Chapter 6
Sequential Voting by Veto

Abstract Sometimes the members of the committee or small group are more interested in avoiding particular outcomes than in reaching their own favourite ones. In such circumstances the sequential voting by veto provides an a priori plausible decision making method. We outline the method and discuss its main properties.

6.1 Introduction

Most procedures discussed in this treatise pertain to settings where all voters submit their preference rankings over alternatives and the outcome is determined by aggregating these rankings according to a specific rule. In small groups where the participants can exchange views on alternatives prior to the preference aggregation, it is conceivable that the alternative set is formed or modified in the course of the discussion preceding the voting. This is quite common in contemporary parliaments, committees and other public sector formal decision making bodies, but it is presumably even more common in boardroom decision making and gatherings of less formal nature such as groups of friends discussing various pastime activities. In this chapter we deal with a method that seems rather promising in these kinds of settings, viz. the sequential voting by veto (SVV, for brevity) introduced by Mueller (1978).

6.2 The Procedure

The background and motivation of this method is in determining the distribution of a given divisible payoff among members of a group in a situation where there is a *status quo* alternative that gives all members a zero payoff. Each member is first asked to make a proposal regarding the payoff distribution. Proposals are then listed and displayed to the group members. The members are then arranged in a random sequence indicating the order in which they will give their votes. The voting takes place by vetoing, not supporting, alternatives. When his/her turn comes, each voter is to veto one and only one alternative. The last un-vetoed alternative is the winner.

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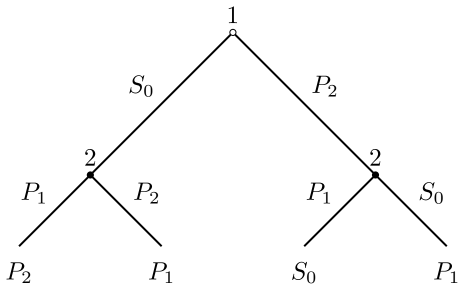
Fig. 6.1 Two-person veto game tree



There are various ways of organizing the proposal submission and balloting stages. Perhaps the most plausible is one where both the proposals and the sequence of balloting are made known to the individuals before the balloting begins. An important ingredient of the procedure is that the order in which the ballots are submitted is random and, thus, not dictated by any member of the group (e.g. the chairperson).

To see how the system works, consider the simplest case of a two-member group. Let S_0, P_1 and P_2 denote the *status quo*, member 1's and member 2's proposal, respectively. It makes sense to assume that both member 1 and member 2 prefer their own proposal to S_0. Moreover, it is plausible to assume that the members prefer their own proposal to that of the other member. Let now the voting order be that member 1 votes first. Obviously, he/she eliminates either S_0 or P_2 depending on which one is worse for him/her. Member 2, then, has the choice either between S_0 and P_1 or between P_1 and P_2. Obviously, for P_2 to have any chance at all of being the final outcome, member 2 has to make it more attractive than S_0 for member 1 or else it will be eliminated on the first ballot. Thus, in order to make it possible that their proposals be adopted in the process, the participants have to consider the preferences of each other *vis-à-vis* S_0 when making proposals: the proposals should be Pareto-improvements over the *status quo*. This by itself does not guarantee the success of a member's proposal, but is a necessary condition for it as will be seen shortly.

The setting for the two-person SVV is depicted in Fig. 6.1. The numbers next to the nodes refer to the players. The symbols next to the edges refer to moves that the players can make at each stage of the game. The time flows from top to bottom, i.e. player 1 moves first. The lower-most symbols refer to the outcomes. Thus, for example, following the left-most sequence, player 1 makes the first move by eliminating S_0 whereupon player 2 chooses to eliminate P_1 so that the end result is P_2.

In order to make predictions about which outcomes are likely to ensue from the calculations of minimally rational players, we can apply the procedure known as backwards induction (also sometimes known as Zermelo's algorithm) (Hamburger 1979; McKelvey and Niemi 1978).[1] We start from the final nodes, i.e. the outcomes and look for the immediately preceding decisions. Thus we notice that the choice between the two left-most outcomes, P_1 and P_2 is actually determined by player 2

[1] For a discussion on Zermelo's game-theoretic work, see Schwalbe and Walker (2001).

who, on the left-most decision node, can choose to eliminate either P_2 or P_1. Since the former is his proposal, it is plausible to assume that he eliminates P_1. So, when pondering upon his/her choice at the first decision node player 1 can safely assume that should he/she eliminate S_0, the outcome would be P_2. By similar reasoning player 1 can assume that by eliminating P_2 at the outset, the outcome will be P_1 or S_0 depending on whether player 2 deems P_1 preferable to S_0 or *vice versa*.

Suppose now that the proposals P_1 and P_2 are not Pareto-improvements over S_0, but the ranking over proposals are as in Table 6.1. Then, if both players have complete information about each other's preferences, player 1 knows that if he/she eliminates S_0, the end result is P_2, while if he/she eliminates P_2 the outcome is S_0. Since the latter outcome is preferred to the former by player 1, we can expect that he/she chooses accordingly, i.e. eliminates P_2.

Suppose now that both proposals are Pareto-improvements over S_0 so that the preferences are as in Table 6.2. Then, by eliminating P_2 at the outset, player 1 can expect to obtain his/her first-ranked alternative – assuming that player 2 acts according to his/her preferences.

We see that in both cases player 1 has an advantage: by eliminating the other player's proposal he/she can force the latter to choose between the *status quo* and player 1's proposal. As long as the latter is somewhat better for player 2 than S_0, it is likely that P_1 emerges as the outcome. Now, at the time of submitting proposals the players are not supposed to know which one of them is player 1, i.e. the first mover. Hence, both have an incentive to 'sweeten' their proposal so that it offers something more than S_0 to the other player as well. Hence, the mechanism is geared towards securing Pareto-optimal outcomes.

It is evident that, given strict preferences over the alternatives (proposals), the SVV yields a unique outcome (Mueller 1978). It can also be shown that the resulting outcome is never Pareto-dominated by another alternative. From the view-point of group decision making it is particularly noteworthy that the SVV outcome is never the lowest-ranked alternative of any participant (Felsenthal and Machover 1992). This feature makes it a plausible decision method in recommendation systems involving group decision making.

Table 6.1 Preference rankings over proposals: I

Player 1	Player 2
P_1	P_2
S_0	S_0
P_2	P_1

Table 6.2 Preference rankings over proposals: II

Player 1	Player 2
P_1	P_2
P_2	P_1
S_0	S_0

Now, the two player, three option case is hardly sufficient to cover all eventualities where SVV could be used. The more general setting involving the set N of n individuals and the set A of $m + s$ alternatives from which s elements have to be chosen can be described using the notation and conceptual apparatus of Felsenthal and Machover (Felsenthal and Machover 1992). The only restrictions on the cardinalities of the sets are that $s > 0$ and $m \geq n \geq 2$. This setting thus allows for situations where the players may make several proposals and where bundles of proposals are to be chosen. Once the order in which the players submit their eliminations has been established, the process begins with player 1 eliminating one alternative, say x_1. Then player 2 eliminates one alternative, say x_2, from $A \setminus \{x_1\}$, etc. until player n eliminates one alternative, say x_n, from $A \setminus \{x_1, x_2, \ldots x_{n-1}\}$. The sequence $x_1, \ldots, x_n$ is called the veto sequence. The alternatives in A left once the elements of the veto sequence have been removed are the selected alternatives. But how to predict the outcomes once the player preferences and the voting order has been established? Generalizing the approaches of Mueller and Moulin (Mueller 1978; Moulin 1983), Felsenthal and Machover introduce the concept of canonical sequence for a voting situation $(X, P_1, \ldots, P_n)$ as a sequence $y_1, \ldots, y_n$, where y_i is the least preferred proposal in the ranking P_i *under the assumption that all players* $j = i + 1, i + 2, \ldots, n$ *have already done their eliminations*. In other words, in forming the canonical sequence one begins with y_n which is the least preferred proposal according to P_n. From y_n we then work our way towards the beginning of the sequence (see Felsenthal and Machover (1992) for a rigorous description of the process). The canonical sequence represents plausible or rational behavior on the part of the players. Combined with the assumption that all player preferences are strict (no ties or incomparable pairs among pairs of proposals) the canonical sequence guarantees a unique solution or prediction for SVV processes involving alternative sets of cardinality larger than that of the player set.

As stated above SVV looks quite plausible system for small committees especially under circumstances where divisive outcomes – those strongly opposed by sizable minorities of players - are to be avoided. Not surprisingly, SVV is non-majoritarian. Thus, it can leave the Condorcet winner unchosen and even result in the choice of the Condorcet loser.

The latter possibility is exemplified in the profile of Table 6.3. Suppose that the order of voting is voter 1, voter 2, voter 3. Then the canonical sequence is: (C, A, B) leaving D, the Condorcet loser, the only un-vetoed alternative. The same outcome ensues under sincere vetoing, whereby voter 1 first eliminates C, then voter 2 eliminates A and finally voter 3 vetoes B.

In a way, SVV represents an extreme version of minority protection since a single individual may exclude a for him/her undesirable outcome. Table 6.4 illustrates the violation of the no-veto condition by SVV. The condition requires that if in a profile all voters except one rank the same alternative first, then this alternative is chosen. In Table 6.4 A is such an alternative and, yet, under whatever voting order, A will be eliminated.

Table 6.3 SVV may choose the Condorcet loser

Voter 1	Voter 2	Voter 3
A	B	C
B	C	A
D	D	D
C	A	B

Table 6.4 No-veto violation of SVV

4 voters	1 voter
A	B
B	C
C	D
D	E
E	F
F	A

6.3 Topics for Further Reflection

1. Consider the elimination game tree of Fig. 6.1. Assume that the preferences of players are as in Table 6.5. What is the likely outcome of the SVV game here?
2. Construct a profile involving four players and six alternatives. (i) Construct a canonical sequence and determine the SVV winner. (ii) Determine the SVV choice set if two alternatives are to be selected.
3. Construct a profile with a Condorcet winner and a sequence of votes such that the Condorcet winner is not selected by SVV.

6.4 Suggestions for Reading

The main sources to be consulted are those that have been referred to above, viz. Mueller (1978), Moulin (1983) as well as Felsenthal and Machover (1992). Mueller (2003) provides a brief analysis of SVV in comparison with a couple of other similar

Table 6.5 Preference rankings over proposals: III

Player 1	Player 2
P_1	P_2
P_2	S_0
S_0	P_1

methods. Yuval's (2002) pioneering study reports on strategies resorted to by individuals in experimental settings.

Answers to Selected Problems

1. P_2
2. Consider the following profile:

Voter 1	Voter 2	Voter 3	Voter 4
A	B	C	D
B	C	D	E
C	D	E	F
D	E	F	A
E	F	A	B
F	A	B	C

   (i) With the vetoing order 4, 3, 2, 1, 4, the winner is D. (ii) With the same order the two winners are D and E.
3. See Table 6.4. Let the right-most voter veto first.

References

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