Solving the Individuation and Counting Puzzle with λ-DRT and MGL

If I Can Get a Book from the Library, It Saves Me from Needing to Buy It in the Bookshop

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Abstract. Individuation and counting present an open puzzle for lexical semantics. The key challenge posed by this puzzle is that polysemous words can be counted according to different facets, using different individuation criteria for each. Several solutions have been proposed and challenged in the past, and the complexity of the responses expected makes it an interesting and pertinent test for formal theories and automated systems. We present a simple solution for this puzzle using an integrated chain of analysis from syntax to semantics and discourse, with a partial implementation that uses publicly available tools and frameworks. This clarifies the status of logical, compositional formalisms for lexical semantics regarding their ability to handle quantification and individuation. We also discuss the ability of the same formalisms to handle the resolution of discourse-level anaphora and correctly parse utterances introducing multiple lexical facets that are later referred to in the discourse.

Keywords: Individuation · Polysemy · Facets · Coercion · Quantification · Counting · Co-predication · Discourse representation theory · Discourse referents · Compositional lexical semantics

1 Compositionality, Polysemy and Counting

Type-theoretic formalisms and frameworks for compositional lexical semantics can produce logical representations for utterances that use polysemous words, based on works by [10] and [28]. These include Type Composition Logic (TCL) given in [1], Dependant Type Semantics (DTS) introduced by [4], Type Theory with Records (TTR) used in [9], Mereological Copredication detailed in [12], Modern Type Theory-based semantics (MTT) given by [20], as well as our proposed framework, the Montagovian Generative Lexicon (MGL), introduced in [3] and detailed in [30].

For our approach, and most of these frameworks, the linguistic and logical compositionality of the semantics is a prominent feature.

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These approaches are often subject to criticism regarding their capacity to provide a full analysis from syntax to semantics integrating discourse and anaphora resolution; there have been several recent progresses in this area, such as [7] and [32]. One of the purposes of this paper is to show how discourse analysis can be integrated within the MGL framework; preliminary work on this issue has been presented in [17] and detailed in [18] in French, for a specialised purpose. The subtitle of the present paper[1], *If I can get a book from the library, it saves me from needing to buy it in the bookshop*, illustrates a common situation in which lexical adaptations (from the lexeme *book* to the physical object and informational content) take place, and are later referred to in the discourse; our goal is not only to extract the correct facets[2] of the lexeme, but also to correctly link their logical representations to the discourse referents, including in complex discourse structures (as the anaphora only takes place on the *informational* facet in this sentence, while the *physical* facets can be different).

Another difficulty for compositional lexical semantics is the ability of the formalisms to correctly individuate and count facets of polysemous entities. This point is illustrated by the "counting puzzle", which was introduced by [1] and is also known as the "quantification puzzle".

The puzzle arises when combining counting and co-predication on polysemous words. Logically, some predicates can select different facets of polysemous words; when the polyseme denotes a plural set of entities, the facets selected by each predicate typically have different individuation criterion, which should be modelled as different quantifications, resulting in different counts for a single referent lexeme. For instance, in utterrances such as *five books, including three copies of the same novel, were on the shelf; they all burnt, but I had already read them*, classic Montague grammar will result in both predicates being asserted on the same entities (five *books*), while taking polysemy into account should result in *burnt* applying to physical objects (five) and *read* to informational contents (three). The difficulty in producing the logical representation for this "puzzling" utterances is thus not only to extract the correct facet for each predication, but also to quantify according to each facet while keeping a single logical referent (as the predications on the different facets may be added at any point int the discourse).

We claim in [30] that the λ-calculus-based MGL framework can be easily adapted to discourse formalisms that extend DRT (presented in [14] and [5]) with Montagovian composition, such as λ-DRT detailed in [6, 11, 26] and [15] for a straightforward integration with tools such as the Grail syntax-semantics parser described in [25]. Other formalisms have also explored these case, with [2] reviewing discourse phenomena and their relationship with TCL and MTT, and claiming the suitability of such frameworks for this purpose.

We want to make good on these claims, detailing both the logical and computational implementation of the discourse-aware MGL mechanisms, while also presenting a simple solution to the objections pertaining to individuation, and our interpretation of the quantification puzzle.

[1] Example sentence found in the collaborative online translation database, tatoeba.org.

[2] The different senses, or meanings, of a single polysemous word which are logically related can be referred to as *lexical aspects*; we use the word "facet" in order to distinguish from the syntactic notion of "aspect".

1.1 Data

We need to be able to treat situations requiring anaphora resolution (as in prototypical DRT test cases) using polysemous words such as *book* which have different facets and different individuation conditions. We will be using the following example sentences and situations:

(1) Oliver stole the books on the shelf, then read them.

(2) If Emma finds a book that she already read, she leaves it in the store.

(3) *On the table are two copies of van Eijck's and Unger's* Computational Semantics *and one of Pustejovsky's* The Generative Lexicon.

Stanley read all the books on the table.

The sentences above are artificial, but such predications upon the physical and informational facets of books can be found in naturally occurring data, as illustrated by our subtitle; this sentence clearly considers a single informational referent, having two possibly different physical copies.

The lexeme *book* is in fact quite specific in presenting these possible co-predications; as discussed for example in [29], many candidates for co-predications are not productive in actual corpus data. *Books* have been used as prototypical examples with intrinsic polysemy; we have remarked that ontological subtypes such as *novels* tend to exhibit a strong bias towards their information facets. We would argue that most polysemic words have *primary* meanings (such as *events* for meals or *informational content* for most readable materials); it does not change our analysis of polysemes, but *books* will probably be the most difficult one to treat as the primary meaning is not decisively clear-cut between physical object and information content, requiring a lexical adaptation to interpret even in simple sentences.

1.2 The Quantification Puzzle

The supposed difficulty of such examples is that they can refer to entities that share a single facet (the *book-as-information*), but differ on another (the *book-as-physical object*); all while sharing a single discourse referent – the *book* lexeme, not the necessarily the same object in a model of the situation. Such sentences, combining co-predication, individuation and quantification, have been used as tests for formalisations of semantics. Paraphrasing the original from [1], the *quantification puzzle* is that a formal system, when asked the question *How many books did I read?* in the situation given in (3) should answer *two*, while correctly keeping track of the three physical objects and the relationships between the facets of the book entities. While [1] uses the quantification puzzle as an argument for preferring TCL to other formalisations of lexical semantics (including early accounts such as [27], as well as TTR and MTT), many of these other frameworks have in fact responded to the quantification puzzle; it is our belief that all basic logic frameworks can be easily adapted to handle the necessary information required to produce suitable analyses of these situations.

An early, straightforward account for these using MGL and quantification has been given in [22]; the present paper does not really depart from this first formalisation, as it incorporates all necessary mechanisms. A complete detailed formalisation built upon these examples has been made in [12] using mereology, and recent dedicated publications such as [13] and [8] abound. A common thread to all of the solutions for the quantification puzzle is to examine the distribution of the quantified sets of arguments when applied to predicates requiring different types, and to illustrate how they are individuated, and thus counted, differently. This can be done in many ways, with TCL providing different quantifiers depending upon different typing presuppositions, MGL quantifying on objects that are accessed through different lexical transformations according to their facets, [13] presenting a component-wise account, and MTT labelling each entity with a setoid, a mathematical object consisting of the pairing of a type and an individuation criterion.

Thus, every approach solves this puzzle by providing a coherent way to count polysemous terms in different ways. Yet, arguments against these accounts persist, such as [19] and [21], and we must thus consider the issue as still unsolved.

2 Anaphora, Coercions and Facets in Compositional Discourse Semantics

Our goal being to have a correct analysis of shifts in lexical meaning and their propagation through discourse references, as well as the ability for the resulting analysis to solve the counting puzzle. We present, in order, the Montagovian Generative Lexicon; its use on λ-DRT output obtained by the *GrailLight* parser; and consequences on individuation and counting.

2.1 MGL: A Montagovian Analysis for Generative Lexical Semantics

In order to compute the correct logical representations and infer the cardinalities of each predication on the different facets, MGL uses a compositional treatment chain (inspired by and extended from Montague Grammar). The syntax of the sentence is analysed using categorial grammars (here, a version of type-logical grammars), and a semantic term is computed according to Lambek calculus, yielding an output in λ-DRT suitable for anaphora resolution. The meaning shifts, known as *linguistic coercions*, that allow access to specific facets of polysemous terms (such as the informational and physical facets of *book*) and co-predications on those facets are inserted according to the mechanisms detailed in [30]:

- each lexical term is associated to a single "main" λ-term, representing its primary denotational meaning (as determined arbitrarily: *book*, being fully polysemous, is given as an entity of a generic *Readable* type),
- each lexeme might also be associated to a number of "optional" λ-terms, the *lexical transformations* that model type coercions, that might be used whenever type mismatches occur (*book* is thus associated to accessors for the two facets, modelled as transformations from *readable* to *physical* and *informational* entities respectively).

This yields logical formulæ typed using terms of a many-sorted logic ΛTY_n, distinguishing between ontological categories (the pertinent sorts in the examples are: *P* for people, R for readable objects including the generic *book*, I for informational content, φ for physical objects, etc). These formulæ will include *type mismatches* whenever polysemous terms are used as arguments for predicates that require a facet that is not their primary denotation (and systematically so with *book*).

The linguistic coercions that enable the disambiguation of polysemous terms are modelled by the optional terms, as transformations lexically associated to the lexemes (f_I, an accessor to the information facet, and f_φ, an accessor to the physical facet, are available for terms representing *book*; there can be many more of these terms including accessors for *qualia* so that sentences such as *The book is finished* can be analysed correctly). Inserting the optional terms in suitable positions guided by the types make the coercions apparent, and the typing of the formula correct: *a book on the shelf* will be analysed as $\exists x^R.\text{on\_shelf}(f_\varphi(x)^\varphi) \wedge \text{book}(x^R)$. Co-predication is made possible by the use of higher-order operators such as the polymorphic conjunction &: *the book is heavy and interesting* is analysed as $\exists x^R.(\&\ \text{heavy}^{\varphi\to\mathbf{t}}\ \text{interesting}^{I\to\mathbf{t}}\ x) \wedge \text{book}^{R\to\mathbf{t}}(x)$, which is resolved as $\exists x^R.\text{heavy}(f_\varphi(x)) \wedge \text{interesting}(f_I(x)) \wedge \text{book}(x)$.

MGL, as its core, is a mechanism modelling the analysis methodology used in lexical semantics; a means to compute the correct logical representation with explicit lexical transformations given the relevant data. It supposes a rich database of lexical information that is challenging to obtain: most of our examples are treated by hand. Another possibility is the conversion of existing resources as described in [16], starting from an extensive crowd-sourced lexicon; however, the diverse nature of the available data makes it difficult to provide a non-supervised automated conversion. MGL provides a series of mechanisms for fixing type mismatches; the variety of types available, dependent on the granularity of the lexicon, force those mismatches to occur. If all sorts are coalesced together into the single Montagovian type **e** for entities, the classic analysis given by Montague grammar is still available – but the disambiguation of the polysemous terms do not occur at all.

Regardless of the mechanisms used to make the lexical transformations (or linguistic coercions) apparent, a common feature for all systems handling lexical semantics is that they can generate many different suitable interpretations. For example, *synecdoche* is a common language usage in which a word denoting a complex object is used to refer to a specific part. In the case of mechanical malfunctions, the word *car* can be commonly used to mean the *tire*, *motor*, *battery* or *tank* of the car (as in *my car won't start* or similar sentences). Lexical semantics predicts that there are many different coercions from vehicles to physical objects, including many parts of the vehicle as well as the vehicle itself; when making a predication on the word *car* using a predicate that take *physical objects* as its arguments, there will be many possible interpretations that should be generated. Which one of those, if any, is correct is determined by *preference*, filtering implausible meanings and scoring the possible ones. This can be done by hand, but also automatically, according to available linguistic data; this process is illustrated in [16], demonstrating how a single source of lexical data can be used both for extracting and ranking the different lexical transformations available for each lexeme.

2.2 Anaphora Resolution, Discourse Analysis and Disambiguation

In summary, the processing of the utterance starts with syntactic analysis, processes with anaphoric resolution in λ-DRT (both steps are automated using *GrailLight*); afterwards, MGL-based disambiguation is applied to the output. The following figures illustrate the processing of the example sentences, detailing first (1) (Figs. 1 and 2).

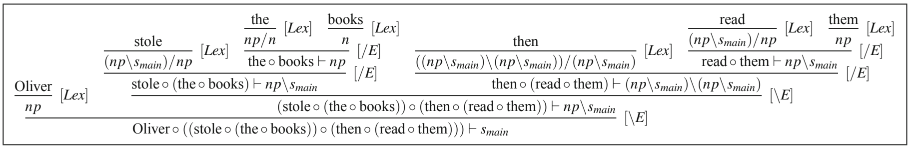

Fig. 1. Automated syntactical analysis for sentence (1) produced by GrailLight [25].

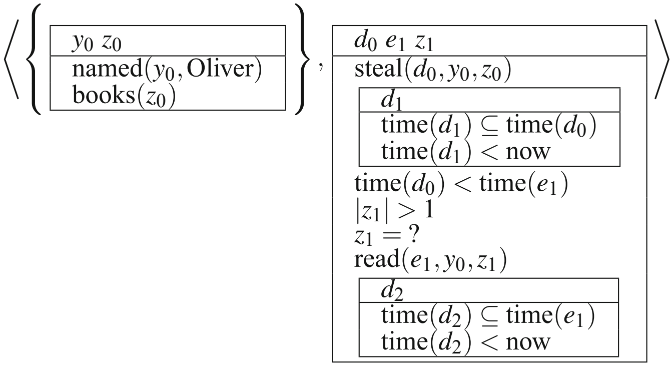

Fig. 2. Resulting DRS for sentence (1) automatically produced by GrailLight [25].

The binding process after this first discourse analysis presents interesting challenges. The anaphoric pronoun *them* (represented by the variable z_1) can safely be bound to the only plural antecedent *the books* (represented by the variable z_0). The event semantics, here given in Davidsonian fashion, summarise the relationship between the tenses of the two predicates and the succession adverb "then". Ignoring the technical sub-events d_1 and d_2 (which represent the past tense), adding missing type information (using the characteristic function to represent the set of readable objects $R \to \mathbf{t}$ for *book* and the person sort P for *Oliver*), a straightforward translation of the DRS above is:

$$\exists d_0^{\mathbf{evt}} e_1^{\mathbf{evt}} z_0^{R\to\mathbf{t}}.\mathrm{books}(z_0) \wedge \mathrm{steal}(d_0, \mathrm{Oliver}^P, z_0) \wedge \mathrm{read}(e_1, \mathrm{Oliver}^P, z_0)$$

As we discussed in [24], the plural predications can be distributed on all members of the set by a functional coercion of the predicates, resulting in:

$$\exists d_0^{\mathbf{evt}} e_1^{\mathbf{evt}} z_0^{R\to\mathbf{t}} \forall z^R.[z_0(z) \Rightarrow \mathrm{book}(z_0) \wedge \mathrm{steal}(d_0, \mathrm{Oliver}^P, z) \wedge \mathrm{read}(e_1, \mathrm{Oliver}^P, z)]$$

That is, the characteristic function z_0 has the property that all items z for which the function is true are books such that Oliver both stole and read them.

At this point, the type mismatch between the *book* argument (z^R) and the predicates (expecting physical and informational arguments respectively) are resolved by inserting the relevant transformations, making the coercions apparent in standard MGL fashion and solving the type mismatches.

An interesting point here is the interaction of the event semantics with the plural transformation, as the act of stealing a number of (physical) books can be done at one (or several) times, but the reading of each (informational) book will typically be accomplished one at a time; stealing and reading events can (and should) be distributed differently. As we suggested in [24], this is treated as a quantifier scope ambiguity where there is either a single stealing event d_0 in which all books z are stolen ($\exists d_0 \forall z$ reading) or there is a possibly different, event for each book ($\forall z \exists d_0$ reading). A similar scope ambiguity exists for the reading event e_1 and the universal quantifier, with the interpretations being one event for every book, and a single reading event for all of the books.

The following is the fully-typed formula for sentence (1), assuming wide scope for the existential quantifiers:

$$\begin{aligned}\exists d_0^{\mathbf{evt}} e_1^{\mathbf{evt}} z_0^{R\to\mathbf{t}} \forall z^R.[z_0(z) \Rightarrow \\ &\mathrm{book}^{R\to\mathbf{t}}(z) \wedge^{\mathbf{t}\to\mathbf{t}\to\mathbf{t}} \\ &\mathrm{steal}^{\mathbf{evt}\to P\to\varphi\to\mathbf{t}}(e_1, \mathrm{Oliver}^P, f_\varphi^{R\to\varphi}(z)) \wedge^{\mathbf{t}\to\mathbf{t}\to\mathbf{t}} \\ &\mathrm{read}^{\mathbf{evt}P\to I\to\mathbf{t}}(d_0, \mathrm{Oliver}^P, f_I^{R\to I}(z))]]\end{aligned}$$

It is correctly asserted here that all physical copies of the books were arguments of *steal*, and all informational content of the same books has been read without duplication.

Example (2) should be treated with DRT-compliant mechanisms for anaphoric resolution, as it has been modelled on classic donkey sentences. The syntax-semantics-discourse analysis proceeds as previously, and there is a single (quantified) event for each predicate.

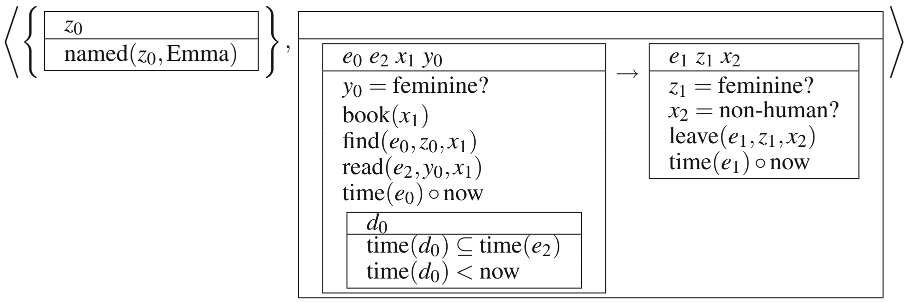

Fig. 3. Initial DRS for sentence (2).

The categorial grammar syntactic analysis ensures the object of *read* must be x_1 (the *book* in the antecedent). Anaphoric resolution binds y_0 and z_1 to the sole female human discourse referent (Emma) and x_2 to the sole non-human, non-event discourse referent x_1, transforming the initial DRS given in Fig. 3 to the structure in Fig. 4:

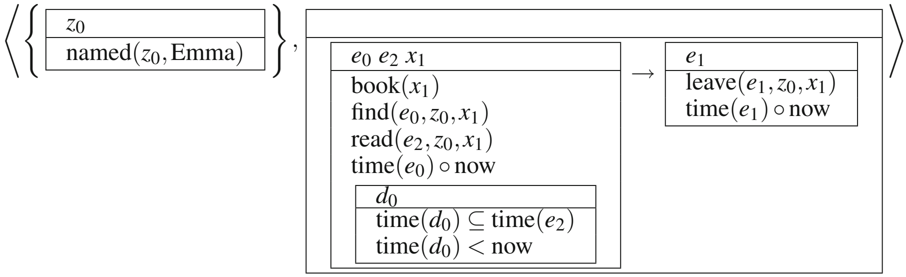

Fig. 4. DRS for sentence (2) after anaphora resolution.

The final interpretation of the sentence as a logical formula is thus:

$$\forall e_0^{\mathbf{evt}} e_2^{\mathbf{evt}} x_1^R.[[\text{book}(x_1) \ \wedge \ \text{find}(e_0, \text{Emma}^P, f_\varphi^{R\to\varphi}(x)) \wedge \text{read}(e_2, \text{Emma}^P, f_I^{R\to I}(x)))] \\ \Rightarrow \exists e_1^{\mathbf{evt}}.\text{leave}(e_1, \text{Emma}^P, f_\varphi^{R\to\varphi}(x))]$$

This correctly asserts that a single book has been found and left as a physical object, and that the informational content of this book has been read, but not necessarily using the same physical object as a support.

2.3 A Partial Implementation of the Treatment Chain

Several tools and frameworks are available for an automated implementation of this formalisation. We have used GrailLight for the production of both the syntactic analysis (using Type-Logical Grammars and variations of the Lambek calculus, with a small English lexicon in the same spirit as the much larger one used for the wide-coverage French parser for GrailLight) and the production of the λ-DRSs for reference resolution and anaphora binding.

Semantic disambiguation via MGL can be performed using techniques described in [23], including the distribution of collective predicates. However, event semantics are not yet implemented in MGL, and neither is scope ambiguity for event variables in GrailLight. Despite our efforts, the biggest roadblock to a fully implemented treatment chain remains the lexicon of polysemous terms with rich types, which has to be done by hand. Our treatment chain, from the text given as plain English utterances and discourse, to λ-DRSs giving a logical representation making the binding of the discourse referents apparent via a Lambek-style proof of the syntax, and then to a disambiguated logical form using MGL to insert the correct lexical transformations, is summarised in Fig. 5 below.

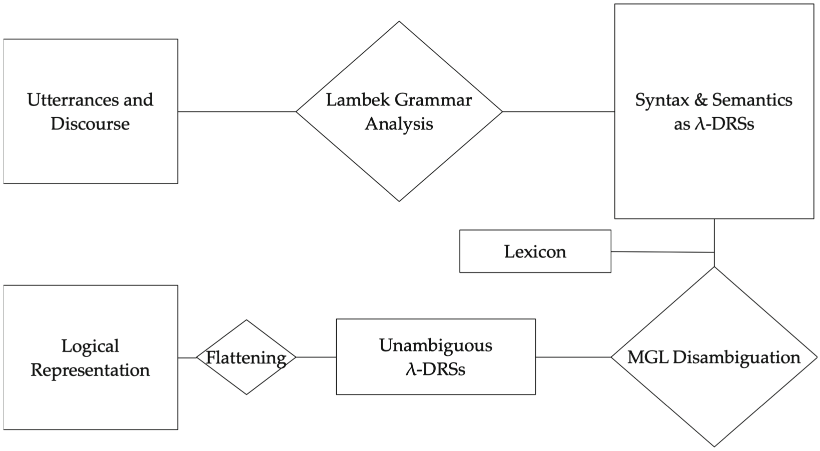

Fig. 5. Our complete treatment chain.

This process supposes that one discourse-level analysis is sufficient in order to resolve the bindings of the referents, and that lexical facets can be determined afterwards. An objection would be that some situations can require the new lexical facets that have been extracted by MGL mechanisms to act as different discourse referents afterwards; while we have no examples of this situation (co-predications being made on a single logic referent), handling it correctly would require to integrate MGL as a GrailLight component.

2.4 Individuation Criteria for Facets

The ability to analyse complex sentences with anaphoric references and co-predications, and to correctly apply lexical transformations that makes predicates access the correct facet of arguments, is sufficient to provide correct individuation criteria and to solve the quantificational puzzle. When making a co-predication on some polysemous term, such as $\lambda x^R.(P^{\varphi\to\mathbf{t}}(f_\varphi(x)) \wedge Q^{I\to\mathbf{t}}(f_I(x)))$ to all books, in a model in which the set of books is given as $\{b_1, b_2\}$, it is easy to have $f_I(b_1) = f_I(b_2)$ while $f_\varphi(b_1) \neq f_\varphi(b_2)$, and thus to have different cardinalities for each set of facets.

In the situation described in (3), there are two copies of van Eijck's and Unger's *Computational Semantics* $\{b_1, b_2\}$ and one of Pustejovsky's *The Generative Lexicon* $\{b_3\}$ on the table. This is modeled simply with two informational contents and three physical objects, such as:

$i_1 = f_I(b_1) = f_I(b_2),\ i_2 = f_I(b_3),\ p_1 = f_\varphi(i_1),\ p_2 = f_\varphi(b_2),\ p_3 = f_\varphi(b_3).$

A simple representation of this situation, specifying the different individuation criteria, is given in Fig. 6 below.

Proceeding as for 1 above, we obtain the following formula:

$$\exists z^{R\to t} \forall z_1^R.$$
$$[z(z_1) \Rightarrow [\mathrm{book}^{R\to\mathbf{t}}(z_1) \wedge \mathrm{on\_the\_table}^{\varphi\to\mathbf{t}}(f_\varphi(z_1)) \wedge \mathrm{read}^{P\to I\to\mathbf{t}}(\mathrm{Stanley}^P, f_I(z_1))]]$$

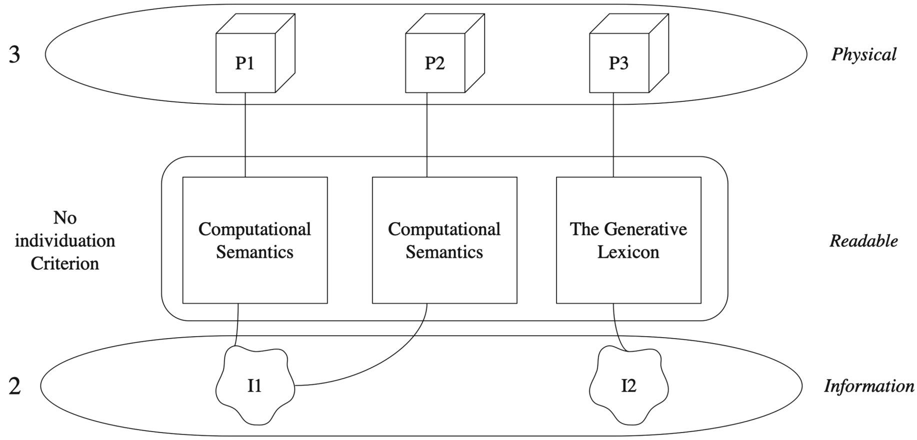

Fig. 6. Individuation summary.

The *books on the table* are the set of (physical) books defined as:

$$\begin{aligned} bt &= \{p^{\varphi} \,|\, \text{on\_the\_table}(p)\} \\ &= \{f_{\varphi}(b) \,|\, \text{on\_the\_table}(f_{\varphi}(b)\} \\ &= \{f_{\varphi}(b) \,|\, b \in \{b_1, b_2, b_3\}\} \\ &= \{p_1, p_2, p_3\} \end{aligned}$$

$|bt| = 3$

The *books Stanley read* are the set of (informational) books such as:

$$\begin{aligned} br &= \{i^{I} \,|\, \text{read}(\text{Stanley}, i\} \\ &= \{f_{I}(b) \,|\, \text{read}(\text{Stanley}, f_{I}(b)\} \\ &= \{f_{I}(b) \,|\, b \in \{b_1, b_2, b_3\}\} \\ &= \{i_1, i_2\} \end{aligned}$$

$|br| = 2$

The quantification puzzle is thus easily solved, as the polysemous terms (*the books*) that are common antecedents to the predicates are not counted as singular entities, but are only individuated with respect to a single, facet-specific predication that does not need to apply the same identity criterion as others.

This is made apparent by the explicit coercions f_{α}, that allow each predication and set-theoretic quantification to access the correct facet of the original term, without modifying that same term so that it can be used in any number of other predications, with or without coercions.

3 The Individuation Controversies

3.1 Books-as-parts

This does not answer all the questions raised by the quantification puzzle. A More difficult version of the quantification puzzle includes physical books that contain several informational parts labelled *books*. The *Bible* is an example, with a varying number of “books” according to different traditions, Tolkien’s *Lord of the Rings* is another, being a single work of six "books" plus prologues and appendices collected as one, three or six volumes depending on the edition or translation. Other examples include collected works from an author, and single-volume compilations of several works, that can be referred to collectively as in the phrase *"three books in one"*.

We feel that such examples depend on a very specific usage of the word *book*: contemporary English speakers would use *part* or *chapter*; this use of the term is also deprecated in French. We treat these cases by associating the word *book* with coercions to *holy text*, *novel* or *work* $f^{R \to I}_{total}$ on the one hand, and to *part* $f^{R \to I}_{part}$ on the other hand.

This does not provide a single straightforward answer pertaining to the individuation condition of such multi-book books, as each of these transformations might be used in case of a type mismatch, everyone being of the same type, $R \to I$. This ties with the fact that *how-many* questions for compound works are difficult with no obvious single answer, as any coherent individuation system may be accepted.

In a situation with two copies of a Roman Catholic *Bible* (73 book-parts) and a copy of the Torah (Pentateuch, 5 book-parts included in the previous 73), possible answers to *how many books are there* include 1 (the complete holy text), 2 (different-looking books), 3 (physical volumes), 73 (books of the bible) and 78 (73+5 when considering the differences between the versions of the texts).

Formally, what happens is that we have an individuation criterion for *physical objects* (identity in the world), and a criterion for *informational content* (identity between texts), but not for polysemous *books* (readable materials should be counted one way or the other). Thus, having to resolve the type for the referent to *books* in *how many books are there*, all combinations of $f^{R \to \varphi}$, $f^{R \to I}_{total}$ and $f^{R \to I}_{part}$ can be used, yielding a number of different possible interpretation combinations.

Every single of these interpretations could be considered as correct; we do not have yet sufficient linguistic data to say which should be eliminated or which one is the "most" correct. We would not thing that a single interpretation would ever be "correct", and suppose that the specific (pragmatic) context of the utterance guides the final interpretation. Giving the list of the possible meanings is the only thing that can be done at the level of semantics, where the necessary information is absent.

3.2 Hyper-Contextuality

Such a view, developed in [19] is that, in these cases, a context can always be found that justifies *any* certain response within the possible combinations of the meanings for *book*. Our intuition is to subscribe to this view, noting that the type-matching interpretations produced by the lexical mechanisms outlined above contain all plausible responses. In our opinion, this does not prove that the meaning of words such as *book* is singular, with no polysemy; this rather illustrates the fact that such words are, as [28] put, *relationally polysemous* terms, presenting different *facets* of a single entity.

There are many possible ways to represent this fact in order to account for the specificities of individuation and quantification, with both the mereological account given by [13] and the definition of entities as setoids including type and individuation criterion given by [8] being correct. We believe that the formalisation presented here is one of the easiest ways to account for this phenomena, and would argue that this contextuality was present in MGL from the start. Including the lexical transformations as pertaining to the words, and thus the specificity of individuation criteria to be different in different contexts, allows us to relativise the identity criteria to a speaker and a situation.

The specifics of *individuation* can be refined further in type-theoretical semantics. As discussed in [31], we subscribe to a view close to Leibniz's equality, that identity and individuation are described according to properties given by predicates; thus, *books-as-information* are quantified by *the books that are the same when being read*. Predicates such as *reading* can thus provide an equivalence relation $=_r$, and, to count the books being read, one may use the *quotient type* given by the sort of readable materials R and this relation.

In one context, a person might be interested in books in general terms and assert that a book and its translation in another language count as a single book, while a translation specialist might count different translations or editions as different books. This is a matter of *preference*, not necessarily related to the *original* sense of a word: *coffee* originally denotes (part of) a plant, but is mostly used as a beverage. This may also not be a cross-linguistic phenomenon, as the French *livre* is more readily associated to the pages and bindings of a physical object, while the English *book* seems to be more polysemous and closer to the information content (or part thereof) of the term. These differences in interpretation between speakers, contexts and languages can be modelled as individuals using different lexica, with different transformations associated to words such as *book*. The basic lexicon, with "common-sense" interpretations and meaning shifts is always available, but additional lexical layers can modify this information in a given context by adding or re-ordering the preferences for some lexical transformations, accounting for the multiple different situations. This mechanism can integrate highly contextual data from the *pragmatics* level of interpretation, and be seamlessly integrated in our account, given for semantics.

As illustrated below in Fig. 7, in a situation where different copies of the same informational part, parts of the same novel and translations of such parts are collected together, different speakers may use different criteria for counting the entities in a way that is relevant to each of them.

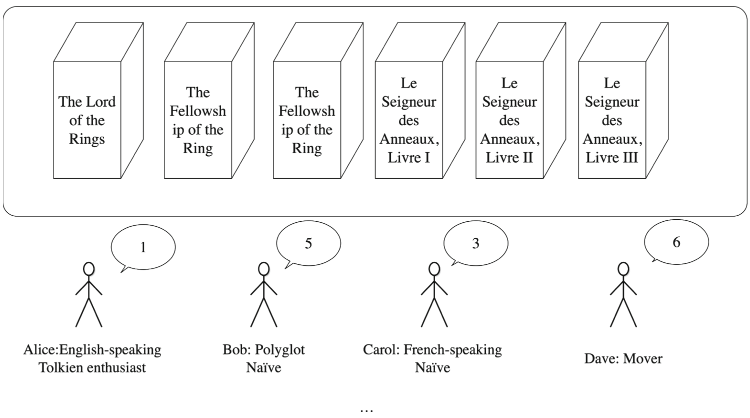

Fig. 7. A few individuals counting parts of *The Lord of the Rings* using different criteria.

4 Conclusion

Discussion – The quantification puzzle has now received responses using at least TCL, Mereology, MTT and MGL. The common elements of these responses clearly indicate that it is possible to quantify, individuate and count different facets of single, polysemous lexical items in different ways, correctly accounting for different contexts and co-predicative utterances. We argue that the differences between these formal treatments of quantifications are fundamentally very small. Our hope is that this paper provides a simple, detailed and correct answer for our framework, with a treatment chain starting from a sequence of words, utterances or sentences, and yielding the proper semantics as DRSs, that is fully mapped and mostly implemented using the Grail platform given in [25]. With this paper, we have thus given an integration of discourse representation theory, anaphora resolution and richly-typed lexical disambiguation, of which the quantification puzzle is just an illustration. Adding lexical transformations and the ability to refer to different facets of polysemous terms across the discourse is a big step towards precise, automated language understanding of real-world texts. Our processing system is quite advanced with a few elements hand-made, and most of the syntactical, semantic, discursive and lexical analysis automated. We are thankful to the LENLS organising team for providing this opportunity to present and discuss this issue, and to the reviewers of this paper for their appreciated input.

Perspectives – As part of the theoretical research on lexical semantics, we would like to explore whether a property-based individuation system, in which predicates yield their own individuations criteria by the means of quotient types as discussed in [31], can help with the more complex cases in which identity is highly dependent on context, speaker and language.

From a practical point of view, while the analysis methodology is nearly complete, we still need to acquire the rich and wide-covering lexical resources necessary for our analytical system; corpus-based approaches are beyond reach for our needs, and we can only hand-code a short lexicon. In [16], we examined suitable crowd-sourced resources to use as a starting point; the extraction of sufficient data is a challenge.

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