Schopenhauer and the Equational Form of Predication

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Abstract Given the common narrative of the history of the nineteenth century logic, it may seem surprising that in some passages of his logic lectures, Schopenhauer invokes an equation sign to express relations of predication as in "A = B". The present paper proposes an assessment of Schopenhauer's use of the equation sign. Departing from an analysis of Schopenhauer's account of concepts and judgments, it offers a survey of logic textbooks which Schopenhauer was acquainted with. The preliminary conclusion will be that for some of Schopenhauer's sources, the equational notation is justifiable as they do suggest certain revisions of logic which point towards the possibility of quasi-"algebraic" models. Schopenhauer's own use of the equation sign, however, fails to come up to the conceptual prerequisites that would allow for an "algebraic" approach. In particular, Schopenhauer does not seem to be aware of the possibility to invoke an equational notation to express implication in the sense of stating equivalences between propositions and their transformations.

Keywords Mathematization of logic · Laws of thought · Syllogistic predication · Interpretation of the copula · Negative terms · Conversion and contraposition · Identity and equivalence

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1 Introduction

It is well-known that around the middle of the nineteenth century, some formative steps were taken towards what came to be named the "algebra of logic." Most of its early promotors are to be found among the British authors of that time. Most

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prominently, we remember George Boole, who tried to cast logical problems into the form of quasi-algebraic equations, invoking notational means borrowed from mathematics, most prominently the equation sign. Even today, Boole's attempts are still alive to the extent that Boole's project is recognizable in today's "equational logics." Given this narrative, it may seem surprising to find that even before Boole's times, there were continental authors whose choice of symbols appears to be comparable to Boole's in spirit. Such cases can be found for the use for plus-signs and minus-signs in various interpretations. The equation sign, however, is usually employed to express either predication, as in "A = B" for "A is B", or a sense of implication by stating an equivalence between several propositional expressions, as in "'All As are Bs' = 'Some Bs are As'".

Interestingly, the equation sign can also be found in Schopenhauer's logic of the *Berlin Lectures*. The present paper will try to give some elucidation and an assessment of Schopenhauer's use of the equation sign. The first step will be to consider equational forms throughout his logic. The second step will be to look at some relevant context within Schopenhauer's logic, namely his account of judgments as being made up of concepts, and his views on the quantity and quality of judgments. The third step consists in a survey of logic textbooks which Schopenhauer was acquainted with, aiming at a reconstruction of where he might have taken his equational notation from. Finally, there remains the question whether Schopenhauer's choice of notation matches the conceptual prerequisites of his logic. The preliminary conclusion will be that it does not. Rather, the equational notation mirrors certain revisions of logic which point towards "algebraic" or algorithmic models, and which are suggested in some of his sources, but which Schopenhauer himself fails to come up to.

2 Schopenhauer's Use of Equality Signs in Logic

Like many of the textbooks of his time, Schopenhauer's logic lectures[1] are composed of a section on concepts, followed by a section on judgments, and a section on inference. Schopenhauer's account of judgments contains a subject matter which more commonly comes under the caption of "immediate inferences," such as inferences by subordination and opposition. A special case of such inferences are those transformations of judgments which are effected by "conversion" and by "contraposition." Schopenhauer concludes his section on judgments with a summary of their admissible forms, in the following arrangement [19, p. 293].

[1]Schopenhauer's logic lectures are contained under Chap. 3 in [19, pp. 234–363].

(a)	*Convertiren simpliciter, lassen sich*		
(1)	Allgemein verneinende	Kein A = B	Kein B = A
(2)	Partikulär bejahende	Einige A = B	Einige B = A
(b)	*Convertiren per accidens*		
(1)	Allgemein bejahende	Alle A = B	Einige B = A
(c)	*Contraponiren simpliciter*		
(1)	Allgemein bejahende	Alle A = B	Kein Nicht B = A
(2)	Partikulär verneinende	Einige A = nicht B	Einige Nicht B = A
(d)	*Contraponiren per accidens*		
(1)	Allgemein verneinende	Kein A = B	Einiges Nicht B = A

Generally speaking, Schopenhauer's account of transformation by conversion and contraposition is not very exceptional. To convert a proposition (*Satz*) means to turn the predicate into the subject and the subject into the predicate (*das Prädikat zum Subjekt und das Subjekt zum Prädikat machen*; [19, p. 289]). In order to preserve their meaning on such an interchange, some kinds of judgments will require a change of quantity and quality (ibid.). The conversion is simple (ibid.) in case neither quantity nor quality are changed. If there is a change in quantity only, the conversion is per accidens (ibid.). If there is a change in quality, a contraposition takes place (ibid.). If in this case, the quantity remains unchanged, the contraposition is, again, simple (ibid.). If, however, both quality and quantity are changed, the contraposition is, again, per accidens (ibid.).

For the admissible transformations, Schopenhauer relies on an extension of the scholastic codification of a set of rules: Universal affirmatives are converted per accidens [19, p. 290] or by simple contraposition (ibid.). Universal negatives are converted *simpliciter* or by contraposition per accidens [19, p. 291]. Particular affirmatives, too, may be converted *simpliciter* (ibid.), except if the predicate happens to be wholly included in the subject. In the latter case, particular affirmatives allow for conversion per accidens (ibid.). However, there is no possibility of contraposition for particular affirmatives (ibid.). Particular negatives, on the other hand, cannot at all be converted, except for by simple contraposition (ibid.). Accordingly, Schopenhauer would have (i) "All A are B" as an "equivalent" (loosely speaking) for "Some B are A," or "No non-B is an A;" (ii) "No A is a B" as an equivalent for "No B is an A," or "Some non-B are A;" (iii) "Some A are B" as an equivalent for "Some B are A;"[2] and, finally, (iv) "Some A are not B" as an equivalent for "Some non-B are A."[3] These are just the forms collected in Schopenhauer's table, (i) being listed

[2]According to Schopenhauer, this may turn out "All B are A" in special cases. One such case would be "Some trees are firs," but "All firs are trees" ([19, p. 291]; hence "Some trees are firs" in a sense turns out a converted universal).

[3]Schopenhauer exemplifies these rules as follows. Universal affirmatives are exemplified by "All rocks are solids," turning into "Some solids are rocks" on conversion, and into "No non-solid is a rock" on contraposition [19, p. 291]. As an universal negative, Schopenhauer gives "No rock is an animal," resulting in "No animal is a rock," or, by contraposition, in "Some non-animals are

under (b)(1) and (c)(1); (ii) under (a)(1) and (d)(1); (iii) under (a)(2), and (iv) under (c)(1).

Thus, the contents of Schopenhauer's table of conversions are not very surprising. What is surprising is the way the contents are presented. Their arrangement results from a classification according to the distinction of kinds of conversion.[4] But what is particularly noteworthy is Schopenhauer's choice of the equality sign "=" to indicate predication, but precisely not to indicate the "equivalence" between the members of the pairs of judgments in each line.

By casting judgments into an "equational" form, Schopenhauer seems to anticipate early proponents of a "mathematization" of logic such as Moritz Wilhelm Drobisch,[5] who in 1836 put it this way:

> To express affirmative judgments by equations seems to be the most appropriate way of signifying that in a certain respect, they always represent an identity of the subject and the predicate, which comes to light most distinctly on conversion."
>
> (Die bejahenden Urtheile durch Gleichungen auszudrücken scheint am zweckmäßigsten, um zu bezeichnen, daß sie in gewisser Hinsicht immer eine Identität des Subjects and Prädicats darstellen, wie aus der Umkehrung am deutlichsten hervorgeht; [1, p. 132].[6])

This quote from Drobisch is representative for the conception of logic which first allowed for a "mathematical" approach. With particular regard to the use of equality signs, one may ascribe to it at least two necessary prerequisites, namely (1) a reduction of forms to the acceptance of affirmative judgments only, and (2) an implicit quantification of concept terms, which allows one to compare their quantities in order to test whether they can be said to "equal" one another. This is because an "equivalence," or, as Drobisch put it, an "identity," just cannot be negative. It is about shared, i.e., "equivalent," or "identical" parts of terms, most naturally taken to be parts of their extensions. But of course, still there remain judgments which express denials. The most convenient approach to deal with them would be to admit negative terms. Another way, however, would be to depart from certain operations of "addition" and "subtraction" in logic: the negative counterpart of a concept term would then be expressible by subtracting the concept term itself

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rocks" (ibid.). The case of particular affirmatives is illustrated by "Some birds a predators"; so "Some predators are birds" (ibid.). For particular negatives, Schopenhauer gives "Some animals are not endothermic"; so "Some non-endothermics are animals" [19, p. 292].

[4]In a footnote, Schopenhauer refers to this listing as a "table," namely one which is of a "peculiar symmetry" (*sonderbare Symmetrie*; [19, p. 293]). This "peculiar symmetry" consists in "the lower part of the table reading just like the upper one" (ibid.). Schopenhauer himself gives no explanation for his comment, and it seems far from clear what he refers to. Moreover, no clue is to be found in any of the textbooks discussed in Sect. 3 of the present article. Therefore, the interpretation of Schopenhauer's sense of symmetry must remain arcane as it is for the present purposes.

[5]For reasons of space, the present article is confined to discuss contributions from the German logic scene at around Schopenhauer's time. Therefore, references to some very important British developments of the early nineteenth century will be omitted.

[6]All translations of German into English are by the author.

from a given larger term extension (cf. ibid.).[7] Schopenhauer, however, gives no clue of what he means by his equational forms of judgments, nor of why he thinks that he is justified in replacing the copula "is" by an equality sign.

It should be noted that up to his table of transformations by conversion and contraposition, Schopenhauer employed an equality sign only four times in his logic lectures. One out of these four instances is within the expression of the arithmetical equation "$3 \times 7 = 21$", cited as an example for what Schopenhauer named "metaphysical truth," i.e., a truth which is independent of experience [19, p. 267]. The remaining three instances of Schopenhauer's use of the equality sign are to be found among his discussion of the so-called laws of thought (*Denk-Gesetzen*; [19, p. 261]), and as in the table given above, they seem to be meant to chiefly mirror predication.

The first instance of an equality sign in Schopenhauer's lectures is to be found within his version of the Principle of Identity, which reads: "a concept is selfsame" (*der Begriff ist sich selbst gleich*; [19, p. 262]), "no matter if I think of it as a whole [. . .] or I dissolve it into all the concepts it contains as predicates" (*ich mag ihn nun [. . .] denken im Ganzen [. . .] oder auflösen in seine sämmtlichen Pr[ä]dikate*; ibid.). In other words: "a concept is equal to the sum of its predicates" (*der Begriff ist gleich der Summe seiner Prädikate*; ibid.). Hence Schopenhauer put the Principle of Identity as "$A = A$" (ibid.).

As the second law of thought, Schopenhauer lists the Principle of (Non-)Contradiction: "The predicate must not annul the subject, neither as a whole nor even partially" (*Das Prädikat darf das Subjekt nicht aufheben, weder ganz noch zum Theil*; ibid.). In other words: "it [the predicate] must not contradict it [the subject], i.e., what is affirmed in the subject must not be denied in the predicate nor vice versa, neither directly nor indirectly" (*d.h. es darf ihm nicht widersprechen, d.h. was im Subjekt bejaht ist darf im Prädikat nicht verneint seyn und umgekehrt, weder mittelbar noch unmittelbar*; ibid.). Interestingly, Schopenhauer abbreviated this principle by a somewhat curious formula, namely: "$A = -A = 0$" (ibid.).

Schopenhauer's third law of thought is the Principle of the Excluded Middle: "Any and every subject either has any one predicate or does not have it; it is to be affirmed or denied of it (*Jedem Subjekt kommt jedes Prädikat entweder zu oder nicht; ist entweder von ihm zu bejah*[*n*] *oder zu verneinen*; [19, p. 263]), i.e., "non datur tertium" (ibid.). Schopenhauer's short-hand formula reads "A aut = b, aut = non b" (ibid.).

Schopenhauer's list of "laws of thought" is then completed by his interpretation of the Principle of Sufficient Reason, which he takes to be the principle of sufficient reason for cognition (ibid.), i.e., for taking cognizance of a judgment being true (ibid.). as warranted by something external to itself [19, p. 263f.]. Hence this

[7] This is Drobisch's option of choice. However, it should be noted that Drobisch does not employ equality signs in negative judgments. Therefore, Drobisch does not face the problem of expressing negatives by "identities" between counterparts of terms.

principle does not relate to predication, taken as a judgment's inner structure. Therefore, Schopenhauer does not make use of any arithmetical signs to state it.

3 Schopenhauer's Account of Quantity and Quality of Judgments

The next interpretive step to Schopenhauer's use of the equality sign should be to consider some relevant parts of his logic as to their conceptual groundwork. Is there anything peculiar about Schopenhauer's views on logic that warrants the use of equality signs to indicate predication? The present section will relate to Schopenhauer's account of judgments as composed of concepts, and his views on quantity and quality of judgments.

Indeed, Schopenhauer's opinion was that "to judge" means "to discern the proportions"[8] between "given concepts" (*Die Verhältnisse gegebener Begriffe zu einander erkennen, heißt urtheilen*; [19, p. 260]; cf. *Jedes Urtheil ist also die Erkenntniß des Verhältnisse*[*s*] *zwischen Begriffen*; [19, p. 261]). To discern proportions between concepts means to discover "their linkage, or lack thereof, respectively" ([*die Erkenntniß*] *ihrer Verbindung oder auch Nicht-Verbindung*; ibid.). But to discover their linkage means to recognize that one concept is thought "within another concept either wholly or partially" (*d.h. die Erkenntniß daß in einem Begriff ein andrer entweder ganz oder zum Theil mitgedacht ist*; ibid.). or that "there is no linkage of this kind at all, to the effect of a negative judgment" (*oder aber umgekehrt daß er gar nicht mit ihm verbunden ist; dann ist das Urtheil negativ*; ibid.).

According to Schopenhauer, it is making and stating such cognitions what is meant by "thinking proper" (*eigent[lich] Denken*; ibid.). Hence in order to think at all, "one starts off with one concept, of which it is to be discovered that it is contained in a second concept, wholly or partially" (*Man geht stets von einem Begriff aus, den man als ganz oder zum Theil im andern enthalten erkennt*; ibid.). These two concepts are what for the purposes of logic are called "subject" and "predicate." The first concept, i.e., the one to start from is the subject; the other one, i.e., the one in which the subject is contained, the predicate (ibid.). However, Schopenhauer pointed out that the second concept, i.e., the predicate, "is just as well contained in the first, i.e., the subject, either wholly or partially" (*allemal ist aber auch der zweite ganz oder zum Theil im ersten enthalten*; ibid.). Therefore, the second can become the subject, and the first the predicate (ibid.). The proportions may differ since of two concepts A and B, "A may be in B wholly, while B is in A only partially" (ibid.). However, a transposition by conversion (*Umkehrung*) should in any case be possible (ibid.).

[8]The German has "Verhältniß," which admits of a broader set of interpretations, such as "relation," but also "ratio."

Now, one might expect Schopenhauer to have thought of the proportion between concepts which are said to be positively related as an overlap between sections of the concepts' "spheres," i.e. their extensions,[9] and hence as a partial identity. Regarding the category of quantity, to determine such an overlap would then imply an explicit consideration of the sizes of the respective portions of both terms. Secondly, as to the category of quality, the most suitable option to express negative judgments by way of determining overlaps of concept spheres would be to confine the form of judgments to that of affirmative categoricals while admitting negative concept terms. However, while Schopenhauer did concentrate on categoricals as to the form of judgments [19, p. 278], a closer look at his account of judgment reveals that as to quality and quantity, neither of the aforementioned suppositions seems to be the case.

From Schopenhauer's account of judgments, it seems fairly clear that he should be committed to conceive of the copula as an indicator of the linkage of concepts by proportion. Indeed, at first sight he did so in defining the copula as "the word which indicates the proportion of the concepts" (*Das Wort, welches das Verhältniß der Begriffe andeutet*; [19, p. 261]). However, Schopenhauer modified this determination in calling it "a trope" (*uneigentlich*; ibid.). The reason is that while the copula is described as a connector, it may also serve the separation of concepts. In this latter case, its expression is not "is" but rather "is not" (ibid.). Hence Schopenhauer seems claim that there are in fact two copulae which differ in quality. Similarly, while discussing the quality of judgments, Schopenhauer noted that quality is either about the union (*Vereinigung*) or about the separation (*Trennung*) of concepts, or rather, their extensions (*Begriffssphären*; [19, p. 274]). But according to Schopenhauer, a judgment's quality is not to be expressed by any of the words which designate a judgment's concepts [19, p. 275]. Rather, it is expressed by "is" or "is not," which indicate two copulae, or, classically speaking: two modifications of the copula: *Affirmatio aut Negatio afficit copulam* (ibid.).

While Schopenhauer holds that a judgment's copula carries the expression of its quality, he is also clear that a judgment's quantity is expressed by the subject term (*Der Ausdruck der Qualität in der Copula, der Ausdruck der Quantität im Subjekt*; [19, p. 278]). A difference in quantity depends on whether the subject term is to be taken by the whole of its extension or by part of it only (*ob das Subjekt in seinem ganzen Umfange genommen werden soll, oder nur ein Theil desselben*; [19, p. 276]). Hence it remains unclear how a copula which can be either affirmative or negative should conform with a single equation sign, and how the idea of an equivalence between the two sides of it should be justified if the quantity of the subject term only is considered.

[9] Schopenhauer is explicit that the notion of a concept's "sphere" is to be interpreted extensionally [19, p. 271].

4 Possible Sources for Schopenhauer's Equational Symbolism

As shown in the previous sections, Schopenhauer himself offers no consistent account of his employing equation signs for predication. Thus, there remains the question if he adopted such means from other works on logic. The following section will serve to make out some possible sources.

In his logic lectures, Schopenhauer seems to relate to three authors from the century before him, notably the philosopher-logician Gottfried Ploucquet (1716–1790), the mathematician Johann Heinrich Lambert (1728–1777), and the physicist-mathematician Leonhard Euler (1701–1783). These references concern questions of diagrammatical representation in logic: While treating of intersecting or nested circles of different sizes to represent proportions between concept "spheres," Schopenhauer adds a marginal note referring to Lambert's employing lines of different lengths to serve the same purpose [19, p. 270], Ploucquet's use of squares and Euler's introducing circles [19, pp. 269–270].[10]

In the introductory part of his lectures, Schopenhauer also makes notice of a more contemporary writer on logic: Johann Friedrich Herbart (1776–1841). Schopenhauer refers to the lengthy logical appendix to Herbart's *Hauptpuncte der Metaphysik* [4, pp. 101–130; 19, p. 248]. Presumably he also knew Herbart's *Lehrbuch zur Einleitung in die Philosophie* [5, §§34–71], which contains some extended sections on logic.

Moreover, some of Schopenhauer's early manuscripts[11] prove that he was at least acquainted with some more logical literature of his time. He refers to Herrmann Samuel Reimarus's (1694–1768) slightly earlier *Vernunftlehre*, published in 1756 [18, 20, p. 52]. There is one reference to Kant's *Logik* [8, 20, p. 53]. Schopenhauer also relates to Johann Gebhard Ehrenreich Maass (1766–1823), who had authored an influential *Grundriß der Logik*, first published in 1793 [13, 20, p. 52], and to Ludwig Heinrich von Jakob's (1759–1827) *Grundriss der allgemeinen Logik* from 1788 ([9]; ibid.). He also refers to Johann Christoph Hoffbauer's (1766–1827) *Anfangsgründe der Logik* ([6]; ibid.).[12] Furthermore, he mentioned Ernst Platner's (1744–1818) *Philosophische Aphorismen* [15, 20, p. 53].[13] There is also a reference to Johann Gottfried Karl Christian Kiesewetter's (1766–1819) *Grundriß der Logik* ([10]; ibid.). Last not least, Schopenhauer refers to Salomon Maimon's (1751–1800) *Versuch einer neuen Logik* [14, 20, p. 52], and to his own former teacher Gottlob Ernst Schulze's (1761–1833) *Grundsätze der allgemeinen Logik* [20, 21, p. 51].[14]

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[10] Schopenhauer's references are [12, 17, pp. 157–204], and [2, vol. 2, p. 106].

[11] Schopenhauer's early manuscripts are contained in [20].

[12] Presumably Schopenhauer also knew Hoffbauer's *Analytik der Urtheile und Schlüsse* [7].

[13] It is probable that Schopenhauer was also acquainted with Platner's *Lehrbuch der Logik und Metaphysik* [16].

[14] All of the listed authors share the opinion that (general) logic is a "formal" science, treating of nothing but the forms, i.e., the necessary conditions and hence "laws" of thought—but abstracting

Now, while Lambert and Ploucquet were of course quite concerned about a reasonable symbolic notation, neither of them employed equality signs. Equality signs are neither to be found in von Jakob, Hoffbauer, Maass, Platner nor in Kiesewetter. However, the remaining authors did give some specimens of a quasi-algebraical notation, some more reasonable than others. These cases will be considered in the following sections of the present paper.

4.1 *Reimarus, Kant, and Fichte*

By the late eighteenth century, it was not quite uncommon to employ an equality sign to express identity in the sense of a concept's self-sameness. An earlier case of equational expression for predication is to be found in Reimarus's *Vernunftlehre*. Reimarus invoked an equality sign in "a = b", as provable by reference to both being "equal" (*gleich*) to a third term c [18, p. 470]. However, in 1794, Johann Gottlieb Fichte famously expressed the Principle of Identity as "A = A" in his *Grundlage der gesammten Wissenschaftslehre*, noting that "this is the meaning of the logical copula" (*denn dies ist die Bedeutung der logischen Copula*; [3, p. 5]). Fichte explicitly contended that "A = A" is the foundational principle of logic (*Grundsaz der Logik*; [3, p. 14]). He also noted that the proposition (*der Saz*) "−A nicht = A" is equally accepted as axiomatic as "−A = −A" is but another way of putting "A = A" [3, p. 18].

Even in the preface to Kant's logic lectures, Kant's student and his lectures' editor Gottlob Benjamin Jäsche invoked an equation sign to express the Principle of Identity by "A = A" within his critique of Fichte [8, p. XVII].[15] Moreover, Jäsche similarly used a minus sign to express negation when positing that besides "A = A", there is an "−A = −A" [8, p. XVII]. But while in Fichte, "−A = −A" looks like another form of the Principle of Identity for negative terms, Jäsche declared that this formula indicates the Principle of Non-Contradiction (ibid.). Thus, Jäsche's version of the Principle of Contradiction is but a way of positing a positive identity for apparently negative terms, which of course is not a contradiction.

In any case it is possible that Schopenhauer took his equational expressions and his minus sign in "A = −A = 0" from Fichte or from Kant's logic as edited by Jäsche.

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from all content whatsoever. (As such a view had been prominently put forward by Kant, they are classified as 'Kantian' in Friedrich Ueberweg's *System der Logik*, cf. [23, pp. 51–52].) Thus, they embrace the premise that the exposition and the justification of the laws of thought are independent of both psychological and ontological or metaphysical considerations. A similar starting point was also shared by Herbart and his followers such as Drobisch, quoted in the introductory section of the present paper.

[15] As to an explicit justification for the use of the equality sign in "A = A" departing from a critique of Fichte and Schelling, more material is to be found in Wilhelm Traugott Krug's *Denklehre oder Logik* [11, pp. 43–60].

4.2 Herbart

Some more specific applications of the equality sign are to be found in Herbart. In Herbart's appendix to his *Hauptpuncte der Metaphysik*, there is an equality sign to identify the minor premise by its subject term, as in the derivation of "S P" from "M P" being the major premise and "S P" being the minor, i.e., the minor "= S" [4, p. 122].[16]

In his *Lehrbuch*, which Schopenhauer does not explicitly refer to, Herbart gave another instance of equality signs by casting the premises of syllogisms into an equational form: Given the major "A B" and the minor "M N," it is possible to distinguish the following "equations" (*Gleichungen*): "1) A = N. 2) B = N. 3) A = M. 4) B = M." [5, p. 59]. Herbart also invoked the equality sign to state that a "series" (*Reihe*) of terms gives another concept, as in "A, B, C, D = p" [5, p. 31]. Furthermore, he employs "A = A" to signify a (semantic) tautology (rather than an ontologically grounded identity), such as "What is evaporated evaporates" (*das Verdunstende verdunstet*; [5, p. 47]). This case of the equality sign is within a more global argument of Herbart's, namely that the predicate is somewhat restricted, depending on what subject it is applied to. Hence as applied in "Water evaporates," "to evaporate" would be taken to mean evaporation as applicable to water, i.e., depending on water being the subject, only a portion of the set of characteristic marks determining the predicate is considered (ibid.). Only in "What is evaporated evaporates," the whole set will be relevant.

While Schopenhauer's use of the equation sign in logic does not quite come up to Herbart's, still he might have adopted Herbart's talk of judgments as "equations."

4.3 Maimon

Herbart's talk of judgments as "equations" seems to have been conceptually anticipated in Maimon's *Versuch einer neuen Logik*. In this work, Maimon explicitly stated that the affirmation of a relation of inclusion between subject and predicate (*die Bejahung, deren Bedeutung ist, daß das Prädikat im Subjekte enthalten ist*) should be indicated by "=" as a sign of equality (*Gleichheit*; [14, pp. 68–69]).[17] Moreover, Maimon held that the affirmation of a relation of agreement "within" an object (*Uebereinstimmung im Objekte*) should be signified by the algebraic "plus" symbol, "+", and the affirmation of a relation of negation within an object by a

[16]"Setzt alsdann der Untersatz [...] einen bestimmten Fall, in welchem das Subject (das *antecedens*) Statt finde, oder das Prädicat (das *consequens*) nicht Statt habe: so gleicht die Conclusion, welchem diesem bestimmten Falle (= S) das andre Glied des Obersatzes zueignet oder abspricht, ganz den gewöhnlichen Schlüssen."

[17]It is noteworthy that according to this passage, what is affirmed is the inclusion of the predicate in the subject, not of the subject in the predicate.

"minus" sign, "−", while "infinity" (*Unendlichkeit*) should be signified by "0" [14, p. 69]. It should be remembered that an "infinite judgment" was one which states the denial of a concept term by its contradictory opposite. Accordingly, Maimon wanted an affirmative judgment to be expressed by "a + b", while a negative judgment should be expressed by " a − b", and an "infinite" judgment by "a 0 b" (ibid.). As in the case of "a 0 b", the conjoined concepts cannot at all determine each other, so their relation does not alter any one of them. Therefore, Maimon thought that their conjunction equals 0: it is "= 0" (ibid.).

As to the meaning of "agreement" between concept terms, Maimon noted that there is a three-fold interpretation of being either "mutually or unilaterally identical" (*wechselseitig oder einseitig identisch*) or jointly determining an object (*zur Bestimmung eines Objekts übereinstimmen*; [14, p. 71]. Mutually identical judgments are such that they give "a = a," "ax = ax," "an = an;" hence they are co-extensive (*von gleichem Umfange*; ibid.). An unilaterally identical judgment, however, is of the form "ax + a," which means that it contains "a" among its determinations, to the effect that as to their consequences, "ax" is equivalent (*einerlei*) to "a," while being of smaller extension (*von kleinerem Umfange*; ibid.). In other words, in this case, "ax = a." Furthermore, there is an equivalence of "a + b = ab," which means that some "a," namely such that are conjoined with "b," are "ab" (ibid.). [The same goes for "a + ab" and "b + ab" (ibid.)].

Moreover, Maimon noted that due to the applicability of algebraic rules, an universal negative judgment should be regarded as equivalent (*gleichgeltend*) with an universal affirmative one, with an opposing predicate [14, p. 72]. Symbolically, this relation is mirrored by "two times minus giving plus" (*da minus minus plus giebt*; ibid.). Hence "ax − (−a)" would be equivalent (*gleich*) to "ax + a" (ibid.). Obviously, Maimon seems to have claimed that there is in fact no distinction of quality in categoricals, at least inasmuch as negative judgments can be translated into affirmative ones and vice versa. Another aspect to the same consequence is that judgments can be translated into one another according to their quantity, as in, e.g., "Some a are b" meaning "Not no a is b" [14, p. 68].

Hence it is imaginable that Maimon is one source of inspiration for Schopenhauer to develop some grasp of the applicability of symbols of algebraic operations to logic. However, of course Schopenhauer's "A = −A = 0" does not quite come up to Maimon's exposition in either the sense of "−" or the sense of "0". Moreover, Maimon clearly employed the equality sign "=" to signify an "equivalence" of different forms of judgment. Schopenhauer, however, fails to do precisely this. Rather, he employs the equality sign to express predication.

4.4 Schulze

Some more clues to Schopenhauer's equational notation are to be found in his teacher Schulze. In his *Grundsätze der allgemeinen Logik*, Schulze made an effort to cast the Principle of (Non-)Contradiction into the form of a pseudo-mathematical equation. As a symbolic paraphrase of "Contradictories are unthinkable," he gave "A = non A = 0" [22, p. 32],[18] which obviously corresponds to Schopenhauer's "A = −A = 0" [19, p. 262].

Now, it is plausible to think of Schopenhauer adapting elements from his teacher's logic into the preparatory manuscript of his own lecture. However, he does not give one word of explanation for his choice of symbols. In Schulze himself, the short-hand "A = non A = 0" is commented on only once, and indirectly. The comment is to be found in an addition to the section on the principles or "laws" of thought. This addition has it that the Principle of Identity should be taken to mean "Everything is what it is" (*Jedes Ding ist das, was es ist*; [22, p. 44])—which, like in Schopenhauer [19, p. 262], should be written as "A = A" [22, p. 44]. Accordingly, its complementary Principle, namely that of (Non-)Contradiction, should be understood as "Nothing is what it is not"; however, at this passage Schulze did not even bother to insert the somewhat clumsy "A = non A = 0" once more (ibid.)

There is only one more instance of Schulze's employing equality signs to express predication. It occurs in Schulze's exposition of a syllogism with two premises and one conclusion. These are represented as "A = B", "C = A", "Ergo C = B" [22, p. 117]. Obviously, Schulze's choice of short-hand for judgments corresponds to that employed by Schopenhauer in his table of conversions. Unfortunately, Schulze gave no more elucidations of his notation than did Schopenhauer, and its scattered use seems similarly nonsystematic. But nonetheless, Schulze's approach to logic allows for an implicit vindication, which seems to be lacking in Schopenhauer.

Schulze's implicit vindication for his casting judgments into an equational form relates to his conceptions of their quantity and quality, as follows. Treating of categoricals, Schulze distinguished between the subject and the predicate of judgments. While he described the subject as a judgment's "fundamental term" (*Grundbegriff*; [22, p. 74]), he considered the predicate as its "appending term" (*Beilegungsbegriff*; [22, p. 75]). The subject is what can be determined (*das Bestimmbare*; [22, p. 74]), but the predicate is the determination (*die Bestimmung*; [22, p. 75]). As related in a judgment, the subject and the predicate enter a certain proportion (*Verhältniß*; ibid.), which itself determines the judgment's form (*Form*; ibid.) Its verbal expression is the "conjunction term" (*Bindewort*), or copula (ibid.).

Departing from such—quite traditional—premises, Schulze noted that the predicate, i.e., the predicated mark of an object to be represented in thought, can be applied to the subject's whole extension ([auf] *den ganzen Umfang des Grundbegriffes*), or to part of it only (*oder nur auf einen Theil davon*; [22, p. 78]). Depending on which of these is the case, the judgment is of universal or particular extension (*Umfang*), i.e., quantity (*Größe, quantitas*; [22, p. 79]). However, Schulze criticized the tradition of attaching quantity to judgments according as the subject term only is considered. Hence while treating of subordination of judgments, he noted that

[18] Schulze's textbook is quoted in the fourth edition from 1822, which of course Schopenhauer could not yet have at his hands while preparing his logic lectures.

> "up to the present time, logicians have without sufficient reasons considered relations of subordination between judgments only as their subject terms are subordinated to one another" (*Die Logiker haben bisher, allein ohne hinreichenden Grund, nur diejenigen Urtheile als im Verhältnisse der Unterordnung zu einander stehend aufgestellt, welche in Ansehung des Grundbegriffes einander untergeordnet sind*; [22, p. 89]).

Departing from this remark of Schulze's, it may be conjectured that he was aware that only if there was a quantitative determination of the predicate, there would be a possibility to compare predicate terms of different judgments as to their extensions' sizes. Moreover, there would be a possibility to compare not only to which portion of the subject the predicate is applied, but also which portion of the predicate is applied. Thus, Schulze's remarks insinuate a quantitative relation of size between the subject's and the predicate's extension, respectively. Therefore, it is imaginable that he (more or less) consistently regarded this relation as one of equating portions of both extensions; hence his possible association of judgments with quasi-arithmetical equations.

Schulze also noted that judgments are normally said to differ in texture (*Beschaffenheit*), or quality (*qualitas*), according as they are classed as affirmative or negative [22, p. 80]. However, Schulze refrained from attaching quality to the copula. In Schulze, it is not the copula which has a double character of either attaching a predicate to a subject or separating them. Rather, Schulze held that the common talk of affirmative and negative judgments concerns one and the same operation of thought (*Handlung des Verstandes*; [22, p. 81]), namely the one of including one concept's extension into another concept's extension: In affirmative judgments, the subject's extension is included into that of a positive term (*wird die Vorstellung, welche dem Urtheile zu Grunde liegt, in den Umfang eines bejahenden Begriffes gehörig gedacht*; [22, p. 80].) In negative judgments, the subject's extension is included into that of a negative term (*wird die Vorstellung, welche dem Urtheile zu Grunde liegt, in den Umfang eines verneinenden Begriffes gehörig gedacht*; [22, p. 81].)[19] But in both cases, one concept is (positively) related to a notion as a mark (*der Begriff als ein Merkmahl mit der Vorstellung verbunden*; ibid.). This is why Schulze explicitly opposed the view that there could be such a thing as a negative copula, which serves a separation of concepts. Rather, a negative copula would be a "conjunction term" (*Bindewort*) to effect a disjunction—which, as Schulze claimed, would be a "logical absurdity" (*eine durch negation affizirte copula wäre eine solche, die nicht verbände, also ein logisches Unding*; ibid.).[20] Rather, it is possible to conceive of negative predicates, i.e., of predicates by which something is denied, or excluded from the subject, as well as of judgments containing such predicates [22, pp. 80–81]. Thus, Schulze seems to have thought of what is expressed by the copula as an unchangeably positive relation of mapping (portions of) concepts' extensions

[19]Extensionally negative terms or concepts are introduced even in the introductory parts of Schulze's textbook [22, p. 28]. Treating of inferences, Schulze also spoke of negative marks of concepts [22, p. 118], which seems to be an intensional equivalent.

[20]Similarly, Krug admits of no negative copula but negative concepts since "a negative copula, i.e., a copula which does not copulate, is a contradiction in itself" [11, p. 206].

onto each other. In the case of the so-called negative judgments, such a mapping would take place onto a privative term, i.e., the opposite of what is spoken of as separated. Again, it is imaginable that Schulze thought of equating portions of both extensions, which may be somehow remind of quasi-arithmetical equations.

5 A Tentative Assessment

Now, how exactly does Schopenhauer's account of judgments and their relations of opposition connect to his table of conversions and contrapositions? In fact, Schopenhauer's table of conversions seems to revoke his account of both quality and quantity. It contains negative terms instead of negative copulae, and his use of equality signs suggests that some more or less definite parts of the subject and predicate terms are positively equaled, even if the determination of the predicate term is omitted.

If Schopenhauer intended to model the proportion between concepts as an overlap between some sections of their "spheres," it is unclear how he should integrate a negative copula into this model. Again, if Schopenhauer meant to assimilate the relation between the subject and predicate of judgments to a partial identity, it is unclear how he could have their quantity depend on the subject term only. Moreover, one might doubt whether an equality sign is the right choice of symbol to express such an overlap or partial identity at all.

It is imaginable that Fichte's *Wissenschaftslehre* or Kant's logic lectures as edited by Jäsche were one source for Schopenhauer's attempt at expressing negation by a minus sign. Schopenhauer's use of the equation sign in logic might relate to Herbart's talk of judgments as "equations." Maybe Schopenhauer also took some inspiration from Maimon, who developed a more consistent way of using "+", "−", "=", and "0". Finally, Schopenhauer's "$A = -A = 0$" seems to come from his teacher Schulze. However, Schopenhauer's applications of such signs remain unjustified. The reason is that unlike his teacher, Schopenhauer proposed a negative copula to the exclusion of negative terms, and to a quantitative determination of judgments by consideration of the subject only—at least explicitly. However, what Schopenhauer *does* in some places opposes to what he *says* about quality and quantity of judgments, and this seems to be the case with his employing equational forms in his table of conversions.

One may conclude that on the whole, Schopenhauer seems to be going in different directions at the same time. On the one hand, what he says explicitly comes close to a typical textbook account of quality and quantity of judgments. But on the other hand, there are some passages where Schopenhauer seems to be pointing towards some revisions of this account, concerning the conceptions of the copula and of the quantities of terms. If Schopenhauer had set out on this track, this would account for his attempt to cast predicative judgments into an equational form, especially as in his table of conversions. However, it seems doubtful that Schopenhauer did so consciously and consistently.

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