CVRPTW Model for Cargo Collection with Heterogeneous Capacity-Fleet

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Abstract. This work shows the application of the Capacitated Vehicle Routing Problem with Time Windows (CVRPTW) to collect different cargo-demand in several locations with low time disponibility to attend any vehicle. The objective of the model is to reduce the routing time in a problem with mixed vehicle-fleet. The initial step is the creation of a distance matrix by using the Google Maps API, then cargo capacities for every vehicle and time-windows for every demand point are included in the model. The problem is solved with Google-OR tools using as firt solution aproximated algoritm and as second solution one metaheuristic algorithm for local search.

Keywords: CVRPTW · Routing · Distribution · Heterogeneous capacity-fleet

1 Introduction

Planning and managing transportation activities plays a fundamental role in the modern supply chain. Two categories are well known to address the problem, those who are related to long-distance cargo transport and other related with pick-up and delivery of packages in short distances [16]. The first category consists most of the cases in assignment related formulations with linear restrictions. By the other hand, the initial formulation of vehicle routing problems is proposed by Dantzing & Ramser as a generalisation of Flood's travelling salesman problem [13] in their work [11] proposed a matching-based heuristic for its solution. The vehicle routing problem objective is finding the best routes for delivery minimising a linear cost function over a set of nodes or clients, usually diverse and disperse forming an integer and combinatorial optimisation-problem. Years later, several heuristic solutions to VRP problem appeared including, savings, proximity, matchings, and intra-route inter-route improvement [20]. The most prominent savings algorithm is the Clarke and Wright savings heuristic, it has remained until now, due to its simplicity and speed for implementation [10]. Exact solution algorithms appear in the 80's whit two works proposed

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by Chistofides et al. the first one is a dynamic programming formulation [8] and the second one a mathematical programming formulation making use of q-paths and k-shortest spanning trees [9]. Later, Laporte et al. propose a VRP algorithm based on the solution of linear relaxation of an integer model [19]. Since then, several exact formulations appear but in most cases are solved by branch-and-cut. Also, its posible to formulate VRP as a partitioning problem like the successfull applications [5,14]. Modern heuristic development is related to the last years, here we highlight the works in tabu search based algorithms [15,28,32]. According with [20] vehicle some algorithms were over-engineered and the best meta-heuristic procedure must have a broad and in-depth search of the solution space and can solve several variants of the problem [23]. This work addresses the problem of a classical formulation of capacitated vehicle routing problem with time windows; the case study refers to a cargo company who have to pick packages in 25 locations or nodes. They also count on 13 vehicles with different capacity. The capacity of the vehicles and the demand show kg as cargo units. The structure of the paper show first a short literature review about the CVRPTW and some applications in a similar context in Colombia, later presents de methodological approach and the model design including mathematical formulation, software and hardware used for solve the model. Finally, shows the results and a short discussion about the benefits and difficulties of this approach to solve CVRTW. The main contribution of this article is to show how a complex problem such as CVRPTW can be solved efficiently and simply, allowing the use of different vehicle capacities, which in the classical formulation is not easy to handle.

2 Literature Review

In this item, the work shows relevant and similar applications of vehicle routing problems in Colombia. As designed, the VRP applications solve instance for develope routes in short-haul transport; the particular characteristics of urban development in Colombia, do not allow parking on the street for a long time and also presents demand points without cargo bays. Along whit, present themselves the recurrent issues in cargo transportation as traffic, road maintenance, new works, small roads, among others. The incremental relevance of the CVRPTW in academic literature shows a rise of publications in this problem among last years, from a search in Scopus®. The results show 1429 documents only in the maim formulation over CVRP; including the time windows restrictions, the number of documents in Scopus® gives 722 document results (Fig. 1).

In Colombia the CVRP and his generalization CVRPTW aplies in many contexts, but in litterature can be highligted the works in collection of food donations [4], also in distribution patterns [7] and school bus routing [26]; also there are interesting works applied in other simmilar countries for pharmaceutical distribution [18]. From the same search in Scopus, notice that there are 10 Documents, 6 original articles and 4 proceedings since 2015 [3,6,12,17,21,22,24,25,27,29].

3 The CVRPTW Model Design

The family of VRP models and his study is extensive. The most study model in their taxonomy is the capacitated vehicle routing problem CVRP; the time windows restrictions form a generalisation of this model named VRPTW. The approximation in this work includes the capacity and time windows constraints simultaneously. Fist, we describe and show the mathematical formulation of the model, later also present the solution algorithms and tools to solve the problem.

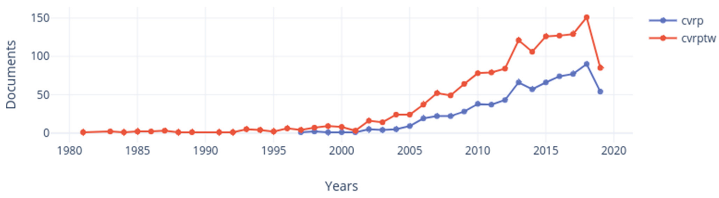

Fig. 1. Documents per year

3.1 Mathematical Formulation to CVRPTW

The mathematical formulation for modelling CVRPTW shows as follows and its described according [30,31,33].

$$Min \sum_{k \in K} \sum_{(i,j) \in A} c_{ij} x_{ijk} \tag{1}$$

subject to

$$\sum_{k \in K} \sum_{j \in \Delta+(i)} x_{ijk} = 1 \qquad \forall\, i \in N, \tag{2}$$

$$\sum_{j \in \Delta+(0)} x_{0jk} = 1 \qquad \forall\, k \in N, \tag{3}$$

$$\sum_{i \in \Delta+(j)} x_{ijk} - \sum_{i \in \Delta+(j)} x_{jik} = 0 \qquad \forall\, k \in N,\ j \in N, \tag{4}$$

$$\sum_{i \in \Delta-(n+1)} x_{i,n+1,k} = 1 \quad \forall\, k \in K, \tag{5}$$

$$x_{ijk}(w_{ik} + s_i + t_{ij} - w_{jk}) \leq 0 \quad \forall\, k \in K, (i,j) \in A, \tag{6}$$

$$a_i \sum_{j \in \Delta+(i)} x_{ijk} \leq w_{ik} \leq b_i \sum_{j \in \Delta+(i)} x_{ijk} \quad \forall\, k \in K,\ i \in N, \tag{7}$$

$$E \leq w_{ik} \leq L \qquad \forall\, k \in K,\ i \in \{0, n+1\}, \tag{8}$$

$$\sum_{i \in N} d_i \sum_{j \in \Delta+(i)} x_{ijk} \leq C \qquad \forall\, k \in K, \tag{9}$$

$$x_{ijk} \geq 0 \qquad \forall\, k \in K,\ (i,j)\ \in\ A, \tag{10}$$

$$x_{ijk} \in \{0,1\} \qquad \forall\, k \in K,\ (i,j)\ \in\ A. \tag{11}$$

The objective function 1 refers to minimize the total cost expressed like distance units in the distance matrix. Constraints 2 restric the assignment of each customer to exactly one vehicle route. Next, constraints 3–5 characterize the flow on the path to be followed by vehicle k. Additionally, constraints 6–8 and 9 guarantee schedule feasibility with respect to time considerations and capacity aspects, respectively. Note that for a given k, constraints 7 force $W_{ik}=$ 0 whenever customer i is not visited by vehicle k. Finally, conditions 11 impose binary conditions on the flow variables.

3.2 Solution Method

- **googleDistance Matrix API:** The CVRPTW is solved using google maps distance matrix API [1], and for solve the algorithm this work use google or-tools [2] for developers.
- **googleORTools:** OR-Tools is open source software for combinatorial optimization, which seeks to find the best solution to a problem out of a very large set of possible solutions.
- **Routing Options:** Time spend in every search of 100 s, fist solution strategy cheapest insertion, local search objective tabu search.
- **IDE:** IPython/Jupyter notebooks.

The distance matrix C_{ij} is created by calling google maps distance matrix API, with units as distance in meters using the directions for every client from the Table 1 and represented in the Fig. 2.

Each client has different demands, the entire fleet is available, but they have different cargo capacity. The first module for the main program developed in python 3.0 is the data creation module, as we say before we use the google distance matrix API, then we add time window constraints with an initial time at 2:00 pm.

The time windows data is a set of pairs with initial time and final time for each time window in every pair into the set.

The vehicle capacities are added as another set of data in the first module of data creation. The second module calculates the distance between directions using the distance matrix created before.

The velocity for every truck is added as a mean velocity for vehicles at the day in Bogota, Colombia (25 km/h). An exciting and useful characteristic of the Google API is the possibility of getting the distance matrix data in real time.

All vehicle capacities must be greater than the sum of every demand nodes. Otherwise, the problem has no feasible solution. The use of heuristics helps to find a feasible (but not always the best) solution with faster development in the time of execution for the algorithm previously selected. The imput information for the first module of data creation in our program is Table 3

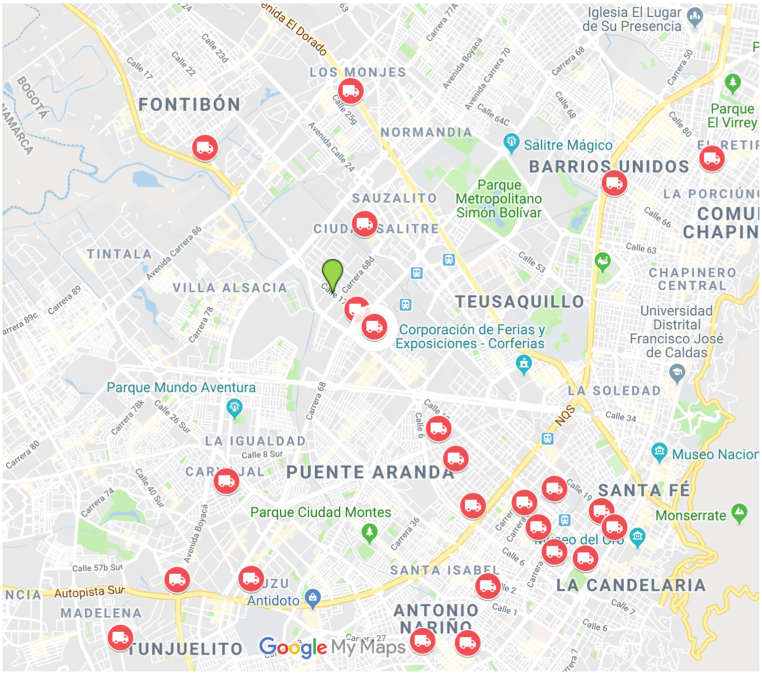

Fig. 2. Clients map - nodes for pick packages

The CVPTW presents in each node a time window expressed as the initial time i_t and the final time f_t when a truck can visit a demand node at a time v_t as we show as follows (Table 2):

The vehicle routing problem with time-windows (CVRPTW) reffers to: As can bee seen in the Fig. 3 a fleet of shipping cars with uniform capability must serve clients with known demand and opening hours for a single commodity.

Table 1. Geographical coordinates for the demand nodes: Lat = Latitude, Lon = Longitude

Node	Lat	Long	Node	Lat	Long	Node	Lat	Long
DEPOT	4,644074	−74,120507	C8	4,61009	−74,090002	C16	4,617141	−74,101125
C1	4,63816	−74,114188	C9	4,588141	−74,106311	C17	4,588499	−74,154815
C2	4,64093	−74,116914	C10	4,597977	−74,133829	C18	4,613504	−74,137859
C3	4,675887	−74,117909	C11	4,609671	−74,09831	C19	4,606171	−74,07572
C4	4,62192	−74,103876	C12	4,612365	−74,085213	C20	4,6012	−74,080275
C5	4,666847	−74,141226	C13	4,661076	−74,075602	C21	4,602244	−74,085227
C6	4,597748	−74,145742	C14	4,654545	−74,1156	C22	4,665044	−74,059959
C7	4,608708	−74,077771	C15	4,596786	−74,095874	C23	4,587774	−74,099263
						C24	4,606273	−74,08787

Table 2. Fleet with heterogeneous capacity

Code	Brand	Class	Capacity
4020	HINO	TRUCK	8500
4033	HINO	TRUCK	5400
4036	CHEVROLET	TRUCK	5400
4037	CHEVROLET	TRUCK	5400
4041	CHEVROLET	TRUCK	5400
4042	HINO	TRUCK	4600
4043	HINO	TRUCK	4200
4044	HINO	TRUCK	4200
4039	HINO	TRUCK	4000
4040	HINO	TRUCK	4000
4035	HINO	TRUCK	4000
4034	HINO	TRUCK	4000
4031	CHEVROLET	TRUCK	3500

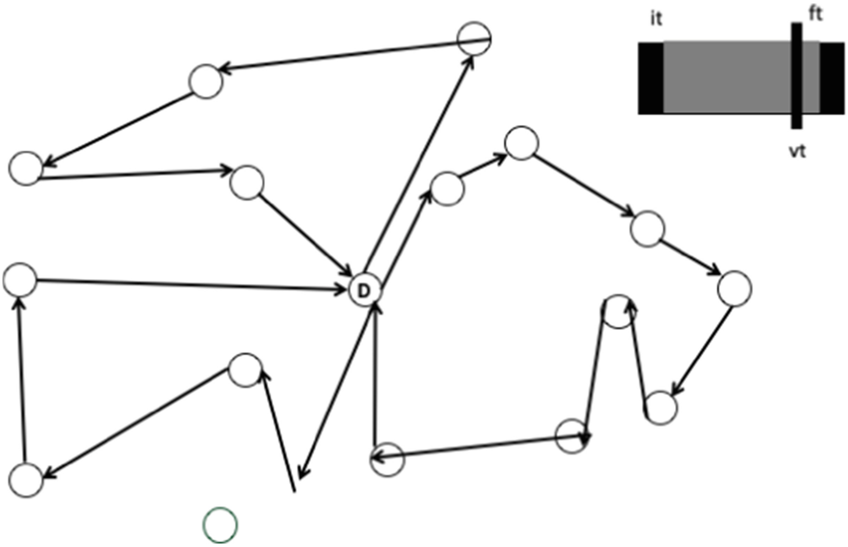

Fig. 3. VRPTW representation of time windows

At a D depot node, the cars begin and end their paths. Only one car can serve each client. The goals are to minimize the fleet size and assign a sequence of customers to each fleet truck minimizing the total distance traveled so that all customers are served and the total demand served by each truck does not exceed its capacity. The time windows are represented as the time interval vt from it to ft in which each customer can receive deliveries (Table 4).

4 Results

The solution of the problem was performed using the algorithm of Objective tabu search and includes time window constraints and capacity constraints simultaneously. The performance of the algorithm was measured, given a solution at a time execution of 0.09 s.

The routes for the Trucks contains load in every node, total time and total distance for each. The total distance for every route was 10044 m, the total time of every route was 2174 min, but the more significant time for every rout was 333 min for truck number 4039. Also, we can plot every route, as can see the first two of them as follows:

Table 3. Time windows in minutes starting at 2:00 pm and demands for everey client

Node (client)	Tw (start)	Tw (end)	Demand
D	0	0	0
C1	0	50	391
C2	30	100	71
C3	60	80	36
C4	120	160	2519
C5	150	270	248
C6	210	227	478
C7	180	200	589
C8	210	235	1105
C9	210	225	55
C10	240	255	237
C11	240	255	235
C12	240	265	908
C13	240	270	1473
C14	240	255	24
C15	270	285	144
C16	270	285	181
C17	270	290	564
C18	270	295	764
C19	300	330	1386
C20	300	345	4034
C21	300	320	669
C22	330	345	361
C23	330	355	1113
C24	360	405	4271

Table 4. Results for the CVPTW

Item	performance results
Total Distance of all routes	10044 m
Total Load of all routes	21856
Total Time of all routes	2174 min
Algorithm performance	0.09801173210144043 s

As can be seen in Table 5a it is possible to obtain the routes of each vehicle where the first row shows the route that each vehicle must follow starting from the deposit and returning to it.

Table 5. Solution for the CVRPTW

(a) Route 1-Truck 4041

Route 1	D	2	12	20	D
Load (Kg)	0	71	908	4034	0
Total Load	5013				
Total Distance	1296				
Total time	302				

(b) Route 2-Truck 4042

Route 2	D	11	24	D
Load (Kg)	0	235	4271	0
Total Load	4506			
Total DIstance	1131			
Total time	362			

(c) Route 3-Truck 4039

Route 3	D	8	9	15	23	D
Load (Kg)	0	1105	55	144	1113	0
Total Load	2417					
Total Distance	1811					
Total time	333					

(d) Route 4-Truck 4040

Route 4	D	1	7	19	21	D
Load (Kg)	0	391	589	1386	669	0
Total Load	3035					
Total Distance	1325					
Total time	302					

(e) Route 5-Truck 4035

Route 5	D	6	10	17	18	D
Load (Kg)	0	478	237	564	764	0
Total Load	2043					
Total Distance	1706					
Total time	272					

(f) Route 6-Truck 4035

Route 6	D	4	16	D
Load (Kg)	0	2519	181	0
Total Load	2700			
Total Distance	761			
Total time	271			

(g) Route 7-Truck 4031

Route 7	D	3	5	14	13	22	D
Load (Kg)	0	36	248	24	1473	361	0
Total Load	2142						
Total Distance	2014						
Total time	332						

The second row shows the load collected at each point, rows 3 and 4 show the total load collected by each vehicle and the total route time (Figs. 4 and figura4).

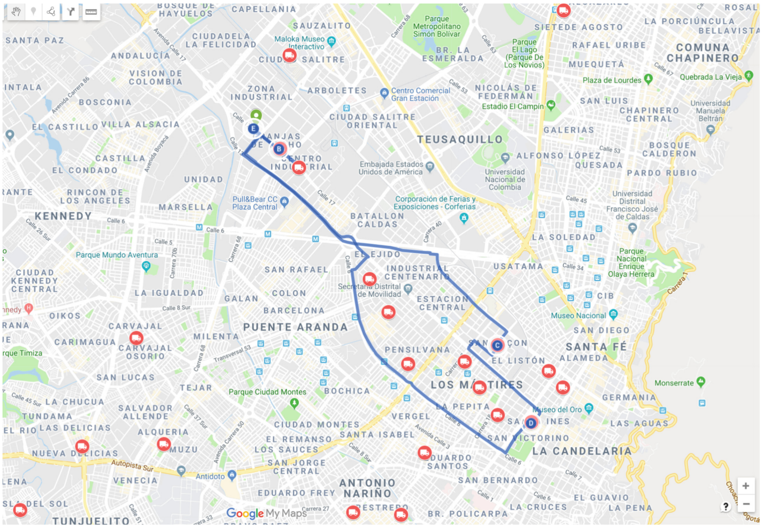

Fig. 4. Route 1 - Route generator in google maps

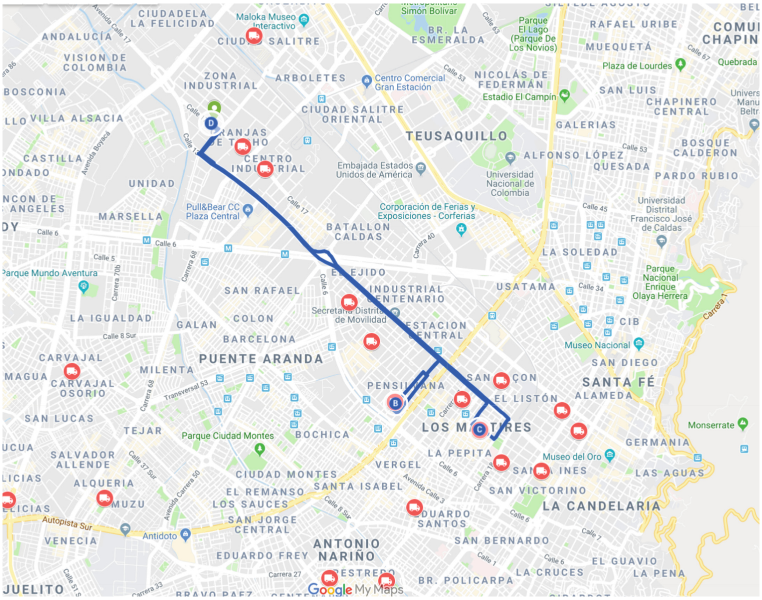

Fig. 5. Route 2- Route generator in google maps

5 Conclusions

The use of google optimization tools allows to solve optimization and metaheuristic problems efficiently and also allows its integration with visualization tools such as google maps for a better understanding of the results.

The routing problem in a real case for a Bogota company is solved by reducing the delivery time and the fleet required to carry out the cargo collections.

The biggest contribution of google tools in this case is the possibility of changing the loading capacity of trucks in a simple way.

The solution time is very efficient, achieving a response in a short time for a complex metaheuristic problem.

The company do not need 6 trucks of 13; the demand is satisfied with 7 trucks as it was described in the last item. The model provides dynamic planning for delivery routes using a heterogeneous capacity fleet.

Future research can include machine learning algorithms to add human preference behaviour to chose how to accomplish the routes, including perception aspects like security or comfort.

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