Hybrid Metaheuristic Optimization Methods for Optimal Location and Sizing DGs in DC Networks

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Abstract. In this paper is proposed a master-slave method for optimal location and sizing of distributed generators (DGs) in direct-current (DC) networks. In the master stage is used the genetic algorithm of Chu & Beasley (GA) for the location of DGs. In the slave stage three different continuous techniques are used: the Continuous genetic algorithm (CGA), the Black Hole optimization method (BH) and the particle swarm optimization (PSO) algorithm, in order to solve the problem of sizing. All of those techniques are combined to find the hybrid method that provides the best results in terms of power losses reduction and processing times. The reduction of the total power losses on the electrical network associated to the transport of energy is used as objective function, by also including a penalty to limit the power injected by the DGs on the grid, and considering all constraints associated to the DC grids. To verify the performance of the different hybrid methods studied, two test systems with 10 and 21 buses are implemented in MATLAB by considering the installation of three distributed generators. To solve the power flow equations, the slave stage uses successive approximations. The results obtained shown that the proposed methodology GA-BH provides the best trade-off between speed and power losses independent of the total power provided by the DGs and the network size.

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1 Introduction

Due to the importance of the DC networks and the need of integrating renewable resources on the electrical systems for reducing the negative impact associated to the fossil fuels [1], different authors have evaluated the integration of distributed generators (DGs) in DC grids. Multiple methods have been proposed to evaluate the DC power flow, such as Gauss-Seidel, Newton-Raphson, linear approximations, successive approximations, among others [1,2]. To evaluate the impact of the power supplied by the DGs into the DC grid, in literature have been proposed optimal power flow methods for finding the power level to be injected by each generator, with the purpose of improving different technical indicators, such as power losses or voltage profiles [3]. An example of this is presented in [4], where a second order cone programming formulation is proposed to solve the problem of optimal power flow (OPF) in stand-alone DC microgrids. In addition, in [5] is proposed a convex quadratic model for solving the OPF problem using, as objective function, the reduction of the power losses. The main problem of those methods is the requirement of specialized optimization software for solving the OPF problem. For this reason, different authors have proposed methodologies based on sequential programming by using metaheuristic optimization techniques; hence solving the OPF problem without specialized software. This is the case of the work presented in [6], where it is proposed a hybrid method with a master-slave structure that uses a continuous approach for the genetic algorithm (CGA) and the Gauss-Seidel method to solve the OPF problem. Similarly, in [7] is addressed the OPF problem in DC microgrids by using a combinatorial optimization technique known as black hole optimization (BHO), where the results shown the effectiveness and robustness of that method. Finally, in [8] is used a PSO algorithm to solve the OPF problem in DC grids by considering DGs and batteries in the electrical network.

To the best of the authors knowledge, the problem of optimal location and sizing of DGs in DC networks has not been explored in the specialized literature. Nevertheless, this problem has been addressed by different authors in AC systems by proposing optimization techniques based on sequential programming and other optimization methods [9]. The effectiveness and robustness of those methods have been evaluate considering different technical and operatives criteria, such as the power losses and voltage profiles, and the processing time required by the solution methods.

The previous review shows that it is necessary to propose optimization methods for solving the problem of optimal location and sizing of DGs in DC networks. Those methods must to ensure an acceptable quality in the solution and short processing times. For those reasons, in this paper is presented a mathematical formulation for solving the problem of optimal integration of DGs in DC grids by using the reduction of power losses as objective function and all the typical constraints associated to this type of electrical networks [5]. In this work the GA is selected as solution method for the optimal location of the DGs, this based on the satisfactory results obtained with this method in AC grids [10]. For solving the OPF problem in DC networks were selected three different metaheuristic techniques: PSO [8], CGA [6] and BH [7]. The main objective of the work is to find the hybrid methodology that presents the best balance between objective function minimization and processing time. The three hybrid solution methods are evaluated in two test systems with 10 and 21 buses, respectively, in which three DGs can be located. All simulations were carried out in MATLAB by using the successive approximation reported in [1] for solving the multiple power flows required in the sizing stage.

The paper is organized as follows: in Sect. 2 is presented the mathematical formulation used for the optimal location and sizing of DGs in DC networks. Section 3 shows the master-slave methodology formed by a GA and the three continuous optimization methods. In Sect. 4 is presented the simulation results and their discussion. Finally, Sect. 5 reports the conclusions and some possible future works.

2 Mathematical Formulation

The mathematical formulation of the problem of locating and sizing DGs in DC networks is described below.

Objective function:

$$\min P_{Loss} = \sum_{i \in \mathcal{N}} \left[\left(\sum_{j \in \mathcal{N}} G_{ij} v_i v_j \right) - G_{i0} v_i^2 \right] \tag{1}$$

Set of constraints:

$$P_i^g - P_i^d = \sum_{j \in \mathcal{N}} G_{ij} v_i v_j \;\; \{\forall i \in \mathcal{N}\} \tag{2}$$

$$v_i^{min} \leq v_i \leq v_i^{max} \;\; \{\forall i \in \mathcal{N}\} \tag{3}$$

$$I_{ij} \leq I_{ij}^{max} \;\; \{\forall ij \in \mathcal{B}\} \tag{4}$$

$$P_i^{g,min} x_i^{DG} \leq P_i^g \leq P_i^{g,max} x_i^{DG} \;\; \{\forall i \in \mathcal{D}\} \tag{5}$$

$$\sum_{i \in \mathcal{N}} x_i^{DG} \leq NDG_{max} \tag{6}$$

$$\sum_{i \in \mathcal{N}} P_i^g x_i^{DG} \leq P_{DG}^{max} \tag{7}$$

$$x_i^{DG} \in \{0, 1\} \;\; \{\forall i \in \mathcal{D}\} \tag{8}$$

In the previous model, Eq. (1) minimizes the power losses on the electrical system associated to the energy transportation, $\mathcal{N}$ is the set of buses that form the DC network, G_{ij} is the ij^{th} component of the matrix of conductances, G_{i0} is the conductance associated to the resistive load connected at bus i, v_i and v_j are the voltages in the buses i and j, respectively. Note that the power losses were selected as objective function, since it is highly used in the specialized literature for evaluating the performance of different methodologies in AC networks [11–13]. The set of restrictions in the problem are shown from (2) to (8). The power balance at each bus is defined in (2), where P_i^g and P_i^d are the power generated and consumed at the bus i, respectively. In (3) are presented the maximum (v_i^{max}) and minimum (v_i^{min}) bounds for the nodal voltages. Expression (4) presents the thermal current bound of each branch in the electrical system, were $\mathcal{B}$ is the set of branches that form the electrical network, i_{ij} the current of the line ij and I_{ij}^{max} the maximum current allowed in that line. The maximum and minimum power bounds to be injected by the DG connected at bus i are show in (5), where x_i^{DG} is a binary variable that takes the value of 1 when a DG is located at bus i, and it takes a value of 0 otherwise; the binary nature of x_i^{DG} is defined in (8), and $\mathcal{D}$ represents the set of buses selected for locating DGs. Finally, constraint (6) limits the maximum number of DGs that can be introduced (NDG_{max}), while constraint (7) imposes the maximum level of penetration (P_{DG}^{max}) allowed into the DC grid.

3 Proposed Methodology

The problem of optimal location and sizing of DGs in DC networks is solved using a master-slave methodology. In the master stage a Chu & Beasley genetic algorithm is used [14], which defines the location of the DGs. In the slave stage are employed three different continuous methods: PSO [8], CGA [6] and BH [7], which solve the OPF problem. In addition, it is used the power flow method based on successive approximations for evaluating all the power flows required in the OPF solution [1]. The master and slave stages are detailed below.

3.1 Master Stage: Chu & Beasley Genetic Algorithm

The master stage is implemented with a GA to determine the best location of the DGs for reducing the total power losses in all the branches of the DC network. This optimization technique works with selection, recombination and mutation operators, to generate each offspring during the searching process [14]. These operators allow replacing the worst individual of the population by the offspring, this in the case that the fitness function is improved, and only for new solutions different from all individuals of the population (diversity criterion). The parameters and characteristics selected for this optimization technique are taken from [10]: population size (40), selection method (tournament), cross over (simple), mutation (random binary simple). In addition, the stop criterion is a maximum generational cycles (iterations) equal to 40 or 10 iterations without improving the fitness function.

3.2 Slave Stage: Continuous Optimization Method

The slave stage is used for dimensioning the DGs and to evaluate the fitness function of the individuals in the initial and descending populations generated by the master stage at each iteration. This process is performed with three different continuous optimization techniques: PSO, CGA and BH. Those methods were selected since they have been used in literature for OPF analysis in DC networks. On the other hand, with the objectives of reducing the processing time and provide a fair comparison between the continuous methods, the successive approximation method reported in [1] was used for solving the power flows required in the evaluation of each continuous optimization method for OPF analysis. The description of each continuous method is presented below.

Particle Swarm Optimization (PSO): the PSO is a bio-inspired metaheuristic algorithm based on the behavior of the flocks of fish and birds, and it was proposed by Eberhart and Kennedy in 1995 [15]. This method takes advantage of the mode used by the groups of animals for exploring a region to find a common source of food for all individuals of the group. By modeling each individual as a particle, it is possible to transform the group of individuals in a particle swarm dispersed over a solution space. This particle swarm is limited by a set of constraints associated with each problem. In the PSO algorithm each step or iteration takes into account the information of each particle, as well as the particle swarm information, for generating the next movement, this to find a good solution for the problem. The application of PSO for solving the OPF problem in DC grids is described in [8].

Continuous Genetic Algorithm (CGA): This optimization method, proposed in [6], is a continuous approach of the conventional GA proposed by Che & Beasley in [14]. It uses the selection, recombination and mutation operators with a continuous representation in order to generate the population representing the sizes of the DGs defined by the master stage.

Black Hole Optimization Method (BH): This is a nature-inspired optimization technique based on the dynamic interaction between stars and black holes [16]. This technique has been used for solving nonlinear optimization problems by implementing a particle swarm (stars) as well as a criterion of elimination and generation of stars through a heuristic approach (event horizon radius). The iterative process of this optimization method for solving the OPF problem in DC grids is reported in [7].

The parameters selected for the sizing techniques are shown in Table 1. Those equivalent values are assigned with the aim of providing a fair comparison between the continuous methods.

Table 1. Parameters of the sizing techniques

Method	CGA	BH	PSO
Number of particles	30	30	30
Selection method	Tournament	Event horizon radius	Cognitive and social component: 1.4
Update population method	Cross over: averaging	Cognitive and social component	Speed/Inertia (max-min): (0.1–0.1)/(0.7–0.001)
Mutation	Random population	Random population	R1 = R2: Random
Stopping criterion	Max. iterations: (200) Iteration without improving: (50)	Max. iterations: (200) Iteration without improving: (50)	Max. iterations: (200) Iteration without improving: (50)

4 Simulation Scenarios and Results

The combination of the GA with the three continuous optimization algorithms produce the following hybrid methodologies: GA/PSO, GA/CGA and GA/BH. The simulations of those methods were carried out on a Dell Precision T7600 Workstation with 32 GB of RAM memory and with an Intel(R) Xeon(R) CPU ES-2670 at 2.50 GHz.

Two DC test systems with 10 and 21 buses were considered for evaluating each hybrid method [17]. These systems have been previously proposed for addressing OPF problems in [6], and [7]. However, some modifications were made to the test systems: a unique slack generator is considered for each system, and only constant power loads are considered, which implies that all the DGs and batteries of the conventional test systems have been replaced by constant power loads. Note that 100 kW and 1 kV are used as power and voltage bases, respectively.

To guaranteed a fair comparison among all of the hybrid optimization approaches, the following assumptions were made: (i) All the nodes are candidates for locating DGs, except the slack node. (ii) A maximum of three DGs can be installed $NDG_{max} = 3$. (iii) Three levels of penetration for the DGs are considered: 20%, 40% and 60%. Finally, (iv) the minimum and maximum power levels able to be generated by each DG in both test systems are 0 and 1.5 p.u, respectively [5]. The previous assumptions are typically used for the optimal location and sizing of DGs in AC grids [10,11].

The fitness function (FF) used in those algorithms is given in (9). This function penalizes when the total power injected by the set of DGs is higher than the maximum power allowed (P_{DG}^{Max}). The expression for penalty (P_{en}) is reported in (10), where P_i^g is the power generated at the bus i.

$$FF = \min\left(P_{Loss} + P_{en}\right) \tag{9}$$

where

$$P_{en} = \max\left[0,\ \left(\sum_{i \in N} P_i^g x_i^{DG}\right) - P_{DG}^{\max}\right] \tag{10}$$

The validations were carried out by testing the same cases with each hybrid optimization technique. The simulation results are presented in Tables 2 and 3, which reports, from left to right, the following information: the hybrid method, the DGs location and size, the power losses (P_{loss}), the square voltage error (V_{error}), the worst voltage profile and the associated bus, and finally, the processing time ($Time$). For the analysis of the results, the scenario without DGs for both test systems is used as reference (base case).

4.1 10 Bus Test System

In this subsection are presented the simulation results associated with the 10 bus test system reported in Table 2. By analyzing the reduction of power losses with respect to the base case (without DGs), it is observed that the GA/PSO method provides the best solution, with an average reduction of 63.18%, i.e. 0.54% and 4.01% higher than GA/CGA and GA/BH, respectively. With respect to the processing time, the shortest time is obtained by the GA/BH, with an average time of 21.98 s, presenting an average reduction of 43.25% and 72.61% when it is compared with the GA/PSO and GA/CGA. In addition, the impact of the optimization methods on the voltage profiles is analyzed using the V_{error}, presented in Eq. (11), and the worst voltage profiles [10]. In Eq. (11) V_{base} is the base voltage assigned to each test system.

$$SVE = \sum_{i \in \mathcal{N}}^{n} (V_i - V_{base})^2 \tag{11}$$

For the reduction of the V_{error} with respect to the base case (0.0075 p.u), the best results are provided by the GA/PSO, with an average reduction of 64.56%; it also exhibiting an additional reduction of 1.99% over the other hybrid methods, with an average worst voltage profile of 0.9823 p.u. The worst result, in terms of voltage profiles, is provided by the GA/BH, with an average reduction of V_{error} of 61.23% and an average worst voltage profile of 0.9814 p.u. Nevertheless, the different methodologies present a voltage absolute error lower than 2% when they are compared with the base voltage (1 p.u), hence satisfying the constraint of $+/-5\%$ around the nominal voltage. This limit is selected according to the load and type of network in order to guarantee a secure and reliable operation [18].

4.2 21 Bus Test System

The results obtained for this test system are presented in Table 3. The highest average reduction of the power losses is achieved again by the GA/PSO with a value of 72.74%, the GA/CGA is at second placed with 71.42%, while the lower impact is given by GA/BH with an average reduction of 65.51%. Concerning the processing time, it was obtained an average time of 28.09 s, 73.78 s and 148.15 s for the GA/BH, GA/PSO and GA/CGA, respectively. The GA/BH method takes the first place, presenting a reduction of 61.92% and 81.03% with respect

Table 2. Results for the 10 bus test system

Method	Location/size [kW]	P_{loss} [kW]	V_{error} [p.u]	Min. vol. [p.u]/bus	Time [s]
Without DG	—	14.3628	0.0075	0.9690/9	—
Maximum level of power injected by the DGs: 20%					
GA/PSO	5/0.3435 9/0.5628 10/0.0879	8.8690	0.00448	0.9768/8	32.26
GA/CGA	4/0.2907 5/0.2095 9/0.4909	8.9117	0.00454	0.9764/8	62.85
GA/BH	5/0.4250 7/0.2035 8/0.3645	8.9262	0.00448	0.9765/9	20.66
Maximum level of power injected by the DGs: 40%					
GA/PSO	5/0.5263 9/0.8744 10/0.5876	4.8656	0.00239	0.9828/4	38.19
GA/CGA	3/0.3586 5/0.8599 9/0.7697	4.9337	0.00243	0.9824/10	75.07
GA/BH	3/0.0225 5/0.9997 8/0.9370	5.2157	0.00238	0.9817/10	21.41
Maximum level of power injected by the DGs: 60%					
GA/PSO	4/1.2347 9/0.9113 10/0.8365	2.1286	0.00106	0.9883/8	45.75
GA/CGA	3/0.8556 4/1.2402 9/0.8860	2.2207	0.00110	0.9882/10	102.86
GA/BH	5/1.1674 6/1.1925 8/0.2745	3.6121	0.00170	0.9843/10	23.88

to the GA/PSO and GA/CGA, respectively. Moreover, the maximum average reduction of the V_{error} was obtained by the GA/PSO with a value of 81.70%, while the minimum average reduction is provided by the GA/BH (76.98%). With respect to the bus voltage profiles, the average absolute error, with respect to the base voltage, is 0.027, 0.029 and 0.033 p.u for the GA/BH, GA/PSO

Table 3. Results for the 21 bus test system

Method	Location/size [kW]	P_{loss} [kW]	V_{error} [p.u]	Min. vol. [p.u]/bus	Time [s]
Without DG	—	27.6034	0.0567	0.9211/17	—
Maximum level of power injected by the DGs: 20%					
GA/PSO	17/0.4979 18/0.4477 21/0.2175	12.8878	0.02100	0.9591/12	68.27
GA/CGA	16/0.9583 20/0.1560 21/0.0494	13.1250	0.02161	0.9591/12	142.10
GA/BH	15/0.1211 18/0.6031 20/0.4143	13.5089	0.02227	0.9547/17	20.45
Maximum level of power injected by the DGs: 40%					
GA/PSO	12/0.8326 15/0.7950 17/0.6987	6.2049	0.00767	0.9747/20	74.18
GA/CGA	11/0.9560 16/0.9338 17/0.4333	6.5654	0.00800	0.9736/20	148.54
GA/BH	12/0.3929 15/0.7937 21/1.0045	8.2473	0.00892	0.9686/17	27.88
Maximum level of power injected by the DGs: 60%					
GA/PSO	8/0.5928 11/1.5 16/1.3968	3.4763	0.00244	0.9834/20	78.89
GA/CGA	11/1.4791 17/0.9870 19/0.9939	3.9743	0.00147	0.9823/9	153.81
GA/BH	11/1.0660 13/0.8375 17/0.7886	6.8031	0.00796	0.9713/18	35.94

and GA/CGA, respectively. It is worth highlighting that the voltage profiles present a voltage absolute error lower than 5%, satisfying the voltage constraint associated with this type of systems. The previous results show that, in both test systems, the GA/PSO obtained the best results in terms of the technical aspects considered in this paper: Power losses and voltage profiles; and the fastest technique is the GA/BH, followed by the GA/PSO and GA/CGA. Furthermore, as the level of power injected by the DGs increases, the technical aspects are improved and the processing time is increased for all the hybrid methods.

Figure 1 was developed to analyze the trade-offs provided by each method, in terms of power losses and processing time, for any network size and level of power injected by the set of DGs. In this figure the Y-axis reports the average value of power losses, in percentage, with respect to the base cases (without DGs); and the X-axis reports the average processing time required by each method, also in percentage, with respect to the hybrid method requiring the longest processing time in all test systems, i.e. GA/CGA. In both axes, the average values were calculated considering the results obtained in all the test systems. In this figure, the best solution is the origin $(0, 0)$, since it represents power losses and processing time equal to zero. Analyzing the global results shown in Fig. 1, it is concluded that the GA/BH is the fastest method, with an average processing time of 23.52%, but with the worst average power losses (37.84%). In this way, the worst solution in terms of processing time is the GA/CGA method, with an average processing time of 114.20s (100%-Base case), obtaining the second place in terms of power losses (32.93%). The GA/PSO is the second faster method with an average value of 49.32%, it also providing the best results in terms of power losses with an average value of 32.03%. On the base of those results it is concluded that the best balance between power losses reduction and processing time is obtained by the GA/BH, followed closely by the GA/PSO.

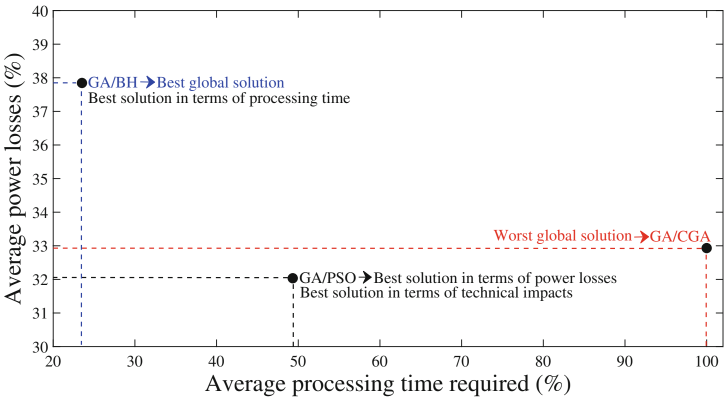

Fig. 1. Average impact of the hybrid methods in all test scenarios proposed

5 Conclusions and Future Works

In this paper was proposed a hybrid method employing a master-salve structure, based on sequential programming, for solving the problem of location and sizing of distributed generators in DC networks. In the master stage a GA was used to define the optimal location of the DGs; while the slave stage was implemented with continuous optimization methods (PSO, CGA and BH) to define the size of each distributed generator. The location and sizing methods were combined to find the hybrid method that provided the best results in terms of power losses and processing times. To evaluate the impact of each hybrid method was proposed a mathematical formulation that analyzed the impact of the distributed generation into the grid. The results shown that as the level of power injected by the DGs grows, the power losses are reduced and the processing time are increased. In addition, the GA/PSO provided the best results in terms of technical impact (reduction of power losses and voltage profiles) with the second shorter processing time. The GA/PSO presented the second and third place in relation to the technical impacts and processing time, respectively; and the GA/BH presented the best performance with respect to the processing time and the worst results in terms of technical impact. Finally, the GA/BH provides the best trade-off in terms of power losses reduction and processing time for locating and sizing distributed generators in DC networks, followed closely by the GA/PSO. As future works it will be considered the evaluation of other location methods applied in AC networks, and the use of parallel processing methods for reducing the processing time.

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