Chapter 26
Facility Location in the Public Sector

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Abstract In this chapter we focus on facility location problems that arise in the public sector. In particular, we consider selected problems in transportation, health care, and education—important sectors of public service. The adequate consideration of demand in these models is of core interest in this chapter. Besides a discussion of selected model formulations we provide a quantitative and qualitative overview of recent publications in the field.

26.1 Introduction

In this chapter, we discuss recent work related to public sector facility location planning. Of course, a location of a public service does not necessarily strictly belong to the public sector. For example, healthcare facilities may also be owned by a private firm while being regulated by a public health agency. The main difference between the planning of public and private facility locations are the objectives that are considered by decision makers. The optimization criteria in private applications are mainly profit and market capture maximization, whereas in public applications social cost minimization, access, efficiency, and equity are the primary goals. Since the measurement of these objectives is relatively difficult, they are frequently simplified by minimizing the locational and operational costs needed for full coverage, or the search for maximal coverage under a given amount of available resources (Marianov and Serra 2002).

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Most of the public facility location models proposed in the selected papers rely on covering problems, p-median problems or a combination of both. They are benchmarks in the development of location models. The public sector applications of covering models are based on the concept of acceptable proximity. If a service is provided by a facility located within a maximum distance or travel time, the service is considered adequate – the client demand is covered. Two major types of formulations can be distinguished in such covering models. Set covering models seek to minimize the number of facilities needed for full coverage of the population. In contrast, maximum covering models are limited by the number of facilities or services and maximize the covered population share. Furthermore, a distinction can be made between fixed servers (e.g., schools, hospitals) and systems with mobile servers (e.g., ambulances, see Nickel et al. (2016)). Additionally, a server can be classified as capacitated or uncapacitated. An example of a capacitated service is a primary school that has a limit on the number of students who can enroll in a particular year (Marianov and Serra 2002; Müller et al. 2009). In the following, we discuss selected areas of application of public facility location planning approaches: Bike sharing systems, simultaneous bus scheduling and depot location planning, electric vehicle charging station planning, healthcare facility location planning, and school location planning. The presented models are classified as discrete location-allocation models as well as location choice models.

26.2 Bike Sharing

Since the political interest in the promotion of cyclists continues to increase, it is important to create enough hubs and parking areas for bicycles. People who do not have a bicycle on their own should have an appropriate possibility to use bicycles within cities. Thus, bike sharing models are becoming more and more popular. Bike sharing is often linked to transport hubs, but there are also stand-alone models for rental stations and also station-less approaches.

With the proposal of Sayarshad et al. (2012), an optimization formula to design a bike-sharing system for small communities is presented. This formula can also be used to extend the public transport with incoming and outgoing bike traffic. They try to find a minimum required bike fleet size that also minimizes the unmet demand, non-utilized bikes, and the need to transport bikes between the stations. The mathematical model maximizes the considered company's total benefit where the objective function consists of six terms: (1) the revenue from rented bikes traveling between network nodes, (2) the cost of moving empty bikes within the network, (3) the cost of processing and maintenance of bikes, (4) the bike holding cost at a station, (5) the bikes' capital cost per period, and (6) the penalty cost of unmet demand.

By combining the models for private cars and public bicycles, in Romero et al. (2012) the goal is to achieve an efficient and sustainable transport system that is also economically and socially efficient at the same time. The choice between motor vehicle and bicycle and route selection is simulated by a user behavior model. Thus, in turn, a combined vehicle-bicycle transport network was created on which a modal split model can be matched. The goal is then to optimize the location of bike stations. The study in Lin and Yang (2011) deals with strategic planning of bike sharing taking into account both the interests of users and investors. Considering those interests, the model attempts to determine the number and location of bike sharing stations, the network structure as well as the travel paths between the stations. Lin et al. (2013) continue expanding this approach by considering the number and location of bicycle stations in the system, the creation of bicycle lanes, and selection of paths between the stations and the inventory levels of the bike sharing facilities. Decisions are made under consideration of total costs and service. An approach to maximize the coverage of a bike sharing facility by also using the available budget as a constraint is proposed by Frade and Ribeiro (2015). They combine the strategic decision for a bike sharing facility and the dimension of the stations with operational decisions. The result is an optimal location as well as the capacity of each station and the number of bikes needed while staying within the budget.

26.3 Location Decisions in Public Transport

Traffic planners face the trade-off between improving accessibility with additional bus stops while simultaneously increasing efficiency so that traffic reach destinations in a reasonable time. Delmelle et al. (2012) address this specific problem with an optimization framework that builds upon facility location coverage models. In contrast to the p-median and maximal covering location problem, the demand can partially be assigned to more than one facility. Furthermore, facility attraction is explicitly integrated. The modeling approach considers the impact of walking distance from a residential location to a stop as well as the transit facility attractiveness (the number of destinations served, for example). Cipriani et al. (2012) deal with the bus network design problem in a multimodal transit context. The approach determines the (near) optimal network configuration regarding bus routes and service frequencies. It aims to minimize the total costs involved in the transport system. A similar method is used by Ciaffi et al. (2012) to solve the feeder-bus network design problem. Their results show that the design procedure could lead to a reduction of the total travel time, an increase in the number of transfers, in a more efficient way.

The locations of bus stops affect travel times and therefore also the expected demand. By using a random utility model (RUM) we can measure the expected impact of travel time and other factors on demand. Klier and Haase (2015) integrate a RUM in line planning that results in a difficult optimization problem. If we assume that distance to the departure stop is the only relevant factor influencing the choice behavior over a given set of potential stop locations, RUM approaches as defined in Haase and Müller (2013, 2014), Müller and Haase (2014) or Ljubić and Moreno (2018) might be appropriate.

Another topic in public transport is the location of bus depots. The depot locations determine the vehicle costs. Therefore, we combine vehicle scheduling and bus-depot location in one integrated approach.

Defining the sets

$\mathscr{N}$ set of nodes representing line trips and potential bus depot nodes,
$\mathscr{M}$ set of potential bus depot nodes,
$\mathscr{I}$ set of nodes representing line trips,
$\mathscr{A}$ set of arcs representing feasible idle trips (compatible with the timetable),

the parameters

c_{ij} costs of idle trip $(i, j) \in \mathscr{A}$,
f_m fixed costs per day of depot m,
k_m maximum number of vehicles in depot m,

and the binary variables

X_{mij} = 1 if a vehicle from depot m serves idle trip $(i, j) \in \mathscr{A}$ (0, otherwise),
Y_m = 1 if depot m is to be established (0, otherwise)

then we formulate the depot location and vehicle scheduling model as follows:

$$\text{Minimize} \quad F = \sum_{m \in \mathscr{M}} \sum_{(i,j) \in \mathscr{A}} c_{ij} X_{mij} + \sum_{m \in \mathscr{M}} f_m Y_m \tag{26.1}$$

subject to

$$\sum_{m \in \mathscr{M}} \sum_{(i,j) \in \mathscr{A}} X_{mij} = 1 \qquad \forall\, i \in \mathscr{I} \tag{26.2}$$

$$\sum_{(i,j) \in \mathscr{A}} X_{mij} - \sum_{(j,i) \in \mathscr{A}} X_{mji} = 0 \qquad \forall\, m \in \mathscr{M};\, j \in \mathscr{N} \tag{26.3}$$

$$\sum_{(m,j) \in \mathscr{A}} X_{mmj} \leq k_m Y_m \qquad \forall\, m \in \mathscr{M} \tag{26.4}$$

$$X_{mij} \in \{0, 1\} \qquad \forall\, m \in \mathscr{M};\, (i, j) \in \mathscr{A} \tag{26.5}$$

$$Y_m \in \{0, 1\} \qquad \forall\, m \in \mathscr{M} \tag{26.6}$$

The objective function (26.1) minimizes the total costs per day. Equation (26.2) ensure that each line trip is operated exactly once. Equation (26.3) are flow conservation constraints and Eq. (26.4) ensure that trips can only start from a depot if it is established and the depot capacity is considered.

26.4 Electric Vehicle Charging Station Location

In the application of electric vehicle (EV) charging station location, we find plenty of recent work attributed to the technical developments in the EV industry and to the rising importance of eco-friendly transport modes in times of climate change (Müller and He 2018). With the increasing demand for vehicles with alternative fuel usage, the demand for their charging or refueling stations is also increasing. The papers in Table 26.1 discuss this topic in several approaches by maximizing the coverage, maximizing the traffic flow, minimizing the costs or by combining of these objectives.

Frade et al. (2011) present a charging location problem for parked cars. For this study area, a slow-charging model is suitable, because parked cars are parked for several hours. The proposed model is based on a maximum coverage location model (MCLP) to optimize the demand coverage by simultaneously keeping an acceptable level of service. They optimize the number of stations and the scale of each station. As input parameters, an estimated refueling demand for the day and nighttime is needed. The approach of Giménez-Gaydou et al. (2016) also covers urban areas. Their models consist of a location-allocation model with detailed analysis of charging needs, charging coverage, and adoption potential. Zheng et al. (2017) investigate a network-design-like problem with a bi-level structure. While the upper level aims for optimal locations with minimized general costs calculated from travel time and energy consumption, the lower level aims at minimized individual costs with traffic equilibrium. By adding the lower level to the upper level, those two levels are then combined to a single level model. The hybrid model from Mozafar et al. (2017) handles the optimal allocation and sizing of either renewable energy sources or electric vehicle charging stations. A multi-objective problem is created to obtain several objective variables such as reducing power losses, voltage fluctuations, charging and demand-supply costs, and battery costs. The location and the dimension of the charging stations are handled as decision variables.

In contrast to public charging stations for private vehicles, Yang et al. (2017) introduce a location model for electric powered taxis. With the goal to minimizing the infrastructure costs, an integer linear program (ILP) is formulated. Their key findings include positioning of the charging stations matching the dwell pattern of the taxis, with the combination of charging and waiting spots, fewer chargers are needed and this compromise can be qualified by the cost of charging spots versus parking spots. Another taxi-based approach is proposed by Tu et al. (2016). In contrast to the approach of Yang et al. (2017), their model's goal is to maximize the charging station service within the taxi network. To achieve this, a spatial-temporal demand coverage location model is proposed and the results are analyzed with respect to spatial coverage, temporal demand availability, and waiting and loading behavior. A bus charging model is proposed by Xiang and Zhang (2017). In contrast to the taxi models, for buses with electric drive it is common to replace the battery instead of charging it. A particle swarm optimization algorithm (PSO) is used to

Table 26.1 Charging and refueling station location papers

Reference	Application	Objective	Modeling approach	Solution method	Demand
Frade et al. (2011)	EV charging stations location	Maximize coverage	MCLP, MIP		Distinguished daytime/nighttime, variable
Giménez-Gaydou et al. (2016)	Charging station location	Maximize coverage	Uncapacitated gradual maximal covering model	XPRESS	Willingness and socio-economic factors, variable
Zheng et al. (2017)	Charging network design	Minimize total cost	Bi-level MILP	CPLEX	Choice decisions, variable
Mozafar et al. (2017)	Allocation and sizing of charging stations	Minimize power loss, voltage fluctuation, power supply and total cost	Multi-objective optimization problem	GA-PSO, MATLAB	Variable
Yang et al. (2017)	Taxi charging stations	Minimize infrastructure costs	ILP	MATLAB, YALMIP, Gurobi solver	Fixed
Tu et al. (2016)	Taxi charging stations	Maximize charging network service	STDCLM	GA	Spatial-temporal dynamic demand, variable
Xiang and Zhang (2017)	Bus charging station location	Minimize total costs	IP	PSO	Fixed
Ghamami et al. (2016a)	Charging facilities located on existing parking lots	Minimize total system cost	Fixed CFM	AMPL, Knitro solver	Considering drivers preference, uncertain market penetration rate, variable
Ghamami et al. (2016b)	Charging facilities location along highways	Minimize total system cost	MINLP	SA, B&B, Knitro solver	Considering flow-dependent charging delay, variable

Jeong (2017)	Charging network planning	Minimize total construction costs	SCRLP, VRCP	CPLEX	
Riemann et al. (2015)	Flow refueling location model	Maximize total captured flow of traffic	MCLP, MINLP	B&B, CPLEX	Travelers route choice behavior, variable
Arslan and Karaşan (2016)	Flow refueling location model	Maximize flow volume; minimize total cost	FRLM, CSLP-PHEV	Arc-cover formulation and Benders decomposition	
Miralinaghi et al. (2017a)	Alternative fuel network system	Minimize construction and operational costs	CFLP	B&B, Lagrangian relaxation	Variation in hydrogen refueling demand, variable
Miralinaghi et al. (2017b)	Refueling demand uncertainty	Minimize total costs	RCPM	GA, CPLEX	Variable
Hosseini and MirHassani (2015)	Refueling station location	Maximize total flow	MIP	CPLEX	Variable
Guo et al. (2016)	Charging infrastructure planning	Maximize profit	MOPEC, CDA	PYOMO, Gurobi solver	Multinomial logic model describes choice, variable

Note: *CFM* cost facility model, *SCRLP* set covering version of the refueling-station location problem, *VRCP* vertex restricted covering problem, *STDCLM* spatial-temporal demand coverage location model, *FRLM* flow refueling location modeling, *CSLP-PHEV* charging station location problem with plug-in hybrid electric vehicles calculate the optimal location for the replacement facilities with a minimum of total costs (transport costs, construction costs, and operating costs).

Another approach is presented by Ghamami et al. (2016a) with the goal to minimize the total system cost. Their idea is to use existing parking lots to install charging facilities. The model becomes complex by introducing costs for uncovered demand and also considering the drivers' preferences for familiar parking lots. Ghamami et al. (2016b) aim to configure charging stations to support long distance intercity travel by using a general corridor model minimizing the total system costs including infrastructure, battery, and user costs. Using a mixed-integer program with non-linear constraints, it is possible to use realistic patterns of origin-destination demands and also considering flow-dependent charging delay caused by traffic jam. With this model, a strategic design of charging stations along highways is possible.

Different to the parking-and-charging models, a wireless-charging model is investigated by Riemann et al. (2015). Based on a mixed-integer non-linear program (MINLP), a method is formulated to find a number of charging facility locations out of a set of candidates and to maximize the total captured flow. Similar to this, a flow refueling location problem for both electric and plug-in hybrid vehicles is introduced by Arslan and Karaşan (2016). With the goal of maximizing the vehicle miles that can be traveled and minimizing the total cost, the presented exact solution is an arc-cover formulation and makes use of a Benders decomposition approach.

To propose a model for locating refueling stations in a transport network, Miralinaghi et al. (2017a) assume that a central planner such as a hydrogen manufacturer or a government agency is planning the locations for refueling stations with alternative fuel type, especially hydrogen. Considering a multi-period travel demand, both the non-linear refueling station operational cost and the deviation of travelers from their shortest routes to refuel are taken into account. The proposed capacitated facility location problem (CFLP) is solved with a combination of Branch-and-Bound and Lagrangian relaxation. Another approach, presented by Miralinaghi et al. (2017b), considers the refueling demand uncertainty with the effect of the deviation of travelers to refuel. A cutting plane algorithm is used to solve the robust centralized planning model (RCPM). The uncertainty model from Hosseini and MirHassani (2015) provides a two-stage stochastic refueling station model for permanent stations in the first stage and portable stations in the second stage. Portable refueling stations are an innovative feature that can be used to close temporary gaps in supply. A business-driven model for charging infrastructure planning is introduced by Guo et al. (2016) by using a multi-agent optimization problem with equilibrium constraint (MOPEC). The goal is to maximize providers' profit. An approach in which a charging network can be planned without existing facilities comes from Jeong (2017). They also provide a dynamic-programming-based algorithm for the case where facilities already exist. The goal is to minimize the total construction costs of the charging network by minimizing the cost of charging stations. In particular, the model underlies the following assumptions: (1) multiple origin-destination round trips along the shortest paths, (2) a single type of alternative fuel vehicle with a constant driving range, (3) uncapacitated stations, (4) possible refueling station locations that are only nodes in the traffic network (i.e., vertex restricted refueling stations), (5) a linear relationship between fuel consumption and driving distance, and (6) fully fueled vehicles at the point of origin.

Defining the sets

$\mathcal{N}$ nodes of the network,
$\mathcal{E}$ existing refueling stations $\mathcal{E} \subset \mathcal{N}$,
$\mathcal{K}$ considered alternative fuel vehicles,
$\mathcal{P}_k$ sequence of arcs (i, j) along path of vehicle k, and

the parameters

d_{ij} (Euclidean) distance from node i to node j,
c_i cost of refueling station at node i,
S maximum vehicle range, and

the variables

X_i = 1 if a refueling station is set up at node i (0, otherwise),
Y_{ik} = 1 if vehicle k is recharged at node i (0, otherwise),
Z_{ik} remaining driving range of vehicle k at node i, and
W_{ik} additional driving range of vehicle k if refueled at node i,

the refueling station location problem is to

$$\text{minimize } F = \sum_{i \in \mathcal{N} \setminus \mathcal{E}} c_i X_i \tag{26.7}$$

subject to

$$Y_{ik} \leq X_i \qquad \forall\, i \in \mathcal{N} \setminus \mathcal{E}, k \in \mathcal{K} \tag{26.8}$$

$$W_{ik} = SY_{ik} \qquad \forall\, k \in \mathcal{K}, i \in \mathcal{N} \tag{26.9}$$

$$W_{ik} \leq S - Z_{ik} \qquad \forall\, k \in \mathcal{K}, i \in \mathcal{N} \tag{26.10}$$

$$Z_{jk} = Z_{ik} + W_{ik} - d_{ij} \qquad \forall\, k \in \mathcal{K}, (i, j) \in \mathcal{P}_k \tag{26.11}$$

$$X_i \geq 0 \qquad \forall\, i \in \mathcal{N} \setminus \mathcal{E} \tag{26.12}$$

$$Y_{ik} \in \{0, 1\} \qquad \forall\, i \in \mathcal{N}, k \in \mathcal{K} \tag{26.13}$$

$$Z_{ik}, W_{ik} \geq 0 \qquad \forall\, i \in \mathcal{N}, k \in \mathcal{K}. \tag{26.14}$$

The objective (26.7) is the minimization of the total set up cost of refueling stations. If there is no refueling station at node i, refueling cannot occur at i by constraint (26.8). The refueling amount at node i is S by constraint (26.9), and this amount must not exceed $S - Z_{ik}$ by constraint (26.10) if the vehicle refuels at node i. Constraint (26.11) defines the remaining distance using the remaining fuel at each node i. Considering arc (i, j) the remaining fuel at node j is the sum of the remaining fuel at node i and the fueled amount at node i minus distance between node i and node j. The authors show that the problem is $\mathcal{NP}$-complete and propose several procedures for its solution. The approach is used to analyze the diffusion of alternative fuel recharging stations in a given market.

26.5 Spatial Planning for Health Care Facilities

One of the key factors to achieve a high standard in healthcare is a systematic and efficient system planning (Shariff et al. 2012). See Chap. 23 for a more detailed discussion. Therefore, it is important to develop methods to facilitate the planners' decision making process in the locating of new healthcare facilities (Zhang et al. 2016) (Table 26.2).

To find a more systematic and efficient way of locating healthcare facilities, Shariff et al. (2012) use a MCLP with capacitated facilities. Zhang et al. (2016) investigate the location problem of healthcare facilities to maximize the equity of accessibility and the total accessibility and to minimize the population outside the coverage range, and minimize the cost of new buildings.

Two location-allocation models to handle the uncertainty in the strategic hospital network planning are proposed by Mestre et al. (2015). The models aim to inform about the (re-) organization of hospital networking systems by improving geographical access (minimize expected travel time) while minimizing costs.

The problem of determining locations for long-term care facilities is investigated in Djenić et al. (2017), where the objective is to minimize the maximum number of patients that are assigned to a single installed facility.

Kim and Kim (2013) focus on public healthcare facilities that can be used by low-income patients. They examine the problem of determining locations of public healthcare facilities within a given budget and allocating the patients to the facilities. The objective is to maximize the number of served patients while considering the patients' preferences of the for the public and private facilities. Basu et al. (2018) focus on socio-economically weaker patients. They aim to quantify the gap in affordable healthcare facilities access. The optimization model shows where new public facilities are required, and the positive impact of the proposed model with increasing coverage is detected.

Besides operations research applications in healthcare operation management, the design of blood supply networks also is important. Hospitals and clinics as demand centers are dependent on blood products and an efficient procurement system is needed. Arvan et al. (2015) intend to locate blood bank components in a network and to determine the allocations among these network components (donation sites, testing and processing labs, blood banks, and demand points). The main objectives are to identify the locations of donation points and central blood banks as well as to decide about the product quantity that is shipped among the facilities. To model the problem a bi-objective approach is proposed not only to minimize the cost but also to minimize the time period in which blood products remain in the network.

Table 26.2 Healthcare facility location papers

Reference	Application	Objective	Modeling approach[a]	Solution method	Demand
Shariff et al. (2012)	Healthcare facility planning	Maximize coverage	CMCLP as a variation of MCLP	GA, CPLEX	Fixed
Zhang et al. (2016)	Healthcare facility location-allocation	Maximize accessibility; minimize inequity of accessibility; minimize the uncovered population, minimize building cost	Multi-objective optimization	GA	Fixed
Mestre et al. (2015)	Location-allocation for hospital planning under uncertainty	Minimize expected travel time, minimize expected cost and capital costs	Model 1: Location as a first-stage decision, Model 2: Location and allocation as first-stage decisions	ε-Constraint method of multi-objective programming	Variable
Djenić et al. (2017)	Long-term care facility location	Minimize the maximum number of patients that are assigned to a single installed facility	LTCFLP LTCFLP-I	Metaheuristic method based on a Variable Neighborhood Search (VNS)	Fixed
Kim and Kim (2013)	Public healthcare facility location	Maximize the number of served patients	IP	Heuristic algorithm based on LR and subgradient optimization methods	Considers patients preference for the public and private facilities, variable
Basu et al. (2018)	Healthcare facility allocation	Maximize the healthcare coverage by minimum number of new public healthcare facilities			Variable
Zhang et al. (2012)	Preventive healthcare facility location	Maximize preventive healthcare program participation	Probabilistic-choice model, optimal-choice model based on MNL, MIP	CPLEX	Variable

(continued)

Table 26.2 (continued)

Reference	Application	Objective	Modeling approach[a]	Solution method	Demand
Haase and Müller (2015)	Preventive healthcare facility location	Maximize preventive healthcare program participation	MNL, MILP, derive lower bound	CPLEX	Variable
Arvan et al. (2015)	Human blood supply chain network	Minimize total cost, minimize times that blood products remain in the network	Bi-objective model, MILP	ε-Constraint method, CPLEX	Deterministic fixed

[a] LTCFLP: long-term care facility location problem

In contrast to immediate medical support, there are also studies regarding preventive healthcare. In this case, clients choose whether to participate in preventive care programs or not. To maximize the total participation in these programs, Zhang et al. (2012) investigate the impact of clients' choice behavior on the preventive care facility network design and the resulting level of participation. They present two alternative models: the probabilistic-choice model and the so-called optimal-choice model. Solving large instances (with CPLEX) can take days. Enhancing the model of Haase (2009), Haase and Müller (2015) show that an alternative formulation of the presented problem can be useful to solve problems considerably faster with commercial solvers. An approach to derive a lower bound to the problem is also presented to accelerate computation time. In the following, we present an extension of this model, which includes variables in patients' utility functions (Krohn et al. 2018).

The locations of client nodes (demand points), the number of eligible patients per node, candidate locations for preventive healthcare facilities and a set of feasible facility modes are given. Different modes represent waiting time for an appointment and quality of care. The problem is to determine the locations and modes of established facilities in a way that maximizes the target population's expected participation in the preventive healthcare program. We integrate quality and waiting time into a deterministic mixed-integer linear problem via discretization of the clients' utility function and consider each combination of a facility's location and its mode as a separate choice alternative, e.g., a single facility with two possible modes results in two alternatives within the client's choice set. The two virtual facilities cannot be established simultaneously, because only exactly one mode is assigned to the facility. Hence, in the solution for this example, only one alternative (the facility located in a specific mode) remains in addition to the no-choice alternative, which is always present. Our approach makes use of the MNL's IIA property (Haase 2009; Aros-Vera et al. 2013; Haase and Müller 2015): The basic idea is to provide in advance calculated choice probabilities as input parameters and to take advantage of their constant ratios.

Defining the sets

- $\mathscr{I}$ set of demand nodes,
- $\mathscr{J}$ set of candidate facility location nodes $\mathscr{J} \subseteq \mathscr{I}$,
- $\mathscr{M}$ set of modes in which a facility can be established (quality of care and waiting time for an appointment), (might also contain capacity levels), and

the parameters

- g_i number of clients in node i that are eligible to require health service,
- p_{ijm} MNL choice probability of clients in i to access service at a facility located at j being in mode m given that (j, m) is the only facility established, i.e. the choice set consists of the two alternatives $\{(j, m); \text{no}\}$, which results in $p_{ijm} = \frac{e^{v_{ijm}}}{e^{v_{i,\text{no}}} + e^{v_{ijm}}}$ where v_{ijm} is the deterministic utility of clients in i going to a facility located at j being in mode m and $v_{i,\text{no}}$ is the deterministic utility

for demand node i of not attending any facility ("no-choice" or "opt-out" alternative)

$\underline{l}_m$ lower threshold for mode m measured in number of clients

$\bar{l}_m$ upper threshold for mode m measured in number of clients,

p total number of available facilities, and

the variables

X_{ijm} choice probability of clients in i to access service at a facility located at j being in mode m,

Z_i cumulative choice probability of clients in i to refuse to access any facility ("no-choice"),

Y_{jm} = 1 if location j is specified to offer healthcare service in mode m (0, otherwise),

we formulate the healthcare facility location problems as follows:

$$\text{Maximize} \quad F = \sum_{i \in \mathscr{I}} \sum_{j \in \mathscr{J}} \sum_{m \in \mathscr{M}} g_i X_{ijm} \tag{26.15}$$

subject to

$$Z_i + \sum_{j \in \mathscr{J}} \sum_{m \in \mathscr{M}} X_{ijm} \le 1 \qquad \forall\, i \in \mathscr{I} \tag{26.16}$$

$$X_{ijm} \le p_{ijm} Y_{jm} \qquad \forall\, i \in \mathscr{I};\, j \in \mathscr{J};\, m \in \mathscr{M} \tag{26.17}$$

$$X_{ijm} \le \frac{p_{ijm}}{1 - p_{ijm}} Z_i \qquad \forall\, i \in \mathscr{I};\, j \in \mathscr{J};\, m \in \mathscr{M} \tag{26.18}$$

$$\sum_{i \in \mathscr{I}} g_i X_{ijm} \ge \underline{l}_m Y_{jm} \qquad \forall\, j \in \mathscr{J};\, m \in \mathscr{M} \tag{26.19}$$

$$\sum_{i \in \mathscr{I}} g_i X_{ijm} \le \bar{l}_m Y_{jm} \qquad \forall\, j \in \mathscr{J};\, m \in \mathscr{M} \tag{26.20}$$

$$\sum_{m \in \mathscr{M}} Y_{jm} \le 1 \qquad \forall\, j \in \mathscr{J} \tag{26.21}$$

$$\sum_{j \in \mathscr{J}} \sum_{m \in \mathscr{M}} Y_{jm} = p \tag{26.22}$$

$$X_{ijm} \ge 0 \qquad \forall\, i \in \mathscr{I};\, j \in \mathscr{J};\, m \in \mathscr{M} \tag{26.23}$$

$$Z_i > 0 \qquad \forall\, i \in \mathscr{I} \tag{26.24}$$

$$Y_{jm} \in \{0; 1\} \qquad \forall\, j \in \mathscr{J};\, m \in \mathscr{M} \tag{26.25}$$

The objective function (26.15) maximizes the expected participation (measured as the number of patients that are expected to access preventive healthcare service). Equations (26.16)–(26.18) in combination with the objective function (26.15) are a linear reformulation of the MNL choice probabilities. Equation (26.16) ensure that a demand node i's final choice probabilities to go to service facilities as well as non-attendance sum up to at most 1. The case where the sum is less than one can be interpreted as rejecting patients at certain facilities. This formulation guarantees feasible solutions if mismatches between mode thresholds and mode demand exist. As an alternative, we might consider a finer mode structure with much more mode levels instead of only a few coarse ones to avoid infeasibility. This is also a possibility to approximate continuous waiting times.

The linking constraints (26.17) allow choice probabilities for a facility to be greater than 0 only if the facility is established. Using p_{ijm} yields a tighter upper bound by the LP-relaxation than just using $X_{ijm} \leq Y_{jm}$ and tighter bounds for X_{ijm} (Haase and Müller 2015), because p_{ijm} is distinctly smaller than 1. Equation (26.18) ensure that the pre-calculated constant substitution ratios between the choice probabilities for any two alternatives are obeyed. They are derived from $\frac{X_{ijm}}{Z_i} = \frac{p_{ijm}}{1-p_{ijm}}$. However, $X_{ijm} \neq p_{ijm}$ and $Z_i \neq (1 - p_{ijm})$ (unless j is the only established facility).

The correct mode in which a facility is established is selected by (26.19) (lower mode interval threshold) and (26.20) (upper threshold). If a certain facility j is established in mode m, $\sum_{i \in \mathscr{I}} g_i X_{ijm}$ has to be between the lower and the upper mode thresholds.

Equation (26.21) ensure that a facility can either only be established in exactly one mode or not at all. Equation (26.22) provides that p facilities are established. We might use a budget constraint instead, with a parameter denoting fixed establishing costs per facility and mode on the left-hand side and replacing the number of desired facilities p with a budget.

26.6 School Location

School networks are expanded or consolidated to meet expected student demand. Müller et al. (2009), Müller (2008) and Delmelle et al. (2014) introduce multi-period capacitated models for school network planning. Müller et al. (2009) consider free school choice and substitution effects between school locations (Müller et al. 2012) whereby school choice probabilities are determined by a mixed multinomial logit model considering scenarios of opened schools. While minimizing total costs, one scenario is selected for each period. Assuming that the students attend the nearest school, the approach of Delmelle et al. (2014) minimizes student travel costs and has the flexibility to modify the maximum capacity of each school, to integrate the minimum facility age closure, and to reflect the uncertainty of demand projections. Considering free school choice, capacity constraints, a budget, and simulated utility values, Haase and Müller (2013) maximize all students' expected utility. To reduce inefficiencies in school facility location such as travel times, Castillo-López and López-Ospina (2015) present a model of location and modification of school capacity, with the objective to maximize utility (minimize operating costs, minimize travel times, maximize average amount of enrolled students per school, minimize number of schools with multi-grade classes). The process of school choice is modeled by including time and income constraints, and the decisions made by other students (segregation).

Now we discuss a school location model that can be used by private school organizations that want to enter a market or to expand their network. Without loss of generality, we assume that there is one private school provider that competes with public schools. Given already existing own and competing public schools, our objective is to find the optimal location for the establishment of new additional private schools to maximize our market share (number of first-year students that apply for our private schools). We propose to utilize the simulation-based approach introduced in Haase and Müller (2013). We generate a spatial representative (location, numbers) sample of first-year students. We simulate their utility values for all schools by applying a random utility model (e.g., multinomial logit model or mixed-logit model). A student chooses a private school if we establish at least one private school with a utility value larger than the utility values for the public schools.

Defining the sets

- $\mathscr{I}$ set of simulated first-year students (spatial representative sample),
- $\mathscr{J}$ set of candidate private schools, and
- $\mathscr{J}_i$ set of candidate private schools of first-year student i, i.e., for student i, the simulated utility value of school $j \in J_i$ is larger than the largest utility value of all public schools,

the parameters

- n number of expected first-year students, and
- r number of private schools to be established, and

the variables

- X_i = 1 if simulated first-year student i chooses a private school (0, otherwise), and
- Y_j = 1 if a private school is to be established at location j (0, otherwise),

we define the following mathematical model:

$$\text{Maximize} \quad F = \frac{n}{|\mathscr{I}|} \sum_{i \in \mathscr{I}} X_i \tag{26.26}$$

subject to

$$X_i \leq \sum_{j \in \mathscr{J}_i} Y_j \qquad \forall\, i \in \mathscr{I} \tag{26.27}$$

$$\sum_{j \in \mathscr{J}} Y_j \leq r \tag{26.28}$$

$$Y_j \in \{0, 1\} \qquad \forall\, j \in \mathscr{J} \tag{26.29}$$

$$X_i \in [0, 1] \qquad \forall\, i \in \mathscr{I} \mid \mathscr{J}_i \neq \emptyset \tag{26.30}$$

The objective function (26.26) maximizes the expected number of all students applying for a private school. Equation (26.27) satisfies that student i selects a private school if at least one of her preferred candidate private schools is available. Equation (26.28) limits the number of private schools to be established. Haase et al. (2018) show that instances with large sample sizes can be solved by this (equivalent) approach within reasonable time.

26.7 Summary

We briefly discussed recent developments in the literature on the public sector facility location planning (2010–2018). They show that a remarkable part of the applications aim at satisfying the needs of the population, minimizing social costs, or ensuring equity. The current focus of the literature particularly lies on topics such as emergency/disaster management and healthcare facility location as well as on transport-related topics like the location of electric vehicle charging stations or bike sharing systems design (Table 26.3). The evolution of approaches enables practitioners to include more and more relevant planning decision factors to build more realistic models. Especially the consideration of stochastic demand modeled with state of the art methods based on behavioral theory is a promising extension of existing facility location proposals.

Table 26.3 Summary of references in public facility location planning 2010–2018

Application area	Number of papers per year									
	2010	2011	2012	2013	2014	2015	2016	2017	2018	Total
General	4	1	2	1				2		10
Hub location			1	2			1			4
Bike sharing		1	2	1		1				5
Bus network design			3							3
Charging and refueling stations		1			2	2	4	7		16
Waste management					1	1	1	2		5
Emergency shelter location	1	1			1	1	1	3		8
Disaster management			2			1				3
Emergency medical services	1		3	2			1	2	1	10
Healthcare facility location			2	1		3	2		1	9
School location				1	1	1				3
Other applications		1						2		3
Total[a]	6	5	15	8	5	10	10	18	2	79

[a] Not all references listed here are discussed in the text

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