Reliability Analysis Based on Incompletely Specified Data

Elena Zaitseva(✉), Vitaly Levashenko, and Peter Sedlacek

Faculty of Management Science and Informatics, University of Žilina, Žilina, Slovakia
{Elena.Zaitseva,Vitaly.Levashenko,Peter.Sedlacek}@fri.uniza.sk

Abstract. In this paper new algorithm for the reliability analysis of system is considered. The reliability analysis consists of 2 essential steps - creation of mathematical model and quantitative analysis. The system mathematical model is constructed depending on the specifics of analysis and properties of the investigated system. In this paper the mathematical model in form of the structure function of non-coherent Multi-State System (MSS) is considered. The non-coherent system is specific group of systems in context of reliability analysis for which for system component degradation does not always lead to a degradation or failure of the system. MSS is mathematical model that allows analyzing of some (not only two) states/performance levels of system reliability. The structure function is type of mathematical model, that express dependency between system behavior and behavior of its components. Structure function can be represented in different forms, for example, as minimal cuts/paths, fault tree, reliability bloc diagrams. One of them used in this paper is Multi-Valued Decision Diagram (MDD). MDD is typical used for representation of data of large dimension. Created mathematical model is then used for quantitative analysis. This analysis includes calculus of different system characteristics such as availability/unavailability and other indices and measures. New methods for the creating of structure function in form of MDD and calculation of some indices for quantitative analysis is proposed in the paper. Important advantage of this method is possibility to use for MDD construction based on incompletely specified data and analysis of non-coherent MSS. Usage of the method is demonstrated on biker crashes survival evaluation.

Keywords: Reliability analysis · Multi-valued decision diagrams · Non-coherent systems · Incompletely specified data · Structure importance

1 Introduction

Reliability analysis is one of system characteristics, that has to be taken into account as failures (system breakdown) can have fatal consequences. Important

This work was supported by the project VEGA 1/0354/17 and the Slovak Research and Development Agency under the contract No. SK-SRB-18-0002.

S. V. Ablameyko et al. (Eds.): PRIP 2019, CCIS 1055, pp. 20–32, 2019.
https://doi.org/10.1007/978-3-030-35430-5_3

step in the system reliability is constructing of mathematical model of system [3, 19, 35]. The system mathematical model is constructed depending on the specifics of system analysis and properties of the investigated system. Depending on number of system performance levels, mathematical models can be divided into two types depending on the detail of the analysis [19]. The first of them is Binary-state systems. This model can be used to analyse system behaviour on two performance levels - system is working or system fails. This model is effectively used to analyse system failures. The other type of systems are Multi-State Systems (MSS). Using this model, we can analyse system behavior on more than two performance levels. This is used to analyse gradual degradation of system performance, however this model is more complex to analyse as binary-state.

Except system type based on number of states, there are different mathematical models depending on mathematical methods used to analyse this system. One of these mathematical methods is Boolean algebra [9]. In this case, structure function is used as mathematical model. Structure function clearly maps system components performance level to system performance [25, 33]. Advantages of using structure function as mathematical model are for example uniform approach to analyse system using Boolean or multiple-valued logic [33]. However in case of real systems, structure function can have large dimension. Therefore it is necessary to represent it in efficient way, such as Multi-Valued Decision Diagram (MDD) [22]. MDD is graphical orthogonal and canonical form for representation of logical function of large dimension. Usage of these diagrams for MSS structure function was considered in [2, 31].

In order to create structure function, full information about system behavior is required. The problem is, in many real systems, there is lack of such information [3, 32]. Therefore different methods have been developed for reliability evaluation based on incompletely specified data, for example, graph databases based methods [8, 27] or datamining based methods [26, 32]. In case of the use of datamining based methods the structure function is interpreted as classification structure that should be created based on incompletely specified [32]. In this paper methods of datamining, specifically decision trees will be used in order to handle this problem [1, 5, 20, 26]. In this case the structure function is represented in form of decision tree that is transformed into MDD [22].

Created mathematical model in form of MDD is in next step of reliability analysis used to calculate system reliability indices and measures, such as availability [13, 35], importance measures [14], minimal cuts and paths set [15] etc. These characteristics can be divided into two types - topological and probabilistic. Topological analysis does not require information about state probabilities and can be used to analyse system behavior based on its topology. Typical example of this characteristic is structure importance [11]. The probabilistic analysis takes into account also probabilities of individual component performance. Typical example of probabilistic importance measure can be Birnbaum's importance. In most of investigations reliability indices and measures are considered for coherent system. But there are special group of non-coherent systems in reliability analysis. The degradation of some of components in such systems does not always lead to a degradation or failure of the system. This fact causes specific in reliability analysis of these system because the structure function of such system is not monotone. There are some investigations for analysis of non-coherent Binary-state systems [4,10,34], but the evaluation of MSS non-coherent systems is not investigated sufficient. In this paper the initial investigation for non-coherent MSS is considered based on calculation of Important measures.

Organization of this paper is following. Section 2 contains detailed information about used mathematical model - structure function together with its representation in form of multi-valued decision diagram. Section 3 contains description of system characteristics and their calculus and calculus of structure importance for multi-state non-coherent systems. These methods are used in Sect. 4 to analyse Bike crashes dataset and finally Sect. 5 concludes previous methods together with plans for next work.

2 Reliability Analysis

Reliability analysis of system consist of 2 essential steps:

1. Creation of mathematical model
2. Quantitative analysis - i.e. calculation of system characteristics and different types of reliability indices. In this paper importance measures will be used, with focus on topological analysis.

There are different mathematical models in reliability analysis. The creation of the mathematical model is caused by system properties and specifics of reliability analysis. For any mathematical model is defined:

- number of system states
- type of mathematical model.

The number of system states or system performance levels is caused by analysis detail. There are two groups of mathematical models:

- Binary-State System (BSS) is modelled in two performance levels - i.e. system is either working or not. This approach is used, if system is binary-state from its nature [6,16], or we are analysing consequences of system failure [35].
- Multi-State System (MSS) is modelled in more than two performance levels. This approach allows us to describe gradual degradation of system performance from fully working to fully broken [18,19,25].

MSS allows the system analysis in more detail but computational complexity of this analysis increases and special methods and algorithms should be developed for quantitative analysis of MSS. The methods of quantitative analysis of MSS and other system associate with the types of mathematical model. The types correlates with background methods used in quantitative analysis. As a rule stochastic methods, methods of Boolean logic or algebra logic are used as background methods in reliability analysis [3]. The type of mathematical model correlated with algebra logic in reliability analysis is named structure function. This mathematical model can be used for representation of system of any topological complexity. It has favorable rules for the construction. But the computational complexity of the structure function analysis increases depending on this function dimensional. The development of special methods and algorithm for the structure function analysis allows creating effective approaches for reliability analysis of system.

2.1 Structure Function

Dependency between system performance and performance of system components can be expressed using Structure Function. It can be defined as following [11]:

$$\begin{aligned}\phi(x_1, x_2, ..., x_n) = \phi(\boldsymbol{x}) : \{0, 1, ..., m_1 - 1\} \times \{0, 1, ..., m_2 - 1\} \\ \times ... \times \{0, 1, ..., m_n - 1\} \rightarrow \{0, 1, ..., m - 1\}\end{aligned} \tag{1}$$

where x_i is i-th system component state, $i \in \{1, 2, ..., n\}$, m is number of system performance levels and m_i is number of states of component i. In case $m = m_i = 2$ system is Binary-state (BSS).

Structure function can be used in topological analysis of system. But in case probabilistic analysis is required, we need to know not only structure function, but also state probabilities of individual system components. The i-th component state probabilities will be denoted as following [11,30]:

$$\begin{aligned} p_{i1} &= \Pr\{x_i = 1\}, ..., \\ p_{i,j} &= \Pr\{x_i = j\}, \\ q_i &= \Pr\{x_i = 0\} \end{aligned} \tag{2}$$

Depending on system behavior, systems can be divided into two types - **coherent** and **non-coherent**. In non-coherent systems there are cases, when degradation of system component leads to increase of system performance level. This is not possible for coherent systems. From structure function point of view according to [11] system is coherent, if system meets 2 conditions:

1. Structure function is non-decreasing - This condition implies, that improving of state of any component does not degrade the performance of system and vice versa.
2. Each system component is relevant - System component is relevant, if there exists at least one case in that the state of component dictates the state of the system.

If at least one of these conditions are not met, the system is non-coherent. Majority of technical systems are coherent in general. Examples of non-coherent systems can be systems including human factor. Case study presented in this paper is also non-coherent system.

The main problem with traditional methods in reliability analysis is the fact, we need all information about system behaviour to create mathematical model.

In many real systems, there is lack of such information, i.e. we either don't know all system components or we don't know system behavior based on these components behavior in some cases [3]. The other problem can be ambiguous data. This can be caused by many factors, for example inaccuracy of measurement [26].

This problem can be handled in different ways, for example usage of graph databases [8,27]. In this paper, we will be used methods of datamining to create decision trees [1,5,20]. This tree will be then reduced into multi-valued decision diagrams using reduction rules described in [21,24,28].

2.2 Multi-valued Decision Diagram

Structure function can be represented in multiple ways such as truth table, reliability block diagram, fault tree, etc. One, effective representation of structure function is Multi-valued Decision Diagram (MDD). According to publications [21,23], MDD is rooted acyclic graph that meets two conditions:

1. graph is canonical - the representation is unique for a particular variable ordering
2. graph is compact - any other graph representation contains more nodes.

There are two type of nodes in MDD - sink and non-sink. Sink nodes, labeled by numbers from 0 to $m - 1$, express system performance levels. There are exactly one sink node for each system state. Non-sink nodes represents system components. Node representing component x_i has exactly m_i outgoing edges expressed state of representing component.

Thanks to fact, MDD is orthogonal form of Structure Function [22], it can be used for the probabilistic analysis. In this case this graph is edge-weighted. Weight of j-th outgoing edge of node representing component x_i is equal to probability $p_{i,j}$, that component x_i is in state j.

3 Quantitative Analysis

Created mathematical model can be in the next step used to calculate system characteristics such as availability and unavailability.

Availability and Unavailability. Availability $\mathrm{A}^{\geq j}$ can be defined as probability system is at least in j-th performance level. Unavailability can be defined similarly - i.e. probability system performance level is lower than j [18,19,25]:

$$\begin{aligned} A^{\geq j}(\boldsymbol{p}) &= Pr\{\phi(\boldsymbol{x}) \geq j\}, \\ U^{\geq j}(\boldsymbol{p}) &= Pr\{\phi(\boldsymbol{x}) < j\}, \\ U^{\geq j}(\boldsymbol{p}) &= 1 - A^{\geq j}(\boldsymbol{p}) \end{aligned} \quad (3)$$

These are characteristics of whole system but give us no information about how system performance in influenced by performance of its components.

For this purpose importance measures has to be calculated. There are 2 types of importance measures - topological and probabilistic. This paper will focus on topological analysis, therefore structure importance will be used.

3.1 Structure Importance

Structure importance can be used to express influence of component to system performance in topological point of view.

$$\mathrm{SI}_{i\downarrow}^{\downarrow} = \mathrm{TD}\left(\frac{\partial\phi(1\to 0)}{\partial x_i(1\to 0)}\right), \tag{4}$$

where TD(.) stands for Truth Density, i.e. function described as relatively number of cases function (.) gets value 1 to all possible cases [15]. Function $\frac{\partial\phi(1\to 0)}{\partial x_i(1\to 0)}$ is partial Boolean derivation and can be defined as following [29]:

$$\frac{\partial\phi(1\to 0)}{\partial x_i(1\to 0)} = \phi(x_1, ..., 1, ...x_n) \wedge \overline{\phi(x_1, ..., 0, ...x_n)}. \tag{5}$$

Equation 4 suppose analysed system is coherent. Therefore it is necessary to investigate only cases, when system component failure leads to system failures. There are multiple structure importance for non-coherent systems. The next one takes into account cases, when system component failure leads to system repair. This can be defined using following equation:

$$\mathrm{SI}_{i\uparrow}^{\downarrow} = \mathrm{TD}\left(\frac{\partial\phi(1\to 0)}{\partial x_i(0\to 1)}\right) \tag{6}$$

The total influence of system component can be calculated as a sum of Eqs. 4 and 6 [12]:

$$\mathrm{SI}_{i}^{\downarrow} = \mathrm{SI}_{i\downarrow}^{\downarrow} + \mathrm{SI}_{i\uparrow}^{\downarrow} = \mathrm{TD}\left(\partial\phi(\boldsymbol{x}/\partial x_i\right) \tag{7}$$

All of previous equations apply for binary-state systems. Structure importance for multi-state systems can be generalized as following:

$$\mathrm{SI}_{i}^{\downarrow} = \mathrm{TD}\left(\frac{\partial\phi(j\to k)}{\partial x_i(r\to s)}\right), \tag{8}$$

where $\phi(j \to k)$ means system change state from state j to state k; $j, k < m$. Expression $x(r \to s)$ means system component change state from r to state s; $r, s < m_i$. m_i is number of i-th system component states.

Formula 8 can be used for both - coherent and non-coherent systems. In case of coherent systems, condition $j > k \wedge r > s$ has to be met.

For non-coherent systems, there are also multiple structure importance. $\mathrm{SI}_{i\downarrow}^{\downarrow}$ is common with previous case. For the next one, $\mathrm{SI}_{i\uparrow}^{\downarrow}$ investigating decrease of system performance caused by increase of system component performance, one of condition $j > k \wedge r < s$ or $j < k \wedge r > s$ has to be met. Total topological influence of component can be calculated as sum of these two values.

4 Case Study

In this section we will demonstrate application of described methods in dataset obtained from [7]. This dataset contains information about bicycle crashes such as driver age, whether ambulance was called or not, what was the weather like etc., together with information if bicycle driver was killed, injured or without injure. Whole dataset consists of 11 attributes and 162 records. In order to simplify manipulation with data, each attribute value was mapped into numeric value. Complete list of attributes together with possible values and their mapping can be seen in Table 1.

Table 1. Details of Bike Crashes Dataset

Attribute name	Number of values	List of values (mapped into state)
Ambulance	2	Yes (1), No (0)
Age	7	0 – 10 (0), 11 – 19 (1), 20 – 29 (2),... 70+ (7)
Bike direction	4	With Traffic (0), Facing Traffic (1), Not applicable (2), Unknown (3)
Bike position	7	Travel Line (0), Sidewalk/Crosswalk/Driveway Crossing (1), Non-Roadway (5), Bike Lane/Paved Shoulder (6), Driveway/Alley (3), Multi-use Path (4), Unknown (2)
Sex	2	Male (0), Female (1)
Biker alcohol	2	Yes (1), No (0)
Driver alcohol	3	Yes (2), No (0), Missing (1)
Driver speed	8	0 – 10 (0), 11 – 20 (1), 21 – 30 (2),..., 61 – 65 (6), Unknown (7)
Light condition	5	DayLight (3), Dark - No Light (1), Dark - Lighted (2), Dusk (0), Unknown (4)
Road surface	5	Croashed Asphalt (4), Smooth Asphalt (3), Concrete (0), Gravel (1), Other (2)
Weather	3	Clear (2), Cloudy (1), Rain (0)

This dataset was analysed using methods of datamining presented in [17, 32]. Using these methods, decision tree was created. This tree was in the next step reduced into MDD using methods described in paper [21,24,28]. Resulting MDD can be seen in Fig. 1. Created mathematical model can be used to perform quantitative analysis as described in previous section.

In the first step, structure importance for each component was calculated according to Eq. 4. This contains only cases, when decrease of system component state leads to decrease of system performance. Results of this calculation can be seen in Fig. 2. The x axis contains individual component and system changes. On the y axis there are values of $\mathrm{SI}_{i\downarrow}^{\downarrow}$ for every component. So for example values

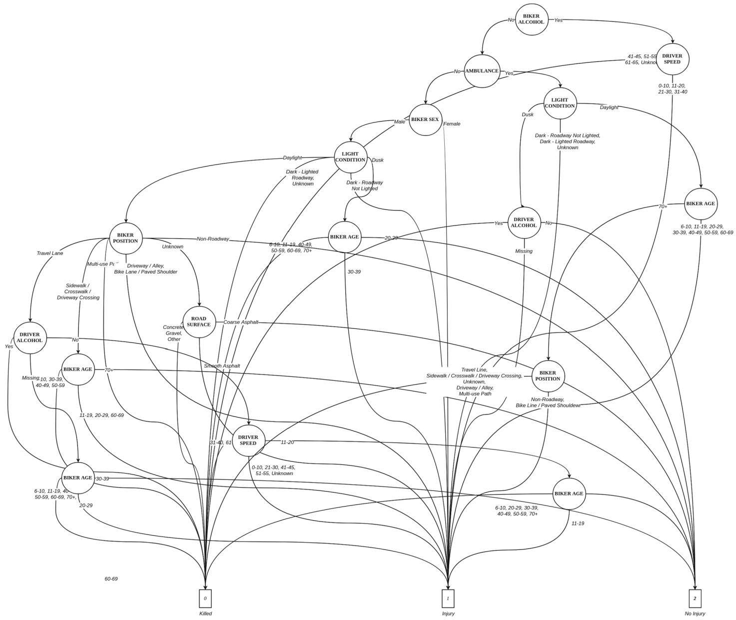

Fig. 1. MDD of Bike Crashes Dataset

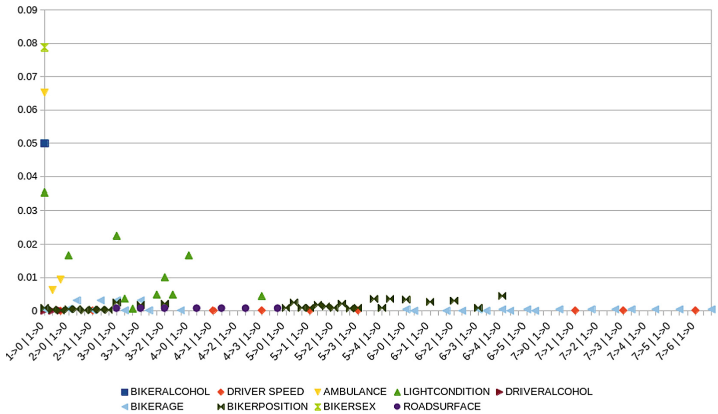

Fig. 2. Results of $\mathrm{SI}_{i\downarrow}^{\downarrow}$ for Bike Crashes Dataset

in column "2 → 0|1 → 0" is equal to value of Structure Importance index, when system component state changes from 2 to 0 and system changes from 1 to 0.

From these results we can identify most crucial components according to system structure. These can be seen in Table 2.

Similarly, second structure importance was calculated according to Eq. 6. This contains only cases, when system component performance decrease leads to case system increase its state. Results of this calculation can be seen in Fig. 3. The most crucial components together with value of its structure importance can be seen in Table 3.

Table 2. Attributes with highest values of $SI^{\downarrow}_{i\downarrow}$ for Bike Crashes Dataset

Component	Component change|System change	Value of $SI^{\downarrow}_{i\downarrow}$
Biker sex	$1 \rightarrow 0 \vert 1 \rightarrow 0$	0.0787426
Ambulance	$1 \rightarrow 0 \vert 1 \rightarrow 0$	0.0651749
Biker alcohol	$1 \rightarrow 0 \vert 1 \rightarrow 0$	0.0499926
Light condition	$1 \rightarrow 0 \vert 1 \rightarrow 0$	0.0354167

Table 3. Attributes with highest values of $SI^{\downarrow}_{i\uparrow}$ for Bike Crashes Dataset

Component	Component change|System change	Value of $SI^{\downarrow}_{i\downarrow}$
Biker alcohol	$1 \rightarrow 0 \vert 0 \rightarrow 1$	0.187783
Driver speed	$6 \rightarrow 0 \vert 0 \rightarrow 1$, $6 \rightarrow 1 \vert 0 \rightarrow 1$, $6 \rightarrow 2 \vert 0 \rightarrow 1$	0.0626488
Light condition	$2 \rightarrow 1 \vert 0 \rightarrow 1$, $4 \rightarrow 1 \vert 0 \rightarrow 1$	0.025

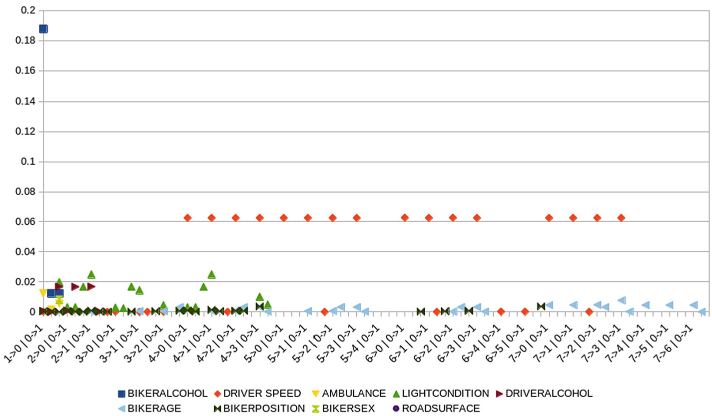

Fig. 3. Results of $SI^{\downarrow}_{i\uparrow}$ for Bike Crashes Dataset

Finally, structure importance according to Eq. 7 was calculated. This includes both cases - when decrease of system component leads to decrease of system state and decrease of system component leads to increase of system state. Results of this case can be seen in Fig. 4. The components with highest structure importance can be seen in Table 4.

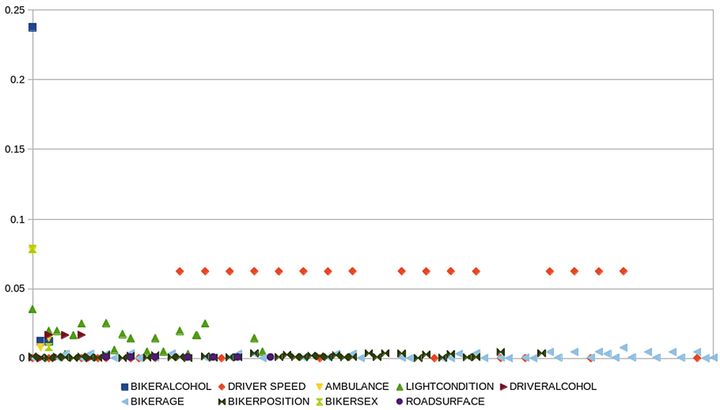

Fig. 4. Results of $\mathrm{SI}_i^{\downarrow}$ for Bike Crashes Dataset

Table 4. Attributes with highest values of $\mathrm{SI}_i^{\downarrow}$ for Bike Crashes Dataset

Component	Component change|System change	Value of $\mathrm{SI}_{i\downarrow}^{\downarrow}$
Biker alcohol	$1 \to 0 \| 0 \leftrightarrow 1$	0.2377756
Biker sex	$1 \to 0 \| 0 \leftrightarrow 1$	0.0787426
Ambulance	$1 \to 0 \| 0 \leftrightarrow 1$	0.0774108
Driver speed	$4 \to 3 \| 0 \leftrightarrow 1$, $5 \to 3 \| 0 \leftrightarrow 1$, $7 \to 3 \| 0 \leftrightarrow 1$	0.06264881
Light condition	$1 \to 0 \| 0 \leftrightarrow 1$	0.0354167

According to Fig. 4, the most crucial attribute of this system is biker alcohol. This can be interpreted as the fact, biker drank alcohol or not is the most significant factor that makes difference between his death or injury. The next crucial attribute is ambulance. The fact ambulance was called to accident has significant impact to result of the accident. The next one is biker sex. The dataset implies, that women has less deaths than men. Significant role in biker survival is caused also by driver speed, where change from 51–55 mps to 31–40 or from 41–50 to 31–40 leads to the fact biker will be alive.

On the other hand, factors as weather or bike direction (whether it is with traffic it) has no impact on result of the accident. However it should be noted, that accuracy of these results depends on amount of available data.

5 Conclusion

This paper contains reliability analysis calculus of multi-state systems based on incompletely specified data. This analysis consists of two steps. The first one is creation of mathematical model. For this purpose we used structure function as it can be used for analysing system behavior based on behavior of its components. Structure function can be represented in multiple ways. In this paper it was represented in form of MDD and it is constructed through the induction of decision tree based on incompletely specified data. MDD can be used for both - topological an probabilistic analysis and can be easily created from decision tree.

The next step in reliability analysis is quantitative analysis using this model. In this paper, we investigate topological analysis of multi-state non-coherent systems, specifically calculus of structure importance of these systems. In difference of other investigations in this paper the non-coherent MSS is considered and mathematical background for calculation of Importance measures is presented.

Mentioned methods was demonstrated on dataset that contains information about bike crashes. Using methods of datamining this dataset was analysed and decision tree was created. This tree was in the next step reduced into MDD from which topological analysis was calculated.

In our future work, we will investigate also probabilistic analysis of multi-state non-coherent systems. This analysis will be based on structure function represented using multi-valued decision diagrams.

References

1. Abdallah, I., et al.: Fault diagnosis of wind turbine structures using decision tree learning algorithms with big data. In: Safety and Reliability – Safe Societies in a Changing World, pp. 3053–3061. CRC Press, June 2018. https://doi.org/10.1201/9781351174664-382
2. Amari, S.V., Xing, L., Shrestha, A., Akers, J., Trivedi, K.S.: Performability analysis of multistate computing systems using multivalued decision diagrams. IEEE Trans. Comput. **59**(10), 1419–1433 (2010). https://doi.org/10.1109/tc.2009.184
3. Aven, T., Baraldi, P., Flage, R., Zio, E.: Uncertainty in Risk Assessment: The Representation and Treatment of Uncertainties by Probabilistic and Non-Probabilistic Methods. Wiley (2013). https://www.amazon.com/Uncertainty-Risk-Assessment-Representation-Non-Probabilistic-ebook/dp/B00HFRG7A8?SubscriptionId=AKIAIOBINVZYXZQZ2U3A&tag=chimbori05-20&linkCode=xm2&camp=2025&creative=165953&creativeASIN=B00HFRG7A8
4. Beeson, S., Andrews, J.: Importance measures for non-coherent-system analysis. IEEE Trans. Reliab. **52**(3), 301–310 (2003). https://doi.org/10.1109/tr.2003.816397
5. Cheushev, V., Simovici, D., Shmerko, V., Yanushkevich, S.: Functional entropy and decision trees. In: Proceedings. 1998 28th IEEE International Symposium on Multiple- Valued Logic (Cat. No.98CB36138), IEEE Computer Society. https://doi.org/10.1109/ismvl.1998.679467
6. Choudhury, M., Mohanram, K.: Reliability analysis of logic circuits. IEEE Trans. Comput. Aided Design Integr. Circuits Syst **28**(3), 392–405 (2009). https://doi.org/10.1109/tcad.2009.2012530
7. Bicycle crashes. https://catalog.data.gov/dataset/bicycle-crashes. Accessed 05 Sept 2019
8. Figueres-Esteban, M., Hughes, P., Rashidy, R.E., van Gulijk, C.: Manifestation of ontologies in graph databases for big data risk analysis. In: Safety and Reliability – Safe Societies in a Changing World, pp. 3189–3193. CRC Press, June 2018. https://doi.org/10.1201/9781351174664-399

9. Gupta, P., Agarwal, S.: A boolean algebra method for reliability calculations. Microelectron. Reliab. **23**(5), 863–865 (1983). https://doi.org/10.1016/0026-2714(83)91014-4
10. Imakhlaf, A.J., Hou, Y., Sallak, M.: Evaluation of the reliability of non-coherent systems using binary decision diagrams. IFAC-PapersOnLine **50**(1), 12243–12248 (2017). https://doi.org/10.1016/j.ifacol.2017.08.2132
11. Kuo, W., Zhu, X.: Importance Measures in Reliability, Risk, and Optimization. Wiley, Hoboken (2012). https://doi.org/10.1002/9781118314593
12. Kvassay, M., Zaitseva, E., Kostolny, J., Levashenko, V.: Reliability analysis of noncoherent systems based on logical differential calculus. In: Risk, Reliability and Safety: Innovating Theory and Practice, pp. 1367–1374. CRC Press, September 2016. https://doi.org/10.1201/9781315374987-205
13. Kvassay, M., Rabcan, J., Rusnak, P.: Multiple-valued logic in analysis of critical states of multi-state system. In: 2017 International Conference on Information and Digital Technologies (IDT), IEEE, July 2017. https://doi.org/10.1109/dt.2017.8024299
14. Kvassay, M., Zaitseva, E., Kostolny, J., Levashenko, V.: Importance analysis of multi-state systems based on integrated direct partial logic derivatives. In: 2015 International Conference on Information and Digital Technologies, IEEE, July 2015. https://doi.org/10.1109/dt.2015.7222970
15. Kvassay, M., Zaitseva, E., Levashenko, V., Kostolny, J.: Minimal cut vectors and logical differential calculus. In: 2014 IEEE 44th International Symposium on Multiple-Valued Logic. IEEE, May 2014. https://doi.org/10.1109/ismvl.2014.37
16. Kvassay, M., Zaitseva, E., Levashenko, V., Kostolny, J.: Reliability analysis of multiple-outputs logic circuits based on structure function approach. IEEE Trans. Comput. Aided Design Integr. Circuits Syst. 1 (2016). https://doi.org/10.1109/tcad.2016.2586444
17. Levashenko, V.G., Zaitseva, E.N.: Usage of new information estimations for induction of fuzzy decision trees. In: Yin, H., Allinson, N., Freeman, R., Keane, J., Hubbard, S. (eds.) IDEAL 2002. LNCS, vol. 2412, pp. 493–499. Springer, Heidelberg (2002). https://doi.org/10.1007/3-540-45675-9_74
18. Lisnianski, A., Frenkel, I., Ding, Y.: Multi-state System Reliability Analysis and Optimization for Engineers and Industrial Managers. Springer, London (2010). https://doi.org/10.1007/978-1-84996-320-6
19. Lisnianski, A., Levitin, G.: Multi-State System Reliability: Assessment, Optimization and Application. Proc. Eng., vol. 6. World Scientific (2003). https://doi.org/10.1142/5221
20. Lloris, A., Gomez, J.F., Roman, R.: Using decision trees for the minimization of multiple-valued functions. Int. J. Electron. **75**(6), 1035–1041 (1993). https://doi.org/10.1080/00207219308907180
21. Miller, D., Drechsler, R.: Implementing a multiple-valued decision diagram package. In: Proceedings. 1998 28th IEEE International Symposium on Multiple-Valued Logic (Cat. No.98CB36138). IEEE Computer Society. https://doi.org/10.1109/ismvl.1998.679287
22. Miller, D., Drechsler, R.: On the construction of multiple-valued decision diagrams. In: Proceedings 32nd IEEE International Symposium on Multiple- Valued Logic, IEEE Computer Society. https://doi.org/10.1109/ismvl.2002.1011095
23. Mo, Y., Xing, L., Amari, S.V.: A multiple-valued decision diagram based method for efficient reliability analysis of non-repairable phased-mission systems. IEEE Trans. Reliab. **63**(1), 320–330 (2014). https://doi.org/10.1109/tr.2014.2299497

24. Mo, Y., Xing, L., Cui, L., Si, S.: MDD-based performability analysis of multi-state linear consecutive- k-out-of-n: F systems. Reliab. Eng. Syst. Saf. **166**, 124–131 (2017). https://doi.org/10.1016/j.ress.2016.08.027
25. Natvig, B.: Multistate Systems Reliability Theory with Applications. Wiley (2011). https://doi.org/10.1002/9780470977088
26. Rabcan, J., Levashenko, V., Zaitseva, E., Chovancova, O.: Generation of structure function based on ambiguous and incompletely specified data using fuzzy random forest. In: 2018 IEEE 9th International Conference on Dependable Systems, Services and Technologies (DESSERT), IEEE, May 2018. https://doi.org/10.1109/dessert.2018.8409170
27. Rashidy, R.A.H.E., Hughes, P., Figueres-Esteban, M., Harrison, C., Gulijk, C.V.: A big data modeling approach with graph databases for SPAD risk. Saf. Sci. **110**, 75–79 (2018). https://doi.org/10.1016/j.ssci.2017.11.019
28. Sedlacek, P., Rabcan, J., Kostolny, J.: Importance analysis of multi-state system based on incompletely specified data by multi-valued decision diagrams. In: 2019 International Conference on Information and Digital Technologies (IDT), IEEE (2019). https://doi.org/10.1109/dt.2019.8813385
29. Tapia, M., Guima, T., Katbab, A.: Calculus for a multivalued-logic algebraic system. Appl. Math. Comput. **42**(3), 255–285 (1991). https://doi.org/10.1016/0096-3003(91)90004-7
30. Zaitseva, E.N., Levashenko, V.G.: Importance analysis by logical differential calculus. Autom. Remote Control **74**(2), 171–182 (2013). https://doi.org/10.1134/S000511791302001X
31. Zaitseva, E., Levashenko, V.: Investigation multi-state system reliability by structure function. In: 2nd International Conference on Dependability of Computer Systems (DepCoS-RELCOMEX), IEEE, June 2007. https://doi.org/10.1109/depcos-relcomex.2007.28
32. Zaitseva, E., Levashenko, V.: Construction of a reliability structure function based on uncertain data. IEEE Trans. Reliab. **65**(4), 1710–1723 (2016). https://doi.org/10.1109/tr.2016.2578948
33. Zaitseva, E., Levashenko, V.: Reliability analysis of multi-state system with application of multiple-valued logic. Int. J. Quality Reliab. Manage. **34**(6), 862–878 (2017). https://doi.org/10.1108/ijqrm-06-2016-0081
34. Zhang, Q., Mei, Q.: Reliability analysis for a real non-coherent system. IEEE Trans. Reliab. **R–36**(4), 436–439 (1987). https://doi.org/10.1109/tr.1987.5222433
35. Zio, E.: An Introduction to the Basics of Reliability and Risk Analysis. World Scientific Publishing Company (2007). https://doi.org/10.1142/6442