Chapter 13
Overview of MCDM/A Methods

Abstract Most decision problems have multiple objectives. The basic ingredients of these problems need to be identified in order to build decision models. There are many MCDM/A (Multi-Criteria Decision Making/Aiding) methods and multi-objective approaches. This chapter places an emphasis on the MCDM/A methods, which are more closely related to rules for making choices. We mainly consider the pros and cons of compensatory and non-compensatory rationality for classifying MCDM/A methods.

13.1 Introduction

MCDM/A is an acronym that stands for a number of methods related to the decision process in multicriteria problems, and was formed by amalgamating the acronyms MCDM (Multi-Criteria Decision-Making) and MCDA (Multi-Criteria Decision-Aiding).

In a classical optimization problem, a maximization (or minimization) procedure is applied to a unique objective function, representing gains (or losses). A multicriteria problem consists of a situation, in which there is more than one objective (each objective being represented by one criterion) and in many situations, these objectives conflict with each other.

MCDM/A methods cope with problems for which there is more than one objective. These methods enable DMs to deal with these objectives simultaneously. The criteria (or attributes) are related to outcomes that may be obtained by choosing an alternative in the decision process. The criteria represent the objectives in the decision-making process.

This vision of dealing with several objectives at once was first put forward many centuries ago. For instance, an evaluation between two sets of criteria for choosing a course of action, was made around 300 B.C., by Socrates, as recorded by Plato in the Protagoras dialogue. The idea is about how to measure two types of criteria (pleasure and pain), as follows (de Almeida et al. 2015):

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> …What measure is there of the relations of pleasure to pain other than excess and defect, which means that they become greater and smaller, and more and fewer, and different in degree? "I should reply: And do they differ in anything but in pleasure and pain? There can be no other measure of them. And do you, like a skilful weigher, put into the balance the pleasures and the pains, and their nearness and distance, and weigh them, and then say which outweighs the other. If you weigh pleasures against pleasures, you of course take the more and greater; or if you weigh pains against pains, you take the fewer and the less; or if pleasures against pains, then you choose that course of action in which the painful is exceeded by the pleasant, whether the distant by the near or the near by the distant; and you avoid that course of action in which the pleasant is exceeded by the painful. Would you not admit, my friends, that this is true? I am confident that they (i.e. the sophists) cannot deny this.

This idea also appears in a text of 1722, by Benjamin Franklin, that deals with analyzing a specific kind of problem, with only one alternative (there are two options: implement it or don't do so), which was expressed in a letter proposing a decision procedure (Hammond et al. 1998; Figueira et al. 2005; de Almeida et al. 2015), as follows:

> In the affair of so much importance to you, wherein you ask my advice, …. […], my way is to divide half a sheet of paper by a line into two columns; writing over the one Pro, and over the other Con. […] When I have thus got them all together in one view, I endeavor to estimate their respective weights; and where I find two, one on each side, that seem equal, I strike them both out. If I find a reason pro equal to some two reasons con, I strike out the three. If I judge some two reasons con, equal to three reasons pro, I strike out the five; and thus proceeding I find at length where the balance lies; and if, after a day or two of further consideration, nothing new that is of importance occurs on either side, I come to a determination accordingly.

Perspectives and historical views for the MCDM/A may be found in several texts (Koksalan et al. 2011; Edwards et al. 2007).

The procedure most frequently applied for aggregating criteria is the additive model, also called the 'weighted sum' model, which is introduced below, in which the global value ($v(x_i)$) is considered for a consequence vector $x_i = (x_{i1}, x_{i2}, \ldots, x_{in})$, for the alternative i, which is the same as the global value $v(a_i)$ for alternative a_i, which has the consequence vector x_i.

$$v(x_i) = \sum_{j=1}^{n} k_j v_j(x_{ij}) \tag{13.1}$$

where:

- x_{ij} is the consequence or outcome of alternative i for criterion j.
- $v_j(x_{ij})$ is the value of consequence for criterion j, for alternative i.
- k_j is the scale constant (weights) for attribute or criterion $j\,(k_j > 0)$, which is usually normalized as follows:

$$\sum_{j=1}^{n} k_j = 1. \tag{13.2}$$

13.2 Basic Ingredients in a Multicriteria Problem

As to the modeling approach for building an MCDM/A model, some basic ingredients are subsequently highlighted that should be applied in the decision process for choosing a voting procedure (DPVP). One of them is the set of alternatives, which consists of the VPs to be evaluated for that particular decision process for the business organization (DPBO). In a business context, decision problems usually have a set of alternatives that is offered to the DM, which consists of a discrete set of elements a_i,. This set is represented by $A = \{a_1, a_2, a_3, \ldots, a_m\}$.

The set of alternatives is associated to the concept of problematic. This is related to the kind of analysis that is to be made of the set of alternatives and therefore, to the format of the recommendation to the DM. A few types of problematic are considered in the literature (Roy 1996; Belton and Stewart 2002), the most relevant two for this text being the problematic of choice and that of ranking. For the latter, the final result consists of ranking all the elements a_i in the set of alternatives.

In the choice problematic, the solution is a subset of chosen alternatives. It would be preferable to have only one alternative in this subset, which corresponds to optimization; a particular situation in this problematic. Nevertheless, it may happen that the procedure applied is not able to achieve optimization. Thus, in this subset, there is more than one alternative, which should be considered non-comparable with each other. Nevertheless, in the end, only one alternative is to be implemented.

Apart from the set of alternatives, another basic ingredient is the consequence x in the decision problem, which leads to a set of consequences X. This set X connects to concepts related to the family of criteria and matrix of consequences.

The set of consequences consists of the possible outcomes that the DM can obtain, after choosing an alternative. Consequences are directly linked with the objectives in the decision problem. A consequence is the result obtained by the DM after implementing a chosen alternative.

In a MCDM/A problem, there is a set of possible consequences for each of the multiple objectives. Therefore, a vector of consequences $x = (x_1, x_2, \ldots x_n)$ is considered, where x_j is the consequence for the criterion j.

Therefore, the alternatives in set A are evaluated by the consequences they can provide to the DM. In fact, the choice is made from among the consequences. These consequences are evaluated by the DM in order to make explicit what the DM's preference structure is over the consequence space. A decision model will recommend an alternative based on this preferential information over the set of consequences. Thus, for each alternative I, a possible consequence x_{ij} can be obtained for the criterion j.

This leads to compiling the matrix of consequences as illustrated in Table 13.1, in

Table 13.1 Consequence matrix

A	Criterion 1	Criterion 2	Criterion 3	…	Criterion j	…	Criterion n
A_1	x_{11}	x_{12}	x_{13}	…		…	x_{1n}
A_2	x_{21}	x_{22}	x_{23}	…		…	x_{2n}
…	…	…	…	…	…	…	…
a_i				…	x_{ij}	…	…
…	…	…	…	…	…	…	…
a_m	x_{m1}	x_{m2}	x_{m3}	…	…	…	x_{mn}

which there is a specific outcome x_{ij}, for each combination of criterion and alternative.

As shown in Table 13.1, since the value of each consequence $v_i(x_i)$ for a given alternative a_i can be obtained, then, the value of alternatives $v_i(a_i)$ can be found.

A dominance relation D between two alternatives a_k and b_l is defined as follows: a_k dominates a_l $(a_k D a_l)$, if $v_j(a_k) \geq v_j(a_l)$, for all $j = 1, 2, 3, \ldots, n$, and for at least one of the criteria j, the inequality is strict (>).

If the dominance relation applies between two alternatives, there is no need to use an MCDM/A method for aggregating all criteria j in order to compare them. Rarely can a solution be found only by applying the dominance relation to all alternatives in the set and therefore, generally speaking, an MCDM/A method is required.

In general, preference binary relations are applied in order to represent the DM's preferences in a preference modeling process. The following basic preference relations can be considered for the subsequent explanations:

- Indifference (I)—zIy means that the DM is indifferent between the two consequences z and y.
- Strict Preference (P)—zPy means that the DM clearly prefers z to y.
- Weak Preference (Q)—zQy means that z is at least as preferable as y to the DM. In other words, the DM can find that zPy or zIy.
- Incomparability (J)—zJy means that the DM is not able to compare the two elements.

13.3 Classifying MCDM/A Methods

MCDM/A methods may be classified in several ways, one of which is to do so according to the nature of the set of alternatives. Since the set of alternatives that is taken into account in the kind of problem analyzed here is a discrete set, only discrete methods are considered. This means that making use of multiobjective mathematical programing approaches is excluded.

One of the classifications for methods most found in the literature (Roy 1996; Vincke 1992; Belton and Stewart 2002; Pardalos et al. 1995; de Almeida et al. 2015) includes two kinds of methods, when only discrete methods are considered:

- Unique criterion of synthesis methods;
- Outranking methods.

These kinds of methods could be classified in another way, in accordance with their rationality, namely, as being either compensatory or non-compensatory (de Almeida et al. 2015). However, it is worth noting that 'unique criterion of synthesis methods' are compensatory, while 'outranking methods' are non-compensatory.

In the unique criterion of synthesis methods, the criteria are aggregated by an analytical model in such a way that a global score for a consequence or alternative is produced. These methods synthesize all the criteria in a unique criterion (global evaluation). They can deal with either deterministic consequences or probabilistic consequences. The former is the scope of Multi-Attribute Value Theory (MAVT) and the latter, Multi-Attribute Utility Theory (MAUT) (Keeney and Raiffa 1976).

The additive model for aggregating criteria, shown in (13.1), is the most commonly applied model for this kind of method within the scope of MAVT or MAUT (Keeney 1992). It is the aggregating model for many additive methods (Keeney and Raiffa 1976; Vincke 1992; Belton and Stewart 2002), especially in MAVT, examples include SMARTS, FITradeoff, MACBETH, AHP, TOPSIS, etc. Even swaps (Hammond et al. 1998).

These methods can be applied if the DM's preference is compatible with the preference structure (P, I), and they can produce a complete pre-order.

Outranking methods usually do not produce a unique criterion of synthesis. Therefore, these methods can produce recommendations with no scores for alternatives. These methods consider the incomparability relation and are compatible with the DM's preference structure (P, Q, I, J). In this case they can produce a partial pre-order. The main methods in this group (Vincke 1992; Belton and Stewart 2002; Polmerol and Barba-Romero 2000) are the ELECTRE (Roy 1996) and PROMETHEE (Brans and Vincke 1985) methods.

As already mentioned, these two kinds of methods differ according to the DM's rationality with regard to the compensation of criteria. These concepts may be rather important in choosing a MCDM/A method as shown in de Almeida et al. (2015). However, how best to analyze this is seldom discussed in the literature (de Almeida et al. 2015). Bouyssou (1986) and others (Munda 2008; Roy 1996; Vincke 1992) have commented on the concepts related to compensation and non-compensation.

The compensation concept between two criteria clearly includes the notion of a tradeoff between these two criteria (Bouyssou 1986). On the other hand, as to non-compensation, there is no tradeoff between the criteria. Additionally to outranking methods, a lexicographical procedure is non-compensatory and illustrates this absence of non-compensation between criteria.

A preference relation P, for comparing consequences x, y, z and w, is non-compensatory if the preference between them only depends on the subset of criteria in favor of x and y (Fishburn 1976). If the DM has a non-compensatory rationality, it does not matter what the level of the performance of x or y in each criterion is. The only information necessary is to know if one is higher or lower than the other, which is directly associated with the strict preference relation P, as follows.

Let $P(x, y) = \{j{:}x_j P_j y_j\}$. That is, $P(x, y)$ is the collection of criteria for which $x_j P_j y_j$. Then, for a non-compensatory rationality (Fishburn 1976):

$$\left\{\begin{array}{l} P(x, y) = P(z, w) \\ P(y, x) = P(w, z) \end{array}\right\} \Rightarrow [xPy \Leftrightarrow zPw] \tag{13.3}$$

That is, in the decision matrix, the only information needed is whether $v_j(x_j) > v_i(y_j)$ or otherwise. This would mean that the $x_j P y_j$. or otherwise, respectively.

On the other hand, it is essential, for a compensatory relation P, to know the level of performance $(v_i(x_j))$ of x_j for criterion j. This is related to the process by which the DM on using a compensatory rationality evaluates how a disadvantage in one criterion may be compensated for by an advantage in another criterion.

The additive model in (13.1) illustrates this notion very well. In order to maintain the same global score in (13.1), if the performance in one of the criteria decreases, the performance in one of the other criteria should be increased. It is valuable to note at this moment what the meaning of the scale constants (k_j) of criteria is. It is not about the degree of importance of the criteria, as one could intuitively think. The scale constants (k_j) indicate in (13.1) by how much the performance of a criterion should increase in order to make the compensation needed to keep the same global score.

Therefore, it is easy to cope with situations in which compensatory rationality applies, when analyzing consequences with multiple criteria. However, for non-compensatory rationality this may be not easy at the beginning, although several real situations may illustrate the use of a non-compensatory rationality. Many of them are found in sports and some of them may be found in voting systems. Examples of these are given below.

Let us visualize a volley-ball game, in which a non-compensatory rationality is applied. That is, the number of sets a team has won indicates the final result. If the total number points were used to determine who has won, that would be a compensatory rationality, similar to the additive model (de Almeida et al. 2015).

In the volley-ball game a non-compensatory rationality is applied. The criteria are represented by the sets, each of which has the same weight. For instance, Table 13.2 shows the results of a volley game between teams A and B (de Almeida et al. 2015). Team A is considered the winner, since it wins three sets and team B wins only two sets. The number of points each team gets in each set is irrelevant. Whoever wins the set gets the whole value of the set. However, in the example, team A wins a total of only 93 points, while team B wins 104 points. If a compensatory rationality were applied, Team B would be the winner, since its total of points is greater than the total points of team A.

Munda (2008) calls attention to an evaluation of students on a course, considering grades in a scale from 0 to 10. This kind of evaluation usually is a compensatory

Table 13.2 A volley-ball game—compensatory or non-compensatory evaluation

Team	*A*	*B*	Set won by
Set 1	25	23	*A*
Set 2	25	20	*A*
Set 3	11	25	*B*
Set 4	17	25	*B*
Set 5	15	11	*A*
Total points	93	104	

procedure. For instance, a student could compensate a grade 4 received for mathematics, with a grade 10 in language, and thus pass the final evaluation. However, an evaluation system could use a non-compensatory rationality, if the system did not wish to allow this kind of compensation amongst different subjects. For instance, each student could be required to have a minimal performance in each subject.

Voting systems may be the source of interesting examples in this regard. A presidential election in the United States of America (USA) is a case in point (de Almeida 2015). In 2016, the USA presidential election gave an interesting result which illustrates this subject. In that system, the candidate has to get 'electoral votes', which are based on winning individual states. Each state has a certain number of 'electoral votes' that are assigned to the candidate who was chosen by the majority of voters in that state. No matter the number of voters in a given state or the winner's margin of victory in that state, whether this is by one vote or by hundreds of thousands of votes, the winning candidate receives all that state's 'electoral college votes'. In 2016, in order to win, a candidate needed 270 'electoral votes'. Candidate Trump got 306 of these votes, while Clinton got only 232. On the other hand, Trump obtained 46.09% of the voters in his favor, while Clinton had 48.18% which means that Clinton 'won' the popular vote by approximately 2.9 million votes. Trump won the election because this decision system uses a non-compensatory rationality. Otherwise, Clinton would have won the election, since she won the popular vote. This is similar to the example above about the volley-ball game, illustrated in Table 13.2.

In the US Presidential voting system, each state (representing a criterion) has a symbolic weight, which is represented by the 'electoral votes'. These 'electoral votes' are related to the number of senators and congress representatives each state has, which is also associated to the population of the state, with some exceptions. Thus, in order to get all the 'electoral votes' (weight) of one state (criterion), a candidate needs to win the majority of the votes cast by the electors registered in that state. For instance, the state of California has 55 'electoral votes'. With 60.4% of the votes, Clinton won all these 55 'electoral votes'. And the state of Florida has 29 'electoral votes'; all of them won by Trump with a small difference of votes in that state (49.1% of the votes cast were for Trump, against the 47.8% for Clinton).

Comparing this electoral system to a multicriteria problem, the states are equivalent to criteria and the outcome for each criterion is the number of votes obtained in that state. A group of states plays the same role as a subset of criteria in an outranking method (Vincke 1992), as a coalition in favor of one of the alternatives (candidates).

In order to win a candidate has to get the best coalition of criteria (states), with the greatest summation of criteria weights ('electoral votes').

It is worth remarking that the presidential election in the USA consists of e elections, in which $e =$ number of states. These e elections are combined with a non-compensatory rationality.

There now follows a brief overview of a few discrete MCDM/A methods. First, we describe some of the methods related to the additive aggregation of criteria, which are classified as unique criterion of synthesis methods. Then, we introduce some outranking methods, which are related to non-compensatory rationality.

13.4 Methods of the Type: Unique Criterion of Synthesis

The additive model is the procedure, within the group of compensatory methods, that is most often applied in order to aggregate criteria. It may be considered in two different contexts (Belton and Stewart 2002; Keeney and Raiffa 1976; de Almeida et al. 2015): Multi-Attribute Value Theory (MAVT), Multi-Attribute Utility Theory (MAUT). This aggregating model is also called the 'weighted sum' model and is shown in Eq. (13.1).

In MAVT, the consequences are deterministic; that is, certainty is assumed. So, the performances of the consequences are assumed to be known. In MAUT the consequences are assumed to be uncertain. Thus, one may either know the probabilities for performances of the consequences in each criterion or these probabilities may be unknown. The former characterize a decision under risk and the latter, decision under uncertainty. In this text an emphasis is given to the MAVT context, since it seems to be more related to analyzing the decision process for choosing rules.

A few properties are assumed for the additive model. A complete pre-order or a complete order is assumed in the DM's preference structure. That is, the DM should be able to compare and order all consequences. The property of transitivity is also assumed for this model. These two properties are assumed for aggregating procedures in this kind of MCDM/A method, the unique criterion of synthesis method. On the other hand, an outranking method, the other kind of MCDM/A method, should not be applied.

A relevant property of the additive model is the mutual preference independence condition amongst the criteria (Keeney and Raiffa 1976), which may not be followed in other aggregating procedures of the unique criterion of synthesis kind of methods. The preference independence between two criteria Y and Z occurs if and only if

$$(y', z')P(y'', z') \Leftrightarrow (y', z)P(y'', z), \text{ for all } z,\ y' \text{and } y''.$$

That is, the preference for the whole space of Y (the marginal value function for different levels of y, e.g., y' and y''), given a level of z (let us say $z = z'$), does not depend on z level. This means that (Vincke 1992) for four consequences (a, b, c and d) with these two criteria, Y and Z are preferentially independent if the following condition holds:

$$If \left\{ \begin{array}{c} v_y(a) = v_y(b) \\ v_y(c) = v_y(d) \\ v_z(a) = v_z(c), \\ v_z(b) = v_z(d) \end{array} \right\} then\, aPb \Rightarrow cPd \tag{13.4}$$

When building a decision model, these properties should be assessed in order to validate the use of such a model. On the other hand, it has been observed that the property of preference independence is not violated in many practical situations. Moreover, Keeney (1992) points out that dependence between criteria in the DM's preference may happen when a criterion is missing from the family of criteria.

Therefore, it is important to check the properties of this model. However, this is not the main issue when building additive decision models. The main concern for this model is related to the DM's preference modeling in order to specify the scale constants, k_i. There are two main issues here: the meaning of these scale constants and how to obtain them in a consistent way.

The meaning of the scale constants (or weights) is related to substitution rates between the criteria (Keeney and Raiffa 1976; Vincke 1992; Belton and Stewart 2002). This issue is well explored by by Keeney and Raiffa (1976) and Keeney (1992). A common mistake is to associate the meaning of a criterion weight with its degree of importance. Actually the name 'weight' may induce this misconception. Maybe the name of scale constants would be more appropriate (de Almeida et al. 2015). Keeney and Raiffa (1976) call attention to the possibility of a criterion having a scale constant larger than another and being of less importance. Also, changing the normalization procedure for consequences with a linear value function v(x) and using different scales, such as a ratio or an interval scale, implies that new values for the scale constant of criteria should be computed. Although this is necessary for the additive model, it is not needed for other methods

Furthermore, behavioral studies have shown the possibility of there being many inconsistences in the elicitation process with the DM in order to obtain the scale constant (Weber and Borcherding 1993). For this reason, although there are many MCDM/A methods based on the additive model in the literature, most of them differ from each other only in the elicitation procedures for obtaining the scale constants. These methods include: SMARTS (Simple Multi-Attribute Rating Technique with Swing) proposed by Edwards and Barron (1994); AHP (Analytic Hierarchy Process) (Saaty 1980); MACBETH Macbeth (Measuring Attractiveness by a Categorical Based Evaluation Technique) (Bana and Costa et al. 2005). Each of these methods is based on one of the basic elicitation procedures (for the scale constants). Amongst these procedures are the swing and the tradeoff procedures which are briefly described below and more fully in de Almeida et al. (2015).

Keeney and Raiffa (1976) presented the tradeoff procedure in detail and Weber and Borcherding (1993) considered this procedure as the one with the strongest theoretical foundation.

This is an algebraic procedure (Weber and Borcherding 1993), since the k_j are calculated from a simple system of equations, including Eq. (13.2). The other equations are based on a set of the DM's $n-1$ judgments over the consequence space. Since the scale constants k_j are not elicited directly from the DM, this procedure is also classified as an indirect procedure. Thus, the scale constants k_j are calculated using information given by the DM regarding the consequence space.

This information is obtained by asking the DM structured questions (Keeney and Raiffa 1976) which he/she answers, thereby identifying consequences which are preferentially indifferent to each other; i.e. trade-offs are identified. This indifference implies the need for an equation, since the value of these two consequences are equivalent.

Another elicitation procedure is called 'swing', which is used by many methods, such as the SMARTS method (Edwards and Barron 1994). Since the scale constants are based on direct information that the DM is asked for, this procedure is classified as being a direct one (Weber and Borcherding 1993). However, it should be noted that this direct information takes into account the range of the consequences, thus avoiding the usual mistake of sampling which is to regard the scale constants as being ratios that represent the degree of importance of criteria.

There is also a sequence of structured questions in this procedure (Edwards and Barron 1994). The first question considers that all criteria have the worst consequence, and the DM is asked to choose only one criterion and to improve its outcome from the worst to the best outcome; that is, to 'swing' from the worst to the best outcome. The chosen criterion should be that with the greatest value of k_j. Then, other similar choices are made in order to identify the ranking of k_j. In the next step of the procedure, the criterion with the largest k_j, is arbitrarily assigned a value of 100. This value acts as a reference for percentages, so that when points are assigned to the other criteria, they express percentages of k_j ranked first. In this case the value of k_j considers the range of each criterion. At the end, in order to produce the final scale constants, these percentages are normalized.

Possible inconsistencies of these elicitation procedures have been evaluated by behavioral studies. For instance, Borcherding et al. (1991) have reported that inconsistencies arise in 50 and 67% of cases, for swing and tradeoff procedures, respectively.

In the tradeoff procedure, the adjustment the DM has to make, in order to obtain two consequences that are preferentially indifferent, is considered to be a critical judgment in the tradeoff procedure and may easily lead to inconsistences. The FITradeoff (Flexible and Interactive Tradeoff) method is based on this procedure (de Almeida et al. 2016) and avoids these adjustments for indifferences by the DM, which ensures this procedure leads to more consistent results and yet it is based on the strongest theoretical foundation. In FITradeoff, the DM does not have to identify consequences with preferential indifferences. Instead, the DM compares consequences and has just to identify which one of them is preferable. This leads to inequalities that are applied as constraints in Linear Programing Problems which are structured to identify Potential Optimal Alternatives (POA). At each question answered, the FITradeoff calculates the current POA, with this partial information. By using these inequalities, the algebraic characteristic of the procedure is maintained. Amongst the many flexibilities of the model, the DM may skip some questions and yet, the method is able to carry on the process in order to identify the best alternative, according to the DM's preference. This method has a Decision Support System (DSS), available for free at www.fitradeoff.org. In order to increase the confidence of the results, the FITradeoff DSS has been improved with behavioral studies using Decision Neuroscience experiments (Roselli et al. 2019).

The use of methods with partial information (Weber, 1987) in the elicitation process, such as FITradeoff, may contribute to minimizing inconsistences (de Almeida et al. 2016). Other advantages of a process with partial information are that this avoids time-consuming and controversial processes (Kirkwood and Sarin 1985; Kirkwood and Corner 1993) and deals with the possibility of the DM being unable to respond specifically and precisely to tradeoff questions (Kirkwood and Sarin 1985).

13.5 Outranking Methods

These methods use a non-compensatory rationality and are completely different from the methods described in the previous section. For instance, in this method, a preference relation of incomparability is allowed in the DM's preference structure.

In outranking methods, the first two properties mentioned for the additive model are not assumed. Therefore, if the DM is not able to compare all consequences and order them, these methods may nevertheless be applied. Also, the transitivity property may not be followed. Consequently, these methods may be able to produce only partial pre-orders.

An important difference between outranking methods and those of the unique criterion of synthesis concerns the meaning given to the criteria weights. For the former, criteria weights mean the degree of importance of the criteria, which can be perceived when analyzing the mathematical structure of those methods and because of how the weights are used.

Amongst the methods in this group we briefly describe two of the most applied:

- ELECTRE (Elimination Et Choix Traduisant la Réalité)—(Roy 1996; Vincke 1992); and
- PROMETEE (Preference Ranking Organization Method for Enrichment Evaluation)—(Brans and Vincke 1985).

Pairwise comparison amongst the alternatives is a common feature of these methods. These pairwise comparisons are not made by the DM but rather the outranking relations between all pairs of alternatives in the set of alternatives are explored.

The notion that the meaning of weights is about the degree of importance of criteria is usually contextualized with a voting process and the weights are compared with votes (Roy 1996; Vincke 1992). The notion of coalition is an interesting feature in these methods. Let us look at this notion of coalition of criteria in order to compare two alternatives a and b. Consider two subsets of criteria G and H so as to compare them. Let us assume that the subset of criteria in G has weights that sum up to a greater value (and so are more important; or combine more votes) than the weights of criteria in the subset in H. If the following conditions hold (Vincke 1992): a is better than b in subset G, b is better than a in subset H, and a and b are indifferent for any other criteria, then: a is globally better than b. This means that the criteria in favor of a have a summation of weights that is greater than the weights for criteria in favor of b. In other words, a has a better coalition of criteria than b.

Two main steps characterize these methods (Roy 1996; Vincke 1992): Building the outranking relation and Exploiting this outranking relation.

The outranking relation is built by making the pairwise comparison for all the set of alternatives. Let us represent the outranking relation by S, and consider applying it to a pair of alternatives a and b. Then, aSb means that a outranks b. This indicates that a is at least as good as b.

These outranking relations are exploited by applying a procedure in order to find recommendations according the problematic in question.

In the ELECTRE methods, the outranking relation aSb, between two alternatives a and b, is based on concordance and discordance concepts, about which the DM gives preference information in the form of thresholds.

The family of ELECTRE methods includes several methods, which differs from the problematic and the kind of criteria (Roy 1996; Vincke 1992; Belton and Stewart 2002; Figueira et al. 2005). Two of these methods are of interest here, since they are related to the choice problematic: ELECTRE I (considering true criteria) and ELECTRE IS (considering pseudo-criteria). However, if the context requires a different problematic to be applied in order to analyze the VPs, then another method may be applied, such as the ELECTRE III for the ranking problematic.

In the ELECTRE methods, the outranking relation between two objects a and b (aSb) is established considering concordance and discordance concepts. The former concept indicates how much the coalition of criteria supports an outranking relation S between two alternatives. If the outranking relation is supported by the former, then, the discordance is applied in order to evaluate other issues and it may disagree with this outranking relation. Therefore, the following indices are applied in order to evaluate the outranking relation aSb: the concordance index $C(a, b)$ and the discordance index $D(a, b)$.

The concordance index $C(a, b)$ is given based on the summation of the weights of criteria in favor of a, as follows:

$$C(a,b) = \sum_{j:v_j(a)\geq v_j(b)} w_j$$

$$\text{with} \sum_j w_j = 1, \text{ for normalization of weights.} \tag{13.5}$$

where: w_j is the weight for criterion j; and $v_j(a)$ and $v_j(b)$ are the values, respectively of alternatives a and b, for criterion j.

Different discordance indices ($D(a, b)$) are proposed by (Roy 1996; Vincke 1992; Belton and Stewart 2002). Let us consider the following:

$$D(a, b) = \max\left(\frac{v_j(b) - v_j(a)}{\max[g_{vj}(c) - v_j(d)]}\right), \quad \forall \mathrm{j}|v_j(b) > v_j(a); \forall j, c, d. \tag{13.6}$$

The DM has to specify threshold levels for both indices; let us say, c' for concordance and d' for discordance. These threshold levels let the outranking relation S be built. Therefore aSb can be established as follows:

$$aSb \text{ if and only if } \begin{cases} C(a, b) \geq c' \\ D(a, b) \leq d' \end{cases} \tag{13.7}$$

It may happen that for a pair of alternatives, there is a simultaneous outranking, such that aSb and bSa. This represents a circuit in a graphical representation of these relations and for ELECTRE I, these alternatives are considered indifferent.

Once the outranking relations are built for all pairs of alternatives, by applying (13.7) with the parameters informed by the DM, then, the step of exploiting the outranking relation can be applied. In this step, a subset of alternatives, called the kernel is obtained in the ELECTRE I method. The kernel consists of the subset of alternatives that are not outranked by any other in the kernel. The kernel is the solution for the choice problematic. If the kernel has only one alternative, this problematic has the same result as an optimization. If the kernel has more than one alternative, this means that those alternatives in the kernel are incomparable.

The PROMETHEE family of methods is based on a valued outranking relation (Brans and Vincke 1985; Vincke 1992; Belton and Stewart 2002). In these methods, the information regarding concordance and discordance are not applied. Therefore, the DM has to provide weights of criteria and information on the indifference or preference thresholds regarding the evaluation of intra-criteria, if any of these thresholds are taken into account.

An outranking degree of a over b, which is denoted by $\pi(a, b)$, is obtained for each pair of alternatives a and b, in order to move forward to the step for building the outranking relation. $\pi(a, b)$ is obtained as follows:

$$\pi(a, b) = \sum_{j=1}^{n} w_j F_j(a, b) \tag{13.8}$$

The weights w_j are normalized, just as in the ELECTRE method. $F_j(a, b)$ is a function that informs the relation between the performance of outcomes for alternatives in criterion j. In the usual case, $F_j(a, b) = 1$, if $v_j(a) > v_j(b)$, which would contribute to a outranking b; otherwise, $F_j(a, b) = 0$. In the usual case, indifference or preference thresholds are not applied for criterion j. Therefore, the outranking degree, $\pi(a, b)$, for $F_j(a, b) = 1$ is the summation of the weights of those criteria, in the usual case, in which $v_j(a) > v_j(b)$.

Regarding other options for the function $F_j(a, b)$, the method has six different patterns, including the one explained above. In the other forms for $F_j(a, b)$, indifference or preference thresholds, or both, are considered. In these cases, $0 < F_j(a, b) > 1$, depending on the difference $[g_j(a) - g_j(b)]$. If this difference is in the range of these thresholds, then, a partial value of the weights is added to the outranking degree $\pi(a, b)$.

The information to be given by the DM regarding the evaluation of intra-criteria is the choice of the form for $F_j(a, b)$, and if applicable, the specification of indifference and preference parameters of thresholds, for each criterion j.

A matrix $\pi(a, b)$ is produced as result of this first step. In the next step, each alternative a is evaluated by exploiting the outranking relation, based on the outranking degree $\pi(a, b)$. The evaluation of these alternatives is based on the outgoing flow $\phi^+(a)$ and on the incoming flow $\phi^-(a)$, as follows:

$$\phi^+(a) = \frac{1}{n-1} \sum_{b \in A} \pi(a, b) \tag{13.9}$$

$$\phi^-(a) = \frac{1}{n-1} \sum_{b \in A} \pi(b, a) \tag{13.10}$$

Since n is the number of criteria, the division by $(n-1)$ in (13.9) and (13.10) implies there is a normalized scale between 0 and 1, for both flows.

The outgoing flow sums up the outranking degree of a over b. for all b, thereby indicating the advantage of the alternative a over all other alternatives (b) in the set of alternatives. The incoming flow sums up the outranking degree of b over a, for all b and thus represents the disadvantage of the alternative a compared with all other alternatives.

The combination of these two indices in (13.9) and (13.10), is applied in the PROMETHEE I method in order to compare all pairs of alternatives and to obtain preference relations (P), indifference (I) and incomparability (J) between them, and thus to produce two pre-orders. Then, a partial pre-order of all alternatives is obtained. (Brans and Vincke 1985; Belton and Stewart 2002).

The PROMETHEE II method uses another index for evaluating the alternatives, namely, the net flow $\phi(a)$ as follows:

$$\phi(a) = \phi^+(a) - \phi^-(a) \tag{13.11}$$

The liquid follow is given in a scale of -1 to 1 and the PROMETHEE II produces a complete pre-order of the set of alternatives.

13.6 Choosing an MCDM/A Method

Choosing a proper MCDM/A method for a particular decision problem is not a topic often found in the literature. On the other hand, this might be changing. The growing interest in using and developing hybrid methods as well as making adaptations in classical methods, has been noteworthy. This may be related to concerns with building methods that have more realistic assumptions, which would be more appropriate for the problem to be tackled. Some studies are concerned with choosing a method (Roy and Słowinski 2013; de Almeida et al. 2015).

There are a few issues to be considered when choosing an MCDM/A method. These include: the DM's preference structure; characteristics of the decision problem itself; contextual features related to the decision problem. The latter may include organizational aspects, and the time available for making the decision. Analysts should avoid using their own preference for choosing a method. This could raise important ethical considerations as discussed in de Almeida et al. (2015).

A framework for building an MCDM/A model, presented in de Almeida et al. (2015), includes how to choose a method. Detailed considerations are given in this regard, which is one of the twelve steps in the process of building a decision model. The main information to be considered is the DM's preference structure.

One of the items to be evaluated is how the DM's preferences fit in with regard to compensatory and non-compensatory rationality. Simon (1955) was one of the first to draw attention to this issue, and before the development of most MCDM/A methods. Bouyssou (1986) presents a discussion on the notion of compensation and non-compensation. Vincke (1992) remarks that choosing an MCDM/A method is equivalent to choosing the type of compensation between criteria. For instance, choosing an additive method is the same as choosing the tradeoff between performances on criteria. Roy and Słowinski (2013) have put forward the question "Is the compensation of bad performances on some criteria by good ones on other criteria acceptable?". This reveals their concern with this issue.

13.7 Challenges in MCDM/A Methods

Challenges in multicriteria decision methods have been pointed out that (de Almeida et al. 2018) considering the question in the previous section, which is related to the process of building decision models, since it includes choosing the most appropriate method. Although some frameworks may be found for building multicriteria models (Polmerol and Barba-Romero 2000; Belton and Stewart, 2002; de Almeida et al. 2015) this topic remains a challenging one.

The techniques that allow the use of partial information (or imprecise/incomplete information) for modeling DM preferences has been proved to be an enrichment for MCDM/A methods. First, surrogate weights has been applied (Edwards and Barron 1994) and still remains as a competitive approach (Morais et al. 2015; Danielson and Ekenberg 2017). There are methods using decision rules, simulation, or linear programing in order to reduce the space of criteria weights, while analyzing the set of alternatives (Weber 1987; de Almeida et al. 2016).

Multi-Criteria Group Decision Making (MCGDM) is another challenge related to the use of these methods. Since, for an individual DM the preferences modeling process is complex task in order to aggregate multiple objectives, adding to this the aggregation of multiple DMs is even more challenging.

All these mentioned questions are relevant issues to be improved in order to help organizations to find consistent decision models.

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