Chapter 7
Criterion Based Choice of Rules

Abstract There are quite a few voting procedures applied for what appears to be a common purpose, viz. to tease out the will of the voters. Despite hundreds of years of study there is no consensus in the scholarly community as to which is the best procedure. The criteria emphasized by different scholars differ to some extent and it turns out that none of the systems satisfies all reasonable desiderata. All procedures are based on some intuitive notion regarding what constitutes the collectively best outcome. In this chapter we discuss the choice of the collective decision procedure on the basis of evaluations in terms of various criteria of performance.

7.1 Introduction

Some years ago a group of voting theorists and electoral experts got together for a symposium in Normandy, France.[1] The proceedings of the symposium were later edited by Felsenthal and Machover (2012). At the end of the symposium an impromptu discussion was held among the participants about the best voting system to be used in a hypothetical situation involving the election of the director of a municipality. In other words, the system should be applicable for electing a single winner. In the discussion various procedures were proposed and the session was concluded with a vote. The alternatives – altogether 18 in number – were the voting systems proposed in the discussion and the ballot aggregation method was the approval voting.[2] The results are reproduced in Tables 7.1 and 7.2.

[1]This chapter is largely based on Nurmi (2015).

[2]The impromptu nature of the proceedings is reflected by the somewhat light-hearted brainstorming debate preceding the vote as well as by the fact that the voters were not asked to reveal anything else but their approved systems. Several weeks after the meeting the participants were asked to disclose their reasons for voting the way they did, but at this time many didn't recall the systems they had approved of, much less the reasons for doing so. Thus, we do not know how much the election outcome depends on the aggregation systems adopted. See Laslier (2012).

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Table 7.1 The number of approved procedures (Laslier 2012)

No. of approvals	0	1	2	3	4	5	6	7	8	9	10	>10	Total
No. of ballots	0	2	7	3	5	2	1	1	0	0	1	0	22

Table 7.2 The procedures and the distribution of approvals (Laslier 2012)

Voting rule	Approvals	Approving %
Approval	15	68.18
Alternative	10	45.45
Copeland	9	40.91
Kemeny	8	36.36
Runoff	6	27.27
Coombs	6	27.27
Simpson	5	22.73
m. judgment	5	22.73
Borda	4	18.18
Black	3	13.64
Range	2	9.09
Nanson	2	9.09
Leximin	1	4.54
Top cycle	1	4.54
Uncovered	1	4.54
Fishburn	0	0
Untrapped	0	0
Plurality	0	0

The former table shows a fairly wide variation in the number of approved systems. Yet, a vast majority of voters approved $2-4$ systems. The procedures are listed in Table 7.2 which also indicates the number of approvals given to each one of them as well as the percentage of voters approving of each system. The reader unfamiliar with the procedures is referred to Laslier's (2012) article which also provides a comprehensive analysis of the voting data. Many of the procedures were also discussed in Chap. 4 above.

A couple of observations about Table 7.2 are in order. Firstly, no procedure was approved of by all participants. Secondly, some proposed systems received no approval votes at all. Thirdly (and related to the preceding point), the most common voting system – the plurality or one-person-one-vote procedure – was voted for by no participant. Fourthly, the winner – the approval voting – was approved of by more than two thirds of the voters.

The first point provides the main motivation for this chapter. It shows that the expert community is not unanimous about the best voting procedure. Glancing at the statements that several voters give to support their ballots one immediately notices that the participants seem to emphasize somewhat different criteria when choosing their favorite systems. It is plausible to think that the very existence of many voting procedures can similarly be explained by the emphasis placed on different criteria of performance of procedures. The next section provides a theoretical reconstruction of some of the best-known systems in terms of this reasoning. Thereafter, we present the main contribution of this chapter, *viz.* a method for choosing a voting procedure on the basis of the participants' priorities regarding the performance criteria.

7.2 The Emergence of Some Voting Procedures

Perhaps the most common of all voting procedures is the plurality rule: each voter has one vote at his/her disposal and the candidate or policy alternative receiving more votes than any of its contestants wins. The rationale of this rule is obvious: no other candidate gets as many votes as the winning one. However, it may happen that the plurality winning candidate gets less than 50% of the votes. Hence it may not always get the support of the majority. To rectify this eventuality the plurality runoff system has been devised. It works precisely as the plurality procedure, but in case the plurality winner receives at most 50% of the votes, a runoff is arranged between the two largest vote-getters. Whichever gets more votes than the other on this second round of voting is the winner. Thus, the winner can always claim to be supported by more than half of the electorate. More importantly, the runoff system guarantees that an eventual Condorcet loser is not elected. This simply follows from the fact that the plurality winner has to defeat by a majority of votes at least one other alternative, viz. its competitor on the second round of voting. If there is a winner already on the first round, i.e. there is a candidate ranked first in the opinion of a majority of voters, then of course the winner would defeat *all* the others in pairwise contests.

Another way of avoiding Condorcet losers being elected was discovered nearly a quarter of a millennium ago by Jean-Charles de Borda and is today known as the Borda count.[3] Thus, we have two solutions to the problem of avoiding the election of a Condorcet loser: the plurality runoff and the Borda count. Yet, the former is accompanied with a new problem, plaguing neither the latter nor, perhaps more importantly, the plurality procedure, viz. nonmonotonicity. In nonmonotonic systems, additional support, *ceteris paribus*, may turn a winning candidate into a non-winning one. Table 7.3 illustrates this problem.

[3]The method was invented already in the 15th century by Nicholas of Cusa, but arguably the latter did not emphasize the particular problem related to the plurality voting, viz. that it may result in the election of a candidate that would lose the pairwise contests against any other candidate. See McLean and Urken (1995) and Hägele and Pukelsheim (2008).

Table 7.3 Nonmononicity of the plurality runoff

6 voters	5 voters	4 voters	2 voters
A	C	B	**B**
B	A	C	**A**
C	B	A	C

Supposing that the voters vote according to their preferences listed in Table 7.3 there will not be a first-round winner, but a runoff that takes place between A and B. In this runoff the winner is A since it is preferred to B by those 5 voters whose favorite didn't make it to the runoff. Suppose now that the two voters on the right with ranking BAC would change their opinion with regard to A and B (marked in bold letters in the table) so that the winner A would be preferred to B by these two voters. I.e. A's support would increase, everything else remaining as before. In this new profile – which differs from the Table 7.3 so that the winner (A) gets more support than originally – a runoff is still needed, but this time one between A and C. This runoff is won by C. This shows that additional support may, indeed, turn winners into non-winners under the plurality runoff system. Hence the procedure is nonmonotonic.

Similarly as plurality runoff can be seen as an attempt to improve upon the plurality system, Nanson's method can be seen – and was in fact seen by its inventor E. J. Nanson – as a way to rectify an apparent flaw in another system, viz. the Borda count (see McLean and Urken 1995). For more than two centuries it has been known that the Borda count does not always end up with the Condorcet winner. Nanson set out to devise a system that would be as similar to the Borda count as possible, but still guarantee the choice of an eventual Condorcet winner. The system is based on the observation concerning the relationship between Condorcet and Borda winners. While it is known that the former winners are not necessarily ones with the highest Borda scores, it is still the case that they never have very low Borda scores. More specifically, an eventual Condorcet winner always has a higher than average Borda score. Nanson's method is based on this observation: it proceeds in rounds whereby the alternatives with an average or lower Borda score are eliminated and new scores are computed for the remaining alternatives until the winner is found. The criterion used in elimination guarantees that an eventual Condorcet winner is not eliminated.

So, the system invented by Nanson was, indeed, capable of solving a specific shortcoming of the Borda count. However, as was the case with plurality and plurality runoff systems, the solution procedure (here Nanson's method) has a flaw that the "flawed" system (here the Borda count) is not associated with. This is nonmonotonicity: while the Borda count is monotonic, Nanson's method isn't (Fishburn 1977, p. 478).[4] This illustrates the nature of many social choice results: they demonstrate

[4]Strictly speaking Fishburn investigates a method he calls the Nanson function which differs from Nanson's method in eliminating only the alternative(s) with the lowest Borda score in each counting round. Fishburn's Nanson function was invented about a hundred years ago by Baldwin (1926). So, strictly speaking Fishburn's example demonstrates that Baldwin's rule is nonmonotonic. It can,

incompatibilities between properties of choice functions. In short, procedures with all desirable properties do not exist. Trade-offs have to be made between desiderata.

This well-known state of affairs suggests a new angle to the problem of choosing a procedure of choice. Instead of fixing specific flaws in the systems that are being used – and thereby conceivably coming up with systems with flaws that the already used ones do not have – one could start from the criteria that one regards of primary importance. Different people may put different value on various criteria. This was clearly exemplified in the introduction of this section. Hence, it would make sense to take into account and make use of the information regarding differences in valuation by different voters when choosing a system to be resorted to in collective decisions. In the next section we outline several ways of going about this.

7.3 From Criterion Preferences to Voting Systems

The most straight-forward way to proceed is to consider the problem as any preference aggregation problem, i.e. to use criterion preferences as inputs and, using some social choice rule, aggregate them into a collective preference ranking. Consider the following set of criteria (Table 7.4).[5]

Let us briefly remind ourselves about the content of these criteria. The Condorcet winner criterion is satisfied by those systems that always elect the Condorcet winner when one exists in the profile. The Condorcet loser criterion, in turn, calls for the exclusion of an eventual Condorcet loser from the choice set. The strong Condorcet winner is an alternative that is ranked first by a majority of voters. The corresponding criterion dictates the choice of the strong Condorcet winner whenever such an alternative is present in a profile. Monotonicity is satisfied by systems where additional support never hurts the winning alternative. Pareto criterion excludes the election of Pareto dominated alternatives. Consistency pertains to results in two or more sub-electorates. In consistent systems, if all sub-electorates elect the same alternative, this should also be elected when the ballots are counted en masse, i.e. without sub-divisions. Chernoff property states that if x is elected in the set of alternatives, it should be elected in all subsets it is an element of as well. Independence of irrelevant alternatives is satisfied whenever in any two profiles where x and y are ranked in an identical manner with respect to each other, they are also ranked in the same way in the resulting outcomes. Invulnerability to the no-show paradox means that the voting procedure never penalizes the voters for voting according to their preferences, i.e. the voters are never better off by abstaining than voting according to their preferences, *ceteris paribus.*

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however, be shown that also the method that Nanson devised is nonmonotonic. See Nurmi (1999, p. 59) and Nurmi (2018).

[5]For further explanation of the criteria, see e.g. Nurmi (2002).

Table 7.4 A set of choice criteria

a	The Condorcet winner criterion
b	The Condorcet loser criterion
c	The strong Condorcet criterion
d	Monotonicity
e	Pareto
f	Consistency
g	Chernoff property
h	Independence of irrelevant alternatives
i	Invulnerability to the no-show paradox

Table 7.4 exhibits but a relatively small subset of criteria discussed in the literature, but arguably some of the most important criteria are included in the list. More extensive sets are introduced and analyzed e.g. in Felsenthal (2012) and Richelson (1979).

To work out a collective preference ranking over these 9 criteria, some aggregation rule has to be used. To do this, one would have to assume what one is aiming at, viz. a suitable choice rule. When due attention is given to their metric representations (see Nitzan 1981; Meskanen and Nurmi 2006), two rules, however stand out: Kemeny's rule and the Borda count. The former chooses the collective ranking that is closest to the reported individual rankings in terms of binary reversals (inversion metric), while the latter counts for each alternative (choice rule) the number of binary preference reversals that are needed to make this alternative unanimously first ranked. Thus, both rules resort to the same metric, but different benchmark state. In the present context Kemeny's rule would perhaps seem more appropriate since the choice procedure is to be chosen using the following performance table (Table 7.5).

The table indicates whether a procedure represented by the row satisfies (denoted by 1) or does not satisfy (denoted by 0) the criterion represented by the column $(a, \ldots, i)$. Again, the procedures are just a sample of those discussed in the literature.

Suppose now that the collective ranking obtained by applying Kemeny's rule to the profile of reported rankings over criteria has criterion l ranked first. One then looks for all procedures that have a unity in the column l. If several procedures satisfy l, one then picks the criterion ranked second in the collective (Kemeny) ranking. Let this criterion be m. One then looks for procedures that satisfy both l and m. Again there may be several procedures, but continuing in this (lexicographic) manner one eventually ends up in a situation where all remaining procedures satisfy all top-most criteria in the collective preference ranking down to a point after which none of them satisfies the next one in the collective ranking. Those remaining procedures then constitute the choice set of procedures. To take an example, suppose that the Kemeny ranking is $d \succ e \succ b \succ f \ldots$. Then the outcome is a three-way tie $\{Copeland, Kemeny, Black\}$ since all these satisfy monotonicity (d), Pareto (e) and Condorcet loser (b) criteria, but none of them is consistent (f).

Table 7.5 A comparison of voting procedures

	Criterion								
Voting system	*a*	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*
Amendment	1	1	1	1	0	0	0	0	0
Copeland	1	1	1	1	1	0	0	0	0
Dodgson	1	0	1	0	1	0	0	0	0
Maxmin	1	0	1	1	1	0	0	0	0
Kemeny	1	1	1	1	1	0	0	0	0
Plurality	0	0	1	1	1	1	0	0	1
Borda	0	1	0	1	1	1	0	0	1
Approval	0	0	0	1	0	1	1	0	1
Black	1	1	1	1	1	0	0	0	0
Pl. runoff	0	1	1	0	1	0	0	0	0
Nanson	1	1	1	0	1	0	0	0	0
Hare	0	1	1	0	1	0	0	0	0
Coombs	0	1	1	0	1	0	0	0	0

Obvious objections can be presented against this system, perhaps the most important being its reliance on lexicographic ordering of criteria. A poor performance on the first ranked criteria cannot be "bought" by good performance on criteria ranked lower in the collective ordering. This can be illustrated by a setting where the collective ranking puts consistency on the first place. It then follows that just three systems are left after the first criterion is considered. If the collective ranking puts the Condorcet loser criterion in the second place, the Borda count emerges as the chosen system. In other words, the other criteria have no role whatsoever in determining the chosen system.

In view of these considerations another set of procedures is suggested. The input is either the set of individual preference rankings over criteria or the distribution of utility values in a fixed interval, say [0, 10], that each voter assigns to each criterion. We illustrate one version of the procedure by using Borda points given by each voter to each criterion. Suppose that there are three individuals and their preference ranking over the 9 criteria are as follows:

individual 1	abcdefghi
individual 2	dcbafeihg
individual 3	ihgfedcba

Criterion a, thus, gets 8 Borda points from 1, 5 points from 2 and 0 points from 3. It would then make sense to argue that procedures satisfying a, get 13 points from these three individuals, while the other procedures get no points. Similarly b gets 7 points from 1, 6 from 2 and 1 from 3. And so on. Those procedures that do not satisfy

Table 7.6 The assignment of points to procedures in the basis of criterion preferences

Voting procedure	Criteria									
	A	B	C	D	E	F	G	H	I	Sum
Amendment	13	14	15	16	0	0	0	0	0	58
Copeland	13	14	15	16	11	0	0	0	0	69
Dodgson	13	0	15	0	11	0	0	0	0	39
Maxmin	13	0	15	16	11	0	0	0	0	55
Kemeny	13	14	15	16	11	12	0	0	0	81
Plurality	0	0	15	16	11	12	0	0	10	64
Borda	0	14	0	16	11	12	0	0	10	63
Approval	0	0	0	16	0	12	8	0	10	46
Black	13	14	15	16	11	0	0	0	0	69
Pl. runoff	0	14	15	0	11	0	0	0	0	40
Nanson	13	14	15	0	11	0	0	0	0	53
Hare	0	14	15	0	11	0	0	0	0	40
Coombs	0	14	15	0	11	0	0	0	0	40

the criterion considered do not get any points from voters on that criterion. In effect, then, for each column of the table the entries are obtained by multiplying the points given by voters to the criterion represented by the column by the corresponding entry of Table 7.5. The results are seen in Table 7.6.

On the basis of criterion preferences and using the Borda count in the point assignment, the winning procedure is Kemeny's rule followed by a tie between Copeland's and Black's procedures.

Given the plethora of voting systems currently in use in various contexts it is arguable that the designers have different desiderata in mind when devising those systems. Focusing on a single desideratum only is bound to cause problems because typically a good performance on one criterion is accompanied with bad performance on some others. Hence we suggest that the opinions regarding the desiderata ought to be made explicit in the choice of the system to be used. We have outlined above a couple of ways of using voter opinions regarding criterion preferences in a systematic way in the choice of a voting procedure.

7.4 Topics for Further Reflection

1. Show by way of an example that the Condorcet winner may not be first-ranked by any voter in a profile.
2. Show by way of an example that a strong Condorcet winner may not be the Borda winner.

3. Discuss circumstances in which consistency is largely irrelevant for the plausibility of the outcomes.
4. Pick your favorite from the systems of Table 7.5. Explain its main advantages and disadvantages.

7.5 Suggestions for Reading

Good introductions to voting procedures and their properties are Riker (1982) and Straffin (1980). Somewhat more concise and advanced texts are Nurmi (1987) and Felsenthal (2012). A profound analysis of voting systems from a geometrical perspective is given by Saari (1995, 2001).

Answers to Selected Problems

1. Consider the following profile:

Voter 1	Voter 2	Voter 3
A	B	C
D	D	D
B	C	A
C	A	B

Here D is the Condorcet winner, but is not ranked first by any voter.

2. Consider the following profile:

5 voters	4 voters
A	B
B	C
C	A

Here A is the strong Condorcet winner, yet B is the Borda winner.

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