Life is Lived on the Edge

By Glenn Webb

The research story

Mathematical models of complex dynamic biological processes sometimes involve systems of partial differential equations for production, growth, decay, interaction, and spatial movement. An example is a nonlinear partial differential equations model for a very small tumor invading surrounding tissue. The outward growth of the tumor results from haptotaxis, which is the directional movement of cells up-gradient a chemoattractant to adhesion sites in the surrounding tissue. The model equations involve five key components of the tumor growth process: proliferating tumor cells (cells progressing through the cell cycle to division), quiescent tumor cells (cells not progressing through the cell cycle, but capable of returning to cell cycle progression), surrounding tissue macromolecules (the cellular environment of the tumor), degradative enzymes (which limit tumor growth), and oxygen (which supports tumor growth).

The image

The images in the figure, obtained from numerical simulation of the model, track the growth of the tumor at six time points. The tumor initially consists of proliferating cells. At time point 0, the tumor is spatially homogeneous and very small. At later times the tumor grows outwardly in an irregular way. As time advances, the interior section of the tumor becomes composed mostly of quiescent cells (black), with more and more proliferating cells (red) at the outer edges.

References

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