Open-Mindedness of Gradual Argumentation Semantics

Nico Potyka(✉)

Institute of Cognitive Science, University of Osnabrück, Osnabrück, Germany
npotyka@uos.de

Abstract. Gradual argumentation frameworks allow modeling arguments and their relationships and have been applied to problems like decision support and social media analysis. Semantics assign strength values to arguments based on an initial belief and their relationships. The final assignment should usually satisfy some common-sense properties. One property that may currently be missing in the literature is *Open-Mindedness*. Intuitively, *Open-Mindedness* is the ability to move away from the initial belief in an argument if sufficient evidence against this belief is given by other arguments. We generalize and refine a previously introduced notion of *Open-Mindedness* and use this definition to analyze nine gradual argumentation approaches from the literature.

Keywords: Gradual argumentation · Weighted argumentation · Semantical properties

1 Introduction

The basic idea of abstract argumentation is to study the acceptability of arguments abstracted from their content, just based on their relationships [13]. While arguments can only be accepted or rejected under classical semantics, gradual argumentation semantics consider a more fine-grained scale between these two extremes [3,6–8,10,16,20,22]. Arguments may have a base score that reflects a degree of belief that the argument is accepted when considered independent of all the other arguments. Semantics then assign strength values to all arguments based on their relationships and the base score if provided.

Of course, strength values should not be assigned in an arbitrary manner, but should satisfy some common-sense properties. Baroni, Rago and Toni recently showed that 29 properties from the literature can be reduced to basically two fundamental properties called *Balance* and *Monotonicity* [8] that we will discuss later. *Balance* and *Monotonicity* already capture a great deal of what we should expect from strength values of arguments, but they do not (and do not attempt to) capture everything. One desiderata that may be missing in many applications is *Open-Mindedness*. To illustrate the idea, suppose that we evaluate arguments by strength values between 0 and 1, where 0 means that we fully reject and 1 means that we fully accept an argument. Then, as we increase the number of

N. Ben Amor et al. (Eds.): SUM 2019, LNAI 11940, pp. 236–249, 2019.
https://doi.org/10.1007/978-3-030-35514-2_18

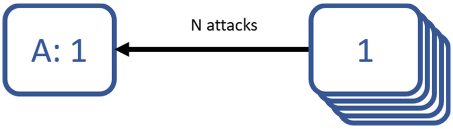

Fig. 1. Argument attacked by N other arguments.

supporters of an argument while keeping everything else equal, we should expect that its strength steadily approaches 1. Symmetrically, as we increase the number of attackers of an argument, we should expect that its strength approaches 0. To illustrate this, consider the graph in Fig. 1 that shows an argument A that is initially accepted (base score 1), but has N attackers that are initially accepted as well. For example, we could model a trial in law, where A corresponds to the argument that we should find the accused not guilty because we do not want to convict an innocent person. The N attackers correspond to pieces of evidence without reasonable doubt. Then, as N grows, we should expect that the strength of A goes to 0. Similar, in medical diagnosis, it is reasonable to initially accept that a patient with an infectious disease has a common cold because this is usually the case. However, as the number of symptoms for a more serious disease grows, we should be able to reject our default diagnosis at some point. Of course, we should expect a dual behaviour for support relations: if we initially reject A and have N supporters that are initially accepted, we should expect that the strength of A goes to 1 as N increases. A gradual argumentation approach that respects this idea is called open-minded. *Open-Mindedness* may not be necessary in every application, but it seems natural in many domains. Therefore, our goal here is to investigate which gradual argumentation semantics from the literature respect this property.

2 Compact QBAFs, Balance and Monotonicity

In our investigation, we consider quantitative bipolar argumentation frameworks (QBAFs) similar to [8]. However, for now, we will restrict to frameworks that assign values from a compact real interval to arguments in order to keep the formalism simple. At the end of this article, we will explain how the idea can be extended to more general QBAFs.

Definition 1 (Compact QBAF). *Let $\mathcal{D}$ be a compact real interval. A QBAF over $\mathcal{D}$ is a quadruple $(\mathcal{A}, \mathrm{Att}, \mathrm{Sup}, \beta)$ consisting of a set of arguments $\mathcal{A}$, two binary relations* Att *and* Sup *called attack and support and a function $\beta : \mathcal{A} \to \mathcal{D}$ that assigns a* base score *$\beta(a)$ to every argument $a \in \mathcal{A}$.*

Typical instantiations of the interval $\mathcal{D}$ are $[0, 1]$ and $[-1, 1]$. Sometimes non-compact intervals like open or unbounded intervals are considered as well, but we exclude these cases for now. We can consider different *subclasses of QBAFs* that use only some of the possible building blocks [8]. Among others, we will look at subclasses that contain QBAFs of the following restricted forms:

Attack-only: $(\mathcal{A}, \mathrm{Att}, \mathrm{Sup}, \beta)$ where $\mathrm{Sup} = \emptyset$,
Support-only: $(\mathcal{A}, \mathrm{Att}, \mathrm{Sup}, \beta)$ where $\mathrm{Att} = \emptyset$,
Bipolar without Base Score: $(\mathcal{A}, \mathrm{Att}, \mathrm{Sup}, \beta)$ where β is a constant function.

In order to interpret a given QBAF, we want to assign strength values to every argument. The strength values should be connected in a reasonable way to the base score of an argument and the strength of its attackers and supporters. Of course, this can be done in many different ways. However, eventually we want a function that assigns a strength value to every argument.

Definition 2 (QBAF interpretation). *Let $Q = (\mathcal{A}, \mathrm{Att}, \mathrm{Sup}, \beta)$ be a QBAF over a real interval $\mathcal{D}$. An interpretation of Q is a function $\sigma : \mathcal{A} \to \mathcal{D}$ and $\sigma(a)$ is called the strength of a for all $a \in \mathcal{A}$.*

Gradual argumentation semantics can define interpretations for the whole class of QBAFs or for a subclass only. One simple example is the *h-categorizer semantics* from [10] that interprets only acyclic attack-only QBAFs without base score. For all $a \in \mathcal{A}$, the *h-categorizer semantics* defines $\sigma(a) = \frac{1}{1+\sum_{(b,a) \in \mathrm{Att}} \sigma(b)}$. That is, unattacked arguments have strength 1, and the strength of all other arguments decreases monotonically based on the strength of their attackers. Since it only interprets acyclic QBAFs, the strength values can be evaluated in topological order, so that the strength values of all parents are known when interpreting the next argument.

Of course, we do not want to assign final strength values in an arbitrary way. Many desirable properties for different families of QBAFs have been proposed in the literature, see, e.g., [2–4,16,22]. Dependent on whether base scores, only attack, only support or both relations are considered, different properties have been proposed. However, as shown in [8], most properties can be reduced to basically two fundamental principles that are called *Balance* and *Monotonicity*. Roughly speaking, *Balance* says that the strength of an argument should be equal to its base score if its attackers and supporters are equally strong and that it should be smaller (greater) if the attackers are stronger (weaker) than the supporters. *Monotonicity* says, intuitively, that if the same impact (in terms of base score, attack and support) acts on two arguments a_1, a_2, then they should have the same strength, whereas if the impact on a_1 is more positive, it should have a larger strength than a_2. Several variants of *Balance* and *Monotonicity* have been discussed in [8]. For example, the stronger-than relationship between arguments can be formalized in a qualitative (focusing on the number of attackers and supporters) or quantitative manner (focusing on the strength of attackers and supporters). We refer to [8] for more details.

3 Open-Mindedness

Intuitively, it seems that *Balance* and *Monotonicity* could already imply *Open-Mindedness*. After all, they demand that adding attacks (supports) increases (decreases) the strength in a sense. However, this is not sufficient to guarantee that the strength can be moved arbitrarily close to the boundary values. To illustrate this, let us consider the Euler-based semantics that has been introduced for the whole class of QBAFs in [4]. Strength values are defined by

$$\sigma(a) = 1 - \frac{1 - \beta(a)^2}{1 + \beta(a) \cdot \exp(\sum_{(b,a)\in \mathrm{Sup}} \sigma(b) - \sum_{(b,a)\in \mathrm{Att}} \sigma(b))}$$

Note that if there are no attackers or supporters, the strength becomes just $1 - \frac{(1+\beta(a))(1-\beta(a))}{1+\beta(a)\cdot 1} = \beta(a)$. If the strength of a's attackers accumulates to a larger (smaller) value than the strength of a's supporters, the strength will be smaller (larger) than the base score. The Euler-based semantics satisfies the basic *Balance* and *Monotonicity* properties in most cases, see [4] for more details. However, it does not satisfy *Open-Mindedness* as has been noted in [21] already. There are two reasons for this. The first reason is somewhat weak and regards the boundary case $\beta(a) = 0$. In this case, the strength becomes $1 - \frac{1-0^2}{1+0} = 0$ independent of the supporters. In this boundary case, the Euler-based semantics does not satisfy *Balance* and *Monotonicity* either. The second reason is more profound and corresponds to the fact that the exponential function always yields positive values. Therefore, $1 + \beta(a) \cdot \exp(x) \geq 1$ and $\sigma(a) \geq 1 - \frac{1-\beta(a)^2}{1} = \beta(a)^2$ independent of the attackers. Hence, the strength value can never be smaller than the base score squared. The reason that the Euler-based semantics can still satisfy *Balance* and *Monotonicity* is that the limit $\beta(a)^2$ can never actually be taken, but is only approximated as the number of attackers goes to infinity.

Hence, *Open-Mindedness* is indeed a property that is currently not captured by *Balance* and *Monotonicity*. To begin with, we give a formal definition for a restricted case. We assume that larger values in $\mathcal{D}$ are stronger to avoid tedious case differentiations. This assumption is satisfied by the first eight semantics that we consider. We will give a more general definition later that also makes sense when this assumption is not satisfied. *Open-Mindedness* includes two dual conditions, one for attack- and one for support-relations. Intuitively, we want that in every QBAF, the strength of every argument with arbitrary base score can be moved arbitrarily close to $\min(\mathcal{D})$ ($\max(\mathcal{D})$) if we only add a sufficient number of strong attackers (supporters). In the following definition, ϵ captures the closeness and N the sufficiently large number.

Definition 3 (Open-Mindedness). *Consider a semantics that defines an interpretation $\sigma : \mathcal{A} \to \mathcal{D}$ for every QBAF from a particular class $\mathcal{F}$ of QBAFs over a compact interval $\mathcal{D}$. We call the semantics open-minded if for every QBAF $(\mathcal{A}, \mathrm{Att}, \mathrm{Sup}, \beta)$ in $\mathcal{F}$, for every argument $a \in \mathcal{A}$ and for every $\epsilon > 0$, the following condition is satisfied: there is an $N \in \mathbb{N}$ such that when adding N new arguments $A_N = \{a_1, \dots, a_N\}$, $\mathcal{A} \cap A_N = \emptyset$, with maximum base score, then*

1. *if $\mathcal{F}$ allows attacks, then for $(\mathcal{A} \cup A_N, \mathrm{Att} \cup \{(a_i, a) \mid 1 \leq i \leq N\}, \mathrm{Sup}, \beta')$, we have $|\sigma(a) - \min(\mathcal{D})| < \epsilon$ and*
2. *if $\mathcal{F}$ allows supports, then for $(\mathcal{A} \cup A_N, \mathrm{Att}, \mathrm{Sup} \cup \{(a_i, a) \mid 1 \leq i \leq N\}, \beta')$, we have $|\sigma(a) - \max(\mathcal{D})| < \epsilon$,*

where $\beta'(b) = \beta(b)$ for all $b \in \mathcal{A}$ and $\beta'(a_i) = \max(\mathcal{D})$ for $i = 1, \dots, n$.

Some explanations are in order. Note that we do not make any assumptions about the base score of a in Definition 3. Hence, we demand that the strength of a must become arbitrary small (large) within the domain $\mathcal{D}$, no matter what its base score is. One may consider a weaker notion of *Open-Mindedness* that excludes the boundary base scores for a. However, this distinction does not make a difference for our investigation and so we will not consider it here. Note also that we do not demand that the strength of a ever takes the extreme value $\max(\mathcal{D})$ ($\min(\mathcal{D})$), but only that it can become arbitrarily close. Finally note that item 1 in Definition 3 is trivially satisfied for support-only QBAFs, and item 2 for attack-only QBAFs.

3.1 Attack-Only QBAFs over $\mathcal{D} = [0, 1]$

In this section, we consider three semantics for attack-only QBAFs over $\mathcal{D} = [0, 1]$. Recall from Sect. 2 that the *h-categorizer semantics* from [10] interprets acyclic attack-only QBAFs without base score. The definition has been extended to arbitrary (including cycles) attack-only QBAFs and base scores from $\mathcal{D} = [0, 1]$ in [6]. The strength of an argument under the *weighted h-categorizer semantics* is then defined by

$$\sigma(a) = \frac{\beta(a)}{1 + \sum_{(b,a)\in \mathrm{Att}} \sigma(b)} \tag{1}$$

for all $a \in \mathcal{A}$. Note that the original definition of the *h-categorizer semantics* from [10] is obtained when all base scores are 1. The strength values in (cyclic) graphs can be computed by initializing the strength values with the base scores and applying formula (1) repeatedly to all arguments simultaneously until the strength values converge [6]. It is not difficult to see that the *weighted h-categorizer semantics* satisfies *Open-Mindedness*. However, in order to illustrate our definition, we give a detailed proof of the claim.

Proposition 1. *The* weighted h-categorizer semantics *is open-minded.*

Proof. In the subclass of attack-only QBAFs, it suffices to check the first condition of Definition 3. Consider an arbitrary attack-only QBAF $(\mathcal{A}, \mathrm{Att}, \emptyset, \beta)$, an arbitrary argument $a \in \mathcal{A}$ and an arbitrary $\epsilon > 0$. Let $N = \lceil \frac{1}{\epsilon} \rceil + 1$ and consider the QBAF $(\mathcal{A} \cup \{a_1, \ldots, a_N\}, \mathrm{Att} \cup \{(a_i, a) \mid 1 \leq i \leq N\}, \mathrm{Sup}, \beta')$ as defined in Definition 3. Recall that the N new attackers $\{a_1, \ldots, a_N\}$ have base score 1 and do not have any attackers. Therefore, $\sigma(a_i) = \frac{\beta(a_i)}{1} = 1$ for $i = 1, \ldots, n$ and $\sum_{(b,a)\in \mathrm{Att}} \sigma(a) \geq \sum_{i=1}^{N} \sigma(a_i) = N$. Furthermore, we have $\beta(a) \leq 1$ because $\mathcal{D} = [0, 1]$. Hence, $|\sigma(a) - 0| = \frac{\beta(a)}{1+\sum_{(b,a)\in \mathrm{Att}} \sigma(a)} < \frac{1}{N} < \epsilon$. □

The *weighted max-based semantics* from [6] can be seen as a variant of the *h-categorizer semantics* that aggregates the strength of attackers by means of the maximum instead of the sum. The strength of arguments is defined by

$$\sigma(a) = \frac{\beta(a)}{1 + \max_{(b,a)\in \mathrm{Att}} \sigma(b)}. \tag{2}$$

If there are no attackers, the maximum yields 0 by convention. The motivation for using the maximum is to satisfy a property called *Quality Precedence*, which guarantees that when arguments a_1 and a_2 have the same base score, but a_1 has an attacker that is stronger than all attackers of a_2, then the strength of a_1 must be smaller than the strength of a_2. The strength values under the *weighted max-based semantics* can again be computed iteratively [6]. Since all strength values are in $[0, 1]$ and the maximum is used for aggregating the strength values, we can immediately see that $\sigma(a) \geq \frac{\beta(a)}{2}$. Therefore, the *weighted max-based semantics* is clearly not open-minded. For example, if $\beta(a) = 1$, the final strength cannot be smaller than $\frac{1}{2}$, no matter how many attackers there are.

Proposition 2. *The* weighted max-based semantics *is not open-minded.*

One may wonder if *Quality Precedence* and *Open-Mindedness* are incompatible. This is actually not the case. For example, when defining strength values by

$$\sigma(a) = \beta(a) \cdot \big(1 - \max_{(b,a) \in \mathrm{Att}} \sigma(b)\big)$$

both *Quality Precedence* and *Open-Mindedness* are satisfied. In particular, the strength now decreases linearly from $\beta(a)$ to 0 with respect to the strongest attacker, which makes this perhaps a more natural way to satisfy *Quality Precedence* when it is desired.

The *weighted card-based semantics* from [6] is another variant of the *h-categorizer semantics*. Instead of putting extra emphasis on the strength of attackers, it now puts extra emphasis on the number of attackers. Let $\mathrm{Att}^+ = \{(a, b) \in \mathrm{Att} \mid \beta(a) > 0\}$. Then the strength of arguments is defined by

$$\sigma(a) = \frac{\beta(a)}{1 + |\mathrm{Att}^+| + \frac{\sum_{(b,a) \in \mathrm{Att}^+} \sigma(b)}{|\mathrm{Att}^+|}}. \tag{3}$$

When reordering terms in the denominator, we can see that the only difference to the *h-categorizer semantics* is that every attacker b with non-zero strength adds $1+\sigma(b)$ instead of just $\sigma(b)$ in the sum in the denominator (attacker with strength 0 do not add anything anyway). This enforces a property called *Cardinality Precedence*, which basically means that when arguments a_1 and a_2 have the same base score and a_1 has a larger number of non-rejected attackers ($\sigma(b) > 0$) than a_2, then the strength of a_1 must be smaller than the strength of a_2. The strength values under the *weighted card-based semantics* can again be computed iteratively [6]. Analogously to the *weighted h-categorizer semantics*, it can be checked that the *weighted card-based semantics* satisfies *Open-Mindedness.*

Proposition 3. *The* weighted card-based semantics *is open-minded.*

3.2 Support-Only QBAFs over $\mathcal{D} = [0, 1]$

We now consider three semantics for support-only QBAFs over $\mathcal{D} = [0, 1]$. For all semantics, the strength of arguments is defined by equations of the form

$$\sigma(a) = \beta(a) + (1 - \beta(a)) \cdot S(a),$$

where $S(a)$ is an aggregate of the strength of a's supporters. Therefore, the question whether *Open-Mindedness* is satisfied boils down to the question whether $S(a)$ converges to 1 as we keep adding supporters.

The *top-based semantics* from [3] defines the strength of arguments by

$$\sigma(a) = \beta(a) + (1 - \beta(a)) \max_{(b,a) \in \mathrm{Sup}} \sigma(b). \tag{4}$$

If there are no supporters, the maximum again yields 0 by convention. Similar to the semantics in the previous section, the strength values can be computed iteratively by setting the initial strength values to the base score and applying formula (4) repeatedly until the values converge [3]. It is easy to check that the *top-based semantics* is open-minded. In fact, a single supporter with strength 1 is sufficient to move the strength all the way to 1 independently of the base score.

Proposition 4. *The* top-based semantics *is open-minded.*

The *aggregation-based semantics* from [3] defines the strength of arguments by the formula

$$\sigma(a) = \beta(a) + (1 - \beta(a)) \frac{\sum_{(b,a) \in \mathrm{Sup}} \sigma(b)}{1 + \sum_{(b,a) \in \mathrm{Sup}} \sigma(b)}. \tag{5}$$

The strength values can again be computed iteratively [6]. It is easy to check that the *aggregation-based semantics* is open-minded. Just note that the fraction in (5) has the form $\frac{N}{1+N}$ and therefore approaches 1 as $N \to \infty$. Therefore, the strength of an argument will go to 1 as we keep adding supporters under the *aggregation-based semantics*.

Proposition 5. *The* aggregation-based semantics *is open-minded.*

The *reward-based semantics* from [3] is based on the idea of *founded arguments*. An argument a is called founded if there exists a sequence of arguments $(a_0, \dots, a_n)$ such that $a_n = a$, $(a_{i-1}, a_i) \in \mathrm{Sup}$ for $i = 1, \dots, n$ and $\beta(a_0) > 0$. That is, a has non-zero base score or is supported by a sequence of supporters such that the first argument in the sequence has a non-zero base score. Intuitively, this implies that a must have non-zero strength. We let $\mathrm{Sup}^+ = \{(a, b) \in \mathrm{Sup} \mid a \text{ is founded}\}$ denote the founded supports. For every $a \in \mathcal{A}$, we let $N(a) = |\mathrm{Sup}^+|$ denote the number of founded supporters of a and $M(a) = \frac{\sum_{(b,a) \in \mathrm{Sup}^+} \sigma(b)}{N(A)}$ the mean strength of the founded supporters. Then the strength of a is defined as

$$\sigma(a) = \beta(a) + (1 - \beta(a)) \big(\sum_{i=1}^{N(a)-1} \frac{1}{2^i} + \frac{M(a)}{2^{N(a)}} \big). \tag{6}$$

The strength values can again be computed iteratively [6]. As we show next, the reward-based semantics also satisfies *Open-Mindedness*.

Proposition 6. *The* reward-based semantics *is open-minded.*

Proof. In the subclass of support-only QBAFs, it suffices to check the second condition of Definition 3. Let us first note that $\sum_{i=1}^{N(a)-1} \frac{1}{2^i}$ is a geometric sum without the first term and therefore evaluates to

$$\frac{1 - \frac{1}{2^{N(a)}}}{1 - \frac{1}{2}} - 1 = 1 - \frac{1}{2^{N(a)-1}}$$

Note that this term already goes to 1 as the number of founded supporters $N(a)$ increases. We additionally add the non-negative term $\frac{M(a)}{2^{N(a)}} = \frac{\sum_{(b,a) \in \mathrm{Sup}^+} \sigma(b)}{N(A) \cdot 2^{N(a)}}$ which is bounded from above by $\frac{1}{2^{N(a)}}$. Therefore, the factor $\left(\sum_{i=1}^{N(a)-1} \frac{1}{2^i} + \frac{M(a)}{2^{N(a)}}\right)$ is always between 0 and 1 and approaches 1 as $|N(A)| \to \infty$.

To complete the proof, consider any support-only QBAF $(\mathcal{A}, \emptyset, \mathrm{Sup}, \beta)$, any argument $a \in \mathcal{A}$, any $\epsilon > 0$ and let $(\mathcal{A} \cup \{a_1, \ldots, a_N\}, \mathrm{Att}, \mathrm{Sup} \cup \{(a_i, a) \mid 1 \leq i \leq N\}, \beta')$ be the QBAF defined in Definition 3 for some $N \in \mathbb{N}$. Note that every argument in $\{a_1, \ldots, a_N\}$ is a founded supporter of a. Therefore, $N(A) \geq N$ and $\sigma(a) \to \beta(a) + (1 - \beta(a)) = 1$ as $N \to \infty$. This then implies that there exists an $N_0 \in \mathbb{N}$ such that $|\sigma(a) - 1| < \epsilon$. □

3.3 Bipolar QBAFs Without Base Score over $\mathcal{D} = [-1, 1]$

In this section, we consider two semantics for bipolar QBAFs without base score over $\mathcal{D} = [-1, 1]$ that have been introduced in [7]. It has not been explained how the strength values are computed in [7]. However, given an acyclic graph, the strength values can again be computed in topological order because the strength of every argument depends only on the strength of its parents. For cyclic graphs, one may consider an iterative procedure as before, but convergence may be an issue. In our investigation, we will just assume that the strength values are well-defined.

Following [8], we call the first semantics from [7], the *loc-max semantics*. It defines strength values by the formula

$$\sigma(a) = \frac{\max_{(b,a) \in \mathrm{Sup}} \sigma(b) - \max_{(b,a) \in \mathrm{Att}} \sigma(b)}{2} \tag{7}$$

By convention, the maximum now yields -1 if there are no supporters/attackers (this is consistent with the previous conventions in that -1 is now the minimum of the domain, whereas the minimum was 0 before). If a has neither attackers nor supporters, then $\sigma(a) = \frac{-1-(-1)}{2} = 0$. As we keep adding supporters (attackers), the first (second) term in the numerator will take the maximum strength value. From this we can see that the *loc-sum semantics* is open-minded for attack-only QBAFs without base score and for support-only QBAFs without base score. However, it is not open-minded for bipolar QBAFs without base score. For example, suppose that a has a single supporter b', which has a single supporter b'' and no attackers. Further assume that b'' has neither attackers nor supporters, so that $\sigma(b'') = 0$, $\sigma(b') = \frac{0-(-1)}{2} = \frac{1}{2}$ and $\sigma(a) \geq \frac{\frac{1}{2} - \max_{(b,a)\in \mathrm{Att}} \sigma(b)}{2}$. Since the maximum of the attackers can never become larger than 1, we have $\sigma(a) \geq \frac{\frac{1}{2}-1}{2} \geq -\frac{1}{4}$, no matter how many attackers we add. Thus, the first condition of *Open-Mindedness* is violated. Using a symmetrical example, we can show that the second condition can be violated as well.

Proposition 7. *The* loc-max semantics *is not open-minded. It is open-minded when restricting to attack-only QBAFs without base score or to support-only QBAFs without base score.*

Following [8], we call the second semantics from [7], the *loc-sum semantics*. It defines strength values by the formula

$$\sigma(a) = \frac{1}{1 + \sum_{(b,a)\in \mathrm{Att}} \frac{\sigma(b)+1}{2}} - \frac{1}{1 + \sum_{(b,a)\in \mathrm{Sup}} \frac{\sigma(b)+1}{2}} \tag{8}$$

Note that if there are neither attackers nor supporters, then both fractions are 1 such that their difference is just 0. As we keep adding attackers (supporters), the first (second) fraction goes to 0. It follows again that the *loc-sum semantics* is open-minded for attack-only QBAFs without base score and for support-only QBAFs without base score. However, it is again not open-minded for bipolar QBAFs without base score. For example, if a has a single supporter b' that has neither attackers nor supporters, then $\sigma(b') = 0$ and the second fraction evaluates to $\frac{1}{1+\frac{1}{2}} = \frac{2}{3}$. As we keep adding attackers, the first fraction will to 0 so that the strength of a will converge to $-\frac{2}{3}$ rather than to -1 as the first condition of *Open-Mindedness* demands. It is again easy to construct a symmetrical example to show that the second condition of *Open-Mindedness* can be violated as well.

Proposition 8. *The* loc-sum semantics *is not open-minded. It is open-minded when restricting to attack-only QBAFs without base score or to support-only QBAFs without base score.*

4 General QBAFs and Open-Mindedness

We now consider the general form of QBAFs as introduced in [8]. The domain $\mathcal{D} = (S, \preceq)$ is now an arbitrary set along with a preorder $\preceq$, that is, a reflexive and transitive relation over S. We further assume that there is an infimum $\inf(S)$ and a supremum $\sup(S)$ that may or may not be contained in S. For example, the open interval $(0, \infty)$, contains neither its infimum 0 nor its supremum ∞, whereas the half-open interval $[0, \infty)$ contains its infimum, but not its supremum.

Definition 4 (QBAF). *A QBAF over* $\mathcal{D} = (S, \preceq)$ *is a quadruple* $(\mathcal{A}, \mathrm{Att}, \mathrm{Sup}, \beta)$ *consisting of a set of arguments* $\mathcal{A}$*, a binary attack relation* Att*, a binary support relation* Sup *and a function* $\beta : \mathcal{A} \to \mathcal{D}$ *that assigns a* base score $\beta(a)$ *to every argument* $a \in \mathcal{A}$.

We now define a generalized form of *Open-Mindedness* for general QBAFs. We have to take account of the fact that there may no longer exist a minimum or maximum of the set. So instead we ask that strength values can be made smaller/larger than every element from $S \setminus \{\inf(S), \sup(S)\}$ by adding a sufficient number of attackers/supporters. Intuitively, we want to add strong supporters. In Definition 3, we just assumed that the maximum corresponds to the strongest value, but there are semantics that regard smaller values as stronger and, again, S may neither contain a maximal nor a minimal element. Therefore, we will just demand that there is some base score s^*, such that adding attackers/supporters with base score s^* has the desired consequence.

Definition 5 (Open-Mindedness (General Form)). *Consider a semantics that defines an interpretation $\sigma : \mathcal{A} \to \mathcal{D}$ for every QBAF from a particular class $\mathcal{F}$ of QBAFs over $\mathcal{D} = (S, \preceq)$. We call the semantics* open-minded *if for every QBAF $(\mathcal{A}, \text{Att}, \text{Sup}, \beta)$ in $\mathcal{F}$, for every argument $a \in \mathcal{A}$ and for every $s \in S \setminus \{\inf(S), \sup(S)\}$, the following condition is satisfied: there is an $N \in \mathbb{N}$ and an $s^* \in S$ such that when adding N new arguments $A_N = \{a_1, \ldots, a_N\}$, $\mathcal{A} \cap A_N = \emptyset$, then*

1. *if $\mathcal{F}$ allows attacks, then for $(\mathcal{A} \cup A_N, \text{Att} \cup \{(a_i, a) \mid 1 \leq i \leq N\}, \text{Sup}, \beta')$, we have $\sigma(a) \preceq s$ and*
2. *if $\mathcal{F}$ allows supports, then for $(\mathcal{A} \cup A_N, \text{Att}, \text{Sup} \cup \{(a_i, a) \mid 1 \leq i \leq N\}, \beta')$, we have $s \preceq \sigma(a)$,*

where $\beta'(b) = \beta(b)$ for all $b \in \mathcal{A}$ and $\beta'(a_i) = s^*$ for $i = 1, \ldots, n$.

Note that if S is a compact real interval, $s \in S \setminus \{\inf(S), \sup(S)\}$ can be chosen arbitrarily close to $\sup(S) = \max(S)$ or $\inf(S) = \min(S)$, so that s in Definition 5 plays the role of ϵ in Definition 3. In particular, if Definition 3 is satisfied, Definition 5 can be satisfied as well for an arbitrary s by choosing base score $s^* = \max(S)$ and choosing N with respect to $\epsilon = \frac{\max(S)-s}{2}$ or $\epsilon = \frac{s-\min(S)}{2}$. Definitions 5 and 3 are actually equivalent for compact real intervals provided that $\max(S)$ corresponds to the strongest initialization of the base score under the given semantics, which is indeed the case in all previous examples.

As an example, for more general QBAFs, let us now consider the *α-burden-semantics* from [5]. It defines strength values for attack-only QBAFs without base score over the half-open interval $[1, \infty)$. As opposed to our previous examples, the minimum 1 now corresponds to the strongest value and increasing values correspond to less plausibility. The *α-burden-semantics* defines strength values via the formula

$$\sigma(a) = 1 + \Big(\sum_{(b,a) \in \text{Att}} \frac{1}{(\sigma(b))^\alpha} \Big)^{\frac{1}{\alpha}}. \tag{9}$$

α is called the *burden-parameter* and can be used to modify the semantics, see [5] for more details about the influence of α. For $\alpha \in [1, \infty) \cup \{\infty\}$, (9) is equivalent to arranging the reciprocals of strength values of all attackers in a vector v and to take the p-norm $\|v\|_p = \Big(\sum_i v_i^p \Big)^{\frac{1}{p}}$ of this vector with respect to $p = \alpha$

and adding 1. Popular examples of p-norms are the Manhattan-, Euclidean- and Maximum-norm that are obtained for $p = 1$, $p = 2$ and the limit-case $p = \infty$, respectively. An unattacked argument has just strength 1 under the *α-burden-semantics*. Hence, when adding N new attackers to a, we have $\sigma(a) \geq 1 + N^{\frac{1}{\alpha}}$ for $\alpha \in [1, \infty)$. Hence, the *α-burden-semantics* is clearly open-minded in this case, even though it becomes more conservative as α increases. In particular, for the limit case $\alpha = \infty$, it is not open-minded. This can be seen from the observation, that the second term in (9) now corresponds to the maximum norm. Since the strength of each attacker is in $[1, \infty)$, their reciprocals are in $(0, 1]$. Therefore, $\sigma(a) \leq 2$ independent of the number of attackers of a.

Proposition 9. *The* α-burden-semantics *is open-minded for $\alpha \in [1, \infty)$, but is not open-minded for $\alpha = \infty$.*

5 Related Work

Gradual argumentation has become a very active research area and found applications in areas like information retrieval [24], decision support [9,22] and social media analysis [1,12,16]. Our selection of semantics followed the selection in [8]. One difference is that we did not consider social abstract argumentation [16] here. The reason is that social abstract argumentation has been formulated in a very abstract form, which makes it difficult to formulate interesting conditions under which *Open-Mindedness* is guaranteed. Instead, we added the *α-burden-semantics* from [5] because it gives a nice example for a more general semantics that neither uses strength values from a compact interval nor regards larger values as stronger.

The authors in [8] also view ranking-based semantics [11] as gradual argumentation frameworks. In their most general form, ranking-based semantics just order arguments qualitatively, so that our notion of *Open-Mindedness* is not very meaningful. A variant may be interesting, however, that demands, that in every argumentation graph, every argument can become first or last in the order if only a sufficient number of supporters or attackers is added to this argument. However, in many cases, this notion of *Open-Mindedness* may be entailed by other properties already. For example, *Cardinality Precedence* [11] states that if argument a_1 has more attackers than a_2, then a_1 must be weaker than a_2. In finite argumentation graphs, this already implies that a_1 will be last in the order if we add a sufficient number of attackers.

There are other quantitative argumentation frameworks like probabilistic argumentation frameworks [14,15,17,19,23]. In this area, *Open-Mindedness* would simply state that the probability of an argument must go to 0 (1) as we keep adding attackers (supporters). It may be interesting to perform a similar analysis for probabilistic argumentation frameworks.

An operational definition of *Open-Mindedness* for the class of modular semantics [18] for weighted bipolar argumentation frameworks has been given in [21]. The *Df-QuAD semantics* [22] and the *Quadratic-energy Semantics* [20] satisfy this notion of open-mindedness [21]. However, in case of DF-QuAD and some other semantics, this is actually counterintuitive because they cannot move the strength of an argument towards 0 if there is a supporter with non-zero strength. Indeed, DF-QuAD does not satisfy Open-Mindedness as defined here (every QBAF with a non-zero strength supporter provides a counterexample). However, the quadratic energy model from [21] still satisfies the more restrictive definition of *Open-Mindedness* that we considered here.

Another interesting property for bipolar QBAFs that is not captured by *Balance* and *Monotonicity* is *Duality* [20]. Duality basically states that attack and support should behave in a symmetrical manner. Roughly speaking, when we convert an attack relation into a support relation or vice versa, the effect of the relation should just be inverted. *Duality* is satisfied by the *Df-QuAD semantics* [22] and the *Quadratic-energy Semantics* [20], but not by the *Euler-based semantics* [4]. A formal analysis can be found in [20,21].

6 Conclusions

We investigated 9 gradual argumentation semantics from the literature. 5 of them satisfy *Open-Mindedness* unconditionally. This includes the *weighted h-categorizer semantics* and the *weighted card-based semantics* for attack-only QBAFs from [6] and all three semantics for support-only QBAFs from [3]. The *α-burden-semantics* for attack-only QBAFs without base score from [5] is open-minded for $\alpha \in [1, \infty)$, but not for the limit case $\alpha = \infty$. The *loc-max semantics* and the *loc-sum semantics* for bipolar QBAFs without base score from [7] are only open-minded when restricted to either attack-only or to support-only QBAFs. Finally, the *weighted max-based semantics* for attack-only QBAFs from [6] is not open-minded. However, as we saw, it can easily be adapted to satisfy both *Open-Mindedness* and *Quality Precedence*.

In future work, it may be interesting to complement *Open-Mindedness* with a *Conservativeness* property that demands that the original base scores are not given up too easily. For the class of modular semantics [18] that iteratively compute strength values by repeatedly aggregating strength values and combining them with the base score, *Conservativeness* can actually be quantified analytically [21]. Intuitively, this can be done by analyzing the maximal local growth of the aggregation and influence functions. There is actually an interesting relationship between *Conservativeness* and *Well-Definedness* of strength values. For general QBAFs, procedures that compute strength values iteratively, can actually diverge [18] so that some strength values remain undefined. However, the mechanics that make semantics more conservative, simultaneously improve convergence guarantees [21]. In other words, convergence guarantees can often be improved by giving up *Open-Mindedness*. The extreme case would be the naive semantics that just assigns the base score as final strength to every argument independent of the attackers and supporters. This semantics is clearly most conservative and always well-defined, but does not make much sense.

My personal impression is indeed that gradual argumentation semantics for general QBAFs with strong convergence guarantees are too conservative at the moment. Some well-defined semantics for general QBAFs have been presented recently in [18], but they are not open-minded. I am indeed unaware of any semantics for general QBAFs that is generally well-defined and open-minded. It is actually possible to define for every $k \in \mathbb{N}$, an open-minded semantics that is well-defined for all QBAFs where arguments have at most k parents. One example is the 1-*max(k) semantics*, see Corollary 3.5 in [21]. However, as k grows, these semantics become more and more conservative even though they remain open-minded. More precisely, every single argument can change the strength value of another argument by at most $\frac{1}{k}$, so that at least k arguments are required to move the strength all the way from 0 to 1 and vice versa. A better way to improve convergence guarantees may be to define strength values not by discrete iterative procedures, but to replace them with continuous procedures that maintain the strength values in the limit, but improve convergence guarantees [20,21]. However, while I find this approach promising, I admit that it requires further analysis.

In conclusion, I think that *Open-Mindedness* is an interesting property that is important for many applications. It is indeed satisfied by many semantics from the literature. For others, like the *weighted max-based semantics*, we may be able to adapt the definition. One interesting open question is whether we can define semantics for general QBAFs that are generally well-defined and open-minded.

References

1. Alsinet, T., Argelich, J., Béjar, R., Fernández, C., Mateu, C., Planes, J.: Weighted argumentation for analysis of discussions in Twitter. Int. J. Approximate Reasoning **85**, 21–35 (2017)
2. Amgoud, L., Ben-Naim, J.: Axiomatic foundations of acceptability semantics. In: International Conference on Principles of Knowledge Representation and Reasoning (KR), pp. 2–11 (2016)
3. Amgoud, L., Ben-Naim, J.: Evaluation of arguments from support relations: axioms and semantics. In: International Joint Conferences on Artificial Intelligence (IJCAI), p. 900 (2016)
4. Amgoud, L., Ben-Naim, J.: Evaluation of arguments in weighted bipolar graphs. In: Antonucci, A., Cholvy, L., Papini, O. (eds.) ECSQARU 2017. LNCS (LNAI), vol. 10369, pp. 25–35. Springer, Cham (2017). https://doi.org/10.1007/978-3-319-61581-3_3
5. Amgoud, L., Ben-Naim, J., Doder, D., Vesic, S.: Ranking arguments with compensation-based semantics. In: International Conference on Principles of Knowledge Representation and Reasoning (KR) (2016)
6. Amgoud, L., Ben-Naim, J., Doder, D., Vesic, S.: Acceptability semantics for weighted argumentation frameworks. In: IJCAI, vol. 2017, pp. 56–62 (2017)
7. Amgoud, L., Cayrol, C., Lagasquie-Schiex, M.C., Livet, P.: On bipolarity in argumentation frameworks. Int. J. Intell. Syst. **23**(10), 1062–1093 (2008)
8. Baroni, P., Rago, A., Toni, F.: How many properties do we need for gradual argumentation? In: AAAI Conference on Artificial Intelligence (AAAI), pp. 1736–1743. AAAI (2018)

9. Baroni, P., Romano, M., Toni, F., Aurisicchio, M., Bertanza, G.: An argumentation-based approach for automatic evaluation of design debates. In: Leite, J., Son, T.C., Torroni, P., van der Torre, L., Woltran, S. (eds.) CLIMA 2013. LNCS (LNAI), vol. 8143, pp. 340–356. Springer, Heidelberg (2013). https://doi.org/10.1007/978-3-642-40624-9_21
10. Besnard, P., Hunter, A.: A logic-based theory of deductive arguments. Artif. Intell. **128**(1–2), 203–235 (2001)
11. Bonzon, E., Delobelle, J., Konieczny, S., Maudet, N.: A comparative study of ranking-based semantics for abstract argumentation. In: AAAI Conference on Artificial Intelligence (AAAI), pp. 914–920 (2016)
12. Cocarascu, O., Rago, A., Toni, F.: Extracting dialogical explanations for review aggregations with argumentative dialogical agents. In: International Conference on Autonomous Agents and MultiAgent Systems (AAMAS), pp. 1261–1269. International Foundation for Autonomous Agents and Multiagent Systems (2019)
13. Dung, P.M.: On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games. Artif. Intell. **77**(2), 321–357 (1995)
14. Hunter, A., Polberg, S., Potyka, N.: Updating belief in arguments in epistemic graphs. In: International Conference on Principles of Knowledge Representation and Reasoning (KR), pp. 138–147 (2018)
15. Hunter, A., Thimm, M.: Probabilistic reasoning with abstract argumentation frameworks. J. Artif. Intell. Res. **59**, 565–611 (2017)
16. Leite, J., Martins, J.: Social abstract argumentation. In: International Joint Conferences on Artificial Intelligence (IJCAI), vol. 11, pp. 2287–2292 (2011)
17. Li, H., Oren, N., Norman, T.J.: Probabilistic argumentation frameworks. In: Modgil, S., Oren, N., Toni, F. (eds.) TAFA 2011. LNCS (LNAI), vol. 7132, pp. 1–16. Springer, Heidelberg (2012). https://doi.org/10.1007/978-3-642-29184-5_1
18. Mossakowski, T., Neuhaus, F.: Modular semantics and characteristics for bipolar weighted argumentation graphs. arXiv preprint arXiv:1807.06685 (2018)
19. Polberg, S., Doder, D.: Probabilistic abstract dialectical frameworks. In: Fermé, E., Leite, J. (eds.) JELIA 2014. LNCS (LNAI), vol. 8761, pp. 591–599. Springer, Cham (2014). https://doi.org/10.1007/978-3-319-11558-0_42
20. Potyka, N.: Continuous dynamical systems for weighted bipolar argumentation. In: International Conference on Principles of Knowledge Representation and Reasoning (KR), pp. 148–157 (2018)
21. Potyka, N.: Extending modular semantics for bipolar weighted argumentation. In: International Conference on Autonomous Agents and MultiAgent Systems (AAMAS), pp. 1722–1730. International Foundation for Autonomous Agents and Multiagent Systems (2019)
22. Rago, A., Toni, F., Aurisicchio, M., Baroni, P.: Discontinuity-free decision support with quantitative argumentation debates. In: International Conference on Principles of Knowledge Representation and Reasoning (KR), pp. 63–73 (2016)
23. Rienstra, T., Thimm, M., Liao, B., van der Torre, L.: Probabilistic abstract argumentation based on SCC decomposability. In: International Conference on Principles of Knowledge Representation and Reasoning (KR), pp. 168–177 (2018)
24. Thiel, M., Ludwig, P., Mossakowski, T., Neuhaus, F., Nürnberger, A.: Web-retrieval supported argument space exploration. In: ACM SIGIR Conference on Human Information Interaction and Retrieval (CHIIR), pp. 309–312. ACM (2017)