



# Block Modelling Based on Grade Domaining: Is It Reliable?

Nursultan Iliyasa<sup>(✉)</sup> and Nasser Madani

School of Mining and Geosciences, Nazarbayev University,  
Nur-Sultan 010000, Kazakhstan  
{nursultan.iliyas,nasser.madani}@nu.edu.kz

**Abstract.** The mineral resource classification is one of the steps in feasibility assessment of a mining project. It involves a quantitative evaluation of hard data obtained from exploratory drilling and sampling procedures. Several approaches are aimed to accurately predict spatial variability and complex relationships between cross-correlated variables. However, addressing the grade constraints, such as mining grade domains arrangement and respective boundaries uncertainty, meaningfully is still doubtful. In this study, continuous-categorical variables relationships were analyzed with respect to a Shubarkol coal deposit located in Kazakhstan. One of the common methodologies in mining is grade domaining, for which the grade of interest should be truncated into sub-domains. Each sub-domain introduces a homogenous area that can be considered as a container for grade estimation. For this study, the main variable of interest is ash content, varying from 0 to 90%. A cut-off value of 45% was set to the ash variable, thus forming two domains: low ash domain with Coal variable, and high ash domain with Waste variable. In this paper, we propose an integrative algorithm as an alternative methodology for coherent mineral resource estimation, comprising of sequential indicator simulation for categorical variables and turning bands simulation for continuous variables modelling. In contrast, a global geostatistical analysis of the deposit without domaining is presented as well. The resulting estimates of this research showed satisfying reproduction of the deposit structure free of grade domaining, which can be adopted to precisely estimate the volume of a coal mine deposit.

**Keywords:** Grade domaining · Continuous-categorical variables · Coal deposit

## 1 Introduction

Modern mining practices involve a comprehensive feasibility assessment of a mineral deposit, including geostatistical analysis. As a relatively novel branch of mining engineering disciplines, geostatistics is aimed to provide a mathematically justified block model of the deposit. The estimation accuracy of the variable of interest in the block model, such as grade, plays a crucial role in mineral resource classification. A common practice in mining industry for mineral resource/reserve estimation, is grade domaining approach, where the deposit is truncated into a number of adjacent domains based on grade variability throughout deposit [1]. This approach has pitfalls, where the precision of estimation/simulation is reduced by a number of limitations, including an

inability to preserve spatial relations among domains and to account for uncertainty in the spatial extent of the domains [2, 3].

This study focuses on examining the reproduction of global variability parameters, such as maximum and mean value, from four different case studies. The objective of this work is to assess the accuracy of grade domaining approach (cases II-IV) in comparison with reference model (case I), i.e. global geostatistical modelling of the deposit irrespective of grade domaining. On top of limitations associated with grade domaining itself, there exists a shortcoming affecting the kriging variance (1, 3), [4]. Simple Kriging is incorporated in each case, being used as conditioning technique in turning bands method and sequential indicator simulation. However, the variance of grade is lowered in Cases II, III, and IV, as a result of forming grade intervals, thus leading to distorted variogram and the kriging variance. The failure to incorporate the boundaries uncertainty into the estimation variance may be critical in reliable grade estimation.

The reliability of the research output is discussed and validated further, showing that unique domaining outperforms the conventional grade based domaining approaches. The suggested methodology is introduced and discussed in this work through a case study from a Kazakhstani coal deposit.

## 2 Methodology

The concept of grade domaining in mineral resource estimation is to truncate the orebody into a number of domains defined by grade ranges [2, 5, 6]. The domains then can be analyzed and undergone geostatistical modelling separately in each domain.

However, this approach for geological modelling has pitfalls, caused by uncertainty in-between the domains and ignoring spatial dependency [2, 7]. The actual domain boundaries are irregular in nature, and the bias interpretation in terms of smooth contouring, coupled with separate analysis of each domain, lead to inaccurate results. The following equation is widely practiced to estimate the final grade:

$$Z^*(X) = \sum_{n=1}^N P_n(X)Z_n^*(X) \quad (1)$$

where  $\{P_n, n = 1 \dots N\}$  is the probability of each grade domain and  $\{Z_n^*, n = 1 \dots N\}$  is the grade at the respective locations.

Kriging interpolator is a geostatistical tool aimed to estimate the value of the variable of interest at unsampled locations throughout the deposit. The interpolator is based on the random function model with a weighted combination of the available data [8]. The weights are assigned according to the distance from the target location to surrounding data, as well as to redundancies within dataset.

Simple Kriging (SK) estimator incorporates the mean value into estimation, and is based on modelling the spatial continuity by variogram. The algorithm assumes stationarity of the continuous variable, with its variability fluctuating around a mean value [9–11]. Despite its straightforwardness, SK may yield negative weights leading to negative estimates and have a smoothing effect, where the tonnage estimation may be

affected by a shrunked dispersion of the estimates, compared to dispersion of the true values. The estimator of the algorithm as a linear combination of the data is following:

$$Z^*(x_0) = a + \sum_{a=1}^n \lambda_a Z(x_a) \tag{2}$$

where  $Z(x)$  is the regionalized variable,  $\{x_a, a = 1..n\}$  is the data location,  $x_0$  is the target location,  $\{\lambda_a, a = 1..n\}$  is assigned weight and  $a$  is an additive coefficient. Indicator Kriging (IK) uses the above formulae for modelling the categorical variables, creating a deterministic model of the geo-domains, where the indicator variables are independently estimated. The output of the algorithm needs to be processed so that the values are non-negative and sum up to 1.

Sequential Indicator Simulation is a probabilistic approach based on IK, yet additionally producing a number of realizations. The realizations are constructed from categorical variables, where each data point is codified into indicator. The categories are mutually exclusive, that's there is only one available category at a given location [9, 12]. Generally, these variables can be expressed as:

$$i(\mathbf{u}; k) = \begin{cases} 1, & \text{if category } k \text{ is present at position } \mathbf{u} \\ 0, & \text{if otherwise} \end{cases}, k = 1, \dots, k \tag{3}$$

Indicator Kriging is incorporated to construct conditional distributions from indicator variograms. [12].

Turning bands method is an algorithm that simulates the variable of interest in random lines in 1D, after that combining them into 3D. Technically, simulation takes place on the lines that are spread throughout the space in different directions. It is based on non-conditional Multi-Gaussian model, further being conditioned by Kriging [13–15].

The computation of random numbers in 3D is carried out by taking the projection of the values simulated in 1D lines:

$$Y(x) = Y^{(1)}(\langle x | \mathbf{u} \rangle) \tag{4}$$

where  $Y^{(1)}$  is a random function in one dimension,  $\mathbf{u}$  is a vector in the  $R^d$ , and  $\langle \cdot | \cdot \rangle$  is a projection of location  $x$  in the line oriented by  $\mathbf{u}$ .

### 3 Case Study

#### 3.1 Presentation of the Dataset

The Shubarkol coal deposit is an important resource of coal in the Kazakhstan located in the Nurinsky district of the Karaganda region of the Republic of Kazakhstan.

The dataset is composed of 89,942 samples belonging to this deposit, where each sample has assay of Ash. Originally, ash content is the only variable of interest within the deposit, varying from 0% to 90%. In this study, for the purpose of grade based domaining the ash variable is divided into Coal, Waste, Indicator Coal and Indicator

Waste variables depending on grade value based on cut-off 45% of ash. These variables constitute a basement for estimation techniques, such as Simple Kriging and Turning Bands Simulation. The data is totally heterotopic, where the first grade domain, Coal variable, ranges from 0 to 45% Ash content, inclusively, and Waste variable includes Ash from 45 to 90% content. Therefore, there are 78,379 samples with ash content less than 45% belonging to Coal, and 11,562 samples with ash content higher than 45% belonging to Waste. Indicator Coal and Waste variables are used to implement simulation techniques, including Indicator Kriging and Sequential Indicator Simulation. The variable present for the given samples is assigned value of 1 if pertaining to coal and 0 if pertaining to waste.

The area of sampling covers approximately  $6300 \times 14000 \times 169$  m, with a typical elevation adjustment of 1 cm between the samples. Table 1 shows the global statistical parameters of all the variables within the entire deposit. Hereby, it is seen that the variance of original data (Ash) is very high due to wide spread of data points from the mean and from one another. The shape and spread of the sample data show a distribution with a positive skewness.

**Table 1.** Statistical parameters of variables under study (%)

| Statistical Parameter | Ash    | Coal  | Waste  | Indicator Coal | Indicator Waste |
|-----------------------|--------|-------|--------|----------------|-----------------|
| Minimum               | 0.05   | 0.05  | 45.02  | 0              | 0               |
| Maximum               | 89.84  | 45    | 89.84  | 1              | 1               |
| Mean                  | 14.04  | 6.74  | 63.56  | 0.87           | 0.13            |
| Variance              | 434.31 | 61.93 | 145.04 | 0.11           | 0.11            |

### 3.2 Geostatistical Modelling

The geostatistical modelling of the available data is carried out by means of conventional Grade Domain and Unique Domain approaches. In Case I, the Unique Domain is meant to represent the whole deposit without being splitted into domains, where the model is built by Turning Bands method. Here, we examine the closeness of the derived statistical parameters to original ones. On the other hand, in Cases II, III and IV grade domaining approach is examined separately through a combination of estimation and simulation techniques, such that we can compare the statistics with Case I and original dataset. Case II incorporates Simple Kriging and Indicator Kriging, Case III- Simple Kriging and Sequential Indicator Simulation, and Case IV- Turning Bands Simulation and Sequential Indicator Simulation. Modelling outputs of Simple Kriging, Indicator Kriging and Turning Bands Simulation are derived by utilizing the ISATIS software. For the purpose of conducting Sequential Indicator Simulation, SGeMS software is used.

#### Case I: Unique Domain by Turning Bands Simulation

Case I of the current study considers the Ash variable only. Due to irregular pattern distribution of data samples across the deposit, the variable should be declustered. This technique places the domain into a grid with a cell size of  $1000 \times 1000 \times 10$  m and

assigns each sample data a weight depending on its proximity to surrounding neighbors within a common cell. The weight is calculated by taking inverse of multiplication of the number of data samples within one cell and overall number of non-empty cells. The assignment of weights makes sure that the spatial representivity of samples is taken into account. Following the declustering of Ash dataset, we should infer normal distribution model of frequency curve to run Turning Bands simulation from the original declustered Ash data. It is based on the normal score Gaussian distribution, with mean 0 and variance 1 at a given set of spatial locations. Next step towards the simulation is to describe quantitatively and model the spatial continuity of the regionalized variable, Ash. The spatial continuity is modelled firstly by calculating an experimental variogram on the basis of the available data, and then by fitting a theoretical variogram:

$$\text{Variogram: } 0.83\text{Sph}(250 \text{ m}, 250 \text{ m}) + 0.14\text{Sph}(4000 \text{ m}, 4000 \text{ m})$$

For the sake of simplicity, only approximately the half of the deposit area is taken for simulation grid, with a cell size of  $50 \times 50 \times 5$  m and centered target point. A moving neighborhood of  $10 \times 10 \times 10$  km size with 100 samples per sector was established. Finally, optimum number of turning bands equal to 1,000 was used in analysis. Figure 1(a) below depicts an average E type map out of 100 realizations, where it can be seen that there are soft boundaries respected in this case, with an approximate layering of coal illustrated in black lines.

### Case II: Grade domains by Simple Kriging and Indicator Kriging

This part of the study gives a start to a grade domaining based mineral resources estimation. Here, we analyze the combination of Simple Kriging as a deterministic methodology for totally heterotopic Coal and Waste variables, and Indicator Kriging to stochastically model the probability of Coal or Waste presence (Indicator Coal and Waste variables). An experimental variogram model for each variable is calculated by 100 lags with a value of 75 m and a tolerance of 0.5 m. Fitted theoretical model is then integrated into grade estimation by Kriging, with a moving neighborhood of  $10 \times 10 \times 10$  km size and 100 samples per sector. Equation 1 is used to calculate the final grade prediction in each block. The following direct variograms are inferred, where the variogram is nested and consists of two structures following spherical model. For example, 1<sup>st</sup> structure of the variogram for direct variogram of coal variable has a sill of 48,96 m and range of 320 m, whereas the 2<sup>nd</sup> structure has a sill of 16,96 m and 5650 m range:

$$\text{Coal variogram: } 48.96\text{Sph}(320 \text{ m}, 320 \text{ m}) + 16.96\text{Sph}(5650 \text{ m}, 5650 \text{ m})$$

$$\text{Waste variogram: } 136\text{Sph}(120 \text{ m}, 120 \text{ m}) + 9\text{Sph}(1600 \text{ m}, 1600 \text{ m})$$

$$\text{Ind. Coal variogram: } 0.102\text{Sph}(150 \text{ m}, 150 \text{ m}) + 0.012\text{Sph}(4200 \text{ m}, 4200 \text{ m})$$

$$\text{Ind. Waste variogram: } 0.103\text{Sph}(250 \text{ m}, 250 \text{ m}) + 0.012\text{Sph}(5000 \text{ m}, 5000 \text{ m})$$

As can be acquired from Fig. 1(b), Kriging yields visible contours of coal layer, but still the abrupt transition between domains remain visible.

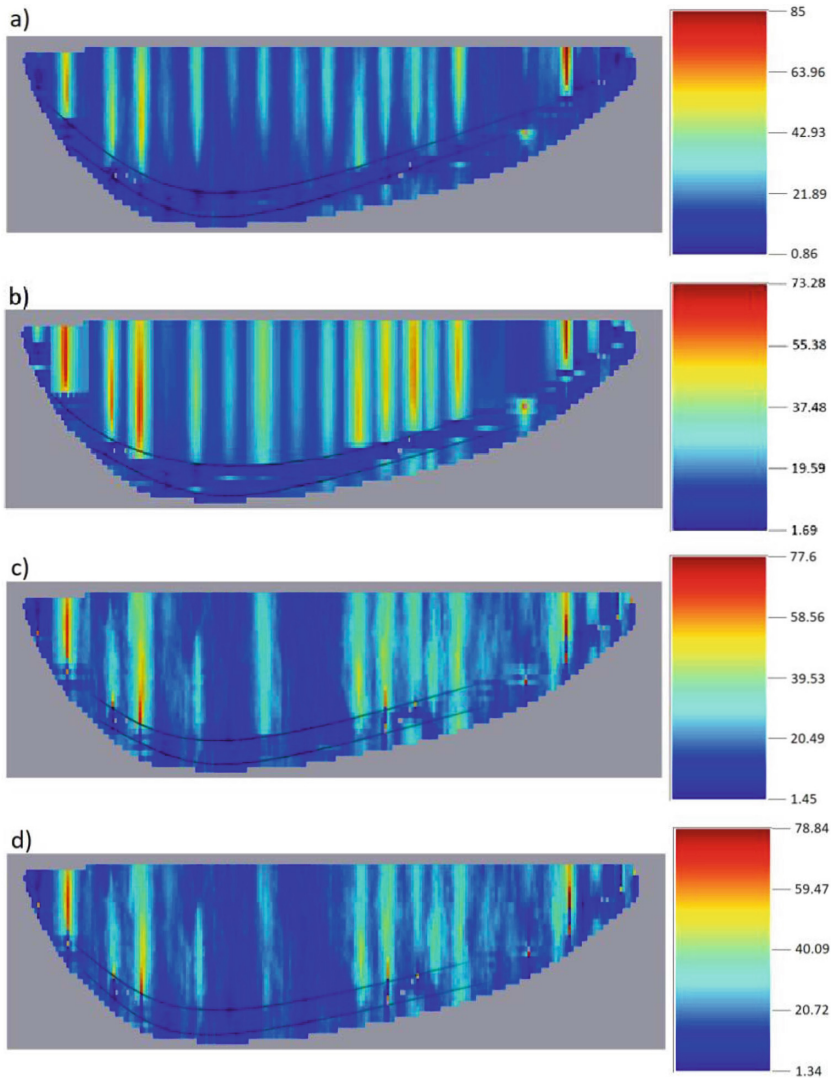
### Case III: Grade domains by Simple Kriging and SISim

The third case examines the combination of Simple Kriging, obtained in Case II, and Sequential Indicator Simulation. SISim algorithm is implemented in SGeMS software, with the same input parameters for direct variograms, as in ISATIS for Indicator Kriging.

The output matrix from SGeMS is then additionally transformed in MATLAB to reflect the probability of variable occurrence from 100 realizations. Figure 1(c) below is an estimation map based on average of 100 realizations, with less observable coal layer, but with a soft boundary, i.e. gradual transition between coal and waste zones.

**Case IV: Grade domains by TBSim and SISim**

Case IV of the research conjoins the two simulation algorithms, TBSim and SI- Sim, to estimate the Ash content throughout the deposit. Turning Bands simulation is taken into account to model the Coal and Waste variables, and Sequential Indicator



**Fig. 1.** (a) Case I (b) Case II (c) Case III (d) Case IV

simulation for probability description of indicator coal and waste variables. While SISim algorithm is already developed in Case III, the TBSim is performed newly. Normal distribution models of empirical density curve are inferred, based on Gaussian distribution (mean of 0 and variance of 1). Direct variograms of Coal and Waste variables are produced independently and respective fitted models are used further in algorithm. The same grid model is used to carry on the simulation with a cell size of  $50 \times 50 \times 5$  m and centered target point. 1000 turning bands are assigned to derive a reasonable simulation output. Figure 1(d) illustrates an average E type map out of 100 realizations with a much less delineation of coal layer and soft boundaries.

## 4 Validation of Algorithms

In this part, in order to make a statistical comparison between the presented cases, Table 2 is given. Based on the table, one can compare the closeness of a global variability indicators from the case studies to the original dataset of Ash. As it can be acquired, Case I yields the closest mean values of estimated ash to the declustered original ones. On the other hand, in terms of maximum and mean value, Case II produces the worst reproduction of borehole data. However, the minimum value of 0 indicates that Simple Kriging implies smoothing effect on grade distribution.

**Table 2.** Reproduced statistical parameters based on different case studies

| Study         | Minimum     | Maximum      | Mean         | Variance     |
|---------------|-------------|--------------|--------------|--------------|
| Original data | 0.05        | 89.84        | 14.04        | 434.31       |
| <b>Case I</b> | <b>0.86</b> | <b>85.00</b> | <b>17.29</b> | <b>32.61</b> |
| Case II       | 0.00        | 74.34        | 19.09        | 44.85        |
| Case III      | 1.45        | 77.60        | 18.59        | 31.75        |
| Case IV       | 1.34        | 78.84        | 17.40        | 24.88        |

## 5 Conclusion

A precise estimation of mineral resources and its classification is of paramount importance in a modern mining practice. A reliable interpretation of borehole samples forms an essential condition for trustworthy mineral resource estimation. Currently, global mining industry prefers estimating grades through the deposit using Grade Domaining approach. In this study, the reliability of this approach was examined in comparison to Unique Domain approach, where the area of interest is analyzed as one domain. A dataset taken from Shubarkol mine deposit was used to examine four case studies, including three examples on Grade Domaining approach and one study on Unique Domain approach. This research work shows that the global statistics of original grade distribution of ash is best reproduced in case of a single unified domain, Case I. It is shown in this study that discretizing the area into domains yield less reliable estimation of grade distribution, as shown in Cases II, III and IV and increases the mean value erroneously that may lead to biased tonnage calculations.

**Acknowledgement.** The authors are grateful to Nazarbayev University for funding this work via “Faculty Development Competitive Research Grants for 2018–2020 under Contract No. 090118FD5336.” The second author acknowledges the Social Policy Grant (SPG) supported by Nazarbayev University. The authors also thank the ERG Company in Kazakhstan for providing the dataset.

## References

1. Emery, X., Ortiz, J.M.: Estimation of mineral resources using grade domains: critical analysis and a suggested methodology. *J. South Afr. Inst. Min. Metall.* **105**(4), 247–255 (2005)
2. Emery, X., Ortiz, J.M.: Defining geological units by grade domaining. Technical report, Universidad de Chile (2004)
3. Ortiz, J.M., Emery, X.: Geostatistical estimation of mineral resources with soft geological boundaries: a comparative study. *J. South Afr. Inst. Min. Metall.* **106**(8), 577–584 (2006)
4. Blackwell, G.: Relative kriging errors; a basis for mineral resource classification. *Explor. Min. Geol.* **7**(1–2), 99–105 (1998)
5. Romary, T., Rivoirard, J., Deraisme, J., Quinones, C., Freulon, X.: Domaining by clustering multivariate geostatistical data. In: Abrahamsen, P., Hauge, R., Kolbjørnsen, O. (eds.) *Geostatistics Oslo*, pp. 455–466. Springer, Dordrecht (2012)
6. Yunsel, T.Y., Ersoy, A.: Geological modeling of gold deposit based on grade domaining using plurigaussian simulation technique. *Nat. Resour. Res.* **20**(4), 231–249 (2011)
7. Madani, N., Ortiz, J.: Geostatistical simulation of cross-correlated variables: a case study through Cerro Matoso Nickel-Laterite deposit. In: *The 26th International Symposium on Mine Planning and Equipment Selection, Luleå, Sweden* (2017)
8. Cressie, N.: The origins of kriging. *Math. Geol.* **22**(3), 239–252 (1990)
9. Deutsch, C.V.: A sequential indicator simulation program for categorical variables with point and block data: BlockSIS. *Comput. Geosci.* **32**(10), 1669–1681 (2006)
10. Emery, X., González, K.E.: Incorporating the uncertainty in geological boundaries into mineral resources evaluation. *J. Geol. Soc. India* **69**, 29–38 (2007)
11. Emery, X., González, K.E.: Probabilistic modelling of lithological domains and its application to resource evaluation. *J. South Afr. Inst. Min. Metall.* **107**(12), 803–809 (2007)
12. Emery, X.: Properties and limitations of sequential indicator simulation. *Stoch. Environ. Res. Risk Assess.* **18**(6), 414–424 (2004)
13. Emery, X., Lantuéjoul, C.: TBSIM: a computer program for conditional simulation of three-dimensional gaussian random fields via the turning bands method. *Comput. Geosci.* **32**(10), 1615–1628 (2006)
14. Brooker, P.I.: Two-dimensional simulation by turning bands. *J. Int. Assoc. Math. Geol.* **17**(1), 81–90 (1985)
15. Mantoglou, A.: Digital simulation of multivariate two-and three-dimensional stochastic processes with a spectral turning bands method. *Math. Geol.* **19**(2), 129–149 (1987)