies inside f, g^* lies inside g, and each edge of P crosses an edge of P^* and vice versa.*

Let z be a vertex of P on the boundary of f. Let y be the face of size 4 that has z on its boundary. Let x be the degree-6 vertex on the boundary of y. We obtain G_ℓ from G' by adding the *special* edge (x, y^*). In the planar embedding of Lemma 2, P^* and thus y^* lies inside face f of P, so (x, y^*) has to cross at least two edges of P; see Fig. 1d. Choosing the face that corresponds to z as the outer face of P^* gives a non-1-planar drawing of G_ℓ with 2 crossings.

Hence, G' has to be drawn in the second way of Lemma 2; see Fig. 1c. Here, the edge (x, y^*) can be added without further crossings. Graph G_ℓ consists of $n = 11\ell + 2$ vertices in total. Both P and P^* have 11ℓ edges, and each of them is crossed, so there are $n - 2$ crossings in total, which is the maximum possible in a 1-planar drawing. Hence, $\mathrm{cr}_{1\text{-pl}}(G_\ell) = n - 2$ and $\mathrm{cr}(G_\ell) = 2$, so $\varrho_{1\text{-pl}} \leq n/2 - 1$.

The construction used in the proof of Theorem 1 can be generalized to k-planar multigraphs. It suffices to replace each edge of G_ℓ, except the special edge, by a bundle of k parallel edges:



(a) $\mathrm{cr}(G_\ell) \le 3$ (b) $\mathrm{cr}_{\text{quasi}}(G_\ell) \le 2\ell+1$ (c) C is not crossed (d) C is crossed

Fig. 2. Illustration for the proof of Theorem 4.

Corollary 3. *For every $\ell \ge 6$, there exists a k-planar multigraph $G_{\ell,k}$ with $n = 11\ell + 2$ vertices and maximum edge multiplicity k such that $\mathrm{cr}_k\text{-}pl(G_{\ell,k}) = k^2\,(n-2)$ and $\mathrm{cr}(G_{\ell,k}) = 2k$, thus $\varrho_k\text{-}pl \ge k\,(n-2)/2$.*

3 The Quasi-planar Crossing Ratio

An n-vertex quasi-planar graph G has at most $6.5n - 20$ edges, thus $\mathrm{cr}_{\text{quasi}}(G) \in \mathcal{O}(n^2)$ [9]. For $\mathrm{cr}(G) < \mathrm{cr}_{\text{quasi}}$ it has to hold that $\mathrm{cr}(G) \ge 2$, and hence $\varrho_{\text{quasi}} \in \mathcal{O}(n^2)$. We show that the quasi-planar crossing ratio is unbounded, even for $\mathrm{cr}(G) \le 3$:

Theorem 4. *For every $\ell \ge 2$, there exists a quasi-planar graph G_ℓ with $n = 12\ell - 5$ vertices such that $\mathrm{cr}_{quasi}(G_\ell) \ge \ell$ and $\mathrm{cr}(G_\ell) \le 3$, thus $\varrho_{quasi} \in \Omega(n)$.*

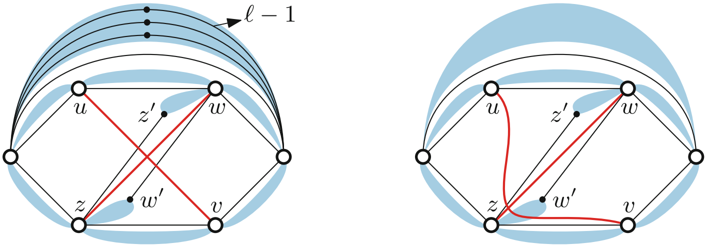
In order to prove Theorem 4, we begin with a technical lemma.

Lemma 5. *Let G be a graph containing two independent edges (u,v) and (w,z). Suppose that u and v (w and z, resp.) are connected by a set Π_{uv} (Π_{wz}, resp.) of $\ell - 1$ paths of length two. Let Γ be a drawing of G. If (u,v) and (w,z) cross in Γ, then Γ contains at least ℓ crossings.*

Proof. Suppose that (u,v) and (w,z) cross. If each of the $\ell - 1$ paths in Π_{wz} crosses (u,v), then the claim follows. Assume otherwise that at least one of these paths does not cross (u,v). This path forms a 3-cycle t with (w,z); the $\ell - 1$ paths of Π_{uv} all cross at least one edge of t, which proves the claim. ◻

Proof (of Theorem 4*).* Let G_ℓ be the graph constructed as follows; cf. Fig. 2a. Start with a 6-cycle $C = \langle u_0, u_1, \dots, u_5\rangle$, and a vertex x connected to each of C, yielding graph G'. *Extend* each edge of G' by adding $\ell - 1$ disjoint paths of length two between its endpoints. Finally, add *special* edges (u_i, u_{i+3}), $i = 0, 1, 2$.

The resulting graph G_ℓ has $n = 12(\ell - 1) + 7 = 12\ell - 5$ vertices and admits a drawing with 3 crossings, so $\mathrm{cr}(G_\ell) \le 3$; see Fig. 2a. Note that G_ℓ admits a quasi-planar drawing with $2\ell + 1$ crossings as shown in Fig. 2b. We prove that $\mathrm{cr}_{\text{quasi}}(G_\ell) \ge \ell$. Let Γ be a quasi-planar drawing of G_ℓ. If there are two edges of G' that cross each other, then the claim follows by Lemma 5.



(a) $\mathrm{cr}(G_\ell) \leq 2$.

(b) $\mathrm{cr}_{\mathrm{fan}}(G_\ell) \leq \ell$.

Fig. 3. Illustration for the proof of Theorem 7.

If no special edge would cross G', they would all be drawn within the unique face of size 6 in G'. They would mutually cross, contradicting quasi-planarity.

Thus, at least one special edge, say $s = (u_0, u_3)$, crosses an edge (a, b) of G'. Consider the closed (possibly self-intersecting) curve $\mathcal{L}$ composed of s plus the subpath of C connecting u_0 to u_3 and containing none of the vertices a and b. This curve partitions the plane into two or more regions, and a and b lie in different regions; see Fig. 2c and d for an illustration. Thus (a, b) and the $\ell - 1$ paths connecting a and b cross $\mathcal{L}$, yielding ℓ crossings in Γ, as desired. □

The above proof can be straight-forwardly extended to k-quasi-planar graphs by using exactly the same construction in which the cycle C has length $2k$. Note that any k-quasi-planar graph has at most $c_k n \log n$ edges, where c_k depends only on k [24], so $\varrho_{\mathrm{quasi}} \leq f(k) \cdot n^2 \log^2 n$.

Corollary 6. *For every* $\ell \geq 2$ *and* $k \geq 3$*, there exists a* k*-quasi-planar graph* $G_{\ell,k}$ *with* $n = 2k(\ell + 1) + 1$ *vertices such that* $\mathrm{cr}_{quasi}(G_{\ell,k}) \geq \ell$ *and* $\mathrm{cr}(G_{\ell,k}) \leq k(k-1)/2$*, thus* $\varrho_{quasi} \in \Omega(n/k^3)$*.*

4 The Fan-Planar Crossing Ratio

An n-vertex fan-planar graph G has at most $5n - 10$ edges, thus $\mathrm{cr}_{\mathrm{fan}}(G) \in \mathcal{O}(n^2)$ [9]. For $\mathrm{cr}(G) < \mathrm{cr}_{\mathrm{fan}}(G)$ it has to hold that $\mathrm{cr}(G) \geq 2$, and hence $\varrho_{\mathrm{fan}} \in \mathcal{O}(n^2)$. We show that the fan-planar crossing ratio is unbounded, even for $\mathrm{cr}(G) = 3$.

Theorem 7. *For every* $\ell \geq 2$*, there exists a fan-planar graph* G_ℓ *with* $n = 9\ell + 1$ *vertices such that* $\mathrm{cr}_{fan}(G_\ell) = \ell$ *and* $\mathrm{cr}(G_\ell) = 3$*, thus* $\varrho_{fan} \in \Omega(n)$*.*

Proof. Let G_ℓ be the graph constructed as follows; cf. Fig. 3a. Start with a $K_{3,3}$. *Extend* each edge of the $K_{3,3}$ by adding $\ell - 1$ disjoint paths of length two between its endpoints, except for two independent edges (u, v) and (w, z). Add vertices

w' and z', edges $\bar{w} = (w, w')$ and $\bar{z} = (z, z')$, ℓ disjoint paths of length two connecting w' and z, and ℓ disjoint paths of length two connecting z' and w.

Graph G_ℓ has $n = 6+7(\ell-1)+2+2\ell = 9\ell+1$ vertices and admits a drawing with three crossings, see Fig. 3a. Recall that we obtain a subdivision of a graph G by subdividing (even multiple times) any subset of its edges. G_ℓ contains three subdivisions of $K_{3,3}$ sharing only edge (u, v), and thus each subdivision requires at least one distinct crossing in any drawing. It follows that $\mathrm{cr}(G_\ell) = 3$. Note that G_ℓ admits a fan-planar drawing with ℓ crossings, cf. Fig. 3b. We prove that $\mathrm{cr}_{\mathrm{fan}}(G_\ell) = \ell$. Let Γ be a fan-planar drawing of G_ℓ. If any two extended edges cross each other, then the claim follows by Lemma 5. Assume they do not:

G_ℓ contains ℓ subdivions of $K_{3,3}$ that share only (u, v) and $\bar{w}$. Since each $K_{3,3}$ subdivision requires at least one crossing, there are either ℓ crossings in Γ (proving the claim), or (u, v) crosses $\bar{w}$. Similarly, G_ℓ contains ℓ $K_{3,3}$ subdivisions that share only (u, v) and $\bar{z}$, and we can assume that (u, v) crosses $\bar{z}$. But fan-planarity forbids (u, v) to cross both $\bar{w}$ and $\bar{z}$. □

5 Open Problems

The main open question is whether there exist fan-planar and quasi-planar graphs whose crossing ratio is $\Omega(n^2)$. In fact, we conjecture that this bound can be reached, but proving our suspected constructions turns out to be elusive. Another natural research direction is to extend our results to further families of beyond-planar graphs, such as k-gap planar graphs or fan-crossing-free graphs (refer to [9] for definitions). Finally, we may ask whether similar lower bounds can be proved in the geometric setting (i.e., when the edges are drawn as straight-line segments).

References

1. Alon, N., Erdös, P.: Disjoint edges in geometric graphs. Disc. Comput. Geom. **4**, 287–290 (1989). https://doi.org/10.1007/BF02187731
2. Batini, C., Furlani, L., Nardelli, E.: What is a good diagram? A pragmatic approach. In: Proceedings of 4th International Conference on Entity-Relationship Approach (ER 1985), pp. 312–319 (1985). http://dl.acm.org/citation.cfm?id=647510.726382
3. Batini, C., Nardelli, E., Tamassia, R.: A layout algorithm for data flow diagrams. IEEE Trans. Softw. Eng. **12**(4), 538–546 (1986). https://doi.org/10.1109/TSE.1986.6312901
4. Bekos, M.A., Kaufmann, M., Montecchiani, F.: Guest editors' foreword and overview - special issue on graph drawing beyond planarity. J. Graph Algorithms Appl. **22**(1), 1–10 (2018). https://doi.org/10.7155/jgaa.00459
5. Bienstock, D.: Some provably hard crossing number problems. Disc. Comput. Geom. **6**, 443–459 (1991). https://doi.org/10.1007/BF02574701
6. Chimani, M., Kindermann, P., Montecchiani, F., Valtr, P.: Crossing numbers of beyond-planar graphs. Arxiv report (2019). http://arxiv.org/abs/1908.03153

7. Chimani, M., Mutzel, P., Bomze, I.: A new approach to exact crossing minimization. In: Halperin, D., Mehlhorn, K. (eds.) ESA 2008. LNCS, vol. 5193, pp. 284–296. Springer, Heidelberg (2008). https://doi.org/10.1007/978-3-540-87744-8_24
8. Di Battista, G., Eades, P., Tamassia, R., Tollis, I.G.: Graph Drawing. Prentice-Hall, Upper Saddle River (1999)
9. Didimo, W., Liotta, G., Montecchiani, F.: A survey on graph drawing beyond planarity. ACM Comput. Surv. **52**(1), 4:1–4:37 (2019). https://doi.org/10.1145/3301281
10. Hliněný, P., Chimani, M.: Approximating the crossing number of graphs embeddable in any orientable surface. In: Charikar, M. (ed.) Proceedings 21sth Annual ACM-SIAM Symposium Discrete Algorithms (SODA 2010), pp. 918–927. SIAM (2010). https://doi.org/10.1137/1.9781611973075.74
11. Hong, S., Kaufmann, M., Kobourov, S.G., Pach, J.: Beyond-planar graphs: Algorithmics and combinatorics (dagstuhl seminar 16452). In: Dagstuhl Reports, vol. 6, no. 11, pp. 35–62 (2016). https://doi.org/10.4230/DagRep.6.11.35
12. Hong, S., Tokuyama, T.: Algoritihmcs for beyond planar graphs (NII shonan meeting 2016–17). NII Shonan Meeting Report 2016 (2016). http://shonan.nii.ac.jp/shonan/report/no-2016-17/
13. Huang, W., Eades, P., Hong, S.: Larger crossing angles make graphs easier to read. J. Vis. Lang. Comput. **25**(4), 452–465 (2014). https://doi.org/10.1016/j.jvlc.2014.03.001
14. Kaufmann, M., Ueckerdt, T.: The density of fan-planar graphs. Arxiv Report (2014). http://arxiv.org/abs/1403.6184
15. Kawarabayashi, K., Reed, B.A.: Computing crossing number in linear time. In: Johnson, D.S., Feige, U. (eds.) Proceedings 39th Annual ACM Symposium Theory Computing(STOC 2007). pp. 382–390. ACM (2007). https://doi.org/10.1145/1250790.1250848
16. Kobourov, S.G., Liotta, G., Montecchiani, F.: An annotated bibliography on 1-planarity. Comput. Sci. Rev. **25**, 49–67 (2017). https://doi.org/10.1016/j.cosrev.2017.06.002
17. Korzhik, V.P., Mohar, B.: Minimal obstructions for 1-immersions and hardness of 1-planarity testing. J. Graph Theory **72**(1), 30–71 (2013). https://doi.org/10.1002/jgt.21630
18. Mutzel, P.: An alternative method to crossing minimization on hierarchical graphs. SIAM J. Optim. **11**(4), 1065–1080 (2001). https://doi.org/10.1137/S1052623498334013
19. Pach, J., Tóth, G.: Graphs drawn with few crossings per edge. Combinatorica **17**(3), 427–439 (1997). https://doi.org/10.1007/BF01215922
20. Purchase, H.C.: Effective information visualisation: a study of graph drawing aesthetics and algorithms. Interact. Comput. **13**(2), 147–162 (2000). https://doi.org/10.1016/S0953-5438(00)00032-1
21. Purchase, H.C., Carrington, D.A., Allder, J.A.: Empirical evaluation of aesthetics-based graph layout. Empir. Softw. Eng. **7**(3), 233–255 (2002)
22. Schaefer, M.: The graph crossing number and its variants: a survey. Electr. J. Comb., Dynamic Surveys, DS21, 113 p. (2017). https://www.combinatorics.org/ojs/index.php/eljc/article/view/DS21
23. Sugiyama, K., Tagawa, S., Toda, M.: Methods for visual understanding of hierarchical system structures. IEEE Trans. Syst. Man Cybern. **11**(2), 109–125 (1981). https://doi.org/10.1109/TSMC.1981.4308636

24. Suk, A., Walczak, B.: New bounds on the maximum number of edges in k-quasi-planar graphs. Comput. Geom. **50**, 24–33 (2015). https://doi.org/10.1016/j.comgeo.2015.06.001
25. Vrt'o, I.: Crossing numbers of graphs: A bibliography (2014). ftp://ftp.ifi.savba.sk/pub/imrich/crobib.pdf
26. Ware, C., Purchase, H.C., Colpoys, L., McGill, M.: Cognitive measurements of graph aesthetics. Inform. Vis. **1**(2), 103–110 (2002). https://doi.org/10.1057/palgrave.ivs.9500013