Chapter 1
Introduction to Location Science

Gilbert Laporte, Stefan Nickel, and Francisco Saldanha-da-Gama

Abstract This chapter introduces modern Location Science. It traces the roots of the area and describes the path leading to the full establishment of this research field. It identifies several disciplines having strong links with Location Science and offers examples of areas in which the knowledge accumulated in the field of location has been applied with great success. It describes the purpose and structure of this volume. Finally, it provides suggestions on how to make use of the contents presented in this book, namely for organizing general or specialized location courses targeting different audiences.

1.1 Introduction

Since the 1960s, Location Science has become a very active research area, attracting the attention of many researchers and practitioners. Facility location problems lie at the core of this discipline. These consist of determining the "best" location for one or several facilities or equipments in order to serve a set of demand points. The meaning of "best" depends on the nature of the problem under study, namely in terms of the constraints and of the optimality criteria considered.

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Location Science is a rich and fruitful field, gathering a large variety of problems. The research conducted in this area has led to the creation of a considerable amount of knowledge, both in terms of theoretical properties and modeling frameworks, together with solution techniques. This knowledge has evolved over time, pushed by the need to solve practical location problems, by technical and theoretical challenges, and often by problems arising in various disciplines. In fact, the interaction with other disciplines such as economics, geography, regional science and logistics, just to mention a few, has always been a driving force behind the development of Location Science. Nowadays, the potential of this field of study in the context of many real-world systems is widely recognized. This book emerges from the need to gather in a single volume the basic knowledge on Location Science as well as from the importance of somehow structuring the field and showing how it interacts with other disciplines.

In this introductory chapter we start by tracing the roots of what is now known as Location Science. This is done is Sects. 1.2 and 1.3. In Sect. 1.4 we present the purpose and structure of this book. Finally, in Sect. 1.5 we provide some suggestions on how to make the best use of the book.

1.2 The Roots

In order to trace the roots of modern Location Science, one must go back to an old geometric problem which is simple to state: What is the point in the Euclidean plane minimizing the sum of its distances to three given points (Fig. 1.1)? This problem is widely credited to the French mathematician Pierre de Fermat (1601–1665)[1] although its origin is a matter of debate (see Wesolowsky 1993).

Since the seventeenth century, different solutions have been proposed for Fermat's problem. There is evidence that the first one is due to the Italian scientist Evangelista Torricelli (1608–1647). The geometric approach proposed by Torricelli is depicted in Fig. 1.2 and can be described as follows: By joining the three given points with line segments, a triangle is obtained. Equilateral triangles can now be constructed on the sides of this triangle, their vertices pointing outwards. A circumscribing circle can then be drawn around each of these three triangles. The circles will intersect in a single point called the Torricelli point or, as some authors call it, the Fermat–Torricelli point. If all the angles in the initial triangle are at most equal to 120°, this point is the optimal solution to the problem; otherwise, the Torricelli point falls outside the initial triangle. In this case, the optimal solution is the initial point located at the apex of the angle greater than 120° (Heinen 1834).

It is interesting to note that nowadays this problem and its extensions still attract the attention of the scientific community (see, for instance, Nam 2013, Görner and Kanzow 2016, Benko and Coroian 2018).

[1]The problem is presented in his famous essay on maxima and minima.

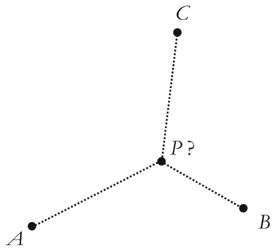


Fig. 1.1 Fermat's problem

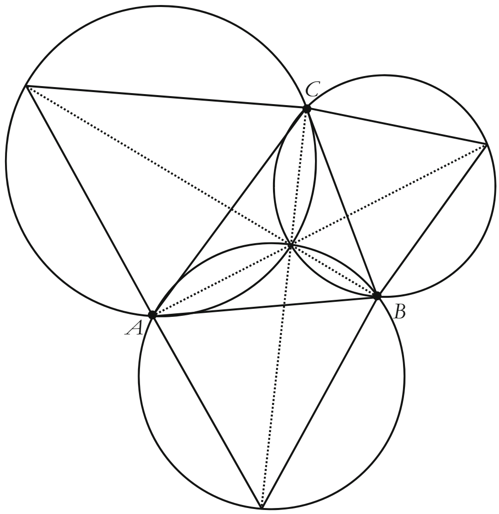


Fig. 1.2 Torricelli's geometric construction for the Fermat's problem

The first documented attempt to position location analysis within an economic context is due to Johann Heinrich von Thünen (1783–1850), an educated landowner in northern Germany. Von Thünen wished to understand the rural developments around an urban center. The results of his analysis were presented in 1826 in a treatise entitled *Der Isolierte Staat in Beziehung auf Landwirtschaft und Nationalökonomie*, which was edited as a book in 1842 and translated into English in 1966 (von Thünen 1842). Figure 1.3 depicts the cover of the 1842 edition. von Thünen (1842) considered an isolated and homogeneous area with an urban center and aimed to discover laws which then governed agricultural prices translating them into land usage patterns. He also considered several types of agricultural activities (e.g., grain farming and livestock) grouped according to their relative economic yield per unit area, their perishability, and the difficulty in delivering the products to the (central) market. His findings led him to postulate that three factors should have a crucial impact on the spacial distribution of the activities: (1) the more perishable a product is, the closer to the market it will be grown; (2) the higher the economic productivity of a product per land area, the closer to the market it will be grown; (3) higher transportation difficulty leads to locating an activity closer to the market.

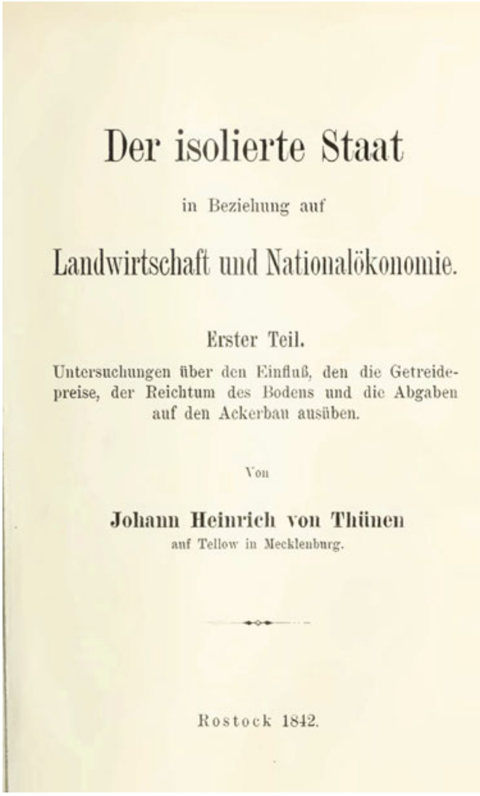


Fig. 1.3 "Der Isolierte Staat" by Johann Heinrich von Thünen, Rostock, 1852 (Source: University of Toronto—Robarts Library, https://archive.org/details/derisoliertestaa00thuoft)

One should therefore expect that the different agriculture activities will evolve in concentric rings around the urban center (Fig. 1.4).

There still exists an intensive debate on the theory of von Thünen (Block and DuPuis 2001). Despite its merit, von Thünen's model is only descriptive, i.e., it is aimed at predicting the behavior of the system. In fact, at the time, models were mostly used to answer to questions such as "why do we do it?". Von Thünen's work can be viewed as fundamental in urban economics and location theory. Nowadays, it is still relevant in areas such as geography, agricultural economics and sociology (Block and DuPuis 2001). These authors emphasize that the centrality theory of von Thünen is still relevant for some dairy products such as milk. Other researchers have pursued von Thünen's centrality idea. The results are reviewed by Fischer (2011).

The first normative location models aimed at determining "what we should do", were proposed by Carl Friedrich Launhardt (1832–1918) and Alfred Weber (1868–1958). Launhardt (1900) introduced the problem of tracing an optimal rail route connecting three points. Interestingly, the author casted this problem within an industrial context. The problem was revisited by Pinto (1977) who stated it as follows: Consider the three nodes depicted in Fig. 1.5. Suppose that w_A tons of iron ore (collected at A) have to be combined with w_B tons of coal (collected at

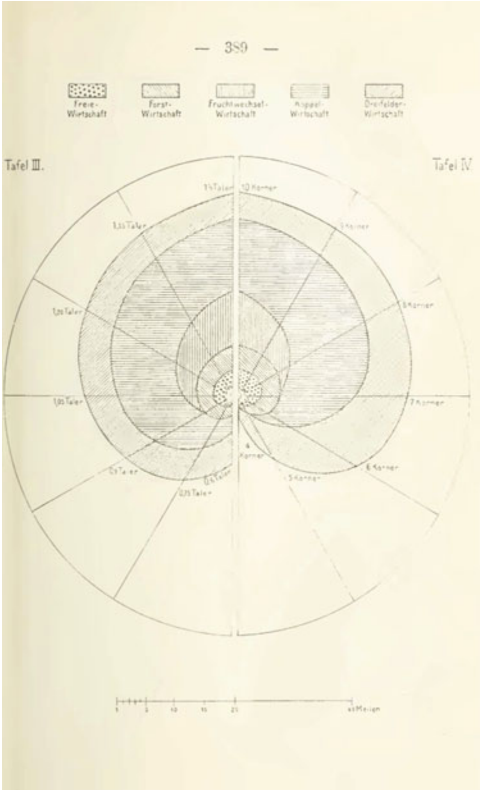


Fig. 1.4 Von Thünen's rings. From "Der Isolierte Staat" by Johann Heinrich von Thünen, Rostock 1842, page 389 (Source: University of Toronto—Robarts Library, https://archive.org/details/derisoliertestaa00thuoft)

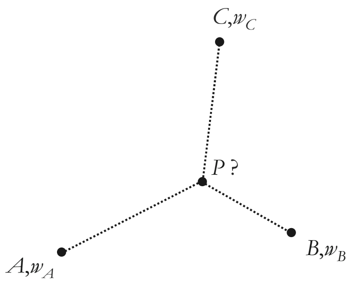


Fig. 1.5 Location problem proposed by Launhardt (1900) within an industrial context

B) to produce w_C tons of pig-iron to be dispatched to C. The problem calls for an industrial facility to be located somewhere between A, B and C. If d_A, d_B, d_C denote the Euclidean distances between the industrial location (to be determined) and nodes A, B, and C, respectively, then the goal is to determine the location of the industrial plant that will minimize the total weighted transportation cost given by $w_A d_A + w_B d_B + w_C d_C$.

This problem introduced by Launhardt is exactly what we now call the 3-node Weber problem. However, as pointed out by Pinto (1977), the problem was

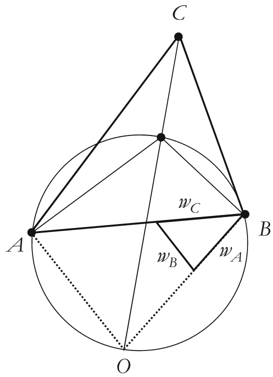


Fig. 1.6 Launhardt's geometric solution

introduced about 10 years before Weber (1909). Indeed, Launhardt (1900) proposed a simple geometric solution scheme for the problem. The solution is obtained as follows (see Fig. 1.6): Consider the triangle ABC defined by the original nodes (the locational triangle) and select one node, say C. Consider another triangle whose sides are proportional to the weights w_A, w_B and w_C—the *weight triangle* as it is referred to by Weber (1909). Draw a triangle AOB similar (in the geometric sense) to the weight triangle but such that the edge proportional w_C has the same length as edge $\overline{AB}$, which is the one opposite to C in the locational triangle. The new triangle AOB is depicted in Fig. 1.6.[2] We can now circumscribe nodes A, B and O, by just touching each point. Finally, a straight line can be drawn connecting O and C. The intersection between the circle and this line yields the optimal location for the industrial facility.

This same problem was treated by Weber (1909) or, to be more accurate, by the mathematician Georg Pick (1859–1942), who is the author of the appendix in which the mathematical considerations of Weber's book are presented. The problem was solved in a different way but this resulted in the same solution. As put by Lösch (1944), the solution to this problem was discovered by Carl Friedrich Launhardt and rediscovered "one generation later" by Alfred Weber. Nevertheless, Weber (1909), presented a deeper analysis of the problem. He first noted that if the geometric construction leads to a point outside the original triangle, then the optimal solution lies on the boundary of the original triangle. Second, he observed that the pole method, which Launhardt (1900) believed should work for polygons with more than three sides, does not necessarily yield the optimal solution when more than three nodes are involved. A practical algorithm for solving the problem with an arbitrary number of nodes was proposed by Weiszfeld (1937).[3] The iterative procedure proposed in this work was revisited in depth more recently by Plastria (2011).

[2]Node O was called by Launhardt the *pole* of the locational triangle.

[3]The author is now known to be Andrew Vázsonyi (1916–2003).

A synthesis of the first steps towards inserting location theory into an economic context is due to Lösch (1944). The importance of this work stems from the fact that, for the first time, location theory and the theory of market areas were connected. This work constitutes the first explicit recognition of the strong link that is often observed between these two areas.

1.3 Towards a New Science

The 1960s set the foundations of Location Science as new scientific area. We first witnessed the natural extension of the Weber problem to the multi-facility case. This was done, among others, by Miehle (1958) and Cooper (1963). In particular, the latter work introduced the planar p-median problem for which each demand node must be served by one out of p new facilities to be located. This became a fundamental problem in Location Science, which still attracts the attention of the scientific community (see the papers by Brimberg and Drezner 2013; Brimberg et al. 2014; Drezner et al. 2015a,b; Drezner and Salhi 2017).

The seminal papers by Hakimi (1964, 1965) opened new important research directions. Hakimi (1964) introduced the concept of absolute median of a graph: a single facility is to be located anywhere in a network so as to minimize the sum of the distances of the nodes of the network to the facility. The author proved that there always exists an optimal solution for which the absolute median is a vertex of the graph. It is also in this paper that the concept of absolute center was first introduced: a single facility has to be located (anywhere in the network) in order to minimize the maximum distance between the facility and all the vertices. This work was extended to the multi-facility case by Hakimi (1965): now, p facilities are to be located. The vertex-optimality property is still valid for the resulting p-median problem. This property is of major importance because it means that many network location problems can be cast into a discrete setting which, in turn, leads to the possibility of using integer programming and combinatorial optimization techniques for tackling these problems.

It is interesting to note that an important step toward the development of discrete facility location problems had been taken the previous year when Manne (1964) proposed the first mixed-integer linear programming (MILP) formulation for a discrete problem which also became classical in Location Science: the uncapacitated facility location problem (UFLP). This model would be revisited later by Balinski (1965) who introduced inequalities of the type $x_{ij} \leq x_{jj}$ ensuring that if a node i is allocated to a node j, then the latter corresponds to a facility and therefore it is assigned to itself. Such inequalities would be later considered by ReVelle and Swain (1970) when formulating the first MILP model for the discrete p-median problem. In the following year, Toregas et al. (1971) introduced the first integer programming formulation for a covering-location problem.

By the early 1970s, the foundations were laid for what would soon become a very active research field. The book by Eiselt and Marianov (2011) describes the works that can be considered to constitute the basis of Location Science.

Within a few decades, significant advances were made in several areas of Location Science, which is attested by several review papers, such as those by Brandeau and Chiu (1989), ReVelle and Laporte (1996), Avella et al. (1998), Hale and Moberg (2003), ReVelle and Eiselt (2005), ReVelle et al. (2008), and Smith et al. (2009).

Initially, the major concern of the researchers had to do with theoretical developments and properties of the problems and their solutions. Much work was developed on continuous and network location problems as well as on fundamental discrete facility location problems. Further links were created with other areas. For instance, the developments in continuous location problems led to the important connection between location analysis and computational geometry. This link remains quite strong to this day. In fact, one of the most relevant structures in computational geometry, the Voronoi diagram (after Georgy Feodosevich Voronoy (1868–1908)), is of major importance in the resolution of many continuous location problems (see, for instance, the review by Okabe and Suzuki 1997). In this volume, we do not focus on computational geometry since there are excellent volumes covering the topic (e.g. Goodman et al. 2017)

Nowadays, location problems can still be categorized according to the location space (continuous, network or discrete), but also according to their context, namely the objectives, constraints or type of facilities involved. Eiselt and Marianov (2011) highlight the three major forms of facility location problems according to the type of objective function: minsum, covering and minmax. For some time, it was also popular to distinguish between public, semi-public and private facility location.

Location Science is highly interconnected with other disciplines and has application in many areas. The theoretical foundations of this area lie in mathematics, economics, geography and computer science. The developments we have observed touch each of these areas.

More recently, stimulated by real-world problems, many areas have emerged where facility location has been applied with great success. Among these, we can point out logistics (see, for instance, Melo et al. (2006), for a problem in the context of logistics network design), telecommunications (see, for instance, Gollowitzer and Ljubić (2011), for a telecommunications network design problem), routing (e.g., in the truck and trailer routing problem introduced by Chao (2002), the location of the trailer-parking places is one of the relevant decisions to make), and transportation (see, e.g., Nickel et al. (2001), for a location problem in the context of public transportation systems). The application of location theory in these areas partially explains why discrete facility location problems have progressively acquired a major relevance when compared with the early developments in Location Science.

Nowadays, Location Science is a very active and well-established research area with its own identity and research community. In addition to the fundamental problems, we observe different research branches being intensively investigated such has multi-criteria facility location, multi-period facility location, facility location under uncertainty, location-routing and competitive location, just to mention a few.

1.4 Purpose and Structure of This Book

As highlighted above, many location problems have applications in other disciplines. Researchers working in these disciplines often encounter location decisions as part of broader problems. From the point of view of researchers coming from the location community, the recent decades have shown that several very successful applications of the knowledge gathered in Location Science require a deep understanding of these disciplines.

In this book, readers will find a full coverage of basic aspects, fundamental problems and properties defining the field of Location Science, as well as advanced models and concepts that are crucial to the solution of many real-life complex problems. The book also presents applications of location problems to several fields. It is intended for researchers working on theory and applications involving location problems and models. It is also suitable as a textbook for graduate courses in facility location. This book is neither a typical textbook with worked examples and exercises, nor a collection of extensive surveys. It is more a book on "what you should know" about various aspects of Location Science; it provides the basic knowledge and structures the field. It is divided into three parts: basic concepts, advanced concepts and applications.

I. Basic Concepts.

   This part is devoted to the fundamental problems in Location Science, which include:

   - Chapter 2: p-median problems;
   - Chapter 3: p-center problems;
   - Chapter 4: Fixed-charge facility location problems;
   - Chapter 5: Covering location problems;

   The goal of this part is to provide the reader with the basic background of location theory. The problems described in Part I serve as a basis for much of the content of Parts II and III.

II. Advanced Concepts

   This part covers models and concepts that aim at broadening and extending the basic knowledge presented in Part I, thus providing the reader with important tools to better understand and solve real-world location problems. The chapters in this part are the following:

   - Chapter 6: Anti-covering problems.
   - Chapter 7: Locating dimensional facilities in a continuous space;
   - Chapter 8: Facility location under uncertainty;
   - Chapter 9: Location problems with multiple criteria;

- Chapter 10: Ordered median location problems;
- Chapter 11: Multi-period facility location;
- Chapter 12: Hub location problems;
- Chapter 13: Hierarchical facility location problems;
- Chapter 14: Competitive location models;
- Chapter 15: Location-routing and location-arc routing;
- Chapter 16: Location logistics in supply chain management;
- Chapter 17: Stochastic location models with congestion;
- Chapter 18: Aggregation in location.

III. Applications

The links between Location Science and other areas are the focus of the third part. By presenting a wide range of applications, it is possible not only to understand the role of facility location in such areas, but also to show how to handle realistic location problems. These applications include:

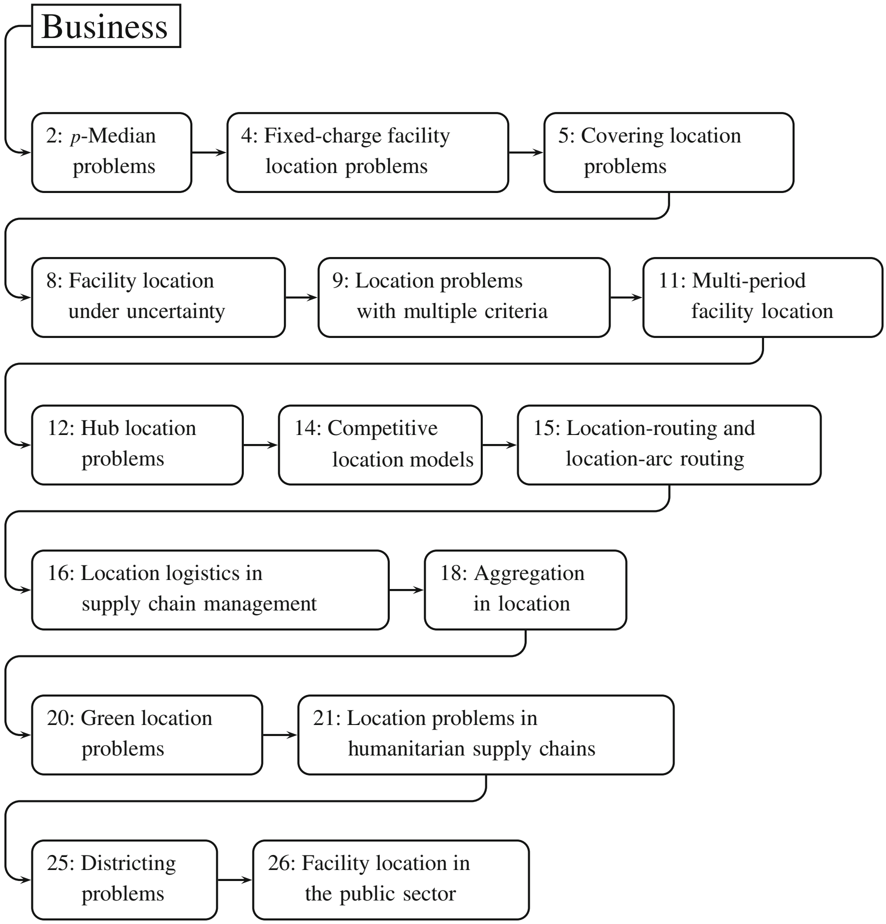
- Chapter 19: Location and geographic information systems;
- Chapter 20: Green location problems;
- Chapter 21: Location problems in humanitarian supply chains;
- Chapter 22: Location problems under disaster events;
- Chapter 23: Location problems in healthcare;
- Chapter 24: The design of rapid transit networks;
- Chapter 25: Districting problems;
- Chapter 26: Facility location in the public sector.

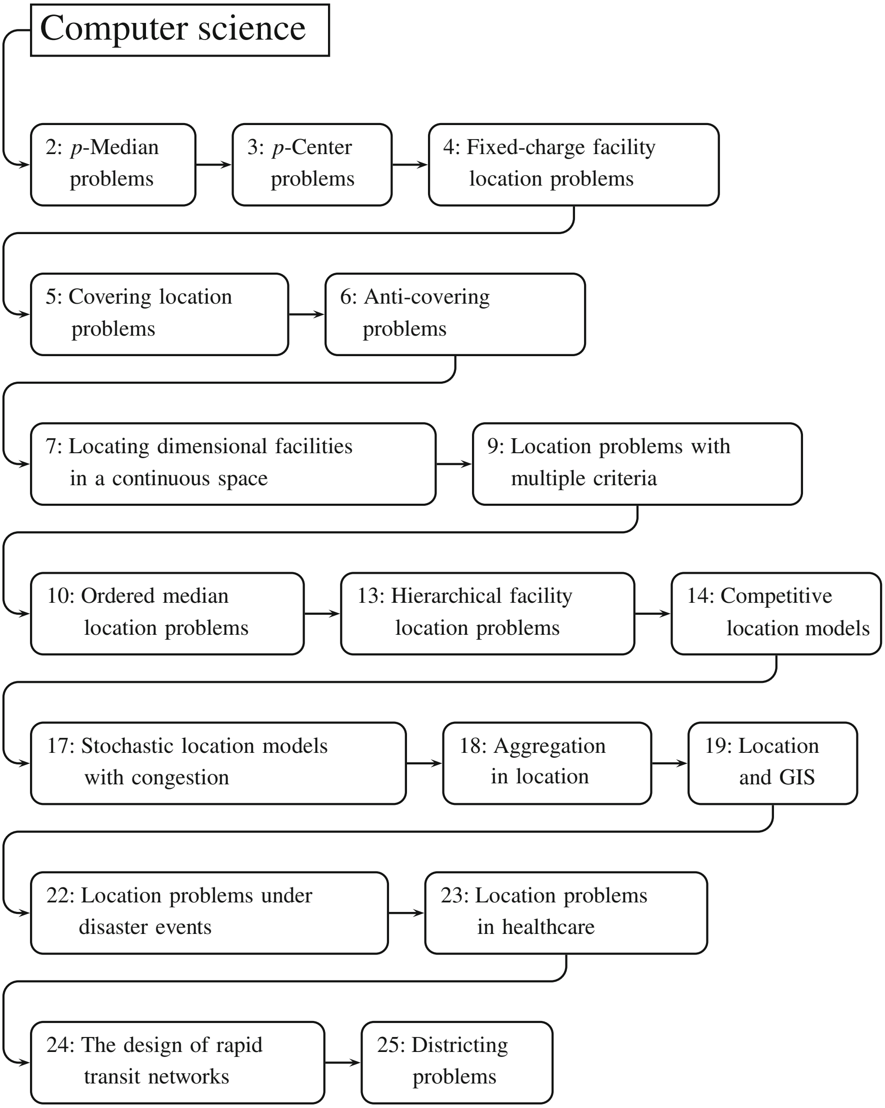
This second edition of *Location Science* should be viewed as a complement to the first edition. It covers topics that were not included in the first edition, such as hierarchical facility location, location problems capturing environmental concerns, location problems in humanitarian logistics, and location problems in the public sector. On the other hand, some topics that were sufficiently covered by the first edition are not part of the current volume. These include quadratic assignment problems and location problems in telecommunications. For such problems the reader should refer to the first edition of the book.

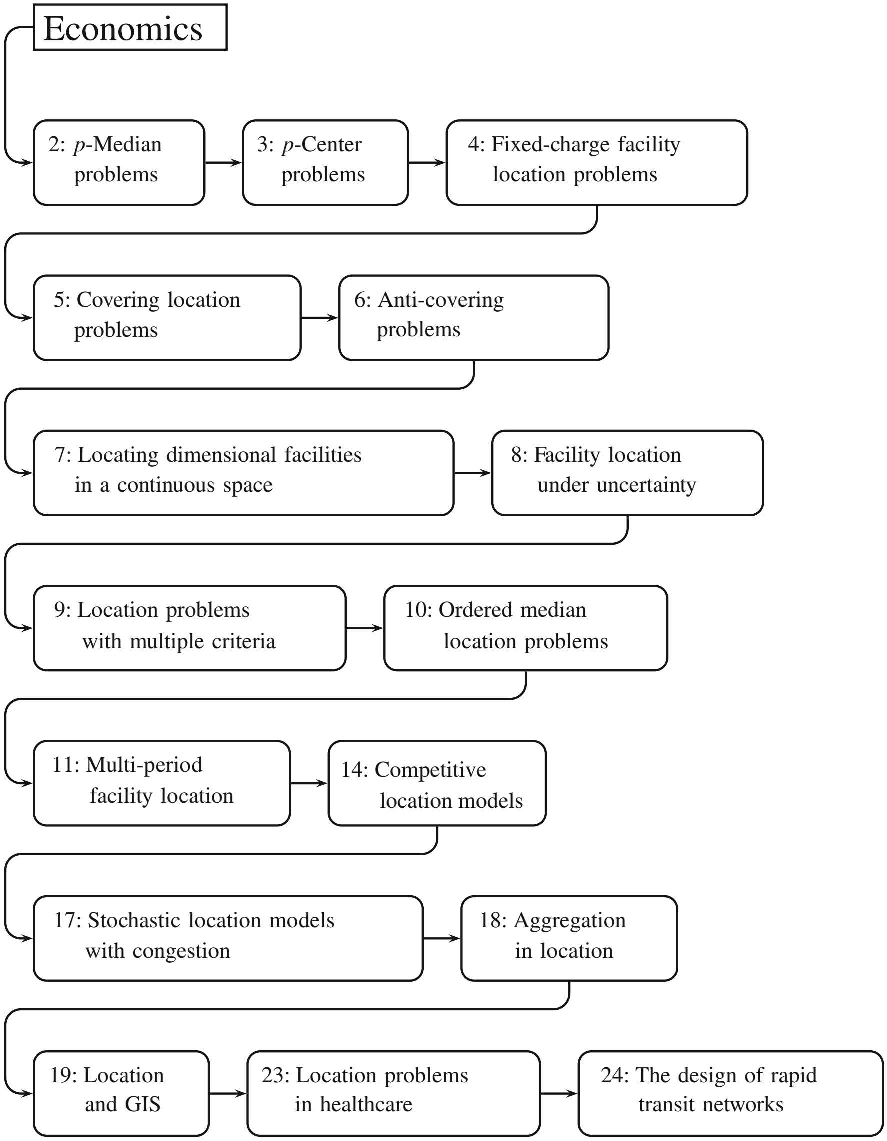
1.5 How to Use This Book

Problems, models, properties, and techniques from Location Science are taught to students enrolled in different programs. We have identified six types of post-graduate curricula having a strong location content: business, computer science, economics, engineering, geography and mathematics.

Depending on the audience, different contents emerge as the most appropriate. This book can be used with the purpose of organizing courses tuned for specialized targets by selecting specific combinations of chapters. Below, we offer some suggestions.


Business
2: p-Median problems
4: Fixed-charge facility location problems
5: Covering location problems
8: Facility location under uncertainty
9: Location problems with multiple criteria
11: Multi-period facility location
12: Hub location problems
14: Competitive location models
15: Location-routing and location-arc routing
16: Location logistics in supply chain management
18: Aggregation in location
20: Green location problems
21: Location problems in humanitarian supply chains
25: Districting problems
26: Facility location in the public sector


Computer science
2: p-Median problems
3: p-Center problems
4: Fixed-charge facility location problems
5: Covering location problems
6: Anti-covering problems
7: Locating dimensional facilities in a continuous space
9: Location problems with multiple criteria
10: Ordered median location problems
13: Hierarchical facility location problems
14: Competitive location models
17: Stochastic location models with congestion
18: Aggregation in location
19: Location and GIS
22: Location problems under disaster events
23: Location problems in healthcare
24: The design of rapid transit networks
25: Districting problems


Economics
2: p-Median problems
3: p-Center problems
4: Fixed-charge facility location problems
5: Covering location problems
6: Anti-covering problems
7: Locating dimensional facilities in a continuous space
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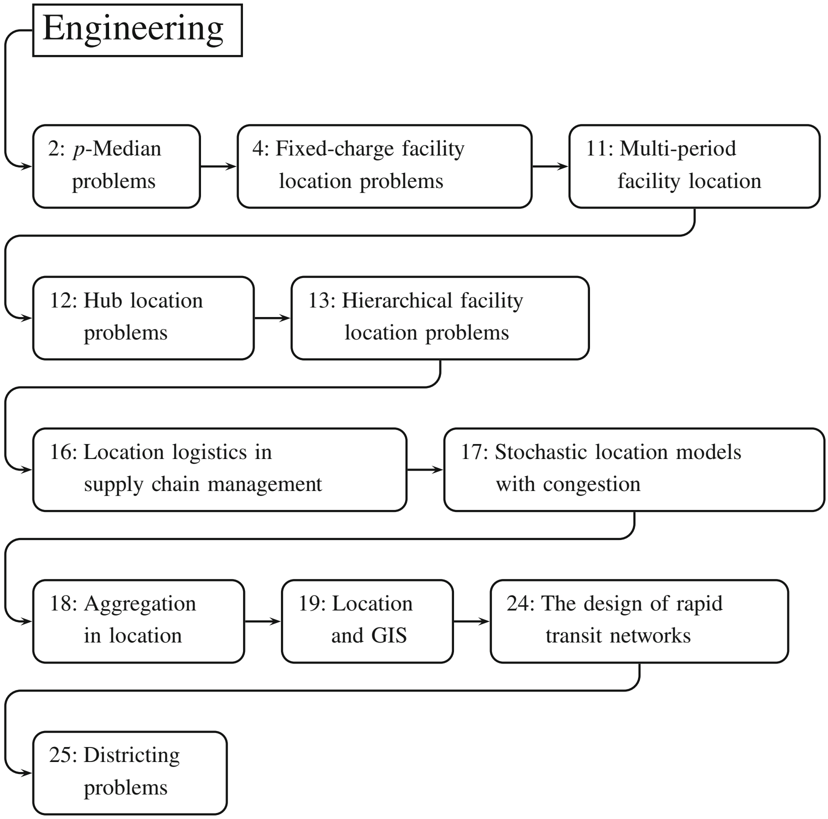
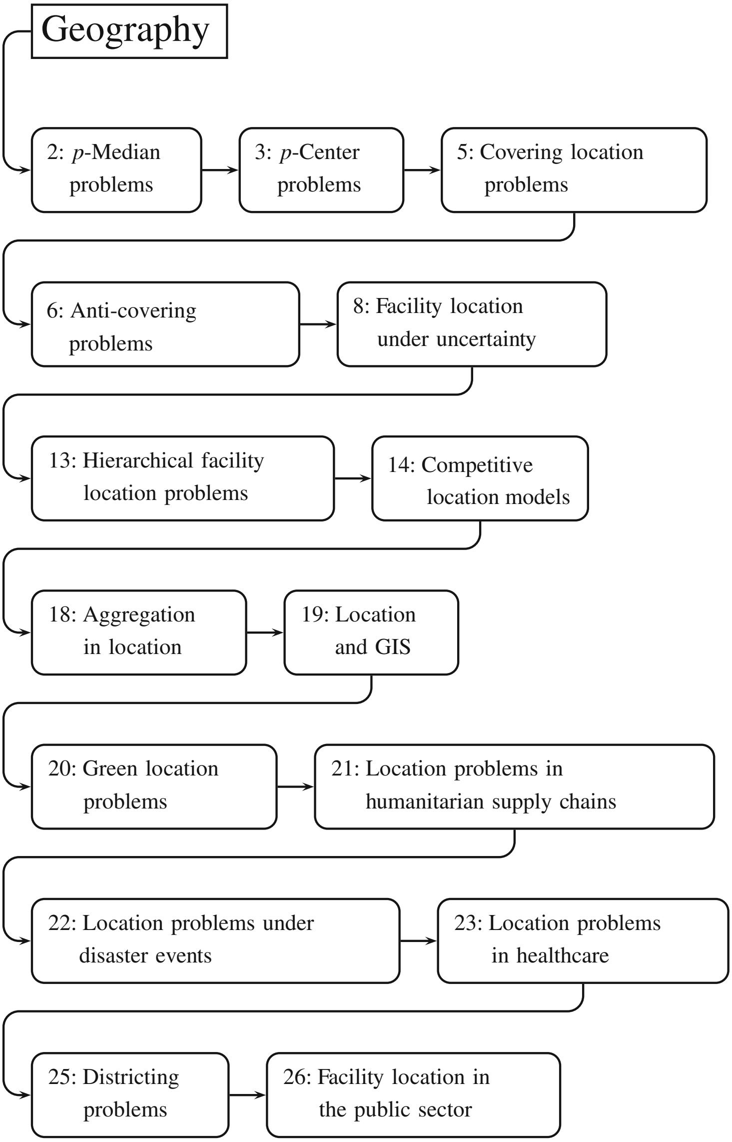
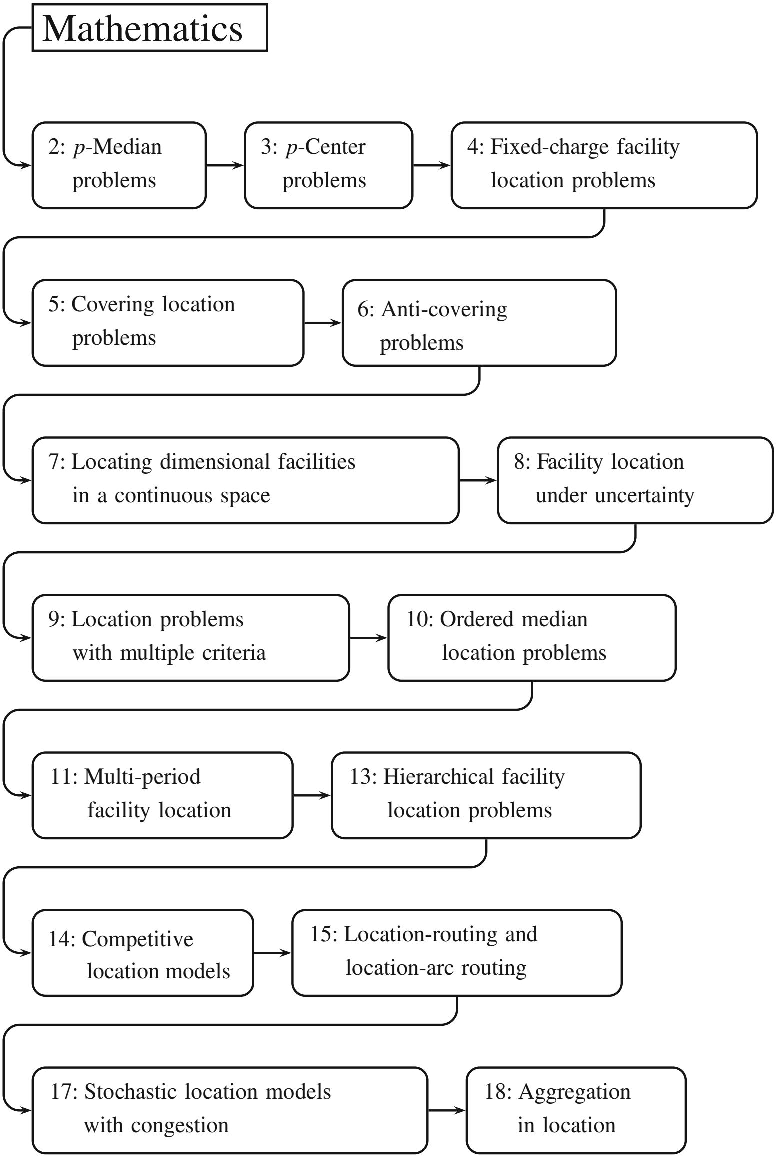
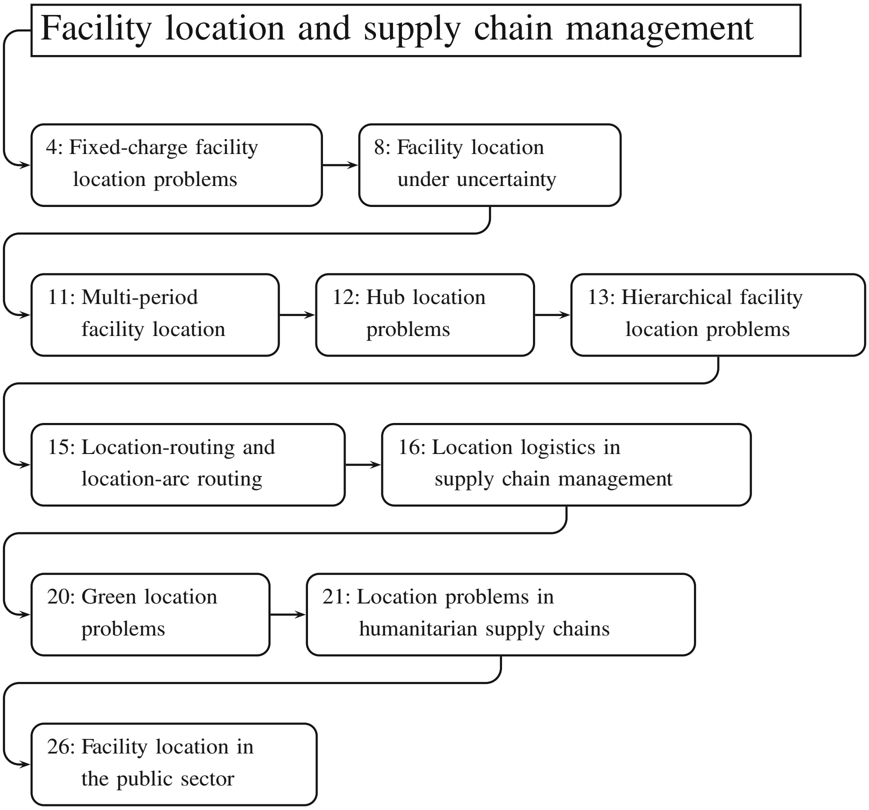
19: Location and GIS
23: Location problems in healthcare
24: The design of rapid transit networks


Engineering
2: p-Median problems
4: Fixed-charge facility location problems
11: Multi-period facility location
12: Hub location problems
13: Hierarchical facility location problems
16: Location logistics in supply chain management
17: Stochastic location models with congestion
18: Aggregation in location
19: Location and GIS
24: The design of rapid transit networks
25: Districting problems


Geography
2: p-Median problems
3: p-Center problems
5: Covering location problems
6: Anti-covering problems
8: Facility location under uncertainty
13: Hierarchical facility location problems
14: Competitive location models
18: Aggregation in location
19: Location and GIS
20: Green location problems
21: Location problems in humanitarian supply chains
22: Location problems under disaster events
23: Location problems in healthcare
25: Districting problems
26: Facility location in the public sector



This book can also be used to build specialized courses in specific areas. Below, we provide examples in four areas: facility location and supply chain management, location of undesirable facilities, location of emergency facilities, and location in transportation systems.


Facility location and supply chain management
4: Fixed-charge facility location problems
8: Facility location under uncertainty
11: Multi-period facility location
12: Hub location problems
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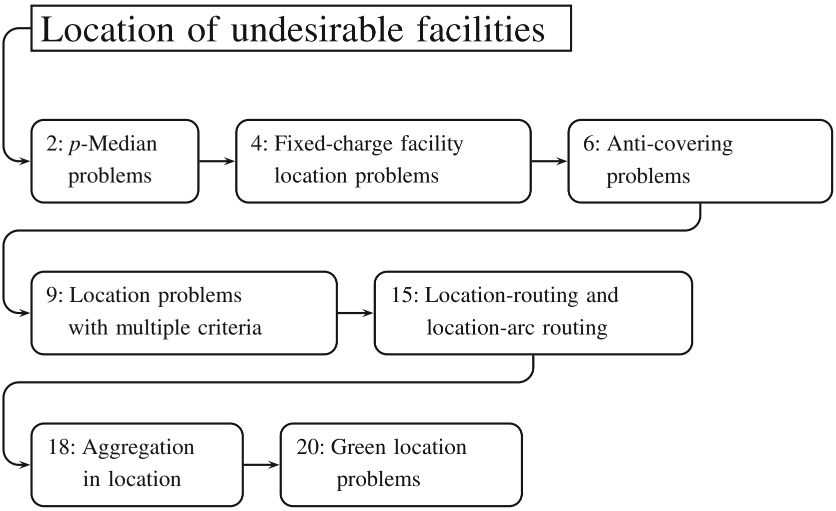
Location of undesirable facilities
2: p-Median problems
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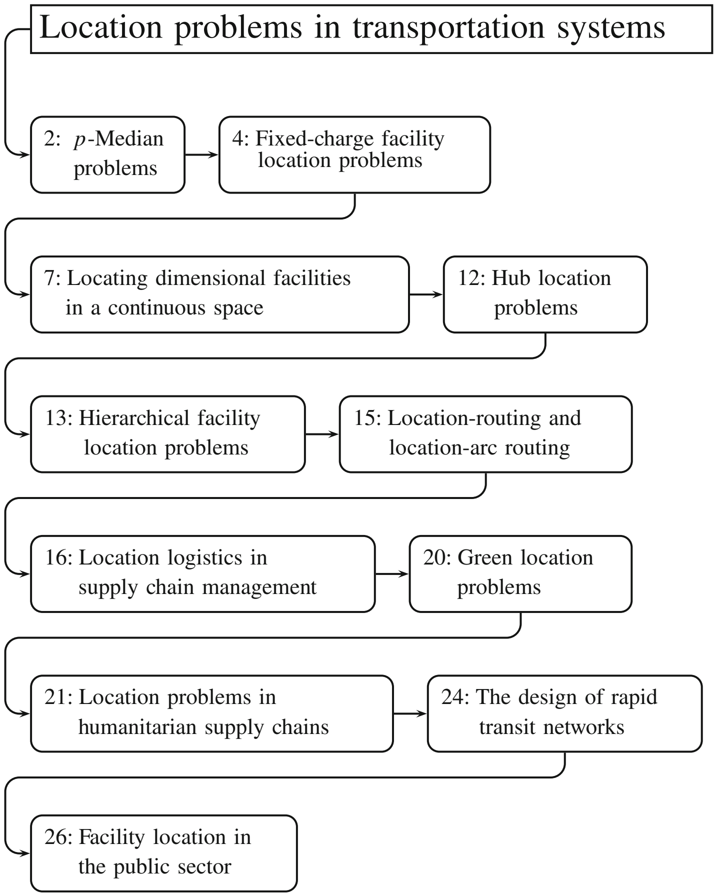
18: Aggregation in location
20: Green location problems


Location problems in transportation systems
2: p-Median problems
4: Fixed-charge facility location problems
7: Locating dimensional facilities in a continuous space
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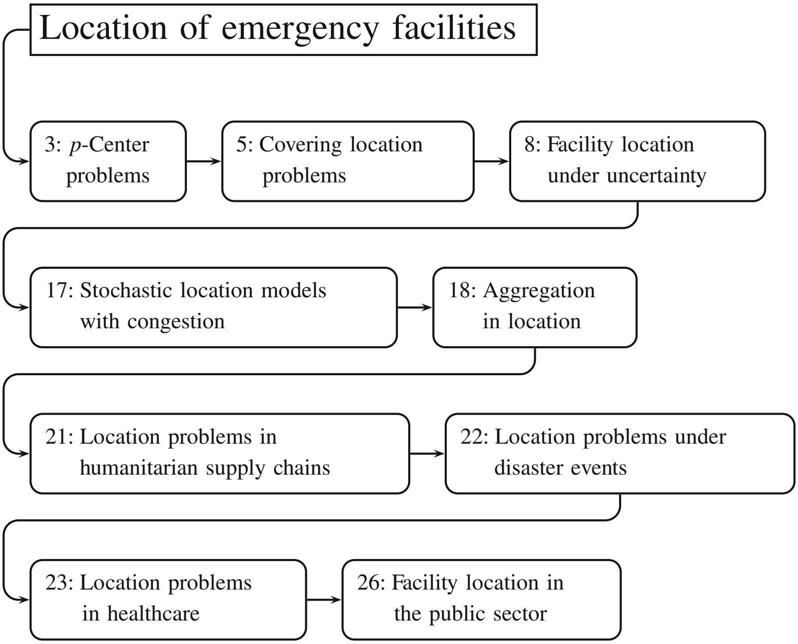

When used for teaching, this book should be complemented with examples and exercises; when used for research, it should be complemented with specialized readings. We found the following comprehensive references particularly relevant: Mirchandani and Francis (1990), Drezner (1995), Drezner and Hamacher (2002), Nickel and Puerto (2005), Eiselt and Marianov (2011), and Daskin (2013).

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