Chapter 14
An MCDM/A Framework for Choosing Rules

Abstract Our focus is on the decision process. A Framework for the DPVP (decision process for choosing a voting procedure) is necessary in order to guide how best to aid DMs. It is assumed that DMs may evaluate the impact of VP (Voting Procedure) properties on their own business decision process. It is assumed that the DMs have agreed on some voting procedure. Choosing the most appropriate MCDM/A (Multi-Criteria Decision Making/Aiding) method is essential to ensure the quality of the decision process. When choosing an MCDM/A method, the DM's preferences should be taken into consideration. A check needs to be made on whether the DM uses compensatory or non-compensatory rationality.

14.1 Introduction

The decision context of a business organization is discussed and a framework is built to deal with the decision process of choosing a VP. This framework is included in an MCDM/A model that can aid this choice.

This framework considers preliminary ideas (de Almeida and Nurmi 2015; de Almeida and Nurmi 2014) for aiding the choice of a voting procedure (VP), by using an MCDM/A model and considering a business organization. As to implementing the framework, it is assumed that an analyst will give some methodological and technical support to the DM.

The framework guides how best to structure the elements to be considered in the process, which include: the MCDM/A method to be used, the criteria to be applied and what the outcomes of these voting procedures for these criteria are likely to be. These elements are not prior specified in a general way, but rather, they will be consolidated according to the business context under analysis.

In the DPVP, it is assumed that there is either: only one DM (a benevolent dictator or a supra-DM) or a group of DMs acting in agreement. In the latter, one of the DMs may be appointed to act on behalf of the others. Otherwise, a more complex group decision process would be conducted and the way in which the DMs interact in order to choose the VP in the DPVP may be an important issue. This situation is discussed in the last section of this chapter.

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14.2 The Framework for the DPVP

As explained in a previous chapter, the DPVP takes place in a phase at the beginning of the whole decision process and is when an MCDM/A method is used to choose a VP. This decision model is built for the particular characteristics of the business context and decision under analysis.

The decision model is built based on a framework presented in this chapter, which is founded on a more general procedure for building MCDM/A decision models (de Almeida et al. 2015), which has three phases: a preliminary phase, preference modeling phase, and a finalization phase.

In the first phase, which has five steps, the basic elements of the decision model e.g. the objectives, the criteria and the set of alternatives are established. The first step is used to identify the DM and other actors in the decision process and whether or not the situation is a group decision problem.

The phase for preference modeling has three steps and includes choosing the MCDM/A method. In the last phase, the alternatives are evaluated, a sensitivity analysis is made and a recommendation is put forward.

The framework for the DPVP is shown in Fig. 14.1. All the steps of the DPVP must be focused on the DPBO (Decision Process for the Business Organization), as it is this which determines the whole decision process.

In the first step, VPs, including those that are technically appropriate for the business decision process, are pre-selected and placed in a set. Most recognized VPs could be included in this step, while those excluded are mainly those that may be incompatible in some way with the decision process. For instance, a VP may require data to be input which it is not feasible to provide for the business decision process.

The second step consists of establishing the criteria, which are associated with the DM's objectives in choosing a VP, including the paradoxes and desirable properties of VPs. Then the consequence matrix and decision matrix are built, since the alternatives (VPs) and criteria are given. The following step consists of choosing the MCDM/A method to be applied for analyzing these VPs according to the criteria given. What MCDM/A method to choose is an important issue to be considered (Roy and Słowinski 2013; de Almeida et al. 2015). The finalization is obtained with the steps for parameterizing and applying the MCDM/A model.

The dashed frame in the middle of Fig. 14.1 shows who makes the decision in each step. Usually the DM gives preference information to be included within the model. The analyst has also to make technical choices; for instance when pre-selecting VPs. There are some steps in which instead of decisions, only technical actions are taken in the modeling process; for instance for building the consequence matrix.

The interactions between the analyst and the DM (or a group of DMs) are shown in Fig. 14.1. The dashed frame on the right side of Fig. 14.1 shows how the analyst supports the decision process. In most of the steps, the analyst conducts a structuring or modeling activity. Some of these activities involve interaction with the DM, e.g. establishing criteria or building the decision matrix.

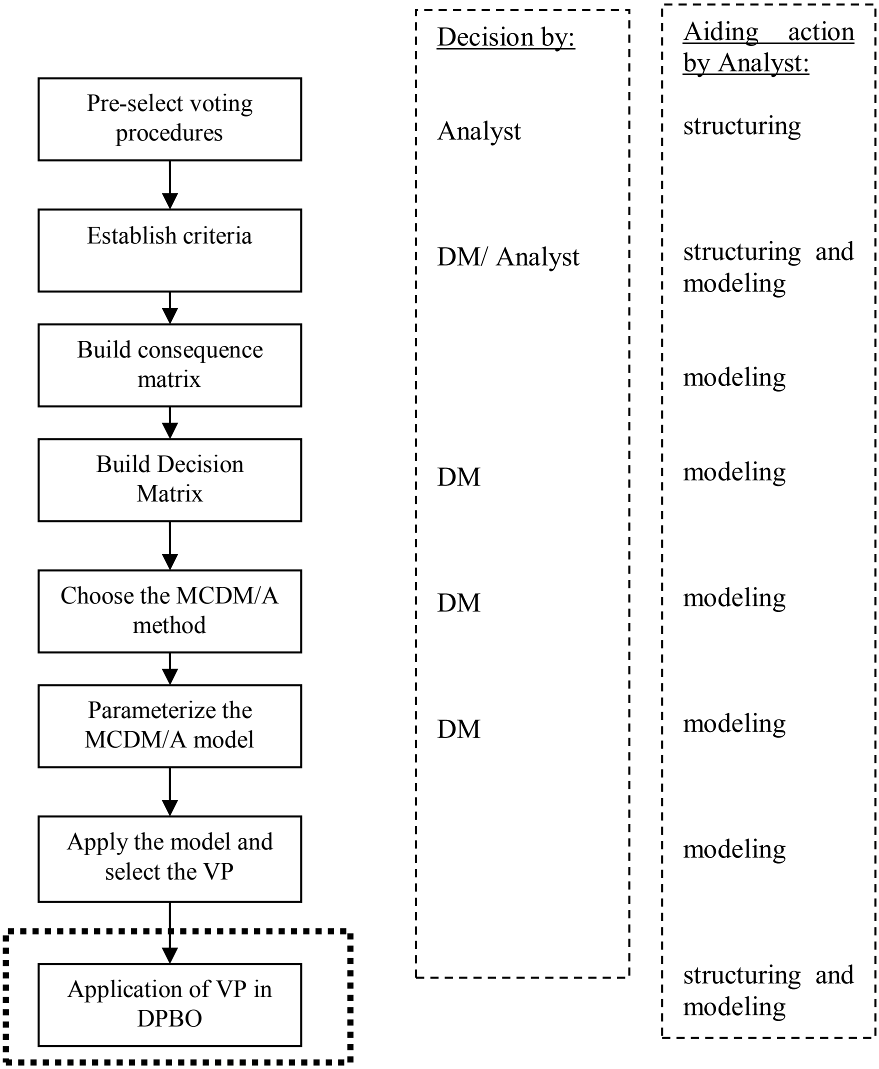

Fig. 14.1 Framework for DPVP—choosing the voting procedure

For the step of pre-selecting voting procedures, many of the considerations given in Part I and II of this book are applied. The main issue to emphasize at this moment is the importance not only of having regard to these but especially also of taking into account the business context of the decision to be made.

The following sections give a few details about other steps of the framework.

14.3 Criteria for Selecting a Voting Procedure

In the DPVP, there are two kinds of criteria to be considered, which are related to two main objectives directly associated with the context of the business decision problem:

- Maximizing the full use of VP's properties that are desirable and appropriate to the DPBO;
- Maximizing the matching between the nature of input required by the VP and its impact on the DPBO.

The latter objective is directly related to the DM's interaction with the DPBO and the way in which the DM supplies information to be processed in the VP. This objective produces a criterion called the 'Input criterion'.

The first objective concerns the characteristics of VPs, associated with their properties and paradoxes, that may have a relevant impact and the way they affect the DPBO. The property or the paradox of each VP may be considered a criterion and the consequence matrix indicates how this criterion affects each VP. This objective produces a criterion called the 'property criterion'.

14.3.1 Criteria Related to the Properties of VPs

The results obtained from Social Choice Theory have shown compatibility issues for choice desiderata. So, a set of intuitively plausible principles of choice has been considered. The incompatibility captured by Arrow's Impossibility Theorem (Arrow 1963) is one of the main issues to be considered in this matter of choosing a VP and becomes an important restriction on establishing preference profiles.

A few studies regarding the analysis of properties and paradoxes of most VPs are reported in the literature (Nurmi 1983, 1987, 2002; Felsenthal and Nurmi 2018). These VPs' features are relevant for choosing one of them for a specific decision problem. For this objective, the selection of a set of relevant properties of the pre-selected VPs may be a set of criteria to be considered. A set of these criteria is shown in Table 7.5. The set consists of the Condorcet winner criterion; the Condorcet loser criterion; the strong Condorcet criterion; monotonicity; Pareto; consistency; Chernoff property; independence of irrelevant alternatives; and invulnerability to the no-show paradox. A wider set of this kind of criteria is introduced and discussed in Sect. 7.3.

There are several criteria for comparing VPs, one of which is particularly important for the business context, since it can be associated with rationality, which is Pareto optimality. It is the collective rationality criterion. It states that an alternative y is not chosen, if each DM strictly prefers alternative x to alternative y. If a VP fails on Pareto optimality, this means that it is collectively irrational.

The Condorcet winner has usually been indicated for social choice rules as a reasonable desideratum. It dictates that an alternative that defeats all the others in pairwise majority comparisons should be chosen. On the other hand, commonly, a VP that satisfies the Condorcet winner criterion, does not satisfy the positional dominance criterion (Fishburn 1982), which is another acceptable criterion. It states that alternative x positionally dominates alternative y, if for each possible rank r, the number of DMs assigning x to rank r or higher is larger than the number of DMs assigning y to rank r or higher. The positional dominance criterion indicates that an alternative may not be chosen if it is positionally dominated by another alternative.

Many paradoxes of VPs have consequences that need to be evaluated with regard to their impact on the DPBO. However, when we try to avoid a paradoxical possibility, this leads to another kind of paradox. Therefore, some trade-offs can be made when dealing with paradoxes. On the other hand, given that the DM's preference in the decision process may lead to a different kind of rationality, in which non-compensatory rationality (de Almeida et al. 2015) takes place, in this kind of situation, other inter-criteria relations are considered, instead of tradeoff.

14.3.2 Criteria Related to Data Input of VP

These kinds of criteria seek to maximize the matching between the nature of input required by the VP and its impact on the DPBO. The input consists of the kind of information the DMs give about the alternatives in order to introduce a chosen VP. An input required by a VP could be, for instance, the ranking of all alternatives by each DM; or the pairwise comparison of all alternatives by each DM; or rating of all alternatives, in such a way that the DM could give a score (for instance, on a five-level scale of 1–5) for all alternatives.

Another classification of input could consider the amount of information. For instance, a VP could require full information; partial information; or the most relevant information (e.g., Approval Voting procedure).

An impact on the DPBO could be, for instance, the effort that the DMs require to make in order to produce the information required. Other aspects may affect the DPBO, such as the ease with which a particular kind of information can be provided. Therefore, the ease of making the input that a particular VP requires may be considered a criterion in this group.

The reliability of information may be also associated with the input required from DMs. For instance, it is well known that choosing alternatives from among a large number of alternatives is affected by the bounded rationality of humans (Simon 1955, 1960, 1982), particularly when the number of alternatives is greater than 7. In this case, it might be useful to consider the style of input: pairwise comparison or ranking, for instance. Another possibility is a grade based on the number of alternatives, for which an exponential function with a negative value could be applied. Also, problems of reliability may arise due to there being a large number (i.e. well over 7) of alternatives in the ranking. For pairwise comparisons, the number of options can be used as a parameter with a function of similar value.

In many circumstances in a business organization, there is a decision process for each DM, which requires another MCDM/A model to be applied to compare all alternatives. In general, this decision model is much more complex than the MCDM/A model for the DPVP. In this case, this kind of criterion may not be relevant in the DPVP, since the MCDM/A model for the DPBO will compare alternatives. The only issue is the compatibility of this information with that required by the pre-selected VPs. On the other hand, this criterion may be rather relevant with a high weight, for problems related to a group of DMs dealing with leisure choices (Naamani-Dery et al. 2014), in which case, the options are evaluated directly.

Of course, the analyst's convenience and personal objectives should not have any kind of influence on this decision process i.e. they should not be included in the set of criteria for comparing the VPs. In other words, only the DM's objectives connected to the business problem should be considered for the set of criteria.

For instance, an analyst may wish to reduce the computational complexity of the VP. However, this criterion should not be accounted for, since the analyst's preference should not be included in the model. On the other hand, if this kind of criterion has some influence on the DPBO, it could be considered. After all, the purpose of the DPVP is to improve the outcomes of the DPBO.

14.3.3 Weights for Criteria

The way in which the weights of criteria are established depends on the kind of MCMD/A method applied. In different methods, these weights have different meanings. For instance, in compensatory methods such as those with additive aggregation for criteria they mean scale constants and will change with the normalization procedure or the scale applied to the input data. For non-compensatory methods such as outranking methods, they mean degree of importance.

Therefore, the elicitation procedure for weights or preference modeling will be conducted in different ways according to the MCDM/A method applied. Also, using partial or incomplete information precise weights may not be needed (de Almeida et al. 2016). In that case, a concept of the space of weights may be applied thereby bringing more robustness to the process. Also, surrogate weights may be applied, when considering partial information (Edwards and Barron 1994).

14.4 The Consequence Matrix

Building the consequence matrix is straightforward and will produce information in the format of Table 14.1.

For the objective related to the nature of the input required by the VP, there are many possibilities for the format of data, and natural attribute (Keeney 1992) should be the first choice.

Table 14.1 Consequence matrix for discrete binary outcome

Voting system	Criteria: Condorcet winner	Condorcet loser	Strong condorcet	Monotonicity	Pareto	Consistency	Chernoff	Independence of IA	Invulnerability to the no-show paradox
Amendment	1	1	1	1	0	0	0	0	0
Copeland	1	1	1	1	1	0	0	0	0
Dodgson	1	0	1	0	1	0	0	0	0
Maxmin	1	0	1	1	1	0	0	0	0
Kemeny	1	1	1	1	1	0	0	0	0
Plurality	0	0	1	1	1	1	0	0	1
Borda	0	1	0	1	1	1	0	0	1
Approval	0	0	0	1	0	1	1	0	1
Black	1	1	1	1	1	0	0	0	0
Pl. runoff	0	1	1	0	1	0	0	0	0
Nanson	1	1	1	0	1	0	0	0	0
Hare	0	1	1	0	1	0	0	0	0

For the first objective, related to a VP's properties, there are two forms of doing this, namely:

- Discrete binary outcome.
- Continuous outcome.

The discrete binary outcome consists of the indication whether some property is violated by a VP. Table 14.1, which is based on Tables 7.4 and 7.5, is the consequence matrix of a type of discrete binary outcome. In that table, '1' indicates that the VP satisfies the property and '0' that it does not. This outcome in the consequence matrix is one of increasing preference; i.e. a score of 1 is preferred to a score of 0.

This kind of information imposes a great limitation on the modeling process, since the information provided is not so rich. That is, the consequence matrix informs only whether the property is satisfied or not. The continuous outcome supplies richer information.

The continuous outcome consists of providing information on how often the property is violated in that VP. If a criterion is not satisfied, we can consider how frequently a VP violates that criterion. This could be represented by the frequency of occurrence in a scale of 0–1. In this case, an indication of '1' in Table 14.1, would have a score of '0' in Table 14.2, and an indication of '0' in Table 14.1 would have a score between 0 and 1 in Table 14.2. This outcome in the consequence matrix is one of decreasing preference; i.e. a score of 0 is preferred to a score of 1. A score of 1 indicates that that property is always violated, while '0' indicates that it is always satisfied. Table 14.2 shows the consequence matrix for the continuous outcome.

In the continuous outcome more information is given, since it can represent that a drawback to a VP may not happen for some contexts of input data in a decision problem. In order to illustrate this situation, let us consider the property of the independence of irrelevant alternatives. Let us consider three different VPs: VP_1, VP_2 and VP_3 and let us make a few simplifications for the sake of clarifying how this kind of outcome may be evaluated. Assuming that for VP_1, the independence of irrelevant alternatives does not hold at all, then the outcome for $x_{18} = 1$, which means that for VP_1 this property is always violated. For VP_2, let us assume that the independence of irrelevant alternatives does hold in 50% of the cases, then $x_{28} = 0.5$. Finally, let us assume that for VP_3, the independence of irrelevant alternatives does hold for more than 90% of the cases, which is assumed to be a limit in this scale and represents a minimum frequency of occurrence, then $x_{38} = 0$. The marginal value function assigns the impact of these outcomes to the DPBO.

This kind of information is richer than that in Table 14.1 and can significantly improve the evaluation process and make it much more reliable. Information related to x_{ij} in Table 14.2 can be found in literature, although this is not available for all VP, considering all properties. Also this information depends on the number of DMs (voters) and the number of alternatives in the DPBO. Simulation studies on this matter might be required in order to complete information on those not found in the literature.

For illustrating this kind of information of x_{ij} in Tables 14.2 and 14.3 provides an example based on studies found in the literature, for the Condorcet winner criterion,

Table 14.2 Consequence matrix for the continuous outcome

	Criteria								
Voting system	Condorcet winner	Condorcet loser	Strong Condorcet	Monotonicity	Pareto	Consistency	Chernoff	Independence of IA	Invulnerability to the no-show paradox
Amendment	0	0	0	0	x_{ij}	x_{ij}	x_{ij}	x_{ij}	x_{ij}
Copeland	0	0	0	0	0	x_{ij}	x_{ij}	x_{ij}	x_{ij}
Dodgson	0	x_{ij}	0	x_{ij}	0	x_{ij}	x_{ij}	x_{ij}	x_{ij}
Maxmin	0	x_{ij}	0	0	0	x_{ij}	x_{ij}	x_{ij}	x_{ij}
Kemeny	0	0	0	0	0	x_{ij}	x_{ij}	x_{ij}	x_{ij}
Plurality	x_{ij}	x_{ij}	0	0	0	0	x_{ij}	x_{ij}	0
Borda	x_{ij}	0	x_{ij}	0	0	0	x_{ij}	x_{ij}	0
Approval	x_{ij}	x_{ij}	x_{ij}	0	x_{ij}	0	0	x_{ij}	0
Black	0	0	0	0	0	x_{ij}	x_{ij}	x_{ij}	x_{ij}
Pl. runoff	x_{ij}	0	0	x_{ij}	0	x_{ij}	x_{ij}	x_{ij}	x_{ij}
Nanson	0	0	0	x_{ij}	0	x_{ij}	x_{ij}	x_{ij}	x_{ij}
Hare	x_{ij}	0	0	x_{ij}	0	x_{ij}	v	x_{ij}	x_{ij}

Table 14.3 Consequence matrix for the continuous outcome with condorcet winner criterion for four VPs

	Criteria								
Voting system	Condorcet winner	Condorcet loser	Strong condorcet	Monotonicity	Pareto	Consistency	Chernoff	Independence of IA	Invulnerability to the no-show paradox
Amendment	0	0	0	0	x_{ij}	x_{ij}	x_{ij}	x_{ij}	x_{ij}
Copeland	0	0	0	0	0	x_{ij}	x_{ij}	x_{ij}	x_{ij}
Dodgson	0	x_{ij}	0	x_{ij}	0	x_{ij}	x_{ij}	x_{ij}	x_{ij}
Maxmin	0	x_{ij}	0	0	0	x_{ij}	x_{ij}	x_{ij}	x_{ij}
Kemeny	0	0	0	0	0	x_{ij}	x_{ij}	x_{ij}	x_{ij}
Plurality	0.21	x_{ij}	0	0	0	0	x_{ij}	x_{ij}	0
Borda	0.11	0	x_{ij}	0	0	0	x_{ij}	x_{ij}	0
Approval	x_{ij}	x_{ij}	x_{ij}	0	x_{ij}	0	0	x_{ij}	0
Black	0	0	0	0	0	x_{ij}	x_{ij}	x_{ij}	x_{ij}
Pl. runoff	0.23	0	0	x_{ij}	0	x_{ij}	x_{ij}	x_{ij}	x_{ij}
Nanson	0	0	0	x_{ij}	0	x_{ij}	x_{ij}	x_{ij}	x_{ij}
Hare	0.16	0	0	x_{ij}	0	x_{ij}	x_{ij}	x_{ij}	x_{ij}

considering 5 DMs (voters) and 7 alternatives (Nurmi 1988, 1992, 1995). Note that the Approval Voting procedure has not be considered in this particular example, since this information is not available in those references.

It might be the case that some information on x_{ij} may be neither available in the literature or there is not a possibility for making the simulations studies in order to found out the values for x_{ij}. In that case, there is a possibility of using Prior probabilities $\pi(x_{ij})$, which is obtained in the framework of Bayesian approach and consists of subjective probabilities, such as it is usually applied in Bayesian Decision Theory (Raiffa 1968; Berger 1985). Actually, $\pi(x_{ij})$ is prior probability density function over the possible 'state of nature' X_{ij} (see Berger 1985). These Prior probabilities $\pi(x_{ij})$, also known as degree of belief, are usually obtained from experts in the subject analyzed; that is the 'state of nature' in consideration.

In some situations, this prior probability might be considered more relevant than conducting simulations studies in order to obtain x_{ij}. In that case, the consequence is probabilistic and MAUT (Multi-Attribute Utility Theory) could be more appropriate than MCDM/A deterministic methods. This is more detailed in the subsequent section. However, it is also possible using an outranking method, when integrated with utility functions, in order to consider the probabilistic information, as it has been shown in de Almeida (2005) and Brito et al. (2010).

For illustrating the use of Prior probabilities $\pi(x_{ij})$, Table 14.4, adds this to Table 14.3, Condorcet winner criterion in Approval Voting procedure and also for all VPs regarding to the Independence of IA criterion. In this illustration, it can observed that the process can be conducted considering a mixing of different kind of information, such as the estimated value for x_{ij}, for some criteria and $\pi(x_{ij})$ for others.

There is an alternative for dealing with this possibility of not having either directly information about X_{ij}, or prior probability $\pi(x_{ij})$. This is the use of a simpler information from experts, which would be based on a score in a discrete scale of at least three levels (0, 1 or 2). Also, a score of five-level scale (0, 1, 2, 3, 4) could be applied, likewise the Likert scale (Likert 1932a, b). This score indicates the influence of that criterion on the VP. For the three-level scale, for instance, score '0' indicates that the VP satisfies the property and '2' that it either rarely satisfies the property or does not satisfies, and score '1' indicates that the VP may satisfy the property with a medium frequency comparatively to the score '2'.

For now, only technical information is provided in this step. The next step associates this information of consequence matrix, such as those of Tables 14.1, 14.2, 14.3 and 14.4, with its impact on the business process, in accordance with the DM's preference.

14.5 The Decision Matrix

The decision matrix provides preferential information over the outcomes in the consequence matrix. This preferential information is given by the DM, according to

Table 14.4 Consequence matrix for the continuous outcome including some prior probability density function

	Criteria								
Voting system	Condorcet winner	Condorcet loser	Strong condorcet	Monotonicity	Pareto	Consistency	Chernoff	Independence of IA	Invulnerability to the no-show paradox
Amendment	0	0	0	0	x_{ij}	x_{ij}	x_{ij}	$\pi(x_{ij})$	x_{ij}
Copeland	0	0	0	0	0	x_{ij}	x_{ij}	$\pi(x_{ij})$	x_{ij}
Dodgson	0	x_{ij}	0	x_{ij}	0	x_{ij}	x_{ij}	$\pi(x_{ij})$	x_{ij}
Maxmin	0	x_{ij}	0	0	0	x_{ij}	x_{ij}	$\pi(x_{ij})$	x_{ij}
Kemeny	0	0	0	0	0	x_{ij}	x_{ij}	$\pi(x_{ij})$	x_{ij}
Plurality	0.21	x_{ij}	0	0	0	0	x_{ij}	$\pi(x_{ij})$	0
Borda	0.11	0	x_{ij}	0	0	0	x_{ij}	$\pi(x_{ij})$	0
Approval	x_{ij}	x_{ij}	x_{ij}	0	x_{ij}	0	0	$\pi(x_{ij})$	0
Black	0	0	0	0	0	x_{ij}	x_{ij}	$\pi(x_{ij})$	x_{ij}
Pl. runoff	0.23	0	0	x_{ij}	0	x_{ij}	x_{ij}	$\pi(x_{ij})$	x_{ij}
Nanson	0	0	0	x_{ij}	0	x_{ij}	x_{ij}	$\pi(x_{ij})$	x_{ij}
Hare	0.16	0	0	x_{ij}	0	x_{ij}	x_{ij}	$\pi(x_{ij})$	x_{ij}

Table 14.5 Decision matrix

A	Criterion 1	Criterion 2	Criterion 3	…	Criterion j	…	Criterion n
a_1	$v(x_{11})$	$v(x_{12})$	$v(x_{13})$	…		…	$v(x_{1n})$
a_2	$v(x_{21})$	$v(x_{22})$	$v(x_{23})$	…		…	$v(x_{2n})$
…	…	…	…	…	…	…	…
a_i				…	$v(x_{ij})$	…	…
…	…	…	…	…	…	…	…
a_m	$v(x_{m1})$	$v(x_{m2})$	$v(x_{m3})$	…	…	…	$v(x_{mn})$

the framework shown in Fig. 14.1. The role of the analyst in this step is to aid the DM in the preference modeling process. At this point, the DM's preferences that are considered are those regarding the evaluation of the intra-criteria, and this leads to producing the marginal value $v(x_j)$ of the outcomes x_j related to criterion j. Table 14.5 shows how this kind of information is considered in an MCDM/A method.

Regarding the DM's preference, it may happen that a property is not satisfied and that this does not matter very much for a particular context decision. Additionally, it may happen that the value for the frequency of occurrence of violation is not a linear function. That is, the value of a frequency of 0.25 might not represent 50% of the value of a frequency of 0.50; i.e. either v(0.25) > 0.5 v(0.5) or v(0.25) < 0.5 v(0.5).

This depends on how the DM evaluates its impact in the business context. In the case illustrated above, the marginal value function would be non-linear. Actually, in most cases, it is expected that a linear function will be found (de Almeida et al. 2015).

For the discrete binary outcome, the following value function may be applied:

$$v_j(x_{ij}) = x_i \tag{14.1}$$

For the continuous outcome, which has an outcome of decreasing preference, such as that shown in Table 14.2 (and Table 14.3), the following value function may be applied, if it is linear:

$$v_j(x_{ij}) = 1 - x_i \tag{14.2}$$

Regarding a non-linear marginal value function, the literature offers many possible analytical forms (Keeney and Raiffa 1976; Belton and Stewart 2002).

Let us give an illustrative explanation of the continuous outcome, based on the example of the preceding section, for three different VPs: VP_1, VP_2 and VP_3. Applying Eq. (14.2) for the value function of x_i gives the outcome values for each VP for this particular criterion, as follows: $v(VP_1) = 0$, $v(VP_2) = 0.5$, and $v(VP_3) = 1$.

Therefore, using this value function, the DM assigns evaluations to these consequences x_i, in accordance with their impact on the DPBO. Of course, the impact on the DPBO could require a non-linear scale for the marginal value of x_i. For instance, considering a scale of 0 to 1, this would keep the values of extreme outcomes. So,

$v(VP_1) = 0$ and $v(VP_3) = 1$. As to VP_2, the value could be 0.5 different, either $v(VP_2) > 0.5$ or $v(VP_2) < 0.5$. If we assume that the harm caused to the DPBO by this drawback has an increasing rate with the outcome score, then $v(VP_1)$ could be higher than 0.5; for instance, $v(VP_2) = 0.7$. The question pointed out here is how often a drawback appears and to what extent this frequency affects each particular DPBO.

For the objective related to the nature of the input required by the VP, a non-linear value function may apply in many cases. Let us consider the reliability of information given by the DM. In this case, a non-linear value function may apply for the reliability criterion; e.g. a negative exponential function on the number of alternatives could represent this situation.

In order to build the decision consequence in Table 14.2, a discrete 5-level scale, such as the Likert (1932a, b) scale, could be used for subjective evaluations related to the marginal value function.

In the case of using prior probabilities $\pi(x_{ij})$, instead of a value function, a utility function is considered and the expected utility is associated with $\pi(x_{ij})$. Therefore, Eq. (14.3) is applied.

$$u_\pi\left(x_{ij}\right) = \int_0^1 \pi\left(x_{ij}\right)u\left(x_{ij}\right) \tag{14.3}$$

For a linear utility function over xij, the Eq. (14.2) can be applied in (14.3), becoming eq (14.4), as follows:

$$u_\pi\left(x_{ij}\right) = \int_0^1 \pi\left(x_{ij}\right)\left(1 - x_{ij}\right) \tag{14.4}$$

For an exponential utility function with parameter *a* (Keeney and Raiffa 1976; de Almeida 2005) over x_{ij}, the Eq. (14.4) becomes (14.5), as follows:

$$u_\pi\left(x_{ij}\right) = \int_0^1 \pi\left(x_{ij}\right)e^{-ax_{ij}} \tag{14.5}$$

The value of expected utilities $u(x_{ij})$ computed for a particular criteria *j* on a specific VP *i*, should be introduced in the Matrix of Table 14.4, for the case in which prior probabilities $\pi(x_{ij})$ are applied, replacing its equivalent value function $v(x_{ij})$ for the case of deterministic consequence.

For the case of the discrete scores applied instead of prior probabilities x_{ij}, the Eq. (14.6) should be applied

$$v_j\left(x_{ij}\right) = (y - x_i)/y \tag{14.6}$$

where y is the highest level in the scale. For the three-level scale, $y = 2$ and for the five-level scale, $y = 4$. The application of this three-level scale in Chap. 17 illustrates its use, with the Eq. (14.6).

14.6 Choice of MCDM/A Method for Comparing VP

After building the decision matrix, an MCDM/A method is applied in order to choose the most appropriate VP for the business decision in question. As shown in the framework of Fig. 14.1, a prior step still had to be followed. This step consists of choosing an MCDM/A method, which is one of the most important steps when building any MCDM/A model (Roy and Słowinski 2013; de Almeida et al. 2015).

The DM's preference is the main contribution to be considered when choosing the MCDM/A method. In this step the analyst should first evaluate the DM's rationality as to compensation amongst criteria, before the method itself is chosen (de Almeida et al. 2015). Then, an inter-criteria evaluation could be developed in order to proceed to the final preference modeling process, and establishing the criteria weights.

As described in the previous Chapter, there are two types of MCDM/A method for discrete action space, such as is the case of the DPVP. The main factor to be analyzed when choosing the MCDM/A method for the DPVP is the compensatory or non-compensatory rationality (de Almeida et al. 2015; Bouyssou 1986; Munda 2008). The question to put forward is: which fits the DM's preference structure for this particular decision problem and context? It should be evaluated if the DM is willing to make compensation amongst the VP's properties, for instance.

However, it should be noted that for a 'discrete binary outcome', it does not matter if the DM's rationality is compensatory or non-compensatory, from the point of view of choosing the method. Applying either method, the unique criterion of synthesis or outranking, would be analytically similar, at least for comparing a method with the additive model with outranking methods. For instance, considering the PROMETHEE method, for the discrete binary outcome, the result is similar to that from using additive models.

This can be easily verified by analyzing Eq. (13.1), which represents the additive model. For all criteria j to which $v_j(x_j) = 1$, the scale constants k_j of those criteria are summed up to the global value of the VP. Similarly the same happens for the PROMETHEE method, for the weights w_j of those criteria, which are going to be combined in (13.8). In this case, following the non-compensatory approach would be easier with regard to eliciting the weights of criteria.

On the other hand, if the consequence matrix is built with a 'continuous outcome', it makes a difference to the results if the DM's rationality is compensatory or non-compensatory. In this case, some reflections on the analysis of the most appropriate rationality to the DMs may be provided.

It does not seem to be natural to a DM to make compensation between the properties of two VPs. It seems more natural that a DM, when comparing two VPs, would be willing to check only which of them has a greater frequency in violating a particular property. It might not be relevant in most cases to consider precisely how frequently violations happened.

That is, it seems reasonable that a DM would compare two VPs for this kind of criterion by considering only which of them has a better performance rather than taking into account the extent of the difference between the two performances. In that case, the assumption of non-compensatory rationality seems to be reasonable and an outranking method could be applied in the DPVP. In this case an ordinal scale would be fine for the decision matrix in Table 14.4 and a linear marginal value function, such as in (14.1) could be applied. Then, the analyst could consider using either the ELECTRE or the PROMETHEE method.

Although the assumption above could be considered reasonable, this should be applied unconditionally. Actually, apart from the preceding considerations, the DM's preference must always be evaluated. It might happen that a DM requires compensation when comparing the properties of two VPs. That is, for comparisons between two VPs, a DM may be willing to take into account the magnitude of frequency in which a property is violated in each VP. In this case, a cardinal scale should be considered for Table 14.2.

If a compensatory rationality is found to be more appropriate, before choosing a method with the additive model, the properties of this model should be evaluated in order to confirm if it is actually adequate. For instance, the mutual preferential independence condition amongst the criteria should be checked. A simple evaluation could be to check the condition in (13.4) with the DM. Although the independence condition may be found in the most partial application, when a preferential dependence occurs between criteria, this model can produce undesirable results. There are some situations in which a preferential dependence may occur in the DM's preference structure. This usually happens when quality and quantity are the criteria to be considered (de Almeida 2013). For instance, in evaluating an Academic Institution, if the quantity and the quality of the degrees awarded are criteria in the evaluation, then, the additive model could make a compensation in such a way that an Institution of poor quality could compensate this and increase its global score in the model (13.1), because it has awarded a higher number of degrees.

Also, the preference structure (P, I) should be compatible with the DM's ability to express preferences. It might happen that the DM is not able to compare all consequences, in which case a preference structure (P, Q, I, J) should be considered.

A first thought in such a situation is to turn to outranking methods, since they first appeared with the possibility of approaching incomparability. However, this would not be a reason for using these methods. Actually, their ability to approach incomparability is an operational advantage in the process of preference modeling. If a compensatory rationality is required, these methods should be considered in the last case as an approximation.

When difficulties are found with regard to using a preference structure (P, I) in the preference modeling process, the additive model can still be considered, if a method with partial information is applied. For instance, the FITradeoff (Flexible and Interactive Tradeoff) method (de Almeida et al. 2016) could be applied in this case. Although this method uses the additive model in the MAVT scope, it is able to model preferences with partial information and has the flexibility of allowing the DM to refuse both to make some comparisons and to answer some typical questions of the elicitation process and yet, even so, a solution may be found. In general, these kinds of questions and other much more difficult ones are essential elements in the classical methods available.

14.7 Parameterization of the MCDM/A Model

Once the MCDM/A method is chosen, the parameterization of the model consists of specifying its parameters with the information collected in the preference modeling with the DM. There are a few methods described in Chap. 13, which also discusses the concerns with this process of making preference modeling in order to obtain parameters.

The main information required here are the scale constants or the weights of the criteria. Other parameters may be required depending on the method applied. In this case, the meaning of these parameters is the main concern, since they may have different meanings in unrelated methods. The main concern here is with the use of the concept of weights of criteria as indicating the degree of importance in additive models, since this is systematically applied in many situations.

For compensatory methods, such as those based on additive models, the elicitation of the scale constants are not applied in an appropriate way, since these parameters do not mean straightaway the degrees of importance of a criterion, as already explained. This is the main issue with regard to these methods.

On the other hand, for the non-compensatory methods, the weights do straightforwardly represent the relative degree of the importance of the criteria. So, when given by the DM, this kind of evaluation may be easier than was the case in the previous additive model. Let us consider two criteria in Table 7.4. For instance, the monotonicity could have a minor impact over the DPBO, compared with the Condorcet winner criterion. In this case, the monotonicity would receive a low weight from the DM, who could consider that the 'Condorcet winner' criterion is twice as important as the monotonicity, with a weight of 0.3. Therefore, the monotonicity criterion would have a weight of 0.15.

As to intra-criteria evaluation, different considerations are also applied. First, for compensatory methods a cardinal scale has to be considered for eliciting the marginal value function. On the other hand, for the non-compensatory methods an ordinal scale may be considered, which can be clearly seen on analyzing Eqs. (13.5) or (13.8), respectively, ELECTRE and PROMETHEE methods. Some exceptions may apply in some of these methods, such as for ELECTRE. The discordance indices may consider an interval scale so as to compute differences between the performance of two alternatives, if the analyst uses this kind of formulation. Regarding the PROMETHEE method, the ordinal scale is applied particularly for $F_j(a, b)$ when in Eq. (13.8). On the other hand, the preference flow consists of a cardinal scale (summation of the criteria weights) for scores of the alternatives. Even, if the usual criterion is not applied to the function $F_j(a, b)$, and indifference or preference thresholds are applied in these methods, the scale is still an ordinal scale, given the meaning of thresholds itself, contextualized for small amounts, with a high degree of approximation. Actually, the thresholds are applied as a benefit for the disadvantage of having less information than necessary as ordinal scales are used for (13.8).

14.8 Applying the Model and Selecting the VP

This step consists of applying the algorithm of the chosen method in order to select the most appropriate VP for the DPBO under analysis. Actually, for some methods the previous step of parameterization includes the process of obtaining the solution that will be recommended, as is the case in interactive methods (e.g., FITradeoff).

This step seems to be relatively simple. However, there is still a concern related to who is going to be the DM of the DPVP.

If there is a supra-DM in the process, this DM assumes the role of making the decision in the DPVP and the process is simpler. That is, the DPVP has an individual decision-making process. Generally, the supra-DM has a hierarchical position in the structure of the business organization that is above that of the other DMs. The concept of the supra-DM is similar to that of the 'benevolent dictator problem' (Keeney 1976).

On the other hand, if the group of DMs for the DPBO is going to play the role of making the decision in the DPVP, then this could bring some modeling difficulties that require additional discussion. In this case, the DPVP is a group decision-making process, in which the DMs act together such as in the DPBO. However, this can be also simplified by a discussion with the group of DMs by using problem structure methods (PSM), with the support of a facilitator (Eden 1988; Eden and Ackermann 2004).

In the case of a group decision process for the DPVP which deals with a more formal analytical approach, one may wonder if this could turn out to be an infinite regress, since a method should be selected in order to solve that second order group decision. Thus, another decision model would be necessary, and so forth. That is why a facilitator is needed in this process, as already mentioned, so that a supposed circle of numerous regresses could be broken.

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