Optimization of Location of Robotic Total Station in 3D Deformation Monitoring of Multiple Points

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Abstract. This paper described an optimization method of deciding the location of RTS (robotic total station) in 3D deformation monitoring of multiple points, which was widely applied in deformation/displacement monitoring in geo-engineering, such as tunnels, excavation, slopes et al. Normally the method of resection and polar coordinate was utilized in deformation monitoring by RTS, in which two datum points should be installed in immovable location and the robotic total stations could be installed in movable location. At the present stage, usually the locations were decided from human experience. This paper presented methods of evaluating the systematic accuracy-functions of the locations of the RTS and two datum points, the idea of which is from the error analysis in the geodesy field. Then the systematic accuracy could be optimized by some optimization methods. In most cases, the problem is a constrained optimization problem with 9 parameters, while in other cases, the problems could be simplified as unconstrained optimization problem, or with only 3 parameters. This method was applicable in installation of the RTS system in tunnel deformation monitoring (usually could be simplified as 2D deformation monitoring optimization with only 3 parameters), and deformation monitoring of excavation engineering.

Keywords: Robotic total station · 3D deformation monitoring · Optimization method

1 Introduction

RTS (Robotic total stations) are widely utilized in the engineering measuring such as excavation engineering, tunneling engineering, and structural engineering [1]. Usually the method of resection and polar coordinate is the method for calculation and installation of the robotic total stations, in which one robotic total station and two reference or datum points are installed. Two datum points should be installed in immovable location and the RTS could then be installed in movable positions. In engineering practice usually the two datum points and the robotic total stations were installed based on human experience and there is no criteria for the decision of position of RTS and datum points. This paper presented a method of evaluating the systematic accuracy of the installation of the RTS and two datum points. Based on the evaluating

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method, a method of minimizing the systematic errors was presented based on an optimization method (in this paper simplex method was utilized).

The position of the RTS and two datum points need to be determined before engineering measuring. For each points there are x, y and z positions, thus there are 9 unknowns in the measuring systems. Generally for a RTS, the measuring error of the distance is Amm ± 1 ppm form and the error of the angle is B″ form (as for one commercial robotic total station, the measuring error of the distance is 0.6 mm ± 1 ppm and the measuring error of the angle is 0.5″). Thus there should be a best position, which has minimum system error during measuring for one measuring project.

There are two possible evaluating methods for the total systematic error of measuring projects. (1) First is that the total adding together error is minimum of all measuring points. (2) Second is that the maximum error of one single measuring point is minimum of the system. This paper give the possible methods of calculating the two different optimized installing points for RTS and datum points.

2 Method

Assume the positions of all the measuring points are already known. The position of the RTS and two datum points needs to be determined. Also assume the total systematic error of the measuring project is only related to the measuring distances and angles, which means other errors such as temperature, sunlight, wind etc. are not considered in this paper. The measuring process of one single point has two steps: (1) Decision of the position of the RTS and the initial direction of the horizontal dial by resection method from two datum points; (2) Decision of the measuring point by polar coordinate method from the RTS. The process is presented in Fig. 1.

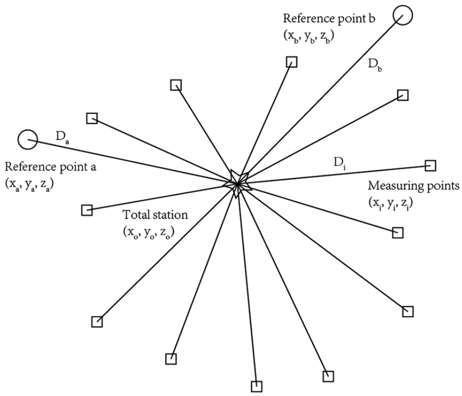


Fig. 1. Position of RTS, two reference/datum points and measuring points.

While S means slope distance and D means horizontal distance. α and β mean horizontal and vertical angles. α_0 means an initial angle and C is a constant. O represented the position of RTS, A and B represented two datum points, and i represented each of the measuring points. The position of all the points was shown in Fig. 1(b). Coordinates of unknowns could be expressed from the known parameters. The total differential could be calculated. The mean square error could be determined by the total differential.

$$\begin{cases} x_o = x_A + D_{OA}\sin(\angle BAO + \alpha_{AB}) \\ \quad = x_A + S_{OA}\sin\beta_{OA}\sin(\angle BAO + \alpha_{AB}) \\ y_o = y_A + D_{OA}\cos(\angle BAO + \alpha_{AB}) \\ \quad = y_A + S_{OA}\sin\beta_{OA}\cos(\angle BAO + \alpha_{AB}) \\ z_o = z_A + S_{OA}\cos\beta_{OA} \\ \alpha_0 = \angle AOy + C \end{cases} \tag{1}$$

$$\begin{cases} dx_o = dx_A + \sin\beta_{OA}\sin(\angle BAO + \alpha_{AB})dS_{OA} \\ \quad + S_{OA}\cos\beta_{OA}\sin(\angle BAO + \alpha_{AB})d\beta_{OA} \\ \quad + S_{OA}\sin\beta_{OA}\cos(\angle BAO + \alpha_{AB})d\angle BAO \\ dy_o = dy_A + \sin\beta_{OA}\cos(\angle BAO + \alpha_{AB})dS_{OA} \\ \quad + S_{OA}\cos\beta_{OA}\cos(\angle BAO + \alpha_{AB})d\beta_{OA} \\ \quad - S_{OA}\sin\beta_{OA}\sin(\angle BAO + \alpha_{AB})d\angle BAO \\ dz_o = dz_A + dS_{OA}\cos\beta_{OA} - S_{OA}\sin\beta_{OA}d\beta_{OA} \\ d\alpha_0 = d\angle BAO \end{cases} \tag{2}$$

$$\begin{bmatrix} m_{x_o}^2 \\ m_{y_o}^2 \\ m_{z_o}^2 \\ m_{\alpha_o}^2 \end{bmatrix} = \begin{bmatrix} \left(\frac{y_{OA}}{S_{OA}}\right)^2 & \left(\frac{z_{OA}y_{OA}}{\rho D_{OA}}\right)^2 & \left(\frac{x_{OA}}{\rho}\right)^2 & 0 \\ \left(\frac{x_{OA}}{S_{OA}}\right)^2 & \left(\frac{z_{OA}x_{OA}}{\rho D_{OA}}\right)^2 & \left(\frac{-y_{OA}}{\rho}\right)^2 & 0 \\ \left(\frac{z_{OA}}{S_{OA}}\right)^2 & \left(\frac{D_{OA}}{\rho}\right)^2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} m_{S_{OA}}^2 \\ m_{\beta_{OA}}^2 \\ m_{\angle BAO}^2 \\ m_{\angle BAO}^2 \end{bmatrix} \tag{3}$$

In which, the mean square error of angle $\angle BAO$ and α_0 ($d\alpha_0 = d\angle BAO$) is unknown and needed to be calculated, the detailed derivation is as follows.

The mean square error of the angle $\angle BAO$, in which O represent the horizontal position of the robotic total station, A represents the horizontal position of one of the datum point and B represents the other.

$$\frac{D_{OB}}{\sin\angle BAO} = \frac{D_{AB}}{\sin\angle BOA} \tag{4}$$

While $\angle BAO$ is an unknown angle and $\angle BOA$ is an angle which could be measured by the RTS.

$$\angle BAO = \arcsin\frac{D_{OB}\sin\angle BOA}{D_{AB}} = \arcsin\frac{S_{OB}\sin\beta_{OB}\sin\angle BOA}{D_{AB}} \quad (5)$$

The total differential of $\angle BAO$ could be calculated as

$$\begin{aligned} d\angle BAO = & \frac{\sin\beta_{OB}\sin\angle BOA}{\sqrt{D_{AB}^2 - D_{OB}^2\sin^2\angle BOA}} dS_{OB} \\ & + \frac{S_{OB}\cos\beta_{OB}\sin\angle BOA}{\sqrt{D_{AB}^2 - D_{OB}^2\sin^2\angle BOA}} d\beta_{OB} \\ & + \frac{S_{OB}\sin\beta_{OB}\cos\angle BOA}{\sqrt{D_{AB}^2 - D_{OB}^2\sin^2\angle BOA}} d\angle BOA \end{aligned} \quad (6)$$

And could be written in matrix form

$$d\angle BAO = K_1[\, dS_{OB} \quad d\beta_{OB} \quad d\angle BOA\,]^T \quad (7)$$

$$K_1 = \begin{bmatrix} \frac{\sin\beta_{OB}\sin\angle BOA}{\sqrt{D_{AB}^2 - D_{OB}^2\sin^2\angle BOA}} \\ \frac{S_{OB}\cos\beta_{OB}\sin\angle BOA}{\sqrt{D_{AB}^2 - D_{OB}^2\sin^2\angle BOA}} \\ \frac{S_{OB}\sin\beta_{OB}\cos\angle BOA}{\sqrt{D_{AB}^2 - D_{OB}^2\sin^2\angle BOA}} \end{bmatrix}^T = \begin{bmatrix} \frac{\tan\angle BAO}{S_{OB}} & \frac{z_{OB}}{D_{OB}}\tan\angle BAO & \frac{\tan\angle BAO}{\tan\angle BOA} \end{bmatrix} \quad (8)$$

Thus the mean square error of the angle $\angle BAO$ could be calculated as

$$m_{\angle BAO}^2 = \left[\left(\frac{\tan\angle BAO}{S_{OB}}\right)^2 \quad \left(\frac{z_{OB}}{\rho D_{OB}}\tan\angle BAO\right)^2 \quad \left(\frac{\tan\angle BAO}{\rho\tan\angle BOA}\right)^2 \right] \begin{bmatrix} m_{D_{OB}}^2 \\ m_{\beta_{OB}}^2 \\ m_{\angle BOA}^2 \end{bmatrix} \quad (9)$$

Then, the error of position of measuring points could be determined,

$$\begin{cases} x_i = x_o + S_i \cdot \sin\beta_i \cos(\alpha_i + \alpha_o) \\ y_i = y_o + S_i \cdot \sin\beta_i \sin(\alpha_i + \alpha_o) \\ z_i = z_o + S_i \cdot \cos\beta_i \end{cases} \quad (10)$$

Also, the mean square errors are,

$$\begin{cases} dx_i = dx_o + \sin\beta_i\cos(\alpha_i+\alpha_o)dS_i + S_i\cos\beta_i\cos(\alpha_i+\alpha_o)d\beta_i \\ \quad - S_i\sin\beta_i\sin(\alpha_i+\alpha_o)d\alpha_i - S_i\sin\beta_i\sin(\alpha_i+\alpha_o)d\alpha_o \\ dy_i = dy_o + \sin\beta_i\sin(\alpha_i+\alpha_o)dS_i + S_i\cos\beta_i\sin(\alpha_i+\alpha_o)d\beta_i \\ \quad + S_i\sin\beta_i\cos(\alpha_i+\alpha_o)d\alpha_i - S_i\sin\beta_i\cos(\alpha_i+\alpha_o)d\alpha_o \\ dz_i = dz_o + \cos\beta_i dS_i - S_i\sin\beta_i d\beta_i \end{cases} \quad (11)$$

$$\begin{bmatrix} m_{x_i}^2 \\ m_{y_i}^2 \\ m_{z_i}^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & (-y_i)^2 & \left(\frac{x_i}{S_i}\right)^2 & \left(\frac{-y_i}{\rho}\right)^2 & \left(\frac{z_i x_i}{\rho D_i}\right)^2 \\ 0 & 1 & 0 & (x_i)^2 & \left(\frac{y_i}{S_i}\right)^2 & \left(\frac{-x_i}{\rho}\right)^2 & \left(\frac{z_i y_i}{\rho D_i}\right)^2 \\ 0 & 0 & 1 & 0 & \left(\frac{z_i}{S_i}\right)^2 & 0 & \left(\frac{-D_i}{\rho}\right)^2 \end{bmatrix} \begin{bmatrix} m_{x_o}^2 \\ m_{y_o}^2 \\ m_{z_o}^2 \\ m_{\alpha_o}^2 \\ m_{S_i}^2 \\ m_{\alpha_i}^2 \\ m_{\beta_i}^2 \end{bmatrix} \tag{12}$$

Thus, the mean square of each direction of position of every single measuring point could be determined. There are two possible methods of evaluation of the overall error of all the measuring points. Assume the mean error of single measuring point is

$$m_{p_i} = \pm\sqrt{m_{x_i}^2 + m_{y_i}^2 + m_{z_i}^2} \tag{13}$$

The overall systematic error of the measuring system could be evaluated by

$$m_{T_1} = \pm\sqrt{\sum m_{p_i}^2} \tag{14}$$

Or by

$$m_{T_2} = \pm\max\left|m_{p_i}\right| \tag{15}$$

Thus, from the two possible systematic error estimation, there are two optimization method: $\min|m_{T_1}|$ or $\min|m_{T_2}|$. In summary, the estimation function could be

$$\min\left(\sum m_{p_i}^2\right) \tag{16}$$

With the name LSM: Least Square Method, Or

$$\min\left(\max\left|m_{p_i}\right|\right) \tag{17}$$

With the name MMM: Minimum Maximum Method.

Basically this optimization problem is a nonlinear constrained optimization problem of 9 degrees of freedom.

3 Optimization

Although the problem is basically a nonlinear constrained optimization problem of 9 degrees of freedom, it could be noticed that the distance measuring error is Amm ± 1 ppm, which will grow with increasing distance. For some cases, the problem could be calculated simply by unconstrained optimization problem, such as the location of RTS measuring multiple points in steel structure engineering, or excavation engineering, or tunnel engineering. Then the problem could be calculated by the network or random method. If the RTS and the datum points are located on a tripod, then the z coordinates of the 3 points are fixed (the height of the tripods). Then the problem could be further simplified as unconstrained optimization with 6 degrees of freedom.

This paper tried the simplex method [2], the simplified calculation process of which is presented as following (Fig. 2).

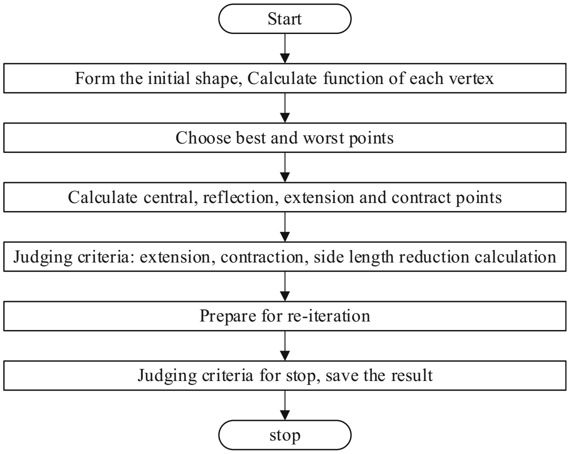
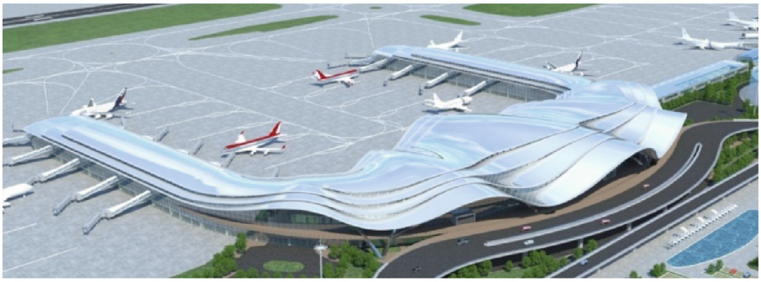


Fig. 2. Simplified process of simplex method.

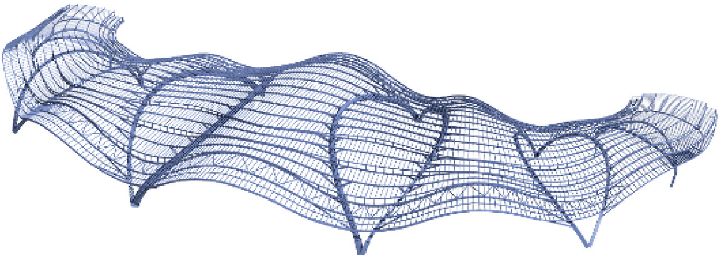
4 Application

Since this paper has not got a chance to test for an excavation engineering project, it has just tried the method for a steel structure, the problem of which is very similar to that of an excavation project in that the positions of all measuring points are above the RTS for a steel structure engineering project while they are below the RTS for an excavation engineering project.

The project is an airport construction project, the designing and 3D modeling of which is presented as Fig. 3. During the construction, some displacement measuring points needed to be controlled as shown in Fig. 4. The length of the structure is 377 m and the width is 118 m. Totally 71 measuring points were on the steel roof structure of the airport terminal. It should be mentioned that the coordinates of all the measuring points are 3D while the RTS and datum points could be only installed on ground with tripods. Thus the problem is reduced to a 6 degree problem, which means on the X and Y coordinates of the RTS and two datum points needed to be calculated.



(a) Designing



(b) 3D modeling of the Steel Structures

Fig. 3. Designing and 3D modeling of the airport steel structure project.

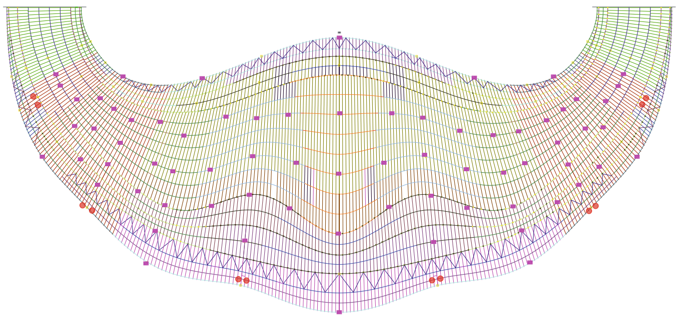


Fig. 4. Controlling measuring points of the projects.

The calculation results are presented as in Fig. 5. It should be noticed that the positions of RTS and two datum points calculated by LSM and MMM are different. If calculated by the LSM, the overall $\min\left(\sum m_{p_i}^2\right)$ is obtained as 11.68 mm^2. If calculated by the MMM, the overall $\min\left(\max\left|m_{p_i}\right|\right)$ is obtained as 0.41 mm^2. Two methods lead to different results and it is the engineers' decision to choose which one to utilize.

It should be noticed that the simplex method is not an optimization method on a global zone and might fall into a local extremum point, which may affect the results from two above different methods. To choose the best point, a global optimization method may be needed in this kind of problem. For engineering practice in multiple point measuring by RTS, the simplex method might be useful such that the total error of the measuring system could be evaluated quantitatively and a relatively more reasonable point could be obtained.

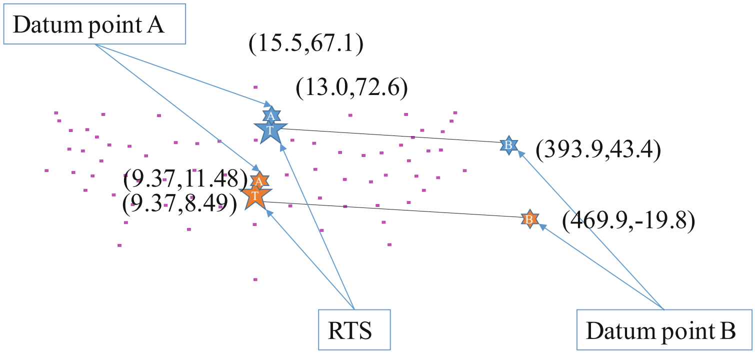


Fig. 5. Results by LSM (blue) and MMM (red) (Relative distance, unit: m).

5 Conclusion

This paper presented methods of quantitatively evaluating the systematic error of deformation measuring by RTS, which were functions of the locations of the RTS and two reference or datum points. The error could be optimized by some optimization methods. This method was preliminary tested in installation of the RTS system in steel structure deformation monitoring and the positions of RTS and datum points could be obtained from two reasonable criterion. This method could be applied in engineering practice.

References

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