Chapter 24
The Design of Rapid Transit Networks

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Abstract Metros and other rapid transit systems increase the mobility of urban populations while decreasing congestion and pollution. There are now over 210 cities with a metro system in the world. The design of a rapid transit system is a hard problem involving several players, multiple objectives, sizeable costs and a high level of uncertainty. Operational research techniques cannot fully solve the problem, but they can generate alternative solutions among which the decision makers can choose, and they can be employed to solve some specific subproblems. The scientific literature on rapid transit location planning has grown at a fast rate over the past 25 years. This chapter provides an account of some of the most important results. It first describes the main objectives and indices used in the assessment of rapid transit systems. It then reviews the main models and algorithms used to design such systems. The cases of a single alignment and of a full network are treated separately. Then follows a section on the location of stations on an already existing network.

24.1 Introduction

Due to the increasing population and the spread of urbanized zones, many cities and metropolitan areas around the world are planning, constructing or extending their transit systems. Among these, metro systems are the most efficient because they consume less energy and are able to transport more passengers per surface unit than any other form of public transport. Metro systems help decrease private

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car traffic, hence reducing congestion and pollution. The term metro is sometimes used synonymously with rapid transit but the latter has a wider acceptance. In the technical literature, rapid transit usually covers not only metro but also commuter train, light metro, light rail, monorail and others urban mass rapid public transport systems. A metro system is independent from other traffic, even though some light metros or German *stadtbahn* are underground in city centers, but at grade with preference level crossings in suburban areas. There are now more than 210 cities with a metro system and this number keeps growing.[1] Bus rapid transit (BRT) systems are sometimes considered as rapid transit systems. They share several characteristics with those using rails but they exhibit several differences, such as slower vehicles, level crossings, and less capacity. They are usually treated separately in planning processes and in academic research, and they will not be covered in this chapter.

In practice, rapid transit planning is a very complex task involving agents with different backgrounds and loyalties (politicians, urban planners, transit agencies, engineers, construction companies, citizen groups, etc.). These players may therefore have different and sometimes conflicting goals. The planning process usually starts by analyzing the area under consideration and the main travel patterns. Then, based on travel patterns codified by origin-destination flow matrices, some broad traffic corridors are identified and combined, giving rise to several network scenarios which can be evaluated from different points of view, often using finite multi-criteria analysis. Since the problem is inherently strategic, this process usually takes a long time.

Rapid transit planning can be broadly classified depending on whether the network is to be constructed from scratch or whether it is to be extended by adding new lines or extending some existing ones. Rail rapid transit planning lies within the broader field of rail network planning. The sequential process of rail planning is based on the knowledge of the travel patterns and starts with network design. Line planning, timetabling and resource scheduling are the subsequent stages in this process. Other related important issues are reliability, robustness, timetabling information, shunting, platforming, etc. However, due to its special characteristics rapid transit planning deserves a particular study. Usually, the tracks of metro lines are not interconnected. There are exceptions to this rule, for example the cases where there is a common trunk for several lines (Los Angeles, Brussels and Bilbao metros), or the case of a line working as a set of lines but most of the lines work independently. This is the case of the London Underground Northern line with three northern termini and two different routes in the city center, see Fig. 24.1. Some lines in commuter systems also share the railway system in the city centre. This implies that network design and line planning (except frequency setting) are considered jointly in the modeling process. A second specific characteristic of metros in that they carry a large number of passengers traveling over short distances compared with medium and long distance railways. This implies that headways are very short

[1] http://mic-ro.com/metro/table.html.

Fig. 24.1 Northern Line, London Underground

(with the new telecommunication technologies, in some cases these are reduced to one minute and a half). Another distinguishing feature is the importance of mode selection due to the fact that in most metropolitan areas where such systems are planned, several competing modes of transportation (bus, private car) are available.

Rapid transit network design is made up of two intertwined problems: the determination of alignments and the location of stations. There exist other related location problems such as those of locating park-and-ride facilities and depots, but usually their corresponding feasible sets are limited to very few possibilities and thus do not give rise to interesting location problems. The location of stations is a typical attractive facility location problem for which several criteria can be applied depending on the goals of the decision maker. However, a station located in a high density area could be non-efficient because of the direction of the line to which it belongs. For example, if the line goes north-south but the people located close to the station work east or west of the station, this station will not be useful for their working trips. Therefore, it is crucial to concentrate on the location of the alignments and not only on that of the stations. Since the facility to be located is a network, and therefore very large with respect to its environment, the problem under consideration is an extensive or multi-dimensional facility location problem (Mesa and Boffey 1996).

Our aim is to review some of the main aspects of rapid transit location. For the sake of readability, we have avoided the use of lengthy formulations and formulas as much as possible, as well as algorithmic details. These can be found in the original sources. We will first describe in Sect. 24.2 the main indicators used to assess the quality of a rapid transit network. Models and algorithms used for rapid transit network design will be described in Sect. 24.3. In Sect. 24.4 we focus on the location of stations. Conclusions follow in Sect. 24.5.

24.2 Objectives and Network Assessment

The main objective of a collective transit system is to improve the population mobility. Since rapid transit systems usually have a high capacity, they extensively reduce traffic congestion, airborne pollution and energy consumption, thus providing sustainable mobility. Moreover, these systems are among the quickest collective mode of ground transportation, and therefore they usually provide the shortest travel times. Another important feature is their structuring influence on cities since they provide the backbone for the development of residential, business and commercial areas. Rapid transit systems require high-level investments, both for construction and maintenance. The initial investment is related to the construction of tunnels, elevated or at grade right-of-ways, communication systems, and the purchase of rolling stocks. Operating cost include fixed and variable costs on a daily basis.

The agents interested in the planning processes can be broadly classified into three groups: the society in general, which is represented by transportation agencies and government sections, the potential riders, and the companies involved in the planning and construction processes, and offering the service. The first group is mainly interested in global advantages such as those mentioned above. A measure frequently used at the planning stage is the population covered by the system, often defined as the population living within a certain distance threshold from stations. This limit has been fixed to 400 m or 5 min walk in dense areas (Vuchic 2005), but it can grow to one km in less populated regions. Moreover, the catchment areas of stations are not always limited to pedestrian traffic but also to combined modes (Mesa and Ortega 2001). However, ridership is not only a function of the distance to the line, but also of the design of the network (Gendreau et al. 1995). A better measure is the predicted trip coverage which can be measured by origin-destination surveys, coupled with traffic equilibrium models. Potential users are mainly interested in reducing their travel time. A secondary objective of the passengers is to transfer between lines as little as possible. Of course this can be included into a more general and difficult to measure concept of comfort. Finally, the third group, that of construction and operating companies, is mainly concerned with fixed and variable construction and operating costs and revenues.

An existing rapid transit network can be evaluated by means of network measures and indicators, but the same measures can also be used to evaluate potential networks, in particular those resulting from the process of combining corridors. To this end graph theory is a useful tool. Furthermore, these measures can be used as objective functions or as constraints in mathematical programming models. Musso and Vuchic (1988) have developed some network topology indicators such as circle availability, network complexity and connectivity. They have also considered service measures and utilization indicators. Laporte et al. (1997) have also measured the efficiency of rapid transit networks via the passengers/network and passengers/plane measures. For example, these authors have shown that in a circular city, triangle and cartwheel designs are preferable to star designs (Fig. 24.2) in terms of connectivity and travel directness. Saidi et al. (2016) developed an analytical model to determine the optimal number of radial lines in a ring-radial configuration, which can be viewed as a generalization of a cartwheel in which the radial lines do not necessarily intersect at a unique point.

Gattuso and Miriello (2005) provide a comparative analysis of 13 existing metro networks with respect to 10 indicators. Lee et al. (2008) analyzed the Seoul metro network with respect to characteristic path length, radius, diameter, clustering coefficient, network efficiency, weight of edges, strength of nodes, and maximum flows spanning tree. Other indicators such as regularity, service availability, punctuality

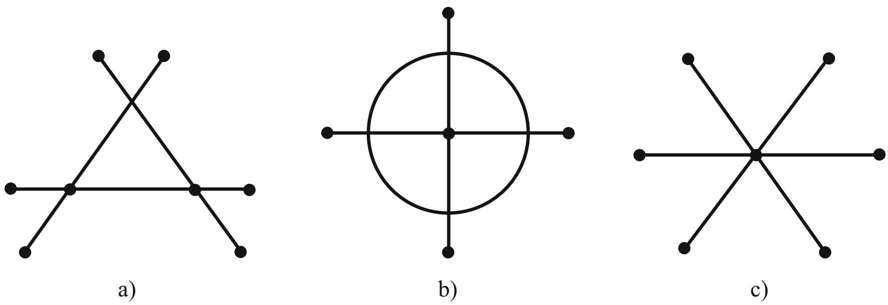

Fig. 24.2 Three basic metro designs. (**a**) Triangle. (**b**) Cartwheel. (**c**) Star

and reliability can be found in UITP (2011). Nowadays, the values of some of these indicators are often presented in the technical reports of operating companies.

Whereas most of the early research on indicators and measures concerns the description and efficiency of the networks with respect to different topological indicators, in recent years we have witnessed the emergence of new indices based on the assessment of transportation networks from the angle of complex network theory and robustness. In accordance with the glossary of IEEE (1990), robustness can be defined as the degree to which a system or component can function correctly in the presence of invalid inputs or stressful environmental conditions. In the case of rapid transit networks planning, future ridership is an uncertainty input variable which also depends on the travel times of alternative transportation modes. As noted by Yang et al. (2017), metro systems that offer a large diversity of routes to passengers, such as the Beijing metro, are also more robust in the presence of disruptions. In a study of the Shanghai metro system, Sun and Guan (2016) found that the metro lines carrying large volumes of passengers have more impact on the system vulnerability, and lines with a circular topological form have a high impact on passenger flow redistribution in the event of disruptions.

Another issue affecting robustness lies in the disturbances of normal operations. The paper of De-Los-Santos et al. (2012) considers robustness from the angle of passengers in the presence of disruptions. The auxiliary function applied to define robustness measures is the total transit time of passengers. Two cases are considered. In the first case, passengers affected by the disruption have to wait for the failure to be repaired or have to take an alternative route in the same network. In the second case, the operator provides a bus-bridge service. An example for the Madrid commuter system illustrates the applicability of the robustness indices developed by the authors.

Several researchers have analyzed rapid transit networks in terms of reliability and robustness in the presence of random failures or deliberate attacks. Thus Zhang et al. (2011) analyzed the effect of attacks on nodes or edges of a network: largest degree node based attacks, highest betweenness node based attacks, and random attacks. In this context, the betweenness of a node or of an edge is the number of shortest paths (defined in number of edges) passing through the edge. They found that the Shanghai subway network is robust against random attacks but vulnerable to highest betweenness node-based attacks. Similar conclusions were later found by Sun et al. (2015) and by Xing et al. (2017) who also studied the Shanghai metro network. Jin et al. (2015) considered the problem of allocating protective resources, such as screening detectors at the entrance of some stations, under the threat of deliberate attacks. They cast this problem in a game theoretical framework and illustrated their methodology on the Singapore network. Yang et al. (2015) assessed the robustness of the Beijing subway system. A related research stream is the study of the resilience of a system, i.e., the speed at which it recovers from a failure. D'Lima and Medda (2015) conducted such a study for the London Underground. For an overview of papers on the vulnerability and resilience of transport system, see the paper of Mattson and Jenelius (2015) which contains a section devoted to rail and public transport networks.

Over the past 20 years there has been an increased research interest in the structural properties of the networks representing complex systems, which is interesting for understanding their functioning. One of the most cited examples in the scientific literature is that of transportation networks and, in particular, metro networks. The concept of small-world phenomenon comes from sociology. The corresponding networks are an intermediate class between regular networks (with equal-degree nodes) and random networks (edge-generated by a given probability). Small-world networks are highly clustered, like regular networks, but they have a low average shortest path length between pairs of nodes (Watts and Strogatz 1998). Let $G = (V, E)$ a graph and let $d_{ij}, v_i, v_j \in V$ be the topological distance between v_i and v_j (the minimum number of edges in a path between v_i and v_j). Then the clustering coefficient C and the characteristic path length L are defined as

$$C = \frac{3 \times \text{ number of triangles}}{\text{number of connected triples}} \quad \text{and} \quad L = \frac{1}{|V|(|V|-1)} \sum_{i \neq j} d_{ij},$$

where triangles are triples of vertices in which each node is connected to each of the other two nodes, and connected triples are sets of three vertices linked to one or two of the other two. In order to adapt these concepts to metric networks and to overcome some indetermination, the average length of shortest paths and clustering coefficients were substituted by global and local efficiency (Latora and Marchiori 2001):

$$E_{glob}(G) = \frac{2}{|V|(|V|-1)} \sum_{i<j} \frac{1}{d_{ij}}, \quad \text{and} \quad E_{loc}(G) = \frac{1}{|V|} \sum_{v_i \in V} E_{glob}(G_i,),$$

where G_i is the subgraph of the neighbors of v_i.

In small-world networks it is easy to travel both at the local and at the global levels. Since such networks are tolerant against disruptions, they are robust. However, metro networks have been shown not to be robust at the local level. Nevertheless, networks of direct connections, where there exists an edge between all pairs of stations for which passengers do not need to transfer to another line, may be seen as small-world networks (Sen et al. 2002; Seaton and Hackett 2004). Other papers dealing with efficiency, robustness, vulnerability and small-world phenomenon of metro networks are those of Latora and Marchiori (2002), Criado et al. (2007), Derrible and Kennedy (2010), Barbadillo and Saldaña (2011) and Zhang et al. (2013). The paper by Roth et al. (2012) also deserves a mention. These authors consider the dynamics of the largest metro networks and prove that they converge to a unique network shape. Xing et al. (2016) studied the connectivity, robustness and reliability of the Shanghai RTS from the viewpoint of complex network theory. Zhang et al. (2016) found that the Minsk and Shanghai metro networks possess the small-world and scale-free properties. They also showed that the hub network is a hierarchical one with a root (the station with the most transfers) which plays an important role controlling some characteristics of the hub network.

A new approach to the study of the connectivity of metro networks and thus their robustness is grounded in the concept of hypergraphs and their associated line graphs. Given a collective transportation network made up of a set of lines $\{L_1, \ldots, L_l\}$, where $L_i = \{s^i_1, \ldots, s^i_{l_i}\}$ is the set of stations of line L_i, the associated hypergraph is the pair $H = (V(H), E(H))$, where $V(H)$ is the set of all stations, and the hyperedge set $E(H) = \{L_1, \ldots, L_l\}$ consists of the network lines. The associated line graphs is $L(H) = (\{L_1, \ldots, L_l\}, E(L(H)))$, where the edge set $E(L(H))$ is the set representing the transfer stations. In Barrena et al. (2013) the indices defined above are extended to collective transportation networks in order to allow them to extract information on the ease of transfer and to compare different metro networks from this viewpoint. In that paper, the notions of clustering, characteristic path length, local efficiency and global efficiency are extended to hypergraphs and are applied to the comparison of several metro networks. Barrena et al. (2015) explore the transfer system of a collective transportation line network taking into account the passenger level by using hypergraphs and their corresponding line graphs. Finally, Criado et al. (2016) define different line graphs for a multiplex network. This concept is useful to study relationships between the edges of a metro network in which each layer of a multiplex network corresponds to a line. It was applied to the computation of local and global efficiency for the light metro of Calgary.

24.3 Location of Rapid Transit Networks: Models and Algorithms

Construction projects for rapid transit networks can be classified into three groups: those in which a single line is planned from scratch (Metro de Granada 2013), those in which several lines are planned from scratch and simultaneously (for example, Sociedad del Metro de Sevilla 2001), and those in which an existing network is to be extended, which corresponds to a conditional network design problem. These problems belong to the class of extensive facility location problems on networks (Puerto et al. 2018).

24.3.1 Location of a Single Alignment

The problem of locating an alignment for a rapid transit system lies within the area of location of one-dimensional structures either in a discrete or in a continuous space (Mesa and Boffey 1996; Díaz et al. 2004), more precisely that of locating paths and networks. Cast in the framework of graph theory, the problem is to select a path between two nodes (which could be fixed a priori fixed) and some of the intermediate nodes to be stations, in order to optimize an objective function subject to certain constraints. In the continuous setting, the problem is that of selecting a straight line, a broken line (a polygonal segment) or a curved segment and some points on it. If the rapid transit line is planned to be at grade, it is almost always necessary to work with a discrete setting, but if the network is to be constructed underground, then a mixed network-continuous space fits better. Here we consider the problem of locating a path and the points on it, leaving the case of locating the stations on a given alignment to Sect. 24.4. Therefore, the decision variables of the problems considered in this section are those of the coordinates of the stations and of the links connecting adjacent stations.

In order to realistically model the problem of locating an alignment, it is necessary to consider several features in addition to those encountered in covering-path problems (Current et al. 1985). These include interstation spacing constraints, competition or intermodality with other means of transportation, demand allocated to pairs of points instead of single point, etc. The early paper of Gendreau et al. (1995) proposes a simple algorithmic approach to the problem of locating a transit line, but without any computational implementation. To our knowledge, Dufourd et al. (1996) provided the first real attempt to solve the problem of locating a transit line taking into account maximum and minimum station interspacing and the number of allowed stations to be located. In this paper, the objective is to maximize the population covered by the stations. This is computed by using several levels of catchment with the use of the Manhattan or ℓ_1 metric. The authors designed a greedy construction procedure to generate an initial solution which was then provided by tabu search. The paper by Bruno et al. (1998) incorporates the more realistic criterion of maximizing trip coverage, as opposed to population coverage. In order to introduce real-world features into their model, the authors consider a private mode of transportation competing with the bimodal pedestrian-public transit mode. Each mode uses its respective network and the demand is assigned to the mode with the least travel cost. The problem consists of computing non-dominated solutions with respect to cost and trip coverage objectives. Bruno et al. (2002) considered the same model as in Dufourd et al. (1996), except for the use of the ℓ_2 metric instead of the ℓ_1 metric for interstation distances. They developed a heuristic consisting of two phases: the construction of the path and the iterative improvement of it. This heuristic was shown to produce better solutions in less time than the tabu search approach of Dufourd et al. (1996).

A similar approach was used in Laporte et al. (2005) to solve the more complex problem of maximizing trip coverage in the presence of an alternative mode of transportation. Instead of considering a binary variable to decide to which mode the demand pair should be allocated, the authors used a continuous variable representing the distribution of the demand of the pair between each mode, according to a logit function which depends on the difference between travel times (or costs) of both modes.

Other objectives have also been employed. For example, in order to avoid possible damage to historical building a modified anticenter path location problem is used in Laporte et al. (2009) to design a metro line as far away as possible from some patrimonial buildings to be protected. The problem was solved with the help of a Voronoi diagram constructed around the protected sites. More recently, Ortega et al. (2018) considered the problem of locating a single alignment in a sprawled city in order to maximize the functional diversity of the districts covered by the alignment and, indirectly, to reduce the need to travel by car in order to satisfy one's current needs. The authors maximized an objective function defined as an entropy measure. They solved the problem by means of a greedy heuristic akin to the construction phase of the Dufourd et al. (1996) heuristic.

24.3.2 Rapid Transit Network Design

We now consider the problem of locating a rapid transit network from scratch, as well as the problem of extending an already located network. The first attempt at modeling and solving the general rapid transit network design problem was presented in the paper of Laporte et al. (2007), which provides a computationally tractable approach consisting of three stages. The first is the selection of key stations, which are the main attraction points: railway or bus stations and airports, hospitals, university campuses, large stores and commercial centers and densely populated areas far away from the central area of the city, etc. The second stage is to connect the key stations to form a core network. Finally, the intermediate stations are located on the alignment resulting from the second stage. In the same paper, a linear integer programming model aiming at maximizing the trip coverage was used in order to solve the core network design problem in presence of an alternative mode of transportation. Later, Marín (2007) relaxed some restrictions on the lines. In his model the number of lines and the extremes of them are not fixed. Cadarso and Marín (2017) later considered transfer effects in rapid transit network design.

With the aim of modeling the user's behavior, Marín and García-Ródenas (2009) introduced a logit function in order to distribute the travelers between the rapid transit and private modes. In order to maintain the linear character of the program, they considered a piecewise linear interpolation of the logit function. In the paper of Escudero and Muñoz (2009) the problem is decomposed into two stages. The first one consists of determining the infrastructure network, and the second one determines the lines. This work was later extended to account for the number of transfers (Escudero and Muñoz 2014, 2016).

A recent methodological contribution to modeling and solving the transit network design problem can be found in Gutiérrez-Jarpa et al. (2013, 2018). These authors take into account the fact that the rapid transit networks are composed of line segments which often have to be constructed within broad corridors defining preassigned configurations. These segments are later assembled into lines. The authors applied two criteria: minimizing construction cost, and maximizing origin-destination traffic capture, and computed Pareto-optimal solutions. Gutiérrez-Jarpa et al. (2017) solved a related problem incorporating three objectives: infrastructure cost, travel time saving yielded by the use of the metro system, and patronage. They performed a study of the trade-offs between these objectives. Laporte and Pascoal

(2015) described a metaheuristic for the solution of a metro network design problem under two objectives: population coverage and construction cost. As in the previous studies, they worked with a predefined configuration defined as a star, a triangle or a cartwheel. They first constructed non-dominated paths corresponding to the segments of the configuration and then assembled them optimally by solving an integer linear programming problem.

Marín and Jaramillo (2008) studied a multi-period capacity expansion problem. In their paper the lines to be opened in each period are determined by taking into account an objective function which is a combination of community, passenger and operator oriented objectives. Since the general problem cannot be solved exactly, a heuristic procedure is designed to solve it. Other approaches to solve the mathematical programming model for the rapid transit network design problems are based on Benders decomposition (Marín and Jaramillo 2009), genetic algorithms (Wang and Lin 2010) and simulated annealing (Fan and Machemehl 2006; Kemanshani et al. 2010). Line configuration with assignment of passengers is studied in Guan et al. (2006).

A recent line of research deals with network robustness aspects. Several ways of treating robustness have been studied: through the application of game theory (Laporte et al. 2010), by providing alternative routes to be used in case of a disruption (Laporte et al. 2011), through the concept of robustness (Cadarso and Marín 2012, 2016; García-Archilla et al. 2013), and of risk aversion (Cadarso and Marín 2016).

Finally, a number of papers now combine two levels of planning in rapid transit network design. The first one, which is strategic, is the location of lines and stations, while the second, which is tactical, is the determination of train capacities and frequencies. These two problems are interrelated because the profitability of a system is a function of the construction and operating costs, and of the revenue related to ridership, which partly depends on travel time and therefore on capacity and frequency. An and Lo (2016) integrate these two planning levels in the framework of stochastic programming. Here the alignments and frequencies are determined in a first phase, and flexible services are offered in a second phase to handle demand overflow. López-Ramos (2014) provides a survey of this line of research. Recent contributions are those of Canca et al. (2016, 2017) and López-Ramos et al. (2017).

24.4 Location of Stations

The problem of locating stations is different in the case of locating a network from scratch than in the case of extending an already existing network. In the first case, several locations attract large volume of passengers and are obvious candidates for stations. The remaining stations must then be located with the help of analytical tools. Assuming that the alignments of the network are given, the problem of efficiently locating the stations arises. The first objective for the community and

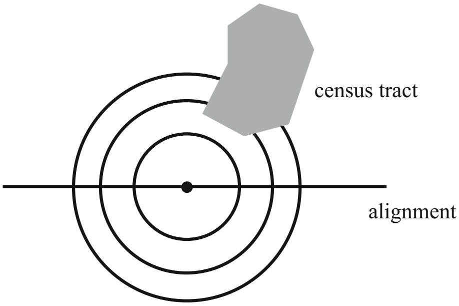

Fig. 24.3 Concentric catchment areas around a station intersecting with a census tract

one of the most important ones for the operating company is to attract as many travelers as possible. To this end, in technical projects the population living in a circle centered at each station is used as an approximation. However, since walking distances are not Euclidean, this is a rough measure for the station attractiveness. In their paper, Laporte et al. (2002) used census tracts coupled with information on population density to estimate the actual walking distances. Different level of attraction were applied in order to obtain a better estimation of the population covered (see Fig. 24.3). For each given location of the stations in a corridor, line coverage was subsequently defined. In that paper, given a discrete set of potential sites for stations, optimal locations are obtained by maximizing the line coverage with the help of an ad hoc defined acyclic graph and a longest-path algorithm.

However, the estimation of future ridership cannot only be based on line coverage since it depends not only on the location of the stations of the line, but also on the overall location of the network. In their paper, De Cea et al. (1986) used origin-destination pairs for computing the total population affected by an improvement of a transportation network. In Laporte et al. (2005), trip coverage was analytically defined and used to compute the network coverage as a good estimate of future ridership. The objective of minimizing the total travel time of passengers was introduced in Vuchic and Newell (1968). These authors considered the case of a population concentrated in a specified area and commuting to a central point. Their aim was to determine an optimal interstation spacing, while taking access time, kinematics of trains, dwell times and intermodal transfer times into account.

There exist a number of papers dealing with the location of new stations on general railway lines. Here we will highlight some of them. Hamacher et al. (2001) studied a problem in which the objective is to maximize the saving in passenger travel time when introducing new stations. Schöbel (2005) considered the maximization of coverage and the minimization of the number of new stations as bicriteria problems. Gross et al. (2009) presented two models combining the number of stations and the distances to them. In the first one, the objective was to minimize the number of new stations assuming that each of these covers a demand located within a predefined distance. The second problem is NP-hard and consists of minimizing the sum of distances from the demand points to the closest (old or new) station under the constraint that the number of new stations is bounded above. The authors have considered two environments for each problem (a planar space with an ℓ_1 metric, and a network), thus giving rise to four cases. For each case, they have identified a polynomial complexity dominating set for the new stations. Körner et al. (2014) have dealt with the problem of locating two new facilities in a mixed planar-network space so that the number of trips between each pair of demand points is maximized. In this paper it is assumed that an alternative mode of transportation exists. The authors have analyzed the cases of segments and tree-networks and have also designed polynomial time algorithms. For the case of more than two facilities to be located on a segment, the big-cube-small-cube method has been shown to be efficient. In a recent paper by Carrizosa et al. (2016), the kinematics of the trains are taken into account in order to minimize the total travel time when a given number of new stops are located, as well as the total travel time of the passengers subject to the coverage of all demand points. Finally, López-de-los-Mozos et al. (2017) recently solved the problem of locating one or two transfer points in a network in such a way that, under various objective functions, the traffic captured by the network is maximized.

24.5 Conclusions

The design of rapid transit systems is a complex process that involves the participation of many players. These projects are fraught with high costs and uncertainty. Formulating models and designing algorithms for such problems is difficult since the objectives and constraints are not as well defined as in many operational research problems. Analytical techniques can be employed to assist decision making or to solve some specific subproblems, but human judgment and intervention remain critical in the planning process. Over the past 25 years we have witnessed a number of important methodological advances in the area of rapid transit location planning. Several quality indices have been developed and mathematical models of increasing realism have been proposed, some of which can be solved directly by off-the-shelf solvers or by powerful heuristics. We expect to see in the near future models and algorithms capable of integrating operational and tactical considerations when solving the problem at the strategic planning level.

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