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Systems, Procedures and Voting Rules in Context

A Primer for Voting Rule Selection

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Preface

This book has been in the making for a long time. Looking back, a first whiff of the idea for it arose at the Group Decision and Negotiation (GDN) 2012 Conference held in Recife, Brazil, which was organized by Adiel de Almeida and Danielle Morais. At this event, Hannu Nurmi gave a plenary presentation on voting theory in which he outlined how it had developed in historical terms and he also set out some of the principal results from social choice theory. The ensuing discussions revealed, somewhat to our surprise, that voting theory is not a topic that is typically mulled over in group decision and negotiation situations. Yet, it was recognized that voting plays an important role in a wide variety of circumstances whether these be for formal reasons (e.g., elections) or for informal purposes (e.g., impromptu gatherings of people at which a decision is taken on an issue of common interest). Business organizations were also known to resort to voting when making important decisions.

The background of our deliberations was punctuated by three puzzling observations. First, it was clear that a large number of different voting procedures are used to reach a seemingly common goal: to find the “will of the group,” a shared view on policies, candidates or similar matters. So, why do we have such a variety of procedures if they all are supposed to deliver the same thing? Second, the properties of the procedures used differ and can lead to strongly contrasting outcomes even when the distribution of opinions is fixed. So, shouldn’t it be asked if some procedures are particularly well suited for some circumstances and work poorly in others, and if this is so, can contexts be sketched out which indicate where each type of procedure works best? Third, since the business decision-making differs in many respects from political decision-making, is it possible to single out procedures that are particularly appropriate for business contexts?

Throughout these discussions, other ideas were put forward about how to go about selecting voting rules, and so, some papers were published by Adiel de Almeida and Hannu Nurmi at GDN 2014 in Toulouse, France, and at GDN 2015 in Warsaw, Poland. Both papers set out preliminary ideas for a framework for choosing a voting procedure, which was applied in different contexts: the former in a business organization and the latter in a leisure context. These papers were based on multi-criteria decision-making/aiding (MCDM/A) models that aim to support decision-makers to select an appropriate voting procedure to reach a decision.

Under that perspective, we resolved to keep working on that issue until we were ready to present every idea coherently. It is now four years later, and this book is the result. Our primary motivation in writing it is to offer basic tools that aid decision-makers to make intelligent choices with regard to selecting voting rules that will be used in business contexts. The tools consist of basic descriptive devices, central results, comparisons of existing procedures and explaining some fundamental paradoxes. Most chapters have appendices which discuss specific problems in order to illustrate the material presented. This is intended to support the eventual classroom use of the book in advanced courses on business administration and management science. Basically, all the chapters are self-contained.

The book is structured into three parts and has a total of 19 chapters.

The first part (Chaps. 1–3) deals with the background of voting procedures. Selecting voting rules is seen as a special case of a more general problem of why and how the rules of cooperation emerge in communities. Since the majority principle is commonly thought to be the cornerstone of group decision-making, it is therefore given particular attention in this part of the book.

The second part (Chaps. 4–11) presents how voting procedures should be evaluated and the justification for this. The main proprieties and strategic aspects of voting procedures are discussed. The discussion in these chapters starts from the classic assumption about the individuals who engage in group decision-making. In particular, these individuals are assumed to be endowed with consistent views about the decision alternatives. More technically, they are assumed to have complete and transitive preference relations over the decision alternatives (policies and candidates). In addition, the criteria of performance of various voting rules are defined and set out in some detail. We deal with Condorcet criteria, monotonicity criteria, the strategic properties of rules and other issues pertaining to standard theory. This part thus builds upon the foundations of standard social choice theory. The aim is to provide tools for rule selection in contexts where the assumptions of standard theory hold.

Then comes the third part (Chaps. 12–19) of the framework and the process for choosing rules. Chapter 12 discusses the decision process in the business context and how to deal with aggregating decision-making preferences. Chapter 13 gives an overview of the MCDM/A methods. Chapter 14 presents the framework for choosing the voting procedure and how it is integrated into the overall decision process in a business organization. Chapters 15–18 present applications of this framework in different contexts, as follows: assessing the readiness of technology for generating energy; tackling a water resource management problem; identifying technology for generating renewable electric power; and evaluating a voting procedure for a Group Decision Support System (GRUS). Finally, Chap. 19 summarizes important issues that should be analyzed when choosing a voting procedure.

Over the long process of writing up this book, the authors have incurred a large number of intellectual debts, in fact too many to be exhaustively enumerated here. We are extremely grateful to everyone whom we have consulted for their advice and opinions and/or who have taken part in trialling our material. We attempt to acknowledge all this below.

Adiel de Almeida and Danielle Morais would like to thank their colleagues and students, who jointly worked with them on modeling MCDM/A problems in diverse contexts and who have taught or studied at the Center for Decision Systems and Information Development (CDSID) of Universidade Federal de Pernambuco (UFPE). From CDSID, they acknowledge, in particular, the valuable contributions made by Ana Paula Costa, Luciana Hazin, Jonatas de Almeida, Eduarda Asfora Frej and Lucia Roselli. They also thank Pascale Zaraté for additional information given in order to organize the application in relation to GRUS (in Chap. 18). They are also grateful to their sponsors (especially CNPq—the Brazilian Research Council) for their partial financial support and also to CGEE (*Centro de Gestão e Estudos Estratégicos*, in English, *the Center for Management and Strategic Studies*) for the valuable information for the application in Chap. 15.

Hannu Nurmi is grateful to Dan S. Felsenthal of the University of Haifa, Israel, for cooperating with him over a long period. Some results of this cooperation are referred to in this book. Our thanks are also due to Professors Maija Setälä and Matti Wiberg of the University of Turku, Finland, for hosting meetings where preliminary versions of this work were discussed and developed.

Adiel de Almeida and Hannu Nurmi also greatly valued the interaction with Lihi Dery of Ariel University, Israel, arising from some discussions on applying the proposed framework in the context of leisure activities.

We would like to acknowledge that some passages in this book have been taken from earlier studies of Hannu Nurmi and Adiel de Almeida, previously published in Springer books or journals. More particularly, they acknowledge that Chap. 5 draws on Nurmi's contribution to Transactions of Computational Collective Intelligence XXXIII, pp. 149–168; Chap. 7 draws on Nurmi's contribution to the GDN 2015 volume (ed. by T. Tzapiro et al.); Chap. 9 (Sects. 9.1–9.3) draws on Nurmi's monograph Voting Procedures under Uncertainty (pp. 49–59); Chap. 13 draws on de Almeida, A. T., et al. (2015) Multicriteria and Multiobjective Models for Risk, Reliability and Maintenance Decision Analysis (see Chaps. 1–2). Full references are given in each chapter.

We would also like to thank the editors of Springer for their professional help and cooperation all during the development of this book, and finally, but most of all, we thank our families, who continuously supported and encouraged us in our research work, despite the stresses and strains that we all went through, and the anonymous referees of this book who gave many important feedbacks.

Recife, Brazil — Adiel Teixeira de Almeida
Recife, Brazil — Danielle Costa Morais
Turku, Finland — Hannu Nurmi

Contents

Chapter 1
Voting Rules in Context

Abstract All societies have rules. Some are explicitly based on them, but in general rules take on many forms. They can strict and formal or ambiguous and informal or something between these extremes. Many rules have a clear-cut motivation. Some pertain to coordination such as the traffic rules. Some have the aim of avoiding collectively irrational or harmful outcomes. The rules prohibiting cartel formation are examples of these. This book deals with the problems of choosing rules. More specifically, our focus is on rules of collective decision making. We study the most common collective decision making rules singling out the advantages and disadvantages using well-defined criteria.

1.1 Introduction

Rules are a pervasive feature in all societies from the primitive hunter-gatherer groupings through medieval city-states to modern industrial and post-industrial societies. Their role is easily recognizable in modern systems with formally regulated ways of rule production and application, but also informal rules constitute an important aspect of living in a community of humans. Rules are often closely related to norms: both map social situations into action patterns. The Ten Commandments of the Holy Bible can be seen both as norms and as rules. Indeed, as a verb 'rule' is basically synonymous with issuing a norm, e.g 'the judge ruled that the defendant be imprisoned for 2 years'. Some rules, however, do not seem to be related to norms, at least not directly. For example, 'people tend to get angry when provoked' seems to refer to rule that expresses what often or almost always happens without stating that this should be the case. Norms typically deal with what ought to be done or ought to be left undone, while rules can have a purely factual meaning.

Rules are basically predictive devices. If you are living in England, you can predict that the vehicles drive on the left-hand side on roads if the traffic is moving in two opposite directions. Similarly, every community of individuals develops rules and/or norms to regulate interactions between its members. Some rules are pretty flexible, e.g. rules concerning how to greet one's fellow group members (although in military units the flexibility is almost nonexistent), while others contain precise instructions,

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such as forms to be filled when applying for a passport. Some rules do not presuppose the existence of a social context. Thus, the rules of rational choice may be applied in settings that can be characterized as games against nature: an individual is making a choice between alternatives using only individual preference information (e.g. in deciding on whether to lie on one's left or right side when trying to get to sleep).

In groups or communities the fact that there are rules implies that one can to an extent predict how people behave in certain types of situations. Rules are often used in coordinating activities, e.g. in setting up meeting times and places. These rules are sometimes called coordination norms in contradistinction to another major class of social rules: the Prisoner's Dilemma norms (Ullman-Margalit 1977). These aim at reaching the cooperative, collectively rational outcomes rather than the individually rational, non-cooperative ones in situations representable as Prisoner's Dilemma games (Rapoport and Chammah 1965; Nurmi 1980).[1] The classic two-person Prisoner's Dilemma payoff matrix is presented in Table 1.1. Here one player's choices are represented as rows and the other's as columns. Both can choose either to cooperate (C) or to defect from cooperation (D). Situations describable as Prisoner's Dilemmas are typically situations involving collective good provision, such as building a bridge, a road or maintaining public safety. The individually best outcome (payoff 4) is obtained when the player defects (D), while the other player cooperates (C). The cooperator then gets his/her worst outcome (1). If both players cooperate the outcome brings the next to highest payoff (3) to both, while if both defect the outcome is next to worst (2) to both. So, there are incentives to cooperate, but if the other player cooperates, it is profitable to defect. This is a typical mixed motive game. Another somewhat less extensively studied game is Chicken (Table 1.2). It is less dramatic than Prisoner's Dilemma, since in Chicken the players do not have dominant choices because C is better than D if the other player chooses D, but D is better than C if the other player chooses C. In Prisoner's Dilemma, on the other hand, D is the dominant choice for both players. Yet, by so choosing they end up with the only Pareto sub-optimal outcome 2, 2.

Now, some Prisoner's Dilemma norms aim at making the cooperative choices more likely, but interestingly there are also rules that aim at exactly the opposite, viz. to guarantee that the actors involved in a Prisoner's Dilemma do not collude, but resort to their individually rational strategies. These kinds of rules are often encountered in arrangements that inhibit the emergence of price-fixing cartels or market sharing collusion of enterprises.

The underlying assumption in these norms is that rules together with principles guiding the behavior of actors determine the social states that prevail under those norms. Some of these may be deemed desirable, others less so. Institutional design deals precisely with these kinds of settings and asks which rules would - either always or often enough - lead to desirable outcomes in equilibrium. In other words, institutional design aims at establishing arrangements that result in desirable outcomes so

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[1] Due to its assumed plausibility as a model of many kinds of social interactions, Prisoner's Dilemma has generated a truly voluminous literature. Much of it is of experimental nature and seeks explanations for the common deviations from individual benefit-maximization.

Table 1.1 Prisoner's Dilemma

	C	*D*
C	3, 3	1, 4
D	4, 1	2, 2

Table 1.2 Chicken game

	C	*D*
C	3, 3	2, 4
D	4, 2	1, 1

that the actors do not have second thoughts about their own strategies. When these kinds of arrangement are found, one can predict that the desired outcomes are likely to emerge barring changes in the principles of behavior.

In this book the focus is on principles of choosing the rules of choice. More specifically we focus on rules that are used in making collectively binding decisions. These decisions are often needed to guarantee the provision of collective or public goods. These goods are by definition ones where the decentralized market mechanism fails to secure optimal provision. Typically, the decentralized supply is grossly sub-optimal as no individual actor has an incentive to contribute to the provision. Given the obvious sub-optimality, the coordination problem emerges. It is, in fact, in the nature of Prisoner's Dilemma. Since the decentralized mechanism is likely to fail, various kinds of non-market rules have evolved. A prominent one among those is voting whereby the alternatives to be voted upon are levels of the public good provision. For voting to succeed in solving the public goods provision problem, the actors have to commit themselves to the resulting voting outcomes. Our focus will be on reasons for actors to agree to such a commitment.

Voting is often used in contexts that are related to public goods in somewhat indirect way, e.g. in parliamentary legislation. The results are then pieces of collectively binding legislation. While public goods provide a standard context for the application of voting rules, many a voting takes place in settings where either private goods, policies, norms or candidates form the sets from which the choice is to be made. Often the act of voting is preceded by some kind of bargaining. If this fails, voting is often deemed the last resort.

Voting may take place in public or private contexts and the methods of voting are often quite similar. That is, while the alternative sets may vary, the voting systems do not necessarily reflect this. E.g. one-person-one-vote rule is commonly applied in both public and private settings. One would, however, expect that the criteria imposed on the voting rules could vary in different settings. This expectation forms the rationale of this work. We consider the duplicate choice of the choice rule problem from the angle of the contextual requirements rather than universally applicable standards of performance. This will hopefully open a novel angle to the problem of rule selection.

The plan of the book is the following. The first part deals with some of the classic treatises in the field. We first discuss the rule choice problem as a cost minimization one: each individual supports the collective decision rule that minimizes the expected costs ensuing from its application. We then turn to the majority rule for it is often considered as the most obvious rule to adopt and, perhaps because of this, it is also widely used. It is also often regarded as the rule that defines democratic governance. This chapter is very much in line with the traditional cost-benefit analysis of public and private decision making. As such it provides a kind of bench-mark for the approach advocated in this book. After this brief discussion we turn to a more general discussion on why so many voting systems exist, i.e what is their primary motivation and what properties they have. The bulk of what follows next focuses on applying the social choice theory to voting procedures. We introduce and evaluate a number of voting systems using the standard social choice desiderata as benchmarks. We also discuss the relevance of strategic behaviour in various voting contexts. Thereafter we introduce a framework for facilitating the choice of a voting rule in business contexts. An important role is played by multiple criterion decision making/aiding (MCDM/A) tools and the standard procedure evaluations, but our aim is to provide a methodology for their systematic application in business contexts. The book presents four applications in different contexts of the framework for choosing rules, such as environmental policy and technological choice. The book is concluded with a comparative assessment of the procedures in various contexts, i.e. a discussion on the advantages and disadvantages of classes of rules. Our contention is that while all voting procedures are vulnerable to some major flaws, there are circumstances that de-emphasize some flaws in the sense of making them less likely to materialize. Hence, one should pay due attention to the contextual factors when making the choice of a procedure.

1.2 Topics for Further Reflection

Consider the following questions from the view point of the organization you are interested in. Elaborate the advantages and disadvantages related to the ways things are arranged in the organization with respect to these questions.

1. What kinds of decisions are made by individuals in the organization?
2. What is the role of bargaining within the organization?
3. Which decisions are normally made by groups?
4. Are there explicit decision rules for group decisions?
5. Do the rules – if they exist – envisage secret balloting or roll-call type of voting where the identity of voters is disclosed?

1.3 Suggestions for Reading

The books of Ullman-Margalit (1977) and Elster (1992) are relatively non-technical introductions and overviews on the emergence and functions of norms. Somewhat more restricted in scope and slightly more technical in presentation is Axelrod's (1984) text on evolution of cooperative strategies in repeated Prisoner's Dilemma games. Voting is intimately linked to democratic theory. The foundations of the modern theory of voting were laid by Black (1958). Comprehensive overviews are provided by Straffin (1980), Riker (1982) and Dummett (1984). One of the writers of the present treatise gave his first contributions in Nurmi (1983, 1987).

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Chapter 2
Calculus of Consent

Abstract We discuss the problem of selecting a decision rule in the simplest possible setting involving dichotomous choice situations. The starting point is individual utility maximization under two types of cost-constraints: one resulting from the collectivity making decisions against the interests of the generic individual and the other associated with resources that are needed to garner enough support for the passage of motions that are in the individual's interest. Our point of departure is the classic decision making calculus envisaged by Buchanan and Tullock.

2.1 Introduction

One of the classics of modern public choice theory is Buchanan and Tullock's (1962) *The Calculus of Consent*. True to the spirit of public choice, it looks at the problem of rule choice from the angle of an individual participant in a fictitious situation where no collective decision making mechanism exists. Each of the n individuals is assumed to be rational in the narrow sense of maximizing his/her (hereinafter his) expected utility. No assumptions regarding the utility of the various outcome states (elected alternatives) are made. Rather, the collective choices are assumed to be costly for each individual. Hence, the rationality in choosing rules amounts to minimizing the cost involved in applying the rule. For this one has to be able to associate an expected cost for each decision rule.

2.2 Individualism and Unanimity

Buchanan and Tullock's account of political constitutions is radically individualistic. As such it may be even more useful in the study of the principles guiding the choice of rules in settings where a purely instrumental role is assigned to the rules. One obvious way to choose rules would be to simply pick those that best serve the public interest. This way is, however, rejected by Buchanan and Tullock because of its explicit reference to the collectivistic notion of public interest. Whatever com-

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Table 2.1 Pairwise majority comparisons may lead to Pareto suboptimal outcomes

Voter 1	Voter 2	Voter 3
A	D	B
B	C	D
D	A	C
C	B	A

parisons between rules are made have to based on individuals, their preferences and beliefs, according to the individualistic approach. A constitutional arrangement can be considered an improvement over another just in case all individuals judge it to be in their interest to replace the latter with the former. So, public interest is in fact reduced to individual interests and the standard of comparison of rules is based on the unanimity rule. The reasons for this standard are two-fold. Firstly, the rule of unanimity enables us to avoid interpersonal comparisons of utilities. If all individuals deem one constitutional arrangement an improvement over another, we can judge the former preferable to the latter without assuming any other than purely ordinal information about individual utilities. Secondly, the unanimity rule avoids the infinite regress regarding the choice of rules of choice since – it is argued by Buchanan and Tullock (1962, p. 15)

> …it is clear that if all members of a social group desire something done that is within their power, action will be taken regardless of the decision rule in operation.

On closer inspection the latter reason for unanimity rule is not correct since it can happen that pairwise comparisons with the simple majority rule may lead to outcomes judged unanimously inferior to some others that have been voted down in earlier pairwise comparisons. For the general result, see (McKelvey 1979), and for a fictitious example involving only four alternatives, see (Nurmi 1983, 196). The latter is reproduced in Table 2.1. The preferences of the three voters (or of three groups of roughly equal size) over four alternatives – *A*, *B*, *C*, *D* – are presented so that the higher up in the list an alternative is placed, the more preferred it is for the voter. Suppose that the agenda of pairwise comparisons is: (i) *B* versus *D*, (ii) the winner of (i) versus *A*, and (iii) the winner of (ii) versus C. Suppose furthermore that each voter votes according to his preference in each pairwise comparison and that the winner is always the alternative receiving more votes than its contestant. Then *B* defeats *D* in ballot (i), *A* defeats *B* in (ii) and finally *C* beats *A* in (iii). Upon looking at Table 2.1 we see that the *C* is Pareto dominated by *D*. Hence, Pareto criterion is violated.[1] We shall return to this subject later on in this book. The point here is that – in contrast to what Buchanan and Tullock suggest – not all voting rules result in Pareto undominated outcomes, at least not in all environments.

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[1] The Pareto criterion amounts to the following requirement on choices regarding any pair of alternatives *X* and *Y*: if all individuals strictly prefer alternative *X* to alternative *Y*, then *Y* is not elected.

2.3 The Cost Calculus

The distinctive feature of Buchanan and Tullock's analysis of the constitutional choice problem is the individual utility calculus. A membership in a collectivity exposes the individual to external costs, viz. costs that the individual incurs as a result of the activities of others. Over these costs the individual has no control. The other type of cost results from the individual's active participation in the collective activity. These are called decision making costs. They are associated with coming to an agreement within the collectivity which the individual belongs to.

The two types of costs play a central role in the constitutional choice. To wit, each constitutional arrangement is assumed to be associated with some amount of expected external and decision making costs to each individual. The simple setting that Buchanan and Tullock focus upon is one where each constitution is essentially a collective decision making rule. In fact, it is reduced to a single number, viz. the amount of individuals needed to support a motion in the collective decision making body for it to pass, i.e to become the collectively binding decision. Obviously, the setting applies to dichotomous (yes-no) voting situations only. Even so, how is it possible to associate a cost or benefit for any individual in any constitution?

The basic assumption is that both external and decision making costs, when summed up, constitute a function that a rational individual aims at minimizing. The summands are assumed to vary from one individual to another. Yet, some general features of the costs can be discerned. First, external costs are presumably at a minimum when no collective action can be taken without the consent of the individual under consideration. The only decision rule that guarantees this is unanimity: if the consent of all individuals is needed for launching collective action, then also the individual under scrutiny has to agree on the action. Hence, the individual whose main concern is to minimize the chance of being overtaken by the action taken in the name of the collectivity can be expected to support the unanimity rule. On the other hand, the external costs can be expected to be at the maximum when any single individual can launch the collective action in the name of the collectivity. Between the minimum and maximum cost, the subjective views of the individual enter the picture, but some qualitative observations can still be made. One can envision that, if unanimity is not a feasible rule, the next best rule in minimizing the external costs is the $n - 1$ one which requires that at least $n - 1$ out of n individuals have to support a motion. This guarantees the individual that – while possible – collective action against his interest requires the consent of all other members of the collectivity. In similar way one may envision that the external cost minimizing individual will always support more inclusive rules against less inclusive ones. Graphically, the external cost function is monotonically decreasing function of the size of the support required for collective action. Within this general qualitative characterization, subjective attitudes and beliefs can be expected to cause variations in the individual external cost functions.

Similarly, the decision making costs would seem to have a 'natural' maximum point, viz. the rule which requires unanimity for collective action to ensue. This follows from the fact that in order to launch collective action on an issue that has significance for him, the individual needs to convert all other members of the group to his side on the issue. The more people he has to persuade, the more time and possibly other resources are needed to accomplish this. On the other hand, if any one member can launch collective action, i.e. the decision rule is one out of n, then there are no decision making costs at all as the individual may act in the name of the collectivity without consulting others. Between these extreme values the decision making cost may vary between individuals. It seems reasonable, though, to assume that it is monotonically increasing with the decision rule: the larger the number of people required for collective action to ensue, the higher the cost of decision making.

Given these two types of costs, it is then straightforward to argue that a rational individual supports the rule where the sum of the costs are minimal for him. Indeed, the sum of costs makes it possible to construct a preference ranking of decision rules for each individual. If the cost functions are identical for all individuals, we can expect that the cost minimizing decision rule be unanimously selected. The assumption of identical cost functions is, however, extremely strong. Not only do individuals differ in the costs assigned to decisions related to any given issue (economic efficiency vs. environmental quality, efficiency vs. equality in resource allocation, public vs. private provision of health and/or education, etc.), but they may also have different expectations regarding the frequency of various types of issues entering the agenda of decision making. Also the salience of the issues typically differs between individuals. Hence, more likely than not, individuals differ in cost minimizing decision rules. How then should one proceed in selecting the collectively binding decision rule?

This crucial issue is all but ignored by Buchanan and Tullock. In a situation where everybody involved knows that there are joint gains to be had through collective action *vis-a-vis* individual decentralized activities, there is a presumption that the individuals get together to outline decision rules to be applied in the future in similar situations. This suggests that in the 'original situation', i.e. before any collective action, the possibility of joint gains could be made by unanimity since everyone expects to benefit from collective action. The specific decision rule that each individual then supports in the forthcoming decisions regarding the collective action can be whatever is unanimously accepted. Under the highly unrealistic assumption that each individual's vision of external and decision making costs under any given rule is identical, there is a rule that is supported by all individuals in paired comparison against any other rule. As said, however, this is an unrealistic assumption. It then follows that in more realistic circumstances it is likely that no decision rule gets a unanimous support against all other rules in pairwise contests. What we learn from Buchanan and Tullock are the intuitively plausible principles that a rational individual has in mind when pondering upon the rules he supports in joining collective action. To wit, he considers the costs incurred by him in case the collective action is taken against his interests and juxtaposes these with the costs of bringing about collective action that furthers his interests.

For the purposes of the present work we can largely overlook the first stage of this two-stage process where the individuals first decide to join a collectivity in order to guarantee the provision of some collective goods, and then decide which particular amount of support is required to launch collective action to produce the collective goods. We can assume that there is a unanimous support for forming of the group or organization. Since unanimous support in this stage exists, could one then resort to the same rule in the second stage, i.e. in deciding whether collective action is warranted under specific circumstances? For individuals stressing the minimization of external costs, this would be welcome, but in general unanimity rule would create an enormous status quo bias. Hence, because of the high decision making costs unanimity rule is rarely resorted to. On the other hand, less than simple majority rules – although associated with relatively small decision making costs – are infeasible for they are bound to result in inconsistent decisions since of two mutually exclusive motions both could receive enough support to pass. This kind of outcomes are clearly unenforceable. Thus, in practice most decision making bodies resort to rules that require at least 50% but less than 100% support for motions to pass. Next we focus on the majority rule which for many commentators is viewed as a basic constituent of democracy.

2.4 Topics for Further Reflection

1. Buchanan and Tullock's view on the choice of rules emphasizes individual rationality. In your opinion, is this the proper way to analyze and evaluate collective decision rules?
2. Compare Buchanan and Tullock's fictitious original situation with the one introduced and elaborated by Rawls (1971) in his classic work on social justice.
3. Do you agree with Buchanan and Tullock in including external and decision making costs as the primary considerations in choosing the decision rule?

2.5 Suggestions for Reading

Calculus of Consent was a starting point of an extensive research program and the original focal point of the public choice scholarly community. The main later works of Buchanan are (Buchanan 1991a, b). Mueller (2003) provides a comprehensive overview of the public choice tradition. Pettit (2002) represents a more general philosophical approach to the problems of rules, norms and constitutional order.

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Chapter 3
The Majority Rule

Abstract We continue the discussion on rule choices in dichotomous settings. An argument for the simple majority principle is built by assuming that each ballot configuration is equally likely. In a situation where just three voters are present it turns out that the probability of each voter being on the winning side is maximized when the simple majority rule is adopted. This highly theoretical insight is then developed by discussing May's axiomatic characterization of the simple majority rule.

3.1 Introduction

The rule of majority is often regarded as a basic ingredient of democratic rule. In particular in dichotomous choice settings – e.g. yes or no decisions regarding policies – the rule of majority is a quite natural consequence of the assumption that each individual is considered equally important. It would seem to guarantee that the majority of individuals is never on the losing side of the collective decision. If the democratic deficit is understood as the difference between the collective decisions and the views of the individuals, the majority rule apparently ascertains that the democratic deficit never afflicts the majority of individuals. It will, however, be seen shortly that these arguments for the majority rule are not necessarily tenable. Before showing this, let us focus on an argument that justifies the rule on different grounds, viz. maximizing the individual's influence over the collective decisions.

3.2 Rae on Binary Decisions

The argument was first presented by Rae (1969). Consider a dichotomous setting with two possible options: yes or no. Let the voting body consist of three individuals I, II and III. Assuming that each voter votes for just one of the two alternatives, eight different ballot configurations are possible. These are listed in the three left-most columns of Table 3.1.

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Table 3.1 Ballot configurations and collective decisions

Voter I	Voter II	Voter III	Majority	Unanimity	k = 1
No	No	No	No	No	No
No	No	Yes	No	No	Yes
No	Yes	No	No	No	Yes
No	Yes	Yes	Yes	No	Yes
Yes	No	No	No	No	Yes
Yes	No	Yes	Yes	No	Yes
Yes	Yes	No	Yes	No	Yes
Yes	Yes	Yes	Yes	Yes	Yes

Table 3.1 lists all conceivable ballot configurations involving three individuals voting on a dichotomous issue. Each configuration (row in the table) assigns either 'yes' or 'no' vote to each voter. Hence there are $2^3 = 8$ different configurations. Assuming that at least two out of three individuals are needed to make a motion pass or be rejected, the right-most column indicates the majority decision under each configuration. If the voting body votes on only those issues that are supported by at least one member of the body, we can exclude the first configuration (no, no, no) from the set of feasible ones. We are thus left with 7 feasible configurations and their associated outcomes. Looking at Table 3.1 from voter I's point of view, we notice that his vote coincides with the majority decision in all but two configurations, viz. those in the middle, i.e. (no, yes, yes) and (yes, no, no) configurations. Hence, if all configurations are equally probable, the probability of voter I's not having his way is $2/7 = 0.29$. Similarly, voter II is on the losing side in configurations (no, yes, no) and (yes, no, yes) and voter III in configurations (no, no, yes) and (yes, yes, no). Obviously, then, the probabilities of not siding with the majority are the same for all voters.

With unanimity rule we assume that 'no' represents the status quo and wins, unless all three individuals support 'yes'. Now, voter I does not get his way in three configurations: (yes, no, no), (yes, no, yes) and (yes, yes, no). Similarly, for voters II and III there are three configurations where their opinion differs from the collective decision under unanimity rule. Therefore, assuming again that all feasible configurations occur with identical probability, the probability of each voter not getting his way in the collective decision under unanimity rule is $3/7 = 0.43$. This 'frustration probability' is thus higher when the unanimity rather than simple majority rule is being applied. With three voters the simple majority and unanimity rules are the only decisive rules in the sense that smaller than majority rules would not always result in consistent outcomes as both 'no' and 'yes' could receive enough support to qualify as collective decisions. Yet, both cannot conceivably be implemented. Assuming, however, that 'no' is the *status quo* position we can envision any number d between 1 and the total size of electorate to be adopted as the decision rule. So, in our three person example, numbers 1, 2 and 3 can each be taken as the decision rule. In

Table 3.2 Probability of not having one's way (Rae 1969)

	Number of voters									
	3	4	5	6	7	8	9	10	11	12
k = 1	.43	.47	.48	.49	.50	.50	.50	.50	.50	.50
k = 2	**.29**	.34	.39	.43	.46	.47	.48	.49	.50	.50
k = 3	.43	**.34**	**.32**	.35	.39	.38	.45	.47	.48	.48
k = 4		.47	.39	**.35**	**.34**	.36	.39	.42	.44	.46
k = 5			.48	.43	.39	**.36**	**.36**	.38	.40	.42
k = 6				.49	.46	.36	.39	**.38**	**.38**	.39
k = 7					.50	.47	.45	.42	.40	**.39**
k = 8						.50	.48	.47	.44	.42
k = 9							.50	.49	.48	.46
k = 10								.50	.50	.49
k = 11									.50	.50
k = 12										.50

fact we have already considered $d = 2$ and $d = 3$ since the first is the simple majority and the latter the unanimity rule. The last column in Table 3.1 indicates the outcomes when a 'yes' vote from any voter results in 'yes' being the collective decision. Comparing the rows of this column with the ballots of each individual reveals that each voter is on the losing side in three of the seven feasible configurations. So, of all three decision rules the simple majority is associated with the smallest probability of not getting one's way.

When more than three voters are introduced there may be several decision rules that require more than a simple majority but less than unanimity. Does the superiority of the simple majority over other rules still prevail? Rae (1969) conjectures that it does. Table 3.2 is borrowed from Rae (1969) and Abrams (1980). In each column the minimum entries are written in bold characters. A glance at the table shows that the simple majority rule is an all cases associated with the smallest probability of not getting one's way. In some cases the minimum value is associated with more than one rule. This is the case when the number of voters is even and the simple majority threshold is accordingly somewhat ambiguous.

Rae's conjecture – which has been proven true first by Taylor (1969) and subsequently in more general terms by Curtis (1972) – constitutes a strong argument for any individual to support the adoption of the simple majority rule. Of course, for an individual who is profoundly suspicious of the decisions that are likely to be made by the collectivity and who thus stresses the external costs in the individual calculus, the unanimity rule is the best guarantee that his views are always represented in the collective decisions.

Apart from the individual calculus aiming at maximizing the individual welfare there is another approach to justify the simple majority rule, viz. to assess it with respect to the theoretical properties of the rule. It turns out that most of these are intuitively quite plausible and, thus, provide a case for the simple majority principle.

3.3 Necessary and Sufficient Conditions

The intuitive notion of the majority rule starts from the idea that of two options available the one supported by more individuals than the other should be selected as the winner. In case both alternatives receive an equal support, there is a tie between the two. In more precise terms, consider the set $\{x, y\}$ of alternatives and a set N of n individuals each endowed with a ballot D_i, $(i = 1, \ldots, n)$ (May 1952). Each ballot has exactly one of three values: $D_i = -1, 0, 1$ with the first value indicating a vote for y, the second abstaining and the third a vote for x.

The collective decision D can have each of these values as well. A group decision function is then

$$D = f(D_1, D_2, \ldots, D_n) \tag{3.1}$$

Let us denote by $N(1)$ the number of 1's in the decision function (i.e. the number of individuals strictly preferring x to y). Similarly, let $N(0)$ and $N(-1)$ be the number of 0's and -1's, respectively, in the decision function. May's definition of the simple majority rule can now be stated:

Definition 1 Simple majority rule is a decision function f that has the values $D = 1, 0$ or -1 according to whether $N(1) - N(-1) > 0, = 0$ or < 0.

May's characterization involves the following properties:

1. Decisiveness: the domain of f consists of the Cartesian product $D_1 \times D_2 \times \cdots \times D_n$. In other words, the function yields a value for any combination of individual preferences over x and y.
2. Anonymity: any permutation of the individuals leaves the value of f unchanged. That is, only the number of 1's, 0's and -1's, not how they are attached to specific individuals, determines the value of f.
3. Neutrality: $f(-D_1, -D_2, \ldots, -D_n) = -f(D_1, D_2, \ldots, D_n)$. In words, if everyone changes his mind so that those strictly preferring x to y now strictly prefer y to x and *vice versa* and, moreover, those who are indifferent between x and y, remain indifferent, then the outcome should change from 1 to -1, from -1 to 1 or remain unchanged if it was a tie.
4. Positive responsiveness: only one voter's change of mind is required to break a tie. More formally, if $D = f(D_1, D_2, \ldots, D_n) = 0$ or 1 and if a new profile is formed so that all individuals except i keep their opinions unchanged and i changes his opinion from -1 to 0, from -1 to 1 or from 0 to 1, then x is chosen under the new profile, i.e. $D' = f(D'_1, D'_2, \ldots, D'_n) = 1$, where D' denotes the new profile.

May's characterization theorem states that a group decision function is the simple majority decision if and only if it satisfies decisiveness, anonymity, neutrality and positive responsiveness. Arguably these properties are quite natural and plausible. The first property pertains to the general applicability of the function in guaranteeing that under all opinion distributions a collective decision can be found. The second and third properties, in turn, exclude discriminating decision functions. The fourth property, finally, states that additional support, *ceteris paribus*, never harms a candidate and that ties can be broken by the change of mind of a single individual.

The theorem gives quite a strong theoretical reason for adopting the simple majority rule. However, the above analysis pertains only to rather simple settings, viz. those dealing with two alternatives. Difficulties arise when more than two alternatives are being considered. In fact, the fourth property can be given at least two different interpretations both of which feature in important theoretical results. We shall return to these later on.

Since many choice situations involve far more than two alternatives, the above results are not immediately applicable. What is needed is a method to narrow down the candidate field to two. In policy choice settings two different methods are widely used, viz. the amendment and successive methods. Their primary forums are in legislative bodies. Both are based on agenda that determines the order of pairwise votes. In the case of amendment method, each vote takes place between two policy (legislative) alternatives. In the successive procedure (to be distinguished from the successive elimination or amendment procedure) each vote compares a specific alternative with all those remaining in the contest. If the former is supported by a majority, it becomes the winner and no further votes are taken. Otherwise, it is eliminated and the next alternative is picked up from the set of remaining alternatives to be confronted with the other non-eliminated alternatives. The procedure continues until one of the alternatives gets the support of more than half of the electorate.

In both amendment and successive systems, the role of agenda becomes very important. Consider the classic Condorcet paradox profile of Table 3.3. Assuming that each voter votes according to the preferences reported in the table and with the following agenda: (i) A versus B, and (ii) the winner of (i) versus C, C becomes the winner since it defeats the winner of (i), viz. A, in the second ballot. Had the agenda been (i') A versus C, and (ii') the winner of (i') versus B, alternative B would have become the winner. Also A can made the winner by constructing the agenda where B and C are first confronted with each other and the winner faces A in the final ballot.

With sophisticated voting determined by backwards induction (a.k.a. Zermelo's algorithm Schwalbe and Walker 2001) the first-mentioned agenda leads to B, the second one to A and the third to C. To illustrate the first outcome, depending on the outcome of (i), C confronts either A or B in (ii). If the former, the result of the second ballot is C. If the latter, the result of (ii) is B. So, the final outcomes that the voters are considering – if sophisticated in the McKelvey and Niemi sense – at the outset are C, if they vote for A, and B, if they vote for B (McKelvey and Niemi 1978). Since in terms of their preferences indicated in Table 3.3 a majority prefers B to C, the sophisticated voting leads to B. A similar argument applies to the two other agendas.

Table 3.3 A Condorcet paradox profile

1 voter	1 voter	1 voter
A	B	C
B	C	A
C	A	B

3.4 Topics for Further Reflection

1. Can you think of settings where having one's way is not the primary reason for advocating a decision rule?
2. Basically all rules for collective decision making envisage that a simple majority or a larger than simple (qualified) majority of individuals have to support a motion in order to get it passed. All dichotomous decision rules can be expressed in terms of a number $q \in (0, 1]$. i.e. at least $q \times n$ individuals have to support a motion to get it accepted. Can you give a reason for the requirement that $q \times n > n/2$, i.e. that $q > 1/2$?
3. Log-rolling or vote trading takes place when an individual or group supports another in one issue in order the secure the latter group's support on another issue. Can you explain how this activity might benefit small groups? What do we have to assume about the setting to guarantee that vote trading works to the benefit of all those involved in it?
4. Consider the following profile (Table 3.4).
   Which alternative is the Condorcet winner?
5. Suppose that the agenda is the following: (i) A versus B, and (ii) the winner of (i) versus C. Which alternative wins under sincere voting in Table 3.4?
6. Which is the result under sophisticated voting in Table 3.4?

3.5 Suggestions for Reading

Dichotomous choice situations have often been discussed in the context of weighted majority voting where each participant is endowed with a fixed weight and the majority rule refers to procedure that results in whichever of the two alternatives is

Table 3.4 A profile with a Condorcet winner

1 voter	1 voter	1 voter
A	B	C
C	C	A
B	A	B

supported by a group of participants with a weight sum larger than the total weight sum divided by two. Usually the pertinent question is the distribution of voting power among the participants. A very useful text in this is (Felsenthal and Machover 1998). A sizable collection of contributions to this research field is compiled in (Holler and Nurmi 2013). An epistemic argument supporting the majority rule in dichotomous settings is Condorcet's Jury Theorem which states that a group of individuals – with the same probability p of being correct with $p > 1/2$ – resorting to majority rule is correct with a higher probability than its individual members and, furthermore, the probability of the majority being correct approaches unity as the size of the group increases. The theorem has spanned an extensive literature, e.g. (Nitzan and Paroush 1982; Ladha 1995). We shall return to this subject later in this book.

Axiomatic characterizations have been given to several other procedures than the binary majority rule. To wit, Fishburn established the axioms for the approval voting and Young for the Borda count (Fishburn 1978; Merlin 2003; Young 1974). An excellent text on agenda-based procedures and solution concepts is (Miller 1995).

Answers to Selected Problems

In Table 3.4 the Condorcet winner is C since it would defeat both A and B is pairwise majority comparison. By the same token, C would win in sincere voting under the agenda envisaged. Also sophisticated voting would result in C since C wins every pairwise comparison.

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Chapter 4
More Than Two Alternatives

Abstract We introduce and discuss the most common voting procedures. Our starting point is the observation that voting rules can make a significant difference in the voting outcomes. First we deal with the ambiguity of the notion of majority outcome in cases involving more than two alternatives or candidates. The concepts of Condorcet winner and core are introduced. We then define the voting procedures and present some descriptive devices for the analysis of voting situations.

4.1 Introduction

In the preceding we have focused on situations involving only two alternatives for the simple reason that it is in these situations that the simple majority decision has an unambiguous meaning. With the advent of a third alternative, the majority decision becomes ambiguous. Consider the following example (Table 4.1). All three alternatives can be considered winners under three different procedures each resorting to some majority-related principle. Firstly, A is the plurality winner, i.e. it is ranked first by more voters than any other alternative. Secondly, B is the plurality runoff winner since no alternative gets the support of at least half the electorate in a one-person-one-vote- election. Hence a runoff between A and B – the two largest vote-getters – is required. In this contest, B wins with 5 votes to 4. Thirdly, C is the Condorcet winner, i.e. it defeats its two competitors with a majority of votes in pairwise comparisons (C beats A with 5 votes to 4 and B with 6 votes to 3).

So, the discrepancy between three majority-related principles is maximal in this setting. Let us see what the arguments against the selection of each alternative might look like. Firstly, one could object the choice of A by pointing out that it is considered the worst alternative by a majority of voters. Secondly, against the choice of B one could argue that a majority of voters prefers another alternative, C, to it. Thirdly, those opposing the choice of C could point out that C is the favorite alternative of the smallest number of voters. So, each choice can be objected to with at least a modicum of plausibility. It is worth observing that all three rules collapse into the same outcome in all profiles where one alternative is ranked first by more than half of the electorate since obviously then the plurality and plurality runoff methods coincide

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Table 4.1 Ambiguous majority principle

4 voters	3 voters	2 voters
A	B	C
C	C	B
B	A	A

as no second round contest is required. At the same time, the candidate ranked first by most voters becomes the (strong) Condorcet winner. Hence, no discrepancy between rules emerges.

Despite the discrepancy exhibited by Table 4.1 and similar settings, the Condorcet winner is often considered a particularly plausible criterion of winning. Hence, the methods that result in a Condorcet winner when one exists – the so-called Condorcet extensions – are often deemed superior to the other main class of procedures, the positional methods (Felsenthal and Machover 1992; McLean 1991; Risse 2001). And indeed, the Condorcet winner criterion is clearly majoritarian in spirit.

Additional advantages have been discovered by Campbell and Kelly (2015). Using their terminology, let us call the method that always chooses the Condorcet winner the Condorcet rule. An important result of Campbell and Kelly states that the Condorcet rule is the only anonymous, neutral and strategy-proof rule in Condorcet domains (Campbell and Kelly 2003, 2015; Merrill 2011). A rule is strategy-proof if and only if there is no situation where it is manipulable by an individual voter. A rule is manipulable by voter i in the preference profile $P = (P_1, \ldots, P_n)$ when by changing his/her preference ranking from P_i to $P*_i$, *ceteris paribus*, the ensuing outcome is preferable by i to the original outcome. Thus, strategy-proof rules are not manipulable by any individual under any profile. Note, however, that the result is restricted to Condorcet domains, i.e. domains where a Condorcet winner exists.[1] Indeed, by a result of Gärdenfors (1976), all Condorcet extensions that are anonymous and neutral are manipulable. The result of Campbell and Kelly rests on a restriction of the domain of social choice functions to the Condorcet domains.

Despite its *prima facie* plausibility it is not difficult to see that Condorcet extensions may lead to severe problems. To wit, as Table 4.1 shows the Condorcet winner may be considered best by a smaller group of voters than any other alternative. In fact, one may envision settings where the Condorcet winner is not ranked first by *any* voter. Table 4.2 is an example of this kind of profile. Here C is the Condorcet winner, but is ranked first by no voter. Thus, if the Condorcet winner is elected, not a single voter – much less a majority – gets his/her favorite alternative elected.

Another argument for choosing a Condorcet winner is presented by Dasgupta and Maskin (2008). Their main result amounts to the statement that the majority rule is the most robust voting rule. This requires some elaboration. Firstly, the majority rule is

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[1] There are a couple of minor restrictions. Firstly, it does not hold when the number of alternatives is 2 and the number of voters is even. Secondly, it is not known if the result holds when the number of voters is a multiple of 4 and the number of alternatives is 3.

Table 4.2 No voter ranks the Condorcet winner first

4 voters	3 voters	2 voters
A	B	D
C	C	C
D	A	B
B	D	A

defined by Dasgupta and Maskin as the rule that elects a Condorcet winner whenever one exists. When it doesn't, the rule ends up with an empty choice set.[2] Secondly, when saying that the rule is most robust these authors mean that it works well in a larger class of environments than any other rule. The environments are basically types of preference profiles under scrutiny. Thirdly, a voting rule works well – by their definition – if it has the following properties: Pareto, anonymity, independence of irrelevant alternatives (IIA) and generic decisiveness.[3] What Dasgupta and Maskin show is that if any rule F works well in a domain in this specific sense (i.e. has the above properties for all profiles in that domain), then the majority rule also works well (i.e has these properties) in that domain. Moreover, there are domains where the majority rules works well, but F does not.

Of the conditions featuring in Arrow's impossibility theorem, the authors have, thus, chosen to retain the most controversial one, viz. IIA (Arrow 1963). Yet it is puzzling that, if the voters are assumed to have ordinal (strict) preferences, the order properties of those preferences are not utilized fully but in pairwise manner only as is done in rules that satisfy IIA. The result states that the majority rule which returns the Condorcet winner when one exists works well in domains where a Condorcet winner exists. Formulated in another way it says that the Condorcet winner is never Pareto dominated by other alternatives and can be determined on the basis of pairwise comparisons without reference to what happens in comparisons between other alternatives, i.e. independently of irrelevant alternatives. That the Condorcet winner is not Pareto dominated by any other alternative is obvious since if an alternative Pareto dominates another alternative, it by the same token would defeat the latter with n votes to 0 in a pairwise comparison. Hence the Pareto dominated alternative cannot be a Condorcet winner.

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[2] The definition of the Condorcet winner differs from the usual one in accepting alternatives that beat *or tie with* all the others as Condorcet winners. This allows for the possibility that there is more than one Condorcet winner in a profile. The concept of Condorcet winner in the sense of Dasgupta and Maskin is thus the set of majority undominated alternatives. This set is generally known as the core of the majority voting game.

[3] Pareto means that whenever an alternative x is strictly preferred to alternative y by all individuals, then y is not elected. IIA requires that the collective preference between any x and y depends on the individual preferences between these two alternative only. In other words, if in two profiles with the same number of voters, each voter has an identical preference for x with respect to y, then the collective preference between these two alternatives is the same in the two profiles. Generic decisiveness means that the rule results in an outcome under all preference profiles.

4.2 Voting Procedures Under Study

The number of voting procedures used in various contexts is large. Our focus here is on the most widely used ones. Some of them have already been discussed in the preceding, but we shall collect all definitions here once more for easy reference.[4]

- Amendment procedure (or successive elimination). This is a relatively common parliamentary voting procedure. It proceeds through $k - 1$ pairwise comparisons of candidates, if the number of candidates is k. The starting point is an agenda determined exogenously e.g. by the speaker or chairperson. Starting from the two first candidates in the agenda, pairwise comparisons are conducted so that the losing candidate is eliminated, whereas the winner proceeds to be confronted with the next candidate. The winner of the $k - 1$th pairwise comparison is declared the overall winner.
- Plurality voting. This is perhaps the most common of all procedures. It is known as one person-one vote or first past the post system. Each voter has one vote at his/her disposal. Once the votes have been cast, the winner is the candidate or alternative that has received more votes than any other. With more than two alternatives, it is quite possible that the winner is a candidate who has less than 50% of the vote total.
- Plurality runoff. This is a variation of the plurality voting. It is based on the intuitive idea that in order to be considered the winner, a candidate needs to have the votes of more than 50% of the voters. Each voter has again one vote. Once the votes have been given, one determines whether any candidate has been given more than 50% of the votes. If such a candidate exists, it is declared the winner. Otherwise, another ballot is taken between those two alternatives that received more votes than others in the first ballot. Again each voter has one vote. Now, barring ties, one of these two gets more votes than the other and becomes thus the winner. The procedure can also be implemented by using the rankings of candidates provided by the voters as input. Then the first round consists of determining if some candidate is ranked first by more than half of the electorate. If such a candidate is found, he/she is the winner. Otherwise, the second counting round compares the relative rankings of those two alternatives that are ranked first by more voters than any other alternative.The one of those two which is ranked higher than the other by more voters is the winner.
- Alternative vote or Hare's system. Starting from the preference profile one first determines whether there is a candidate that has been ranked first by more than half of the electorate. If there is, then this candidate is elected. Otherwise, one eliminates from consideration the candidate that has been ranked first by the smallest number of voters and modifies the original profile as if this candidate had not been in the ballot at all. A new determination is then made to ascertain if any candidate has now been ranked first by more than 50% of the voters in the modified profile.

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[4] For a more extensive discussion, see e.g. Felsenthal and Nurmi (2018, 15–23) and Nurmi (1987).

If some candidate is, then this candidate is elected. Otherwise the procedure is repeated until a winner is found.

- Anti-plurality voting. In this system all voters assign one vote for every alternative ranked higher than the least preferred one. The votes are summed up and the winner is the candidate with the largest sum of votes.
- The Borda count. This is based on rankings over candidates. If the number of candidates is k, a candidate ranked first by a voter gets $k - 1$ points from that voter, a second ranked candidate gets $k - 2$ points etc, the last-ranked one gets 0 points. Summing up the points gives the Borda score of the candidate. The candidate with the largest score is the winner in the Borda count.
- Nanson's rule. This is a sequential elimination procedure based on the Borda scores of candidates as follows. On the first round, the Borda scores of all candidates are determined whereupon those with the average or smaller Borda score are eliminated. In the resulting reduced profile the Borda scores of remaining candidates are re-computed and again those with at most the average Borda score are eliminated. The procedure is repeated until such time when only one candidate is not eliminated. This is the winner by Nanson's rule.
- Copeland's method. Given a preference profile, one determines for each candidate the number of candidates it defeats by a majority of votes in pairwise comparisons. This number is the Copeland score of the candidate. The scores range from 0 to $k - 1$, if there are k candidates. The score $k - 1$ indicates that the candidate in question defeats all the other and is, thus, the Condorcet winner. In any event, Copeland's method elects the candidate with the largest Copeland score.
- Black's procedure. Given a preference profile, one first determines if there is a Condorcet winner. If there is, then it is the winner of Black's procedure. Otherwise, the winner is the candidate with the largest Borda score (i.e the Borda winner).
- Coombs' procedure. Given a preference profile, one determines if there is candidate ranked first by more than half of the electorate. If such a candidate exists, it is the winner under Coombs' procedure. Otherwise, one eliminates the candidate who is ranked last by a larger number of voters than any other candidate. One then examines the resulting reduced profile to find out if some candidate is now ranked first by more than half of the electorate. If such a candidate is found, it is the Coombs winner. Otherwise one continues eliminating candidates until a winner is found.
- Kemeny's median. Given a preference profile, one looks for the ranking over the candidates that is closest to the profile in the sense of requiring the smallest number of binary preference switches of candidates in individual rankings to become universally adopted. Alternatively, given a preference profile and a fixed ranking, one computes the support of this ranking by adding up the number of voters whose preferences over all pairs of candidates coincide with those in the ranking under consideration. The ranking with the largest support is the Kemeny ranking. Its top-ranked candidate is the Kemeny winner.
- Maxmin procedure. Let the set of alternatives be A. Given a preference profile one conducts for each alternative x all pairwise comparisons and determines for each pair (x, y) the support $s(x, y)$ of x. Let $min(x) = min_{y \in A} s(x, y)$ be the minimum

support for x is all pairwise comparisons. The maxmin winner M is the alternative with the largest minimum support, i.e. $M = \{z \in A | min(z) \geq min(y)$, for all $y \in A\}$.

- Dodgson's rule. One determines for each candidate the minimum number of preference switches between two adjacent candidates in the voters' rankings for this candidate to become the Condorcet winner. The candidate associated with the smallest minimum number is then the winner. Obviously, Dodgson's rule necessarily elects the Condorcet winner when one exists as it requires no pairwise preference switches at all to become the Condorcet winner.

The above procedures are but a sample of those discussed in the literature. Nonetheless they display the wide variety among the opinion aggregation methods applicable in ranking environments. Together with the few methods applicable in non-ranking settings discussed in this book they provide a rich menu for choice. The ranking procedures fall quite naturally into three classes: binary, positional and hybrid systems. The first class consists of systems that determine the winner on the basis of pairwise comparison of candidates. Positional procedures, in turn, use information about the positions of candidates in individual rankings, while hybrid systems use both types of information or repeat the computations in reduced alternative (or voter) sets. In the above list maxmin and Copeland's methods are clear examples of binary systems, but a closer inspection reveals that also Kemeny's median and Black's procedure are based on pairwise comparisons. So is in principle also the Borda count, although it is commonly deemed a positional system. Plurality and anti-plurality procedures are purely positional as are Nanson's and Coombs' systems. Black's procedure is a standard example of a hybrid system.

4.3 Some Descriptive Devices

The analysis and evaluation of voting procedures requires descriptive instruments. Our starting point is the preference profile, i.e. a collection of complete and transitive preference relations by voters. The most useful device to use in analyzing profiles is the outranking matrix. If the number of candidates is k, this represents the preferences in a $k \times k$ matrix. Each row and column represents a candidate and the entry x in the row representing candidate i and in the column representing candidate j indicates that according to the profile x voters prefer candidate i to candidate j. The diagonal entries are marked with "–". Table 4.3 gives the outranking matrix of Table 4.2. In the three columns on the right separated by vertical line we list the minimum and maximum entries of each row and well as the row sum. In other words, the three columns indicate the minimum and maximum support given to the candidate represented by the row in all pairwise contests with the others as well as the sum total of its support in all pairwise comparisons. The last entry in each row is always identical with the Borda score of the candidate represented by the row.

Table 4.3 Outranking matrix of Table 4.2

	A	B	C	D	min	max	sum
A	–	4	4	7	4	7	15
B	5	–	3	3	3	5	11
C	5	6	–	7	5	7	18
D	2	6	2	–	2	6	10

Several things can be discovered when looking the outranking matrices. Firstly, if there is a Condorcet winner, all entries, including the minimum one, on the row corresponding to this candidate have values larger than $n/2$, with n denoting the number of voters. Since $n = 9$ in the present example, the minimum entry of the eventual Condorcet winner should be at least 5. C satisfies this criterion and is thus the Condorcet winner in Table 4.2. Secondly, the eventual Condorcet loser can also be spotted by looking the outranking matrix. For a Condorcet loser, the entries must all be less than $n/2$. None of the candidates here does so poorly in pairwise comparisons and, hence, we can conclude that there is no Condorcet loser in this example. In other words, all candidates beat at least one of their contestants in pairwise comparisons. Thirdly, the row sums indicate the Borda scores of candidates. Thus, we see that C is the Borda winner and the overall Borda ranking is $C \succ A \succ B \succ D$. Fourthly, the maxmin winner is the candidate with the largest minimum entry in its row. Here it is C which is also the Condorcet winner. The coincidence of the Condorcet and maxmin winners is not accidental, but follows from the fact that at most one candidate can defeat all others by strictly more than $n/2$ votes. All other candidates then have at least one entry (viz. that corresponding to the Condorcet winner) in their row with a value strictly less than $n/2$. Hence, no other candidate than the Condorcet winner can become the maxmin winner in profiles where a Condorcet winner is present.

In some circumstances another descriptive device is useful, viz. the tournament matrix. This is also a $k \times k$ matrix. Its entries are either 0 or 1, the latter indicating that the candidate represented by the row defeats the candidate represented by the column. When this is not the case, the entry has the value 0. The criterion of defeat is usually that of relative majority, i.e. X defeats Y just in case more voters prefer X to Y than Y to X. The tournament matrix corresponding to Table 4.2 is presented in Table 4.4.

Table 4.4 Tournament matrix of Table 4.2

	A	B	C	D
A	–	0	0	1
B	1	–	0	0
C	1	1	–	1
D	0	1	0	–

The tournament matrix enables us to discover the eventual Condorcet winners and losers immediately. The row corresponding to the former has the sum of entries equal to $k - 1$, while the latter has the row sum equal to zero. The Copeland winner is the candidate with the largest row sum.

All preference profiles can be transformed into outranking matrices. Similarly, all outranking matrices can be transformed into tournament ones once the criterion of winning (or defeat) is given. Both transformations lose some information: the first transformation loses the information about the ordinal properties of preference rankings, the second transformation loses the information on margins of victory or defeat of candidates.

4.4 Topics for Further Reflection

1. Construct a preference profile of three voters over four alternatives where there is a Condorcet winner which is ranked first by no voter.
2. Consider the following profile (Fishburn 1973, 147):

1 voter	1 voter	1 voter	1 voter	1 voter
D	E	C	D	E
E	A	D	E	B
A	C	E	B	A
B	B	A	C	D
C	D	B	A	C

   Form the corresponding outranking matrix, i.e. a 5×5 matrix where the entry on the i'th row and j'th column represents the number of voters preferring i'th alternative to the j'th one.
3. Determine the Condorcet winner, i.e. look for the row in the outranking matrix where all non-diagonal entries are greater than the number of voters (5) divided by two.
4. Determine the Borda winner, i.e. look for the row with the largest sum of non-diagonal entries.
5. Which one looks more plausible as the winner? Please explain.

4.5 Suggestions for Reading

The Condorcet extension systems – i.e. those that always elect the Condorcet winner when one exists – enjoy a widespread support among the social choice community. See e.g. Felsenthal and Machover (1992), McLean (1991). A strong case for the Borda count, on the other hand, has been built by Saari (1995) and (2003). A comprehensive overview of positional methods is Pattanaik (2003).

Answers to Selected Problems

1. An example is the following profile:

Voter 1	Voter 2	Voter 3
A	C	B
D	D	D
B	A	C
C	B	A

   Here D is the Condorcet winner, but not ranked first by any voter.

2. the outranking matrix is:

	A	B	C	D	E
A	–	3	3	2	0
B	2	–	3	2	0
C	2	2	–	2	1
D	3	3	3	–	3
E	5	5	4	2	–

3. The Condorcet winner is D since all elements in its row are larger than 2.5.
4. The row sums are 8, 7, 7, 12 and 16, respectively for A, B, C, D and E. Hence, E is the Borda winner.
5. Arguably E is more plausible choice than D since E is ranked first by as many voters as D, first or second by strictly more voters than D. In fact, all voters rank E first, second or third, while D is ranked last or next-to-last by one voter each. Positionally E is thus a better choice than D.

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Chapter 5
Strategic Aspects

Abstract This chapter deals with the concept of manipulation, understood as preference misrepresentation, in the light of the main theoretical results focusing on their practical significance (This chapter is largely based on Nurmi (Transactions on computational collective intelligence XXIII. Springer, Berlin, pp. 149–161, 2016)). Manipulability is a pervasive property among choice rules. However, its practical importance hinges on several things. The information requirements of successful misrepresentation can be very demanding and suitable situations may not be common. We also review some indices measuring the degree of manipulability of choice functions. Moreover, the results on complexity of manipulation as well as on safe manipulability are briefly touched upon.

5.1 Introduction

The voting rules are mappings from individual preferences to outcomes (candidates, policy options, rankings). The rules specify which choices will ensue once the individuals report specific preferences. In real world these preferences are not necessarily those that the individuals have regarding the alternatives. It may be beneficial for some individuals to report preferences that are not precisely identical to their true preferences. Thus, for example, in one-person-one-vote elections individuals who deem their true favorites having no chance of winning will quite understandably vote for one of the realistic candidates rather than their true favorite. The same holds for other voting systems as well, as will be seen below. Yet, sometimes this deviation does not improve the outcome from what sincere voting would have resulted in. Indeed, it may even make the outcome worse from the individual's point of view. In any event, the possibility of the individuals deviating from their true preferences in voting makes it difficult to consider voting rules as mappings from true preferences to voting outcomes. Rather the rules should be seen as ways of mapping reported preferences to voting outcomes.

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5.2 The Concept of Manipulation

Ever since the publication of Farquharson's (1969) seminal work in late 1960s the concept of manipulation has played an important role in the social choice and voting theory. Stemming from the Latin word 'manipulus' (handful, bundle or, as a military term, maneuverable formation) it refers to 'handling or using, esp with some skill, in a process or action: to manipulate a pair of scissors' (Collins English Dictionary). It also denotes 'falsification for one's own advantage'. It is in the latter meaning that 'manipulation' is being used in the social choice theory. In short, in refers to activity whereby an individual or group gives an incorrect report on its preferences in order to change the voting outcome to his/her or its advantage. It is quite common to speak about manipulation just in those cases where the intended result is achieved, i.e. when the falsification succeeds in bringing about an improvement in the outcome reached. Let us now make this concept a bit more precise.

Let X be the set of alternatives, N the set of voters, $\mathcal{R}$ the set of n-person preference profiles over X and $F : \mathcal{R} \times A' \rightarrow 2^X$, for any $A' \subseteq X$, a social choice function. I.e. F associates with any subset of X and preference profile over it, a subset of X called the winners or the social choice set. A pair consisting of a set of alternatives and a preference profile over this set is called a situation.

Definition 1 F is manipulable (by individuals) if and only if (hereafter iff) there is a situation and an individual so that the latter can bring about a preferable outcome for himself/herself by preference misrepresentation than by truthful revelation of his/her preference ranking, *ceteris paribus*.

More formally, F is manipulable iff there is a situation (X, R), where $R = (R_1, \ldots, R_n)$, so that there is a pair $x, y \in X$ with $x P_i y$ and $F(X, R_1, \ldots, R_n) = y$ and $F(X, R_1, \ldots, R_{i-1}, R_i', R_{i+1}, \ldots, R_n) = x$.

In other words, F is manipulable iff there is at least one such situation where the n-tuple of sincere voting strategies does not lead to a Nash equilibrium (in pure strategies).[1]

Definition 2 F is non-trivial (non-degenerate) iff for each alternative x, there is a preference profile so that x is chosen.

Table 5.1 illustrates manipulation in the the widely used plurality runoff system. Here $X = \{A, B, C\}$ and $|N| = 17$. With sincere voting the runoff contestants are A and C, whereupon C wins. Should now the 2 right-most voters switch their preference between A and B, the runoff would take place between A and B, whereupon A, their favorite, would win.[2]

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[1] A Nash equilibrium in pure strategies is an n-tuple of strategies (one for each individual) such that no individual is better off deviating from his/her strategy in this n-tuple, provided that the others stick to their strategy choices. So, in Nash equilibrium unilateral deviations do not bring benefit to the deviator. The same idea extends to mixed strategies, i.e. probability distributions over pure strategies. In this work we focus on pure strategies only.

[2] With sincere voting the outcome is the worst for the 2 voters, with strategic voting their most preferred alternative wins.

Table 5.1 Manipulation in plurality runoff system

6 voters	5 voters	4 voters	2 voters
A	C	B	A
B	A	C	B
C	B	A	C

Table 5.2 Manipulation in the amendment procedure

2 voters	3 voters	2 voters	2 voters
A	B	C	C
B	C	A	B
C	A	B	A

Table 5.2 gives another example of manipulation, this time in the case of the amendment procedure used e.g. in U.S. Congress and well as in Finland and Sweden in parliamentary decision making. Here $X = \{A, B, C\}$ and $|N| = 9$.[3] Since the procedure is based on an agenda, we use the following agenda to illustrate manipulation: (1) A versus B, (2) the winner versus C. With sincere voting B wins. Suppose that the 2 right-most voters vote *as if* their preference were: $C \succ A \succ B$. Then the winner is C, their first ranked alternative.

In the definition above as well as in the examples just discussed, manipulation takes the form of misrepresentation of preferences, i.e. reporting in the ballots cast a preference order that does not correspond the preferences one holds with regard to the alternatives or candidates at hand. Manipulation in this sense does not, however, cover the entire spectrum of strategic behaviour in voting contexts. In particular, it does not cover manipulation through agenda control. Our primary aim is, however, to assess the significance of the results achieved in the field of preference misrepresentation.

5.3 Is the Condorcet Winner Criterion Plausible?

In the debate concerning the relative plausibility of Condorcet winner versus Borda winner, an often stated claim is that the Borda winner is crucially dependent on the alternative set under consideration. More importantly, a removal of an alternative may dramatically change the Borda ranking between the remaining alternatives. Similarly, adding an alternative may essentially change the Borda ranking among the rest of the

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[3] It should be mentioned that in the profiles constructed in this work, the number of voters having each preference ranking can be multiplied by any integer without changing the outcomes as long as the same integer is used as the multiplier in all preference rankings. This follows from the homogeneity of nearly all procedures discussed here. The only exception is Dodgson's rule, but its non-homogeneity is not relevant in the examples discussed in this book.

Table 5.3 Subset choices by Borda count

2 voters	2 voters	2 voters	1 voter
D	A	B	D
C	D	A	C
B	C	D	B
A	B	C	A

Table 5.4 Instability of plurality procedure

Voters							
3	6	3	5	2	5	2	4
A	A	B	B	C	C	D	D
C	D	C	D	B	D	B	C
D	B	D	C	D	B	C	B
B	C	A	A	A	A	A	A

alternatives. These findings were made by Fishburn in two articles (Fishburn 1974, 1981). Consider the 7-voter, 4-alternative profile of Table 5.3. The alternatives might be the candidates in the athlete of the year contest where prominent sport journalists vote on four main candidates by indicating their ranking over these athletes.

Borda count results in the ranking $D \succ A \succ B \succ C$. However, before the results are made known, some evidence turns out suggesting that D is guilty of using illegal performance-enhancing drugs. D is, therefore, found ineligible in the contest at hand. Since nothing else has changed in the circumstances, it is decided that the submitted rankings be used in determining the Borda ranking over the remaining three candidates. Upon computing the new scores it is found that the new ranking is $C \succ B \succ A$, i.e. a complete reversal of collective preference over A, B and C. Fishburn's result states that if alternative x is the Borda winner in set X, there are such profiles that x wins in only one proper subset of X. Clearly then widening or narrowing the alternative set opens avenues for outcome control. Could one then find some other positional procedure that provides more stable outcomes under variations of the alternative set? Saari's (2001, p. 70) answer is a resounding: no. Table 5.4 of Saari shows the extreme instability of the plurality procedure.

In Table 5.4 the collective ranking in terms of the plurality votes is: $A \succ B \succ C \succ D$. Strike now the last alternative D out and recompute the plurality scores for the remaining three alternatives to get $C \succ B \succ A$, which reverses the previous ranking among A, B and C. Finally, let us now eliminate the lowest-ranked alternative A and recompute the plurality scores to get $B \succ C$, which again reverses the previous ranking over B and C.

These examples come nowhere near in describing the profound instability possibilities underlying the positional procedures. These are captured in Saari's (2001, p. 72) theorem.

Theorem 1 *Saari. Consider a setting with at least three candidates. Then proceed as follows:*

- *rank the candidates in any desired way and choose procedure to be used in this set of candidates,*
- *eliminate one candidate, place the remaining ones into any ranking (independently of the preceding ranking) and choose again a positional procedure to be applied to this set of candidates,*
- *continue in this way until just two candidates are left and rank these two using the majority rule.*

There exists a profile which produces exactly the outcomes described above when the voters vote in each subset using the designated positional procedure.

These profiles are thus completely chaotic: the result in the superset of alternatives in no way enables one to predict the collective choices in the subsets. There is no systematic connection between winning in subsets and in the outcome rankings in the supersets – or subsets, for that matter.

In defense of the Condorcet extensions one could, however, maintain that removing non-winning alternatives from consideration because of ineligibility does not change Condorcet winners. If an alternative wins all the others in binary contests by a majority of votes in a set of alternatives, surely it will also win all the remaining ones if an ineligible alternative is removed. So, Condorcet extensions would seem to be immune to removing alternatives from the alternative set, while this is clearly not always the case with positional procedures. There are, however, modifications in the choice setting that change the Condorcet winner in a implausible way – while not changing the Borda winner. Consider the profile of Table 5.5.

Ignoring the thick vertical line for a moment, we have a 23-voter profile over three alternatives. There is a Condorcet winner, viz. B which is also the Borda winner. Observe that the 12-voter sub-profile on the right-hand side of the thick vertical line constitutes a Condorcet-paradox profile: A defeats C, C defeats B and B defeats A, all with an 8 to 4 margin. Focusing on this sub-profile only, there is no reason – based on the ranking information only – to put one alternative ahead of another since each alternative is ranked first, second and third equally many (4) times. A perfect tie, then, prevails in this sub-profile. Focus now on the left side sub-profile of 11 voters. There A is a strong Condorcet winner, but B is the Borda winner. Now, if the 12-voter sub-profile is a perfect tie, its addition to some profile should, intuitively, make no

Table 5.5 Condorcet instability

Voters				
7	4	4	4	4
A	B	A	C	B
B	C	C	B	A
C	A	B	A	C

difference to the outcome in the latter: if there was a winner, it should not change by the addition of a perfectly tied sub-profile. And, indeed, this is the case if the Borda count is applied; B wins both in the 11- and 23-voter profiles. For Condorcet extensions, in contrast, the addition of the tied sub-profile changes the Condorcet winner from A to B.

So, both Condorcet extensions and the Borda count are subject to instabilities as the result of various modifications in the choice settings. What makes the former methods more questionable, however, is the existence of several results showing the incompatibility of the Condorcet extensions with some other social choice desiderata. Of particular interest in the democratic deficit discussion are results showing that all Condorcet extensions are vulnerable to the no-show paradox (Moulin 1988). A no-show paradox occurs when a group of voters with identical opinions is better off (in terms of their own preferences) by not voting at all than by voting according to their preferences (Fishburn and Brams 1983).

In determining whether the paradox can occur one has to compare two profiles: (i) one where everyone submits a ranking and the result is determined on the basis of this, and (ii) one which is otherwise the same as in (i), but a group of identically minded voters abstains. If the outcome in (ii) is ranked higher in the preference of the abstainers than the outcome of (i), then an instance of the no-show paradox has occurred. The no-show paradox comes in two versions: the ‘plain’ one, just defined, and the strong version. The latter occurs when the outcome in (ii) consists of the alternative ranked first by the abstainers. In other words, the result of abstaining is not only required to be preferable to the one in (i), but the best one for the abstainers.

Table 5.6 presents an instance of the strong version under a specific Condorcet extension, Black’s method. As was pointed out earlier, the method is a combination of two principles: (i) if a Condorcet winner exists, it is elected, otherwise (ii) the Borda winner is chosen.

In Table 5.6 alternative D is the Condorcet winner and is, therefore, the winner of Black’s method. Now, consider the same profile modified so that the right-most voter abstains. This is now the profile (ii) in the above definition. In this (ii) profile there is no Condorcet winner. Accordingly, the Borda winner E is elected by Black’s method. A glance at Table 5.6 reveals that E is the first ranked alternative of the abstaining voter. We therefore have an instance of the strong no-show paradox.

Table 5.6 Black’s method and the no-show paradox

1 voter	1 voter	1 voter	1 voter	1 voter
D	E	C	D	E
E	A	D	E	B
A	C	E	B	A
B	B	A	C	D
C	D	B	A	C

Table 5.7 No voter ranks the Condorcet winner first

4 voters	3 voters	2 voters
A	B	D
C	C	C
D	A	B
B	D	A

Obviously the strong no-show paradox is more dramatic failure of responsiveness of a voting system than the ‘plain’ version of the paradox. It is therefore worth asking what kind of procedures are vulnerable to this stronger version. Pérez gives an answer to this question (Pérez 2001): nearly all Condorcet extensions are vulnerable to the strong no-show paradox. The only commonly known exception is the maxmin rule (Kramer 1977).

From the vantage point of minimizing democratic deficit, the positional procedures would seem preferable to Condorcet extensions. At least situations resembling that depicted in Table 5.7 can be avoided by plurality-related systems. This is, admittedly, not a conclusive argument in favour of positional systems, but in the restricted domain of minimizing democratic deficit it should have some bearing. By adopting Condorcet extensions one runs the risk of encountering no-show paradoxes and these undermine the very rationality of ‘going to the people’, i.e. turning to the electorate for advice. The Borda count, on the other hand, can be directly related to the minimization of democratic deficit (Nitzan 1981).

Consider a profile of individual preferences over a set of alternatives. Take now any alternative, say x, and a voter, say i, into consideration and determine the number of pairwise preference switches that are needed to make x the first ranked by i. Obviously this is the same as counting the number of alternatives ranked higher than x by i. Considering all voters gives us the sum measure of how far from the observed profile is from one where everybody ranks x the first. Comparing these sum measures of all alternatives suggests a reasonable way of electing the winner, viz. the alternative which has the smallest sum. It has been shown by Nitzan that is precisely the Borda winner. This gives us a pretty strong case for using Borda count as a method: it minimizes the democratic deficit when the latter is measured as the distance from a consensus where all individuals have the same alternative ranked first. Admittedly, this argument against Condorcet extensions rests to some extent on the definition of the democratic deficit as the difference between collective outcomes and the individual preference rankings. Should one adopt a different approach to describing voter opinions, the conclusion might also be different.

5.4 Principal Results

The best-known result in manipulation literature is undoubtedly the theorem proven by Gibbard (1973) and Satterthwaite (1975). In contradistinction to the social choice function defined above, the theorem deals with another formal counterpart of voting rule, viz. the resolute social choice function, sometimes also known as the social decision function. This concept refers to mechanisms that in every situation end up with one and only one alternative specified as the winner.

Theorem 2 (Gibbard-Satterthwaite 1973–75). *Every universal and non-trivial resolute social choice function is either manipulable or dictatorial.*

One strategy of proof is the following (Feldman and Serrano 2006):

1. It is shown that any universal, non-trivial and non-manipulable SCF must satisfy the Pareto condition if the number of voters is two.
2. One goes through all 36 different preference profiles of two voters and three alternatives and determines the winners that are possible under the Pareto principle. It turns out that the possible outcomes make either the rule manipulable at some profile or one of the voters is a dictator (the outcome is always his first ranked alternative).
3. The argument is extended to larger electorates and larger alternative sets.

The theorem is *prima facie* very damaging to the view that voting procedures always reveal 'the will of the people'. After all, what it says is that no reasonable voting rule can be expected to accomplish this under all circumstances. But does it apply to all reasonable voting rules? It does not. In fact, it applies directly to very few since very few systems are resolute. By far the most procedures may end up in a tie between two or more alternatives. These are then broken in various ways to select the winner. Nonetheless the rules themselves are typically not resolute.

It is, however, relatively straight-forward to show by way of examples that all systems used in practice are – while not resolute – still manipulable through preference misrepresentation. Two examples were shown in the first section of this chapter (Nurmi 1984). What should be observed, though, is that in both examples above, a coordination of several like-minded voters is required for successful manipulation. By glossing over the possibility of ties in outcomes, the Gibbard-Satterthwaite theorem also overlooks the distinction one could make between procedures manipulable by individuals and those manipulable by coalitions. Similarly, it overlooks the distinction between outcomes that result from manipulation in cases where there are ties in manipulated and non-manipulated outcomes. These distinctions are taken into account in Taylor's (2005) comprehensive analysis. The overall conclusion remains that a vast majority of voting systems is manipulable in some sense. So, the Gibbard-Satterthwaite theorem seems to extend far wider than the concept of resolute social choice function would envisage.

Yet, some of the dramatic effect of the Gibbard-Satterthwaite theorem is lost once one realizes that it applies directly mechanisms that are not used. In the context of this observation Gärdenfors' (1976) theorem seems a significant step forward in applied social choice theory.

Theorem 3 (Gardenfors 1976). *If a social choice function is anonymous and neutral and satisfies the Condorcet winning criterion, then it is manipulable.*

As pointed out in the preceding, the Condorcet winning criterion is satisfied by all voting systems that always result in the Condorcet winner when one exists in the observed profile. A noteworthy aspect of this theorem is its wide range of applicability: it covers all social choice functions, not just resolute ones. Specifically, it covers basically all voting procedures that single out a set of winners once the ballots have been cast.

Strategy of proof of this theorem - as envisaged by Gärdenfors - is the following:

- One begins with a specific 3-voter, 3-alternative profile, where the one specific alternative is ranked first by two voters. One postulates that this specific alternative is chosen in this profile.
- Another specific 3-voter, 3-alternative profile is then focused upon and all logically possible choices from this profile are analyzed.
- For each choice from the latter profile, one shows that if this were the actual choice, then the social choice function applied would be manipulable by some voter at some other profile. Since the Condorcet winner is chosen in the first profile, the conclusion is that all Condorcet extensions are manipulable.

It is well-known that not all voting systems are Condorcet extensions. Of those that are not, the theorem, of course, says nothing, but again a more detailed analysis reveals that manipulability is a pervasive property among these as well. Gärdenfors points out, however, two choice functions that are not manipulable:

- If every voter's preference ranking is linear or strict (no ties), then a social choice function that chooses the Condorcet winner when one exists and all alternatives otherwise, is non-manipulable.
- Under the same assumption concerning voter preferences a social choice function that chooses the Condorcet winner when one exists and the set of Pareto undominated outcomes otherwise, is also non-manipulable.

Pareto domination is defined as follows. An alternative x Pareto dominates another alternative y iff x is ranked at least as high as y by all voters, and strictly higher by at least one voter. The set of Pareto undominated alternatives consists of those that are not Pareto dominated by any others. Typically this is a very large set and, hence, the improvement in terms of discriminating power of the latter function is not typically much greater than that of the former.

The outlook for finding a system that would encourage sincere preference revelation from voters is, thus, not promising in the light of these results. On a more positive side the following theorem is worth mentioning.

Theorem 4 (Campbell and Kelly 2015). *Let n be the number of voters and m the number of alternatives. (i) For $n = 4$ or $n = 4k + 2$ with $k \geq 0$ and $m \geq 3$, if F is* *anonymous, neutral and strategy-proof social choice function on Condorcet domain, then F is the Condorcet rule (i.e. selects the Condorcet winner). (ii) For* $n = 4k$ *with* $k \geq 1$ *and* $m \geq 4$*, if F is anonymous, neutral and strategy-proof social choice function on Condorcet domain, then F is the Condorcet rule.*

Condorcet domain is the class of profiles where there is a Condorcet winner. Campbell and Kelly's theorem thus essentially states that all Condorcet extensions are immune to manipulation as long as we allow only those profiles where a Condorcet winner exists (Campbell and Kelly 2015). As will be seen shortly, the restriction envisaged is important.

5.5 The Practical Significance of the Results

The example of Table 5.1 shows that manipulable systems can present some of the voters with a dilemma: (1) to vote according to their true preferences, thereby contributing to their favourite's possible victory on the first round and at the same time risking its loss on the second round by not voting for a weaker contestant in the first round. Or (2) to use their vote to contribute to the success – on the first round – of a candidate that is a weaker competitor to their own favourite on the second round – assuming there is going to be one. This is a quandary that faces those voters who can reasonably expect their favourite to make it to the second round, but to still fall short of the 50% required for overall victory on the first round. Similar incentives are faced by small-party supporters in two-party systems: should one reveal one's true preferences in voting or should one support 'the lesser of two devils'? These dilemmas are well known.

Table 5.1 is instructive in a another sense as well. To wit, the two voters whose strategic behaviour has been in the focus of our interest are in fact making a choice between their best and worst alternative: with sincere voting, their worst alternative wins, while by misrepresenting their preferences, *ceteris paribus*, their best alternative gets elected. It would seem that the supporters of A would have strong incentives to vote for B rather than A on the first round. However, not *all* supporters of A are advised to do so, since should this happen, the outcome would be the victory of B in the first round since it would get more than 50% of the votes. Not a disastrous outcome but not optimal either. To get the desired result the supporters of A need coordination in order to avoid overshooting – and ending up with B – and undershooting – and ending up with the worst possible outcome C.

One of the factors restricting the practical significance of the general manipulability results is the fact that, although the system may be manipulable, the difference between the manipulated and sincere voting result is small and certainly not of the order of magnitude of Table 5.1 example. Moreover, the *ceteris paribus* clause embedded into the manipulability results is to be taken seriously. The reason is simple: if the other parties get a hint that some party aims at strategic misrepresentation of its preferences, they may resort to misrepresentation counter-measures themselves.

Table 5.8 Manipulation of Copeland's rule

1 voter	1 voter	1 voter		1 voter	1 voter	1 voter
A	B	E		A	B	E
C	C	D	⇒	D	C	D
B	A	C		B	A	C
D	E	A		E	E	A
E	D	B		C	D	B

Thus, for example in Table 5.1, if the supporters of B suspect that the two A supporters intend to vote for B in the first round to get it defeated by A on the second one, they might strategically vote for C in the first round so that C would become the overall winner. This is better than A for the supporters of B. Thus, the counter-measures may well frustrate the efforts of the manipulators. In other words, manipulability of a system in a situation does not mean that strategic misrepresentation would be plausible or likely. Indeed, preference misrepresentation may conceivably lead to better, worse or equal outcome with respect to the sincere voting outcome. More recent research has, accordingly, focused on these aspects as will be discussed later on.

Of the results discussed in the preceding section, the theorem of Campbell and Kelly is certainly the most positive one. On closer inspection it is, however, of very restricted applicability (Mayer 2015; Napel 2015). Consider the example devised by Alexander Mayer on the Copeland rule applied to the following pair of profiles (Table 5.8).

On the left, C is the Condorcet winner and is thus elected by Copeland's rule (and by Condorcet's rule). The right-side is a result of first person's manipulation. There A, his first ranked alternative, wins with Copeland. Thus the manipulation is beneficial to the voter. Note, however, that the right profile is not in the Condorcet domain. So, by excluding profiles without Condorcet winner, the theorem in fact disregards the most obvious ways of manipulating Condorcet extensions. This, of course, doesn't undermine the validity of the result itself.

5.6 Difficulty of Manipulation

Anyone who has worked on providing examples of various criterion violations in social choice theory knows that coming up with such examples can, in cases they are theoretically possible, be exceedingly difficult for some criteria and procedures, while for others it can be relatively straight-forward. The same applies to demonstrating the manipulability of voting rules: for some rules it is easy to find profiles where voters can benefit from preference misrepresentation, while for other rules such profiles are more difficult to find. This intuitive observation suggests that perhaps it would make sense to consider the manipulability of voting rules as a matter degree rather than a dichotomy. Various ways of measuring the degree have, indeed, been devised.

- Kelly's index: $K = \frac{d_0}{(m!)^n}$, where d_0 is the number of profiles that are manipulable by at least one voter (Kelly 1993).
- Kelly index as modified by Aleskerov and Kurbanov: let λ_k = number of profiles that precisely k voters can manipulate (Aleskerov and Kurbanov 1999). Then $J_k = \frac{\lambda_k}{(m!)^n}$ is the share of profiles manipulable by k voters. The Aleskerov-Kurbanov index is the vector $J = (J_1, \ldots, J_n)$.[4]
- three indices of freedom of manipulation $I+$, I^0, and $I-$ (Aleskerov et al. 2011, 2012).

In any profile of m alternatives, each voter has $m! - 1$ possibilities for preference misrepresentation. Let k_{ij}^{+} be the number of cases where misrepresentation improves the outcome to the voter i in profile j. Similarly, k_{ij}^{0} = the number of cases where misrepresentation makes no change in the outcome for voter i in profile j and k_{ij}^{-} = the number of cases where preference misrepresentation makes the outcome worse for i in profile j (Aleskerov and Kurbanov 1999).

- $I+ = \frac{\sum_{j=1}^{(m!)^n} \sum_{i=1}^{n} k_{ij}^{+}}{(m!)^n \times n \times (m!-1)}$
- $I^0 = \frac{\sum_{j=1}^{(m!)^n} \sum_{i=1}^{n} k_{ij}^{0}}{(m!)^n \times n \times (m!-1)}$
- $I^- = \frac{\sum_{j=1}^{(m!)^n} \sum_{i=1}^{n} k_{ij}^{-}}{(m!)^n \times n \times (m!-1)}$

Suppose that with sincere voting the outcome occupies kth position in individual i's ranking. After i's misrepresentation the outcome occupies the position s in his ranking. Let $\theta = k - s$. θ thus shows how much – in terms of ranks – difference i's misrepresentation has made for him/her in this single case. Summing up these θ's over cases and dividing the sum by k_{ij}^{+} (the number of successful misrepresentations by i in profile j) one obtains Z_{ij}. This is then used to define efficiency index

$$I_2 = \frac{\sum_{j=1}^{(m!)^n} \sum_{i=1}^{n} Z_{ij}}{(m!)^n \times n}$$

Let $Z_{ij}^{max} = max(\theta_1, \ldots, \theta_{k_{ij}^{+}})$. Then

$$I_3 = \frac{\sum_{j=1}^{(m!)^n} \sum_{i=1}^{n} Z_{ij}^{max}}{(m!)^n \times n}$$

On the basis of the results of Aleskerov and Kurbanov regarding 3-alternative settings the following conclusions can be made (Aleskerov and Kurbanov 1999):

- the likelihood of a manipulable profile depends on the assumptions regarding extended preferences (over subset of alternatives)

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[4] $K = \sum_j J_j$.

- for small number of voters and alternatives, threshold rule and Borda count seem most manipulable[5]
- for medium range, plurality gets highest values of the index
- Black's procedure has the smallest values over most of the range of voters
- some index values (esp. for Black) depend on the parity of the number of voters

To a large extent the same conclusions extend to 4- or 5-alternative settings (Aleskerov et al. 2012).

The main problem related to practical use of the above measures of the degree of manipulability is the fact that typically not all preference profiles are equally likely. This restricts the applicability of these measures as direct guidelines for selecting voting rules. This problem pertains also to the other main approach to measuring difficulty: the computational complexity of manipulation. This approach builds on and expands the results of algorithmic complexity theory, a well-established field within computer science (Hemaspaandra and Ogihara 2002). The basic classification of computational tasks is the following:

- computationally tractable problems: those that can be computed by polynomial time algorithms of order $O(n^k)$, where k is a fixed constant and n the size of input (e.g. number of alternatives or voters). This class of problems is denoted by P.
- problems in *NP* (non-linear polynomial time): no polynomial time algorithm is known, but given a solution proposal, its correctness can be verified in polynomial time.
- *NP*-complete problems: if any of these are shown to be computable in polynomial time algorithm, all others can be similarly computed. Then $P = NP$.

It is generally believed – although this hasn't been proven – that $P \neq NP$.

Computational complexity relates to voting rules in several ways. Firstly, the computation of the election results once the ballots have been cast may, depending on the rule being applied, require varying amounts of computing resources (time, memory-space). This problem was first addressed by Bartholdi et al. in the context of Dodgson's rule (Bartholdi et al. 1989b). More specifically, the problem addressed was: given the set C of candidates, the set V of preference rankings over C and a positive integer K, is the Dodgson score of candidate c in C less than or equal to K? It was proven that the Dodgson score is NP-complete. The proof proceeds by reducing the score problem to another problem known to be NP-complete, viz. exact cover by 3-sets.

A related problem, viz. Dodgson ranking problem is the following: given sets C and V as above with two distinguished members c and c' in C, one asks: did c defeat c' in the election? The result is that Dodgdon ranking is NP-hard, i.e. easy for a good guesser, but in general not solvable in polynomial time. In contradistinction to the

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[5] In 3-alternative contexts the threshold rule ranks alternative x ahead of y if and only if the number of individuals giving y the lowest grade is strictly larger than the number of individuals assigning x the lowest grade. In case x and y are given the lowest grade by the same number of individuals, their collective ranking is determined by the number of individuals assigning x and y the middle grade.

Dodgson score problem this one is not NP-complete, i.e. does not imply anything with respect to the canonical quandary: is $P \neq NP$? In addition to these now classic problems, Bartholdi et al. prove similar results for the Kemeny rule, i.e. Kemeny score is NP-complete, Kemeny ranking and Kemeny winner NP-hard.

Complexity theory has also applications in the study of preference misrepresentation. In this context the problem takes the following form: given a profile Π of votes cast by everyone else but the manipulator, and a preferred alternative x, is there a vote that the manipulator can cast so that x wins? This problem is typically in NP as the yes or no answer can be checked (normally) in polynomial time. Sometimes (e.g. plurality voting) even the solution can be computed in polynomial time (in which case even the problem is in P) (Conitzer and Walsh 2016). Bartholdi et al. prove the following important theorem (Bartholdi et al. 1989a).

Theorem 5 (Bartholdi et al. 1989a): *the manipulation problem can be solved in polynomial time for all rules that satisfy the following:*

1. *the rule can be run in polynomial time*
2. *the rule is scoring rule*
3. *the following type of monotonicity holds, i.e. for all profiles* Π *and* Π' *and for all* $a \in X$ *and for all* $i \in N : \{b : a \succ_i b\} \subseteq \{b : a \succ_i' b\}$ *implies that* $S(\Pi, a) \leq S(\Pi', a)$.

It should be emphasized that the type of monotonicity featuring in the theorem is not equivalent to the standard concept of monotonicity. This can be seen e.g. in the following example (Table 5.9) where it turns out that while the Borda count satisfies the latter, it does not satisfy the former.

In the 3-person profile in the left, the subset of alternatives regarded inferior to D by all voters is $\{A, B\}$, and in the right-hand profile $\{B\}$. So, the Bartholdi monotonicity would require that the score of D be larger on the left than on the right profile. This is not the case if the Borda count is applied: the score of D is 8 on the left and 9 on the right.

From the practical point of view the complexity results should be understood in their proper role: they are based on worst-case settings. In other words, if a result implies that manipulating a given system is computationally intractable, this does not mean that this should always or even in a majority of situations be so. It only says

Table 5.9 Two concepts of monotonicity

1 voter	1 voter	1 voter	1 voter	1 voter	1 voter
C	E	E	C	D	D
E	D	D	A	E	E
D	C	A	E	C	A
B	B	B	D	B	B
A	A	C	B	A	C

that there are situations in which manipulating successfully confronts the voter with an computationally intractable problem. These kinds of situations may be extremely rare in practice.

5.7 Safe and Unsafe Manipulation

Preference misrepresentation does not always succeed. The most obvious explanation for a failure is that the *ceteris paribus* condition that is used in defining manipulability does not hold in the situation at hand. Other participants may resort to counter-measures so that the preference misrepresentation backfires. Obviously the possibility of such failures plays a significant role in the calculus of any voter pondering upon the choice of the voting strategy. Consider the following example devised by Slinko and White (2014) where uncoordinated manipulation may backfire (Table 5.10).

With sincere voting B wins in Borda count. If either of the two left-most voters votes $A \succ C \succ B$ and ties are broken alphabetically, A wins. However, if they both manipulate, C (their worst) wins. The necessity (and precariousness) of coordination is even more evident in Table 5.11, also devised by Slinko and White.

The Borda count yields B as the sincere voting outcome. If $4 - 8$ of the first 17 voters vote $A \succ C \succ B$, *ceteris paribus*, A wins. If $10 - 17$ of the same voters vote as indicated, the winner is C.

These considerations motivate the introduction of the concept of safe manipulation (Slinko and White 2014).

Definition 3 A strategic vote L is safe, iff for any subset of like-minded (identical preferences) voters the outcome resulting from their choosing L (rather than their true preference) is better (in terms of their true preferences) than sincere voting.

Table 5.10 Manipulation of Borda count may backfire

1 voter	1 voter	1 voter	1 voter
A	A	B	C
B	B	C	B
C	C	A	A

Table 5.11 Precariousness of manipulation

17	15	18	16	14	14
A	A	B	B	C	C
B	C	A	C	A	B
C	B	C	A	B	A

In other words, manipulation is safe whenever no harm is done to the voter by resorting to it. One could say that the manipulation is the dominant strategy for the voter. In line with the standard definition one again assumes that outside the group of would-be manipulators the behaviour remains fixed, i.e. no counter-measures are resorted to.

Theorem 6 *Slinko and White. Let a nondictatorial and resolute social choice function F be applied to a choice set of at least three alternatives. Then there exists a profile and an individual so that the individual can safely manipulate F in the profile.*

This theorem quashes the hopes of finding a reasonable sub-class of voting rules that would be immune to the Gibbard-Sattertwaite result when the additional condition that manipulation be safe is imposed. Thus, manipulability – even safe manipulability – seems to be a pervasive feature of voting rules.

Lest too drastic conclusions be drawn, it is worth emphasizing that the Slinko-White theorem is an existence result. It states that for each nondictatorial and resolute rule a situation can be found where it is safely manipulable barring counter-measures. No estimate of the probability of such situations is given in the theorem.

Finally, an important assumption underlying the above manipulability results should be made explicit: the results assume that the voters have complete information about the preference profile. Together with the assumption of no counter-measures by other voters the complete information requirement glosses over many considerations that the real world manipulability would seem to depend upon. Which is another way of saying that the theoretical results are precisely what they should be, viz. theoretical.

5.8 Topics for Further Reflection

1. Compare plurality voting with Condorcet extensions in terms of difficulty of manipulation
2. Construct a profile showing that the plurality voting is manipulable in three-party contests
3. Show by way of an example how the Borda count can be manipulated by adding new alternatives to a profile
4. Consider the organization you are interested in. What types of manipulation – if any – you think might be important in its decision making? Are there safeguards to prevent or discourage preference misrepresentation?

5.9 Suggestions for Reading

The classic text on manipulability is Farquharson (1969). A more recent and comprehensive text is Taylor (2005). Also Moulin's work is well worth studying (Moulin 1983). Nitzan's excellent textbook provides a concise and lucid discussion on preference misrepresentation (Nitzan 2010).

Answers to Selected Problems

1. A successful preference misrepresentation in plurality voting typically requires information about the first ranked candidates only, while Condorcet extensions can be successfully manipulated if enough information is available on the pairwise comparisons. Hence, typically more detailed information about the preference profile is required for manipulating Condorcet extensions.
2. The following example illustrates:

3 voters	2 voters	2 voters
A	B	C
B	A	B
C	C	A

With sincere voting, A wins by plurality of votes. Since A is the worst candidate for the two right-most voters, it makes sense, *ceteris paribus*, for them to vote for B (and not C) thereby bringing about the victory of B which for them is better than the victory of A.

3. Consider first the profile on the left-hand side of the arrow:

3 voters	2 voters		3 voters	2 voters
A	B		A	B
B	A	→	B	C
			C	A

Under the Borda count A wins with 3 points against 2. Introducing an alternative like C on the right-hand panel makes B the Borda winner.

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Chapter 6
Sequential Voting by Veto

Abstract Sometimes the members of the committee or small group are more interested in avoiding particular outcomes than in reaching their own favourite ones. In such circumstances the sequential voting by veto provides an a priori plausible decision making method. We outline the method and discuss its main properties.

6.1 Introduction

Most procedures discussed in this treatise pertain to settings where all voters submit their preference rankings over alternatives and the outcome is determined by aggregating these rankings according to a specific rule. In small groups where the participants can exchange views on alternatives prior to the preference aggregation, it is conceivable that the alternative set is formed or modified in the course of the discussion preceding the voting. This is quite common in contemporary parliaments, committees and other public sector formal decision making bodies, but it is presumably even more common in boardroom decision making and gatherings of less formal nature such as groups of friends discussing various pastime activities. In this chapter we deal with a method that seems rather promising in these kinds of settings, viz. the sequential voting by veto (SVV, for brevity) introduced by Mueller (1978).

6.2 The Procedure

The background and motivation of this method is in determining the distribution of a given divisible payoff among members of a group in a situation where there is a *status quo* alternative that gives all members a zero payoff. Each member is first asked to make a proposal regarding the payoff distribution. Proposals are then listed and displayed to the group members. The members are then arranged in a random sequence indicating the order in which they will give their votes. The voting takes place by vetoing, not supporting, alternatives. When his/her turn comes, each voter is to veto one and only one alternative. The last un-vetoed alternative is the winner.

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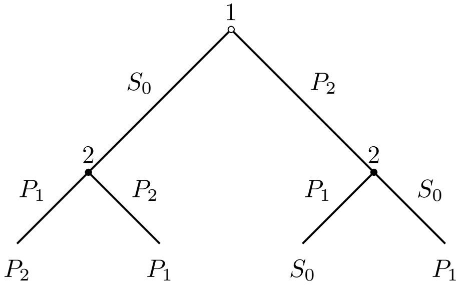
Fig. 6.1 Two-person veto game tree



There are various ways of organizing the proposal submission and balloting stages. Perhaps the most plausible is one where both the proposals and the sequence of balloting are made known to the individuals before the balloting begins. An important ingredient of the procedure is that the order in which the ballots are submitted is random and, thus, not dictated by any member of the group (e.g. the chairperson).

To see how the system works, consider the simplest case of a two-member group. Let S_0, P_1 and P_2 denote the *status quo*, member 1's and member 2's proposal, respectively. It makes sense to assume that both member 1 and member 2 prefer their own proposal to S_0. Moreover, it is plausible to assume that the members prefer their own proposal to that of the other member. Let now the voting order be that member 1 votes first. Obviously, he/she eliminates either S_0 or P_2 depending on which one is worse for him/her. Member 2, then, has the choice either between S_0 and P_1 or between P_1 and P_2. Obviously, for P_2 to have any chance at all of being the final outcome, member 2 has to make it more attractive than S_0 for member 1 or else it will be eliminated on the first ballot. Thus, in order to make it possible that their proposals be adopted in the process, the participants have to consider the preferences of each other *vis-à-vis* S_0 when making proposals: the proposals should be Pareto-improvements over the *status quo*. This by itself does not guarantee the success of a member's proposal, but is a necessary condition for it as will be seen shortly.

The setting for the two-person SVV is depicted in Fig. 6.1. The numbers next to the nodes refer to the players. The symbols next to the edges refer to moves that the players can make at each stage of the game. The time flows from top to bottom, i.e. player 1 moves first. The lower-most symbols refer to the outcomes. Thus, for example, following the left-most sequence, player 1 makes the first move by eliminating S_0 whereupon player 2 chooses to eliminate P_1 so that the end result is P_2.

In order to make predictions about which outcomes are likely to ensue from the calculations of minimally rational players, we can apply the procedure known as backwards induction (also sometimes known as Zermelo's algorithm) (Hamburger 1979; McKelvey and Niemi 1978).[1] We start from the final nodes, i.e. the outcomes and look for the immediately preceding decisions. Thus we notice that the choice between the two left-most outcomes, P_1 and P_2 is actually determined by player 2

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[1] For a discussion on Zermelo's game-theoretic work, see Schwalbe and Walker (2001).

who, on the left-most decision node, can choose to eliminate either P_2 or P_1. Since the former is his proposal, it is plausible to assume that he eliminates P_1. So, when pondering upon his/her choice at the first decision node player 1 can safely assume that should he/she eliminate S_0, the outcome would be P_2. By similar reasoning player 1 can assume that by eliminating P_2 at the outset, the outcome will be P_1 or S_0 depending on whether player 2 deems P_1 preferable to S_0 or *vice versa*.

Suppose now that the proposals P_1 and P_2 are not Pareto-improvements over S_0, but the ranking over proposals are as in Table 6.1. Then, if both players have complete information about each other's preferences, player 1 knows that if he/she eliminates S_0, the end result is P_2, while if he/she eliminates P_2 the outcome is S_0. Since the latter outcome is preferred to the former by player 1, we can expect that he/she chooses accordingly, i.e. eliminates P_2.

Suppose now that both proposals are Pareto-improvements over S_0 so that the preferences are as in Table 6.2. Then, by eliminating P_2 at the outset, player 1 can expect to obtain his/her first-ranked alternative – assuming that player 2 acts according to his/her preferences.

We see that in both cases player 1 has an advantage: by eliminating the other player's proposal he/she can force the latter to choose between the *status quo* and player 1's proposal. As long as the latter is somewhat better for player 2 than S_0, it is likely that P_1 emerges as the outcome. Now, at the time of submitting proposals the players are not supposed to know which one of them is player 1, i.e. the first mover. Hence, both have an incentive to 'sweeten' their proposal so that it offers something more than S_0 to the other player as well. Hence, the mechanism is geared towards securing Pareto-optimal outcomes.

It is evident that, given strict preferences over the alternatives (proposals), the SVV yields a unique outcome (Mueller 1978). It can also be shown that the resulting outcome is never Pareto-dominated by another alternative. From the view-point of group decision making it is particularly noteworthy that the SVV outcome is never the lowest-ranked alternative of any participant (Felsenthal and Machover 1992). This feature makes it a plausible decision method in recommendation systems involving group decision making.

Table 6.1 Preference rankings over proposals: I

Player 1	Player 2
P_1	P_2
S_0	S_0
P_2	P_1

Table 6.2 Preference rankings over proposals: II

Player 1	Player 2
P_1	P_2
P_2	P_1
S_0	S_0

Now, the two player, three option case is hardly sufficient to cover all eventualities where SVV could be used. The more general setting involving the set N of n individuals and the set A of $m + s$ alternatives from which s elements have to be chosen can be described using the notation and conceptual apparatus of Felsenthal and Machover (Felsenthal and Machover 1992). The only restrictions on the cardinalities of the sets are that $s > 0$ and $m \geq n \geq 2$. This setting thus allows for situations where the players may make several proposals and where bundles of proposals are to be chosen. Once the order in which the players submit their eliminations has been established, the process begins with player 1 eliminating one alternative, say x_1. Then player 2 eliminates one alternative, say x_2, from $A \setminus \{x_1\}$, etc. until player n eliminates one alternative, say x_n, from $A \setminus \{x_1, x_2, \ldots x_{n-1}\}$. The sequence $x_1, \ldots, x_n$ is called the veto sequence. The alternatives in A left once the elements of the veto sequence have been removed are the selected alternatives. But how to predict the outcomes once the player preferences and the voting order has been established? Generalizing the approaches of Mueller and Moulin (Mueller 1978; Moulin 1983), Felsenthal and Machover introduce the concept of canonical sequence for a voting situation $(X, P_1, \ldots, P_n)$ as a sequence $y_1, \ldots, y_n$, where y_i is the least preferred proposal in the ranking P_i *under the assumption that all players* $j = i + 1, i + 2, \ldots, n$ *have already done their eliminations*. In other words, in forming the canonical sequence one begins with y_n which is the least preferred proposal according to P_n. From y_n we then work our way towards the beginning of the sequence (see Felsenthal and Machover (1992) for a rigorous description of the process). The canonical sequence represents plausible or rational behavior on the part of the players. Combined with the assumption that all player preferences are strict (no ties or incomparable pairs among pairs of proposals) the canonical sequence guarantees a unique solution or prediction for SVV processes involving alternative sets of cardinality larger than that of the player set.

As stated above SVV looks quite plausible system for small committees especially under circumstances where divisive outcomes – those strongly opposed by sizable minorities of players - are to be avoided. Not surprisingly, SVV is non-majoritarian. Thus, it can leave the Condorcet winner unchosen and even result in the choice of the Condorcet loser.

The latter possibility is exemplified in the profile of Table 6.3. Suppose that the order of voting is voter 1, voter 2, voter 3. Then the canonical sequence is: (C, A, B) leaving D, the Condorcet loser, the only un-vetoed alternative. The same outcome ensues under sincere vetoing, whereby voter 1 first eliminates C, then voter 2 eliminates A and finally voter 3 vetoes B.

In a way, SVV represents an extreme version of minority protection since a single individual may exclude a for him/her undesirable outcome. Table 6.4 illustrates the violation of the no-veto condition by SVV. The condition requires that if in a profile all voters except one rank the same alternative first, then this alternative is chosen. In Table 6.4 A is such an alternative and, yet, under whatever voting order, A will be eliminated.

Table 6.3 SVV may choose the Condorcet loser

Voter 1	Voter 2	Voter 3
A	B	C
B	C	A
D	D	D
C	A	B

Table 6.4 No-veto violation of SVV

4 voters	1 voter
A	B
B	C
C	D
D	E
E	F
F	A

6.3 Topics for Further Reflection

1. Consider the elimination game tree of Fig. 6.1. Assume that the preferences of players are as in Table 6.5. What is the likely outcome of the SVV game here?
2. Construct a profile involving four players and six alternatives. (i) Construct a canonical sequence and determine the SVV winner. (ii) Determine the SVV choice set if two alternatives are to be selected.
3. Construct a profile with a Condorcet winner and a sequence of votes such that the Condorcet winner is not selected by SVV.

6.4 Suggestions for Reading

The main sources to be consulted are those that have been referred to above, viz. Mueller (1978), Moulin (1983) as well as Felsenthal and Machover (1992). Mueller (2003) provides a brief analysis of SVV in comparison with a couple of other similar

Table 6.5 Preference rankings over proposals: III

Player 1	Player 2
P_1	P_2
P_2	S_0
S_0	P_1

methods. Yuval's (2002) pioneering study reports on strategies resorted to by individuals in experimental settings.

Answers to Selected Problems

1. P_2
2. Consider the following profile:

Voter 1	Voter 2	Voter 3	Voter 4
A	B	C	D
B	C	D	E
C	D	E	F
D	E	F	A
E	F	A	B
F	A	B	C

   (i) With the vetoing order 4, 3, 2, 1, 4, the winner is D. (ii) With the same order the two winners are D and E.
3. See Table 6.4. Let the right-most voter veto first.

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Chapter 7
Criterion Based Choice of Rules

Abstract There are quite a few voting procedures applied for what appears to be a common purpose, viz. to tease out the will of the voters. Despite hundreds of years of study there is no consensus in the scholarly community as to which is the best procedure. The criteria emphasized by different scholars differ to some extent and it turns out that none of the systems satisfies all reasonable desiderata. All procedures are based on some intuitive notion regarding what constitutes the collectively best outcome. In this chapter we discuss the choice of the collective decision procedure on the basis of evaluations in terms of various criteria of performance.

7.1 Introduction

Some years ago a group of voting theorists and electoral experts got together for a symposium in Normandy, France.[1] The proceedings of the symposium were later edited by Felsenthal and Machover (2012). At the end of the symposium an impromptu discussion was held among the participants about the best voting system to be used in a hypothetical situation involving the election of the director of a municipality. In other words, the system should be applicable for electing a single winner. In the discussion various procedures were proposed and the session was concluded with a vote. The alternatives – altogether 18 in number – were the voting systems proposed in the discussion and the ballot aggregation method was the approval voting.[2] The results are reproduced in Tables 7.1 and 7.2.

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[1] This chapter is largely based on Nurmi (2015).

[2] The impromptu nature of the proceedings is reflected by the somewhat light-hearted brainstorming debate preceding the vote as well as by the fact that the voters were not asked to reveal anything else but their approved systems. Several weeks after the meeting the participants were asked to disclose their reasons for voting the way they did, but at this time many didn't recall the systems they had approved of, much less the reasons for doing so. Thus, we do not know how much the election outcome depends on the aggregation systems adopted. See Laslier (2012).

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Table 7.1 The number of approved procedures (Laslier 2012)

No. of approvals	0	1	2	3	4	5	6	7	8	9	10	>10	Total
No. of ballots	0	2	7	3	5	2	1	1	0	0	1	0	22

Table 7.2 The procedures and the distribution of approvals (Laslier 2012)

Voting rule	Approvals	Approving %
Approval	15	68.18
Alternative	10	45.45
Copeland	9	40.91
Kemeny	8	36.36
Runoff	6	27.27
Coombs	6	27.27
Simpson	5	22.73
m. judgment	5	22.73
Borda	4	18.18
Black	3	13.64
Range	2	9.09
Nanson	2	9.09
Leximin	1	4.54
Top cycle	1	4.54
Uncovered	1	4.54
Fishburn	0	0
Untrapped	0	0
Plurality	0	0

The former table shows a fairly wide variation in the number of approved systems. Yet, a vast majority of voters approved $2 - 4$ systems. The procedures are listed in Table 7.2 which also indicates the number of approvals given to each one of them as well as the percentage of voters approving of each system. The reader unfamiliar with the procedures is referred to Laslier's (2012) article which also provides a comprehensive analysis of the voting data. Many of the procedures were also discussed in Chap. 4 above.

A couple of observations about Table 7.2 are in order. Firstly, no procedure was approved of by all participants. Secondly, some proposed systems received no approval votes at all. Thirdly (and related to the preceding point), the most common voting system – the plurality or one-person-one-vote procedure – was voted for by no participant. Fourthly, the winner – the approval voting – was approved of by more than two thirds of the voters.

The first point provides the main motivation for this chapter. It shows that the expert community is not unanimous about the best voting procedure. Glancing at the statements that several voters give to support their ballots one immediately notices that the participants seem to emphasize somewhat different criteria when choosing their favorite systems. It is plausible to think that the very existence of many voting procedures can similarly be explained by the emphasis placed on different criteria of performance of procedures. The next section provides a theoretical reconstruction of some of the best-known systems in terms of this reasoning. Thereafter, we present the main contribution of this chapter, *viz.* a method for choosing a voting procedure on the basis of the participants' priorities regarding the performance criteria.

7.2 The Emergence of Some Voting Procedures

Perhaps the most common of all voting procedures is the plurality rule: each voter has one vote at his/her disposal and the candidate or policy alternative receiving more votes than any of its contestants wins. The rationale of this rule is obvious: no other candidate gets as many votes as the winning one. However, it may happen that the plurality winning candidate gets less than 50% of the votes. Hence it may not always get the support of the majority. To rectify this eventuality the plurality runoff system has been devised. It works precisely as the plurality procedure, but in case the plurality winner receives at most 50% of the votes, a runoff is arranged between the two largest vote-getters. Whichever gets more votes than the other on this second round of voting is the winner. Thus, the winner can always claim to be supported by more than half of the electorate. More importantly, the runoff system guarantees that an eventual Condorcet loser is not elected. This simply follows from the fact that the plurality winner has to defeat by a majority of votes at least one other alternative, viz. its competitor on the second round of voting. If there is a winner already on the first round, i.e. there is a candidate ranked first in the opinion of a majority of voters, then of course the winner would defeat *all* the others in pairwise contests.

Another way of avoiding Condorcet losers being elected was discovered nearly a quarter of a millennium ago by Jean-Charles de Borda and is today known as the Borda count.[3] Thus, we have two solutions to the problem of avoiding the election of a Condorcet loser: the plurality runoff and the Borda count. Yet, the former is accompanied with a new problem, plaguing neither the latter nor, perhaps more importantly, the plurality procedure, viz. nonmonotonicity. In nonmonotonic systems, additional support, *ceteris paribus*, may turn a winning candidate into a non-winning one. Table 7.3 illustrates this problem.

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[3] The method was invented already in the 15th century by Nicholas of Cusa, but arguably the latter did not emphasize the particular problem related to the plurality voting, viz. that it may result in the election of a candidate that would lose the pairwise contests against any other candidate. See McLean and Urken (1995) and Hägele and Pukelsheim (2008).

Table 7.3 Nonmononicity of the plurality runoff

6 voters	5 voters	4 voters	2 voters
A	C	B	**B**
B	A	C	**A**
C	B	A	C

Supposing that the voters vote according to their preferences listed in Table 7.3 there will not be a first-round winner, but a runoff that takes place between A and B. In this runoff the winner is A since it is preferred to B by those 5 voters whose favorite didn't make it to the runoff. Suppose now that the two voters on the right with ranking BAC would change their opinion with regard to A and B (marked in bold letters in the table) so that the winner A would be preferred to B by these two voters. I.e. A's support would increase, everything else remaining as before. In this new profile – which differs from the Table 7.3 so that the winner (A) gets more support than originally – a runoff is still needed, but this time one between A and C. This runoff is won by C. This shows that additional support may, indeed, turn winners into non-winners under the plurality runoff system. Hence the procedure is nonmonotonic.

Similarly as plurality runoff can be seen as an attempt to improve upon the plurality system, Nanson's method can be seen – and was in fact seen by its inventor E. J. Nanson – as a way to rectify an apparent flaw in another system, viz. the Borda count (see McLean and Urken 1995). For more than two centuries it has been known that the Borda count does not always end up with the Condorcet winner. Nanson set out to devise a system that would be as similar to the Borda count as possible, but still guarantee the choice of an eventual Condorcet winner. The system is based on the observation concerning the relationship between Condorcet and Borda winners. While it is known that the former winners are not necessarily ones with the highest Borda scores, it is still the case that they never have very low Borda scores. More specifically, an eventual Condorcet winner always has a higher than average Borda score. Nanson's method is based on this observation: it proceeds in rounds whereby the alternatives with an average or lower Borda score are eliminated and new scores are computed for the remaining alternatives until the winner is found. The criterion used in elimination guarantees that an eventual Condorcet winner is not eliminated.

So, the system invented by Nanson was, indeed, capable of solving a specific shortcoming of the Borda count. However, as was the case with plurality and plurality runoff systems, the solution procedure (here Nanson's method) has a flaw that the "flawed" system (here the Borda count) is not associated with. This is nonmonotonicity: while the Borda count is monotonic, Nanson's method isn't (Fishburn 1977, p. 478).[4] This illustrates the nature of many social choice results: they demonstrate

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[4] Strictly speaking Fishburn investigates a method he calls the Nanson function which differs from Nanson's method in eliminating only the alternative(s) with the lowest Borda score in each counting round. Fishburn's Nanson function was invented about a hundred years ago by Baldwin (1926). So, strictly speaking Fishburn's example demonstrates that Baldwin's rule is nonmonotonic. It can,

incompatibilities between properties of choice functions. In short, procedures with all desirable properties do not exist. Trade-offs have to be made between desiderata.

This well-known state of affairs suggests a new angle to the problem of choosing a procedure of choice. Instead of fixing specific flaws in the systems that are being used – and thereby conceivably coming up with systems with flaws that the already used ones do not have – one could start from the criteria that one regards of primary importance. Different people may put different value on various criteria. This was clearly exemplified in the introduction of this section. Hence, it would make sense to take into account and make use of the information regarding differences in valuation by different voters when choosing a system to be resorted to in collective decisions. In the next section we outline several ways of going about this.

7.3 From Criterion Preferences to Voting Systems

The most straight-forward way to proceed is to consider the problem as any preference aggregation problem, i.e. to use criterion preferences as inputs and, using some social choice rule, aggregate them into a collective preference ranking. Consider the following set of criteria (Table 7.4).[5]

Let us briefly remind ourselves about the content of these criteria. The Condorcet winner criterion is satisfied by those systems that always elect the Condorcet winner when one exists in the profile. The Condorcet loser criterion, in turn, calls for the exclusion of an eventual Condorcet loser from the choice set. The strong Condorcet winner is an alternative that is ranked first by a majority of voters. The corresponding criterion dictates the choice of the strong Condorcet winner whenever such an alternative is present in a profile. Monotonicity is satisfied by systems where additional support never hurts the winning alternative. Pareto criterion excludes the election of Pareto dominated alternatives. Consistency pertains to results in two or more sub-electorates. In consistent systems, if all sub-electorates elect the same alternative, this should also be elected when the ballots are counted en masse, i.e. without subdivisions. Chernoff property states that if x is elected in the set of alternatives, it should be elected in all subsets it is an element of as well. Independence of irrelevant alternatives is satisfied whenever in any two profiles where x and y are ranked in an identical manner with respect to each other, they are also ranked in the same way in the resulting outcomes. Invulnerability to the no-show paradox means that the voting procedure never penalizes the voters for voting according to their preferences, i.e. the voters are never better off by abstaining than voting according to their preferences, *ceteris paribus.*

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however, be shown that also the method that Nanson devised is nonmonotonic. See Nurmi (1999, p. 59) and Nurmi (2018).

[5] For further explanation of the criteria, see e.g. Nurmi (2002).

Table 7.4 A set of choice criteria

a	The Condorcet winner criterion
b	The Condorcet loser criterion
c	The strong Condorcet criterion
d	Monotonicity
e	Pareto
f	Consistency
g	Chernoff property
h	Independence of irrelevant alternatives
i	Invulnerability to the no-show paradox

Table 7.4 exhibits but a relatively small subset of criteria discussed in the literature, but arguably some of the most important criteria are included in the list. More extensive sets are introduced and analyzed e.g. in Felsenthal (2012) and Richelson (1979).

To work out a collective preference ranking over these 9 criteria, some aggregation rule has to be used. To do this, one would have to assume what one is aiming at, viz. a suitable choice rule. When due attention is given to their metric representations (see Nitzan 1981; Meskanen and Nurmi 2006), two rules, however stand out: Kemeny's rule and the Borda count. The former chooses the collective ranking that is closest to the reported individual rankings in terms of binary reversals (inversion metric), while the latter counts for each alternative (choice rule) the number of binary preference reversals that are needed to make this alternative unanimously first ranked. Thus, both rules resort to the same metric, but different benchmark state. In the present context Kemeny's rule would perhaps seem more appropriate since the choice procedure is to be chosen using the following performance table (Table 7.5).

The table indicates whether a procedure represented by the row satisfies (denoted by 1) or does not satisfy (denoted by 0) the criterion represented by the column $(a, \ldots, i)$. Again, the procedures are just a sample of those discussed in the literature.

Suppose now that the collective ranking obtained by applying Kemeny's rule to the profile of reported rankings over criteria has criterion l ranked first. One then looks for all procedures that have a unity in the column l. If several procedures satisfy l, one then picks the criterion ranked second in the collective (Kemeny) ranking. Let this criterion be m. One then looks for procedures that satisfy both l and m. Again there may be several procedures, but continuing in this (lexicographic) manner one eventually ends up in a situation where all remaining procedures satisfy all top-most criteria in the collective preference ranking down to a point after which none of them satisfies the next one in the collective ranking. Those remaining procedures then constitute the choice set of procedures. To take an example, suppose that the Kemeny ranking is $d \succ e \succ b \succ f \ldots$. Then the outcome is a three-way tie $\{Copeland, Kemeny, Black\}$ since all these satisfy monotonicity (d), Pareto (e) and Condorcet loser (b) criteria, but none of them is consistent (f).

Table 7.5 A comparison of voting procedures

	Criterion								
Voting system	*a*	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*
Amendment	1	1	1	1	0	0	0	0	0
Copeland	1	1	1	1	1	0	0	0	0
Dodgson	1	0	1	0	1	0	0	0	0
Maxmin	1	0	1	1	1	0	0	0	0
Kemeny	1	1	1	1	1	0	0	0	0
Plurality	0	0	1	1	1	1	0	0	1
Borda	0	1	0	1	1	1	0	0	1
Approval	0	0	0	1	0	1	1	0	1
Black	1	1	1	1	1	0	0	0	0
Pl. runoff	0	1	1	0	1	0	0	0	0
Nanson	1	1	1	0	1	0	0	0	0
Hare	0	1	1	0	1	0	0	0	0
Coombs	0	1	1	0	1	0	0	0	0

Obvious objections can be presented against this system, perhaps the most important being its reliance on lexicographic ordering of criteria. A poor performance on the first ranked criteria cannot be "bought" by good performance on criteria ranked lower in the collective ordering. This can be illustrated by a setting where the collective ranking puts consistency on the first place. It then follows that just three systems are left after the first criterion is considered. If the collective ranking puts the Condorcet loser criterion in the second place, the Borda count emerges as the chosen system. In other words, the other criteria have no role whatsoever in determining the chosen system.

In view of these considerations another set of procedures is suggested. The input is either the set of individual preference rankings over criteria or the distribution of utility values in a fixed interval, say [0, 10], that each voter assigns to each criterion. We illustrate one version of the procedure by using Borda points given by each voter to each criterion. Suppose that there are three individuals and their preference ranking over the 9 criteria are as follows:

individual 1	abcdefghi
individual 2	dcbafeihg
individual 3	ihgfedcba

Criterion a, thus, gets 8 Borda points from 1, 5 points from 2 and 0 points from 3. It would then make sense to argue that procedures satisfying a, get 13 points from these three individuals, while the other procedures get no points. Similarly b gets 7 points from 1, 6 from 2 and 1 from 3. And so on. Those procedures that do not satisfy

Table 7.6 The assignment of points to procedures in the basis of criterion preferences

Voting procedure	Criteria									
	A	B	C	D	E	F	G	H	I	Sum
Amendment	13	14	15	16	0	0	0	0	0	58
Copeland	13	14	15	16	11	0	0	0	0	69
Dodgson	13	0	15	0	11	0	0	0	0	39
Maxmin	13	0	15	16	11	0	0	0	0	55
Kemeny	13	14	15	16	11	12	0	0	0	81
Plurality	0	0	15	16	11	12	0	0	10	64
Borda	0	14	0	16	11	12	0	0	10	63
Approval	0	0	0	16	0	12	8	0	10	46
Black	13	14	15	16	11	0	0	0	0	69
Pl. runoff	0	14	15	0	11	0	0	0	0	40
Nanson	13	14	15	0	11	0	0	0	0	53
Hare	0	14	15	0	11	0	0	0	0	40
Coombs	0	14	15	0	11	0	0	0	0	40

the criterion considered do not get any points from voters on that criterion. In effect, then, for each column of the table the entries are obtained by multiplying the points given by voters to the criterion represented by the column by the corresponding entry of Table 7.5. The results are seen in Table 7.6.

On the basis of criterion preferences and using the Borda count in the point assignment, the winning procedure is Kemeny's rule followed by a tie between Copeland's and Black's procedures.

Given the plethora of voting systems currently in use in various contexts it is arguable that the designers have different desiderata in mind when devising those systems. Focusing on a single desideratum only is bound to cause problems because typically a good performance on one criterion is accompanied with bad performance on some others. Hence we suggest that the opinions regarding the desiderata ought to be made explicit in the choice of the system to be used. We have outlined above a couple of ways of using voter opinions regarding criterion preferences in a systematic way in the choice of a voting procedure.

7.4 Topics for Further Reflection

1. Show by way of an example that the Condorcet winner may not be first-ranked by any voter in a profile.
2. Show by way of an example that a strong Condorcet winner may not be the Borda winner.

3. Discuss circumstances in which consistency is largely irrelevant for the plausibility of the outcomes.
4. Pick your favorite from the systems of Table 7.5. Explain its main advantages and disadvantages.

7.5 Suggestions for Reading

Good introductions to voting procedures and their properties are Riker (1982) and Straffin (1980). Somewhat more concise and advanced texts are Nurmi (1987) and Felsenthal (2012). A profound analysis of voting systems from a geometrical perspective is given by Saari (1995, 2001).

Answers to Selected Problems

1. Consider the following profile:

Voter 1	Voter 2	Voter 3
A	B	C
D	D	D
B	C	A
C	A	B

   Here D is the Condorcet winner, but is not ranked first by any voter.
2. Consider the following profile:

5 voters	4 voters
A	B
B	C
C	A

   Here A is the strong Condorcet winner, yet B is the Borda winner.

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Chapter 8
Two Procedures Based on Ratings

Abstract While most social choice results pertain to ranking environments where the individuals submit their preference relations (and these only) to the balloting procedure, there are procedures that require a slightly different kind of input from the voters. We discuss two such systems: the majority judgment and the range voting. These are relatively recent entrants in the social choice field. As all procedures they have their advantages and disadvantages, but deserve attention is some decision situations.

8.1 Introduction

The standard assumptions of voting theory include complete and transitive preference relations or rankings. In Hillinger's (2005) view this assumption is at least partly to blame for the paradoxes of voting as well:

> ... a new 'paradox of voting': It is theorists' fixation on a context dependent and ordinal preference scale; the most primitive scale imaginable and the mother of all paradoxes.

In any event, one could argue that there are settings in which people are capable of submitting not only rankings but ratings of alternatives in a meaningful way. Accordingly some methods have been devised to aggregate ratings of alternatives into social ratings, rankings and/or choices. In this section we discuss briefly two such methods.

8.2 Majority Judgment

Rating that is familiar to most people is the grading of essays, exams, presentations etc. in institutions of learning. The assessors, judges or instructors are asked to evaluate the participants or their works using predetermined scales, such as numbers from 0 to 5 or letters from A to F or similar ratings that are supposed to reflect

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the superiority of items with respect to each other. Since we are very familiar with these kinds of performance assessments in schools, it has been suggested that this familiarity should be expected to extend to political contexts as well. If we can assign grades to student papers reflecting their academic quality, what objections are there to our using the same principles in assessing the qualities of candidates or policy alternatives?

Probably none at all, apart from the general idea that academic merit is a more 'objective' notion than political merit. Yet, one can envision that also academic assessments are based on some more or less well-defined notion of an ideal – be it of an essay, oral presentation or written exam paper. Similarly our assignment of political merit to alternatives may also be based on some subjective notion of political quality, such as agreement with our own political values. No doubt assessments of political merit of any given candidate are bound to vary a great deal more among the assessors than the assessments of their academic merits. This doesn't mean that the grades or other ratings could not be used in political elections. Indeed, the method of majority judgment (MJ, for short) introduced and elaborated by Balinski and Laraki is based on aggregating ratings, numerical or non-numerical (Balinski and Laraki 2007, 2010).

To illustrate, consider the profile of Table 8.1. It is consistent with the profile of Table 8.3 in the sense that the preference rankings of the 9 voters are the same in both tables. Thus in Table 8.1 we have assumed that the 4 voters ranking A first give it the grade 'excellent', C the grade 'good' and B 'reject'. The 3 voters with $B \succ C \succ A$ ranking give the grades "very good', 'good' and 'reject', respectively. Finally, the 2 voters with $C \succ B \succ A$ ranking assign these alternatives the grades 'very good', 'satisfactory' and 'reject', respectively. Now, the MJ method focuses on the median of the grades assigned to alternatives. To determine the median, let the number of voters be n (9 in our example). Denote the grades from the lowest to the highest by $g_1, \ldots, g_h$ and the number of voters assigning an alternative the grade g_j by n_j. Then the median grade g_{med} is defined by the following two properties:

$$\sum_{j=1}^{med} n_j > n/2$$

$$\sum_{j=med}^{n} n_j > n/2$$

The MJ choice set consists of those alternatives associated with the highest median grade. In Table 8.1 the MJ winner is C as its median grade is the highest. Thus, the Borda and Condorcet winner C of Table 8.3 is chosen. There is, however, also an assignment of grades such that C is not chosen. This is shown in Table 8.2. So, despite the fact the distribution of voter rankings remains the same, there is some variation in MJ outcomes due to the more detailed information on voter opinions utilized in determining the outcomes. It is to be noted, however, that MJ makes a

Table 8.1 Majority judgment example

Alt.	Reject	Satisfactory	Good	Very good	Excellent	Median
A	5	0	0	0	4	Reject
B	4	2	0	3	0	Satisfactory
C	0	0	7	2	0	Good

Table 8.2 MJ does not elect the Condorcet and Borda winner

Alt.	Reject	Satisfactory	Good	Very good	Excellent	Median
A	5	0	0	0	4	Reject
B	4	0	0	2	3	Very good
C	0	7	0	0	2	Satisfactory

Table 8.3 Ambiguous majority principle

4 voters	3 voters	2 voters
A	B	C
C	C	B
B	A	A

very limited use of the detailed information in focusing on the median grade only. Thus, the median grade of *A* in Table 8.2 is unaffected if all those voters giving it the grade ‘excellent’ were to assign it the grade ‘reject’. By definition all modifications in the distribution of opinions that leave the median grades of alternatives unaffected result in the same MJ choices.

The fact that several grade assignments can correspond to a given ranking profile – such as in Tables 8.1 and 8.2 – would seem to suggest that the grading methods call for different performance evaluation criteria than the ranking ones. Be that as it may, the grading methods can be – and have been – evaluated in terms of ranking criteria (see Felsenthal and Machover 2008; Felsenthal and Nurmi 2016). For example, as we have just seen in Table 8.2, MJ does not always result in a Condorcet or Borda winner.

8.3 Range Voting

Range voting (RV) is in some respects similar to MJ: each voter gives a grade or value out of a set of grades or values to each alternative. In contradistinction to MJ, the winner in RV is determined on the basis of the grade averages of alternatives: the alternative with the highest average wins. This means that RV is applicable only in those settings where the grades are numerical, while MJ is applicable also in situations

Table 8.4 Range voting fails on Condorcet criteria

4 voters	3 voters	2 voters
A (100)	B (30)	C(20)
C (20)	C (20)	B(10)
B(10)	A (10)	A(0)

where the grades have only ordinal significance. In contradistinction to most other voting systems, the advocates of RV have established a web site – RangeVoting.org – with frequent updates on issues related to RV and its competitors.

RV enables (or requires) the voters not only to express their preferences in terms of an ordinal ranking, but also to indicate for any pair of alternatives how much they prefer one to the other. The grades can, thus, be viewed as values or utilities of alternatives from the voters' point of view. Hence, RV is sometimes called utilitarian voting. Summing up the grades given by voters to an alternative in a way reflects its collective utility or value. It has been shown that RV satisfies a number of social choice desiderata (e.g. monotonicity and consistency), but fails on Condorcet winner and loser criteria. To see this, consider the profile of Table 8.3 and assign the alternatives grades from the interval 0–100 (the larger the more preferred) as in Table 8.4.

There is a Condorcet winner, C, in this profile, but the Condorcet loser A emerges as the RV winner under the grade assignment of Table 8.4.

In contrast to all ranking based procedures, RV seems to satisfy independence of irrelevant alternatives (IIA) condition that is one of those desiderata that Arrow's impossibility theorem shows to be mutually incompatible (Arrow 1963). In ranking context this requirement states that if for any pair of n-person profiles, R and R', over a fixed set of alternatives A, the profiles agree on the relative ranking of any two alternatives $x, y \in A$, then so must the relative ranking of x and y also be identical in the collective ranking that ensues from applying the procedure to R and R'. Strictly speaking, IIA is not applicable to RV since the latter is a rating, not ranking based system. However, it is obvious that the collective RV rankings of x and y depend only on the individual ratings assigned to them in R and R', not on whatever grades are assigned to other alternatives.

8.4 Topics for Further Reflection

1. Construct an example where MJ ends up with a Condorcet winner.
2. Construct an example where RV elects the Condorcet loser.
3. Let us define Nash's method as follows: each voter assigns each alternative a utility value from the [0.5, 1.0] interval. The Nash score of each alternative is the product of the utility values assigned to this alternative by all voters. The winner is the alternative with the highest score (Riker 1982). Construct an example where Nash's method does not result in a Condorcet winner.

8.5 Suggestions for Reading

The best available exposition of MJ is (Balinski and Laraki 2010). RV, introduced by Warren D. Smith, is explained, illustrated and compared with other voting rules on the web site maintained by RangeVoting.Org.

Answers to Selected Problems

1. Consider the following setting with three voters, three candidates and grades from a (worst) to d (best):

	Voter 1	Voter 2	Voter 3	Median
A	b	c	b	b
B	c	c	c	c
C	a	b	d	b

   Here B has the highest median and is thus the MJ winner. At the same time it is the Condorcet winner.
2. Consider the above MJ example and let $a = 0, b = 1, c = 2, d = 10$. Then C, the Condorcet loser, is the RV winner.
3. Consider again the above MJ example and let all voters assign the same values to grades so that $a = 0.5$, $b = 0.55$, $c = 0.6$, $d = 1.0$. Now C, the Condorcet loser, is elected.

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Chapter 9
Qualified Majorities and Expert Choice

Abstract What if the decision makers have different degrees of expertise and the aim is to maximize the probability of a correct decision? (The first three sub-sections are largely based on Nurmi, Voting procedures under uncertainty. Springer, Berlin-Heidelberg, pp 49–59, 2002) This possibility has been considered for a long time. We shall describe the main results in this field of inquiry where the degrees of competence play a crucial role. We begin with a classic result that is based on the assumption that the individual decision competences are equal and representable by the probability that the decision made by the individual is correct. The issue of where the competence probability comes from is left open. We also discuss epistemic paradoxes, i.e. peculiarities encountered when aggregating premises of an argument separately from the conclusions.

9.1 Condorcet's Jury Theorem

While the modern social choice theory deals mostly with elections and other opinion aggregation contexts, the earlier results of the theory focus on somewhat different settings, viz. jury decision making. Marquis de Condorcet dealt with the problem of amalgamating the opinions of several jurors into a just or correct collective decision or verdict (McLean and Urken 1995). More specifically, Condorcet was looking for an answer to the following question: assuming that each individual has a given probability of being right, what is the probability that the majority of a group consisting of such individuals is right? Although related to the modern social choice theory, this question invokes considerations that are absent in the modern theorizing, viz. the notion that there is a correct decision and that collective decision making procedures are to varying degrees capable of resulting in those correct decisions.

Condorcet's starting point is, thus, that every individual has a fixed probability of being right on the issue to be decided. Whether this probability is determined on the basis of success rates in similar previous decision settings or on the basis of formal or practical training or some other factors is left open. The simplest situation would seem to be one in which each individual has an identical probability p of being right. The probability could be interpreted as the relative frequency of right "yes" or "no" answers to a long sequence of questions for which the correctness of the answers can

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be determined. Let us focus on a question that calls for either "yes" or "no" answer and assume that the number of individuals who have given the right answer is x. To simplify the setting even further, let us assume that the persons vote independently of each other. In other words, the voters make their decisions without consulting each other or knowing each other's decision. Under these assumptions we can apply the binomial probability formula to express the probability that among n individuals exactly x have given the right answer:

$$f(x) = p^x(1-p)^{n-x}.$$

Let P denote the probability that the group using the simple majority rule gives the right answer. In other words, P is the probability that more than 50% of the group members will vote "yes" ("no", respectively) when "yes" ("no") is the right answer. For any given size of majority x, this probability equals the number of different ways of picking exactly x individuals times the probability of exactly x individuals being right. Thus, the probability is the sum of these products over the sizes of majority. In symbols,

$$P = \sum_{x=n'}^{n} \binom{n}{x} p^x (1-p)^{n-x}. \tag{9.1}$$

Here $n' = (n+1)/2$. With the exception of those values of p which are very close to 1 or 0, the distribution of P can be approximated by the normal distribution with mean np and variance $np(1-p)$. Thus, we obtain

$$P = 1 - G\left(\frac{n/2 - np}{\sqrt{np(1-p)}}\right) = G\left(\frac{p - 0.5}{\sqrt{p(1-p)/n}}\right).$$

Here $G(y)$ is the area under the density curve of normal distribution from $-\infty$ to y. Condorcet's jury theorem can now be stated (Miller 1986).

Theorem 1 (Condorcet) *The probability P of the majority being right depends on the individuals' probability p of being right as follows:*

1. *If $0.5 < p < 1$ and $n > 2$, then $P > p$, P increases with n and when n approaches infinity, P converges to 1.*
2. *If $0 < p < 0.5$ and $n > 2$, then $P < p$, P decreases with the increase of n and P approaches 0 when n approaches infinity.*
3. *If $p = 0.5$, then $P = 0.5$, for all values of n.*

Table 9.1 gives an idea of how fast P approaches 1 when n increases for various values of p.

The first two parts of Condorcet's jury theorem contain two statements: one pertaining to probability of the majority being right *vis-à-vis* the individual probability of being right, and the other indicating the limiting probability value of the majority being right. The former is called non-asymptotic and the latter the asymptotic part of the theorem. The non-asymptotic part can be proven by showing that

Table 9.1 Probability of being right. *Source* Miller (1986)

Value of n	Value of p				
	.5050	.5500	.7500	.9000	.9750
3	.5075	.5748	.8438	.9720	.9982
5	.5094	.5931	.8965	.9914	.9998
7	.5109	.6083	.9294	.9973	.9999
9	.5123	.6214	.9510	.9991	.9999
15	.5154	.6514	.9873	.9999	.9999
25	.5199	.6924	.9981	.9999	.9999
75	.5345	.8079	.9999	.9999	.9999
250	.5628	.9440	.9999	.9999	.9999
1000	.6241	.9993	.9999	.9999	.9999

$$p < \sum_{i=n'}^{n} \binom{n}{i} p^i (1-p)^{n-i}$$

for groups of any size n. Here $n' = (n+1)/2$ and by assumption n is odd. Similarly, the asymptotic part, which states that the right hand side of the preceding inequality approaches unity as the group size approaches infinity, follows from the observation that the limiting value of the sum is, indeed, unity (Ben-Yashar and Paroush 2000).

The message of the theorem is clear: the majority is more reliable than the average citizen if the latter is more often right than wrong and if the probability of being right is the same for all citizens. Indeed, the majority becomes omniscient when the number of individuals increases. The assumption that the probability of each citizen's being right is larger than $1/2$ is essential: should the probability be strictly less than $1/2$, then P approaches 0, i.e. it becomes certain that the majority is wrong. The applicability of Condorcet's jury theorem is, however, seriously limited by the assumption that each individual has the same competence, i.e. the same probability of being right.

Various generalizations of the above theorem have been discussed in the modern literature. Of particular interest is one proven by Owen et al. (1989). Suppose that each individual i is characterized by probability p_i of being right. Let $\bar{p} = \sum_i p_i/n$, i.e. $\bar{p}$ is the average competence of the individuals or the average probability of their being right. If now $1/2 < \bar{p} < 1$ and $n > 2$, then $P > \bar{p}$ and P approaches 1 as n approaches infinity. In this theorem the individuals do not necessarily have identical competences. Furthermore, they are not all required to be more often right than wrong. What is assumed instead is that the arithmetic mean of the individual competences is larger than $1/2$.

The non-asymptotic part of Condorcet's theorem, thus, holds in the generalized setting in the sense that the competence of the majority always exceeds that of the average competence. In another sense, when it is asserted that the majority be more competent than each of the individuals, the theorem does not always hold. Consider, for example, a group consisting of three individuals with $p_1 = 0.6$, $p_2 = 0.7$, and $p_3 = 0.9$. Here we get:

$$\begin{aligned} P &= 0.6 \times 0.7 \times 0.1 + 0.6 \times 0.3 \times 0.9 \\ &\quad + 0.4 \times 0.7 \times 0.9 + 0.6 \times 0.7 \times 0.9 = 0.834. \end{aligned} \tag{9.2}$$

Thus, the majority is more competent than the average competence (0.73), but less competent than one of the individuals. On the other hand, in a three-person setting where $p_1 = 0.6$, $p_2 = 0.7$ and $p_3 = 0.7$, the majority competence exceeds that of the most competent individual since $P = 0.742$. In other words, the majority can be, but is not always, more competent than every individual when the average competence exceeds $1/2$.

This result significantly qualifies Dahl's contention (Dahl 1970, 34):

> ... whenever you believe that 1 is significantly more competent than 2 or 3 to make a decision that will seriously affect you, you will want the decision to be made by 1. You will not want it to be made by 2 or 3, nor by any majority of 1, 2, and 3.

Suppose that person 1's competence is 0.8, person 2's 0.7 and person 3's 0.7 (Ben-Yashar and Paroush 2000, 192). Person 1 is, thus, significantly more competent than 2 and 3. Yet, $P = 0.826$ which exceeds person 1's competence. Hence, *pace* Dahl, one might well prefer the decision to be made by a majority of the three persons rather than by the most competent person 1.

The generalized Condorcet theorem demonstrates that one should not perhaps be overly concerned about the use of referenda in matters which in other times and places may have been decided by experts, e.g. joining military or economic alliances. Adding a sufficient number of minimally competent decision makers improves the quality of decision making in the sense that the competence of the majority exceeds the average individual level of competence. One should observe, though, that if the added decision makers are just barely competent, they may lower the prevailing average competence. Anyway, the nonasymptotic part of Condorcet's jury theorem is not always valid in the sense that the majority would be more competent than any individual. In fact, there is a result which states under which conditions the asymptotic part is not valid (see Nitzan and Paroush 1982 as well as Shapley and Grofman 1984).

Theorem 2 *Let there be an odd number n of voters who vote independently of each other. Assume that $p_i > 0.5$ for all voters and that the voters are labeled in non-increasing order of competence, i.e. $p_i \geq p_j$ if $i < j$. The non asymptotic part of Condorcet's jury theorem does not hold, if*

$$\frac{p_1}{1 - p_1} > \prod_{i=2}^{n} \frac{p_i}{1 - p_i}.$$

The expression $p_i/(1-p_i)$ indicates the odds regarding voter i's competence. By assumption it is larger than unity for all voters with values increasing with the competence of the voter. The result thus states that if the most competent voter has higher odds than the product of the odds of the other voters, then voter 1 is more competent than the collective choice made using the majority rule.

9.2 Relaxing the Independence Assumption

One of the assumptions underlying Condorcet's jury theorem is that the voters act independently of each other. Intuitively this is somewhat implausible. More often than not in politics people take their cues from other people's actions and plans. It is, however, difficult to find an alternative modeling assumption that would at the same time be more plausible in taking into account the intuitively frequent interdependencies of people's behaviors and be general enough to cover a wide variety of voting situations. Nevertheless, it is important to get even a rough idea of the importance of the independence assumption. Some results achieved in system reliability theory are pertinent here.

This theory aims at estimating the probabilities for proper functioning of systems under the assumption that certain portion of their components break down or otherwise fail. The majority systems model is constructed assuming that the system is composed of several components so that it works if and only if the majority of its components works. Given that each component has a fixed probability of working properly, we can analyze the reliability of majority systems under various assumptions concerning the interdependence of components (Boland 1989; Boland et al. 1989). The components can be viewed as voters or jurors and the proper working of a component as the event that the juror is right.

Let us assume that there is an odd number $2m+1$ of components. We label them Y, X_1, ..., X_{2m}. For our purposes it is convenient to interpret Y as a prominent individual or opinion leader whose lead is followed by several other individuals X_i. Each component is interpreted as a dichotomous variable so that e.g. $X_i = 1$ means that the component X_i works properly, $X_i = 0$, in turn, means that it fails. We assume that $p(Y=1) = p(X_i = 1) = p$, for all $i = 1, \ldots, 2m$. In other words, every component has the same probability p of working properly or every juror has the same probability of being right. Let $q = 1-p$. The conditional probability of each X_i working properly, given that Y does, is:

$$p(X_i = 1|Y = 1) = p + rq$$

and working properly, given that Y fails, is:

$$p(X_i = 1|Y = 0) = p - rp,$$

where $i = 1, \ldots, 2m$.

The conditional probabilities are, thus, assumed to be identical for each X_i. The parameter r measures the interdependence or correlation between X_i, on the one hand, and Y, on the other. Obviously, with $r = 1$, the probability that X_i gets the value 1 when Y gets the value 1, is 1. On the other hand, when $r = 0$, the conditional probabilities of X_i equal their absolute probabilities, i.e. they are independent of Y. The parameter r thus allows us to describe positive association between X_i and Y. It is noteworthy, however, that this model cannot accommodate negative dependence. Thus, we can deal with voters who imitate each other, but not with voters who wish to "cancel" each other's votes.

One of Boland's results states that the probability of the majority of components working properly decreases with the increase of correlation. In other words, the larger r, the larger the probability of the majority system failure. Applying this result to voting contexts we can argue that the probability that the majority is right decreases when the dependence of voters on one "leader" (variable Y) increases. However, as long as the correlation between the voters and the leader is less than 1, the probability that the majority is right exceeds that of a single voter. Hence, in Boland's model the interdependence between voters does not affect the essence of Condorcet's theorem.

More general approach to modelling interdependence is developed by Berg who replaces the binomial distribution with Pólya-Eggenberger or beta-binomial distribution (Berg 1993). This distribution is a generalization of the binomial one. In the model a parameter h is introduced so that $h/(h + 1)$ is the correlation between any two voters. Thus, h can be interpreted as a dependence parameter.

Table 9.2 of Berg reports the variation of the majority competence for small values of h (Berg 1993). We see that, at small absolute values of the interdependence parameter, the majority competence increases if the interdependence is negative, whereas it decreases if the dependence is positive. Berg shows that this is the case whenever $p > 1/2$ (Berg 1993, 92–93). Thus, we may conclude that positive interdependence between voters decreases the majority competence from its value under independence assumption.

Despite this observation the main content of Condorcet's jury theorem remains intact also under beta-binomial distributions. Thus, with $p > 1/2$ and for fixed value of h, the probability that the majority decision is right increases with the increase of

Table 9.2 The majority competence (mc) for individual competence value $p = 0.6$ for varying group sizes and dependence values (Berg 1993)

n			
5	h = −0.08	h = 0	h = 0.08
	mc = 0.7221	mc = 0.6826	mc = 0.6587
9	h = −0.04	h = 0	h = 0.04
	mc = 0.7784	mc = 0.7334	mc = 0.7084
41	h = −0.01	h = 0	h = 0.01
	mc = 0.955	mc = 0.905	mc = 0.867

the number of voters. Moreover, whenever $1/2 < p < 1$ the majority competence always exceeds that of individual p.

The preceding discussion on the variations of Condorcet's jury theorem reveals that even in contexts where one can meaningfully speak about correct and incorrect decisions the group choice using majority rule is not necessarily inferior to expert choice, unless the expert is perfect and the group consists of individuals who are not even minimally competent. The main conclusion, however, is that Condorcet's jury theorem is relatively robust under modifications regarding the independence of voters. What is perhaps of more interest is that positive association between voters does not increase the majority competence, but rather diminishes it from the level that is achieved by independent voters.

9.3 Optimal Jury Decision Making

Although the setting analyzed in the preceding sections pertains to making correct decisions and seems thus somewhat distant from political decision making where subjective values play a major role, it is well worth studying since, if it turns out that significant results with regard to optimal decision making principles can be found in these settings, we might then try to introduce additional political realism into the model and possibly end up with feasible solutions to the design of political institutions. One of the potentially significant results deals with principles of designing optimal jury decision procedures under the assumption that jurors have different degrees of expertise in matters to be decided.

In Theorem 2 we have already touched upon a corollary of the most important result in this genre. This corollary states the conditions under which the most competent individual is more competent than the majority of voters. In other words, the result tells us in an abstract manner when it is advisable - from a consequentialist point of view - to bestow the decision making authority upon a single individual rather than the group, provided that the latter makes decisions using the majority rule. The theorem follows from a deeper result which pertains to maximizing the probability of making correct decisions by a group of voters. The result is due to Nitzan and Paroush (1982). Before spelling it out, let us consider an example.

Suppose we have a group of five individuals with individual competences: 0.9, 0.8, 0.8, 0.6, 0.6. The average competence then is 0.74. The majority competence, in turn, is 0.897, which clearly exceeds the average, but falls slightly short of the most competent individual. What happens when we increase the weight of the most competent individual? In weighted voting each voter is assigned a weight that reflects his relative influence on the voting outcomes. Typically weights are normalized so that each voter i gets the weight w_i which behaves like a probability, i.e. $\sum_i w_i = 1$ and $0 \leq w_i \leq 1$. In order for a motion to pass, it has to be supported by voters whose weights sum to a number that exceeds a given quota of weights, e.g. 50% of total weights. If the quota is set at 50%, as often is the case, then we are dealing with weighted majority rule.

To continue our example, let the first individual with competence value of 0.9 be assigned the weight of 0.4, while the other voters have equal weights of 0.15 each. Suppose that the required quota is 50% of the total weight. We notice that now any pair that the most competent individual forms with some other individual exceeds the quota. On the other hand, not all groups consisting of three individuals exceed the weight quota. Computing the competence of the weighted majority voting results in value 0.919 which exceeds that of the most competent individual. So, it seems that increasing the weight of the most competent individual increases the group's competence if the group makes its decisions using the weighted majority rule. This is intuitively plausible. But is there a general method for assigning weights to individuals that results in the best achievable group competence? There is and that is provided by Nitzan and Paroush (1982) theorem (see also Grofman et al. 1983 as well as Shapley and Grofman 1984).

Theorem 3 *Given a group of minimally competent individuals (i.e. $p_i > 0.5$, for all i), the decision procedure that maximizes the probability that the group decision is right is weighted majority rule where each individual i is assigned a weight*

$$w_i = \log\left(\frac{p_i}{1 - p_i}\right).$$

In other words, weighted majority voting with weights assigned to individuals in proportion of the logarithm of their competence odds, is the answer to the above question. Since the odds are larger than unity for all voters by assumption, this means that the logarithms in question are real numbers larger than zero.

In our example, the odds of the voters are: $0.9/0.1 = 9, 0.8/0.2 = 4, 0.8/0.2 = 4, 0.6/0.4 = 1.5$ and $0.6/0.4 = 1.5$. Since the Briggs' logarithms of these numbers are: 0.954, 0.602, 0.602, 0.176 and 0.176, the optimal weights are 0.380 to individual 1, 0.240 to individuals 2 and 3 and 0.07 to individuals 4 and 5.[1] Computing the group competence under the assumption that weighted majority rule is being used in decision making, we get the group competence value 0.984, well above any of the values discussed above and quite close to unity.

So, there is an apparently plausible method of making decisions in a way that not only improves upon the competence of the average group member, but even that of the most competent member. Now, the natural question to ask is how does one go about applying this apparently useful result. The main restriction to its applicability in business and politics is, of course, the fact that very few relevant issues pertain to competence in the sense of knowing true answers to questions. Rather the bulk of business and political decision making deals with values, goals and other desiderata. But even in those hypothetical situations where the competence in the sense of probability of being right is a reasonably meaningful notion, one faces a severe

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[1] Nitzan and Paroush (1982) express the theorem in natural logarithms, i.e. logarithms to base $e = lim_{n \to \infty}(1 + (1/n))^n = 2.718\ldots$, while Shapley and Grofman (1984) use Briggs' logarithms or logarithms to base 10. These are equivalent in the present setting, since $ln\ x/ln\ y = lg\ x/lg\ y$ for all real numbers x and y.

application problem, to wit, how to find out the competence values of individuals. A remarkable result of Feld is one plausible way of proceeding (Grofman et al. 1983, p. 275).

Theorem 4 *The optimal individual weights can be approximated by assigning each individual i the weight $r_i - 0.5$ where r_i is the proportion of times that i has been in agreement with the majority decision in the past.*

This theorem enables us to sidestep the issue of determining what is the right decision in any given situation. Instead we can determine the optimal weights by counting the relative number of times the individual has been in agreement with the majority. This theorem should, however, not be read as a solution to the philosophical problem of induction. What it states is that, assuming that the future decision settings do not essentially differ from those of the past, the agreement with the majority works well as a determinant of the optimal weight.

The above theorem can be utilized in designing institutions which provide incentives for consensus. To wit, by assigning each decision maker a weight in accordance with the theorem, i.e. $r_i - 0.5$, one gives larger weights to persons with larger conformity to majority decisions. If the individuals want to maximize their weight, then the way to proceed is to stick with the majority. Not a recipe for innovation, a critique could say.

9.4 Epistemic Paradoxes and Their Relevance

In the same way as the social choice theory deals with aggregation of individual preferences, we can study the rules used in aggregating judgments. This is the case, for example, in jury decision making or any situation involving arguments developed to justify conclusions. Similarly in expert groups one often aggregates judgments, not opinions of the experts. The judgments may concern various states of affairs, e.g. whether a given occurrence has taken place, whether a certain assessment is reliable, whether a given applicant has sufficient skills for a given task, etc. So, it is not the values of the experts that count, but their judgments regarding facts. Furthermore, many expert views involve not only the statement regarding the facts, but also an argument relating those facts to each other so that – together with some logical statements – they form a sequence where some sentences are premises leading to other statements, viz. the conclusions. For example, in an economic policy advisory group, an expert might suggest that since the inflation rate, unemployment, foreign trade balance and immigration have reached a given level, certain economic policies ought to be resorted to by the government. The suggestion thus lists specific facts and reaches its policy recommendation or conclusion resting on the premises and some general principles reflecting the views of the expert about the causal relationships prevailing in the economy. So, when a group of experts is drafting a policy recommendation, it basically aggregates the judgments of its members regarding the

Table 9.3 Doctrinal paradox

Judge	Prop. p	Prop. q	Prop. r
Judge 1	True	True	True
Judge 2	True	False	False
Judge 3	False	True	False
Majority	True	True	False

facts and general principles that rule in the economy. This differs from aggregating opinions, *simpliciter*.

The classic example in this literature is the doctrinal paradox introduced by Kornhauser (1992) (an early precursor is Vacca 1921).[2] This paradox involves a jury of three jurors and a case where the issue is whether the defendant has breached a contract with another party. The legal doctrine has it that a breach of contract has occurred if and only if there is an act A such that the defendant is contractually obliged not to do A and, yet, the defendant did A. Otherwise, no breach has occurred.

For the jury decision three propositions are relevant:

1. p: the defendant was contractually obliged not to do A
2. q: defendant did A
3. r: the defendant breached the contract

All judges are adhering to the prevailing legal doctrine, i.e. r is true if and only if both p and q are true. Even though they agree on the doctrine, they may disagree on the truth value of the three propositions. Suppose that their truth value assignments are those presented in Table 9.3. Thus, for example, judge 2 sees that the defendant was, indeed, contractually obliged to refrain from doing A, but he/she did not do A. Since judge 2 acts in accordance with the legal doctrine, his/her view is that the defendant was not in breach of the contract. Similarly the other two jurors can be seen to adhere to the prevailing legal doctrine. When looking at the judgments of the majority of jurors it, however, turns out that this doctrine is no more valid: the majority deems propositions p and q true, but – in contrast to what the doctrine dictates – judges r to be false.

It is easy to see the similarity of the doctrinal paradox with the Condorcet one: a principle characterizing each individual does not extend to the majority of those individuals. In the case of the doctrinal paradox the principle is the adherence to the legal doctrine, while in Condorcet's paradox it is the completeness and transitivity of preferences.

What is called legal doctrine above is basically a specification of admissible ways of combining propositions, i.e. a rule guiding allowable inferences. One might say that the doctrine here is a kind of constraint that any legitimate reasoning has to satisfy. Thus, we can generalize the Table 9.3 setting to any situation where certain types of logical constraints are imposed on individual judgments and, yet, these

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[2] See Kornhauser and Sager (1986) and List (2012).

constraints are not satisfied by the reasoning of the majority. In this more general setting where issue-wise majority voting leads to an inconsistent outcome is known as discursive dilemma (Pettit 2001; List and Pettit 2002).

The dilemma basically undermines the possibilities to make consistent arguments by aggregating proposition-wise judgments using majority. This leaves open two possibilities for handling judgment aggregation in group choice: (i) to impose restrictions to distributions of inputs, i.e. the individual judgments, or (ii) accept either the premise-based or conclusion-based majority as the decisive one. The former possibility could involve ruling out inputs that lead to inconsistent majority arguments, while the latter would essentially rule out paradoxes by *assuming* that the majority decision on premises or conclusions is paradox-free. Although the latter might seem impossible to accept, it is in fact common practice in preference aggregation settings where the successive elimination method is resorted (e.g. in the U.S. Congress). This method conducts $k - 1$ pairwise majority votes if the alternative set consists of k elements. In each vote, the losing alternative is eliminated and the winner is confronted with the next one until all alternatives have been present in at least one pairwise comparison. The winner of the final pair is the overall winner. This method is based on the incorrect assumption that the group preference relation formed through pairwise majority votes is transitive, e.g. if z wins the winner of the x, y pair, it also wins the loser of the pair. As Condorcet's paradox shows, this is not guaranteed by the majority rule. Indeed, it may well be that x defeats y and z defeats x, and yet y defeats z. The successive elimination solves the Condorcet paradox by fiat. Hence, it is in general impossible to find out on the basis of pairwise voting records whether the successive elimination system results in a robust (i.e. Condorcet) winner or one whose victory is merely due to the order of voting since the underlying majority preference relation is cyclic.

The relevance of the doctrinal paradoxes or discursive dilemmas is in settings where one is not simply aggregating opinions regarding the desirability of policies or candidates, but the voters are expected to be able and willing to formulate or accept arguments in support of certain conclusions. It is, of course, possible and, indeed, likely that the voters think in terms of arguments also in those settings where they are not expected to present them. This possibility widens essentially the domain of relevance of these paradoxes. We simply do not know what kind of arguments underlie conclusion-based aggregations. An analogous observation can be made regarding the likelihood of the Condorcet paradox or related anomalies in preference aggregation. What, on the other hand, restricts the relevance of judgment aggregations paradoxes is the intuitive observation that the arguments underlying the choice of a policy alternative or candidate have a wide variety of factual statements built into them. E. g. in political competitions one voter may regard economic self-interest as the primary consideration, another might have social justice considerations in mind, while a third voter could deem religious variables the most important ones. It is in fact not common to encounter settings where all voters would base their conclusions on the same propositions and their truth-value combinations. The same holds for expert bodies composed of representatives from different areas of knowledge, say, finance, customer relations, technical expertise, marketing. It is quite natural to expect that these experts build their arguments on different kinds of propositions.

9.5 Topics for Further Reflection

1. Bovens and Rabinowicz (2006) discuss the following example: an item of technical equipment is to be purchased for a specific purpose. Consider three propositions:
   - p: the item meets the safety standards
   - q: the item is economically feasible
   - r: the item should be purchased

   Three persons are in charge of the purchasing decision. Each thinks that the item should be purchased if and only if both p and q are true. Construct a table similar to Table 9.3 that does not exhibit the doctrinal paradox.
2. Consider a country that has cumulated a huge amount of foreign debt. It turns to a coalition of international actors for an economic aid package in the form of additional loans. The coalition consists of three equal-sized groups: A, B and C. Within each group there is a unanimity that the country ought to be given the requested aid if and only of if propositions p and q are true. Here p: the government of the country is able to execute economic policies that enable it to pay back – with interest – the borrowed funds within a 30-year period, q: the government's policies are acceptable enough to the population for the government to stay in power for an adequate period of time to launch the policies. Construct a table similar to Table 9.3 so that the doctrinal paradox occurs. Then construct another table where is doesn't occur.

9.6 Suggestions for Reading

Very useful accounts of the epistemic paradoxes are Bovens and Rabinowicz (2006), Dietrich and List (2013) and List (2011). The earlier contribution by Kornhauser and Sager (1986) set the stage for later developments in this rapidly expanding field.

Answers to Selected Problems

1. Suppose that two individuals think that both p and q are true and therefore the equivalence is true as well. Since these two person constitute a majority, their opinion coincides with the collective opinion. Hence, no paradox ensues.

2. A situation where the doctrinal paradox appears (*A* believes that *p* is true, but *q* is not, *B* believes that both *p* and *q* are true and *C* believes that *p* is false, but *q* is true):

Country	Prop. p	Prop. q	Prop. r
A	True	False	False
B	True	True	True
C	False	True	False
Majority	True	True	False

3. A situation where the doctrinal paradox does not appear (country *B* changes its mind regarding the truth value of *q* with respect to the preceding table:

Country	Prop. p	Prop. q	Prop. r
A	True	False	False
B	True	False	False
C	False	True	False
Majority	True	False	False

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Chapter 10
Representativeness

Abstract A referendum paradox occurs when a collective decision by a majority in a representative body contradicts the majority opinion in the electorate at large. We discuss this paradox as an introduction to the problems of constructing optimally representative committees. Two important studies are reviewed and the notion of a Condorcet committee introduced. We also deal with power distribution in committees *vis-à-vis* the electorate at large.

10.1 The Referendum Paradox

Since direct democracy is for several reasons impossible in contemporary political systems, some principle of representation has to be resorted in those systems that call themselves democratic. Rightly or wrongly, most current systems of governance of political entities declare themselves democratic. Yet, a wide variety of principles are being used in composing the representative bodies making decisions on behalf and in the name of the populations at large. In business environments, the highest decision making bodies are typically not envisaged to be democratic in the same sense as in political contexts, but quite often we encounter the representation problem when various working groups or task forces are being set up. The population or electorate in those contexts typically consists of boards or councils or plenary assemblies. In what follows we draw upon some results from political science to shed light to the process of rational composition of representative bodies. We start with the description of a puzzling – but at the same time quite understandable – phenomenon sometimes called the referendum paradox.

There are 10 single-member districts, each with 100 voters. The districts are not necessarily geographical entities, but may constitute partitioning of the electorate on other – age, occupation, ethnicity, income, stock share – grounds. In any case, it is here assumed that each district, however defined, sends one representative to the task force, parliament or working group to be elected. The representative body is then expected to vote on a dichotomous issue, i.e. one that results either in a 'yes' or 'no' stand depending on which position has more votes in the body. An illustrative distribution of voters over positions and districts is exhibited in Table 10.1.

A. T. de Almeida et al., *Systems, Procedures and Voting Rules in Context*,
Advances in Group Decision and Negotiation 9,
https://doi.org/10.1007/978-3-030-30955-8_10

Table 10.1 Referendum paradox

Opinion	District 1	…	District 9	District 10	Total
Yes	45	…	45	100	505
No	55	…	55	0	495

The paradox consists of the fact that direct and representative majority voting lead to opposite outcomes with a clear margin. To wit, if we assume that each representative votes according to what s/he believes (correctly) to be the opinion of the majority of voters in his district, the outcome in the representative body is 9 to 1 in favour of ‘no’, but if the voters vote directly on the issue, the winner is ‘yes’ with a 505 to 495 margin. The observers of the U.S. presidential elections are, of course, familiar with this paradox in a slightly different disguise.

The referendum paradox is just another way of saying that moving from direct to indirect (representative) decision making comes with a price. Even if the representatives aim at faithfully representing their electors by reflecting the majority opinion of the latter, the outcome in the representative body may contradict the view of the majority of the electorate. This possibility is unavoidable as long as the simple majority principle is being utilized. It becomes, however, less likely in homogeneous electorates. There is another reason for not being overly worried about the referendum paradox, viz. it deals with a single issue, while one could perhaps expect that in the long run, i.e. with a long sequence of issues, the majorities in the representative body and in the electorate at large are more likely to coincide on the average. This is, course, a conjecture.

Another conjecture suggests that when choosing representatives the voters are not primarily interested in having their own views reflected in the final outcomes, since most of the time they simply do not have them. One could argue that in modern democracies very few if any voters have an opinion on every issue on the legislative agenda of the parliament. The same presumably holds for many other representative bodies. Hence, the voter’s main interest is in the qualities – personal or professional – of the candidates competing for representative positions.

10.2 Optimal Committees

A pioneering paper on optimal committees from the social choice perspective is the article of Chamberlin and Courant (1983). It starts from assuming that each individual has a preference ranking over the candidates competing for available committee (working group, council, board) membership. The optimality of the ensuing committee is defined in terms of these preferences. The second crucial assumption is that for each possible committee and each voter, the latter is represented in the former by one committee member, viz. that member which has the highest position in the voter's ranking. Thus, for example, if the committee consists of three members and voter i ranks them in third, fifth and tenth position, then i is represented by the member whom s/he ranks third. This highest ranked member will be called i's representative in the committee. Now, the desirability of any given committee to any voter, is defined in a natural way: the higher the representative is positioned in the voter's ranking, the better s/he is represented. Or, stated in another way, the degree of misrepresentation of the voter is the smaller, the higher his/her representative in this committee is ranked. An obvious way of measuring the degree of misrepresentation of a committee for a voter is count the number of candidates that are ranked higher than the voter's representative in the committee. E.g. if the voter's representative is ranked first by him/her, the degree of misrepresentation of this committee is 0 for this voter, if his/her representative is ranked third, the degree of misrepresentation is 2 and so on. The determination of the best or optimal committee in this sense is basically straight-forward: one generates all possible committees and sums up the voters' degrees of misrepresentation for each committee. The committee with the smallest sum is the optimal one. This way of computing optimal committees is linearly related to the Borda scores of each committee member.

The procedure outlined above differs from extant methods of proportional representation, but captures important features that underlie the concept of proportionality. At the same time it is subject to the criticism that in the optimal committee the members may represent voters in very different ways since the average Borda score sum of the members allows for a wide variation in the scores of individual members. Stated in a another way, the optimal committees in the Chamberlin-Courant procedure typically consists of members with widely varying constituencies or support sizes. The constituency $N_j(c)$ of a candidate j in a committee c is defined as the number of voters who rank j higher than any other candidate in c. That is,

$$N_j(c) = |\{i \in N | r_{ij} \leq r_{is}, \forall s \in c\}|$$

Here N is the set of voters and r_{ij} denotes the rank assigned to member j by voter i.

To see how the Chamberlin-Courant procedure works, consider the following example involving nine voters, five candidates and the committee of three members. The voter preferences over the candidates are presented in Table 10.2. There are 10 possible ways of selecting a 3-member committee out of the five candidates. One of them is ABC. Its associated degree of misrepresentation is computed as follows: there are three voters each of whom assigns A the degree of misrepresentation of 4 and two voters assigning A the degree 3, while four voters assign A a zero degree of misrepresentation. Similarly, B is assigned 1 degree by four voters and 2 degrees by two voters. C, in turn, is associated with the degree 2 by seven voters. Hence the degree of misrepresentation associated with the committee ABC is 40. The degrees of the other nine committees can be computed in the same manner. It turns out that the committee BCD has the smallest degree of misrepresentation, viz. 39.

The example shows that the constituencies of the committee members in the optimal committee are far from being equal-sized: seven voters are associated with

Table 10.2 The Chamberlin-Courant procedure illustrated

4 voters	3 voters	2 voters
A	B	C
B	D	D
C	C	B
D	E	A
E	A	E

B, two with C and none with D. On the other hand, the candidate ranked first by more voters than any other candidate, viz. A, is not present in the optimal committee. It is difficult to say which is more counterintuitive: either an optimal committee where the candidate ranked first by more voters than any other candidate is not present or an optimal committee that includes a member with an empty constituency. Both these counterintuitive features are results of the fact that in defining the optimal committee Chamberlin and Courant resort to the Borda count, while in defining the assignment of voters to various committee members, another positional criterion is being applied.

Viewed from the representation point of view the Chamberlin-Courant method has a flaw that is related to what was just said, viz. it does not guarantee that each representative stands for equally many voters. To rectify this, Monroe devised a technique that guarantees that each representative represents the same number of voters Monroe (1995).

The fundamental idea is to divide the electorate of size n into segments of equal size, that is, each segment consists of n/k voters where k is the size of the committee. Here each segment represents the same number of voters. The way to achieve a fully proportional representation in Monroe's sense is to start from the voters' preference profile over all candidates and to construct every possible k-member committee where each voter is assigned to the member that minimizes his/her misrepresentation under the constraint that the number of voters assigned to each member is the same, viz. n/k. For a given committee this is done by first assigning every voter to the candidate that best represents him/her. Suppose that voter i assigns candidate j the rank r_{ij}. Monroe suggests the misrepresentation measure of Chamberlin and Courant, viz. $\mu_{ij} = r_{ij} - 1$. In other words, the misrepresentation related to a candidate is simply his/her rank minus unity. Thus, for any given committee one first assigns each voter to the committee member for whom his/her misrepresentation is smallest (ties are broken randomly). Since this does not in general lead to a uniform distribution of voters over committee members, one proceeds by transferring voters from one candidate to another to achieve a situation where each member represents n/k voters. The criterion for transfer is the following: of any two voters, say i and l, the one that suffers less from the transfer is re-assigned. This means that those voters who are indifferent or nearly indifferent between the candidates they are assigned to before and after re-assignment are transferred first.

Once the transfers required to make each constituency of equal size have been performed, the degree of misrepresentation associated with the committee is computed as the sum of the individual misrepresentations of the voters. The committee with the smallest value of misrepresentation is the fully proportional committee. This description of the construction process is basically one that was presented in Monroe's article. For large candidate sets it is very tedious. Fortunately, Potthoff and Brams (1998) have devised an integer programming algorithm for computing fully proportional committees (Brams 2008).

Let there be m candidates. Define the value of the variable x_j to be 1 if candidate j is a winner (i.e. belongs to the fully proportional committee) and $x_j = 0$, otherwise. The sum over all candidates of x_j's thus indicates the number of candidates in the committee. Now define $x_{ij} = 1$, if candidate i is assigned to voter j, and $x_{ij} = 0$, otherwise. For a fixed value of the variable i the sum over j indicates the number of voters associated with candidate i. The objective function to be minimized is the sum of misrepresentation values:

$$z = \sum_i \sum_j \mu_{ij}$$

The constraints under which the minimization is to be done are

$$\begin{aligned}
\sum_i x_i &= k \\
\sum_i x_{ij} &= 1, \text{ for each } \quad j = 1, \ldots, n \\
-Lx_i + \sum_j x_{ij} &\geq 0, \text{ for each } \quad i = 1, \ldots, m \\
-Ux_i + \sum jx_{ij} &\leq 0, \text{ for each } \quad i = 1, \ldots, m \\
&x_i \text{ is an integer less than or equal to 1, for each } \quad i \\
&x_{ij} \text{ is an integer less than or equal to 1, for each } \quad i \quad \text{and} j
\end{aligned}$$

Here L is the lower bound of the number of voters assigned to each candidate, while U is the upper bound of this number. Since the constituencies in fully proportional committees are of equal size, L is equal to the largest integer smaller than n/k, while U is the smallest integer larger than n/k. Should n/k be an integer the constraints involving L and U can be replaced by a single constraint:

$$-\frac{n}{k}x_i + \sum_j x_{ij} = 0, \text{ for each } \ i$$

Monroe's idea of full proportionality and its precise formulation and solution through integer programming as suggested by Brams and Potthoff allow for various kinds of extensions. To wit, the measure of misrepresentation, μ_{ij}, can be defined in ways that differ from the one described above. Moreover, the voters can be assigned to several candidates instead of just one. Some of these variations can be easily accommodated by the integer programming approach.

10.3 Topics for Further Reflection

1. Is the referendum paradox possible in the presidential election system of the United States? If it is, can you suggest a way of avoiding it? If it isn't, explain why.
2. The Chamberlin-Courant procedure for determining optimal committees uses all rank positions to determine the degree of misrepresentation associated with a candidate. Suppose a different count were used; to wit, suppose all first ranks were associated with 0 degree of misrepresentation while all lower positions had the same misrepresentation value, viz. 1. Compute the optimal 3-member committee in the Table 10.2 profile using this alternative count.
3. A strong Condorcet committee is a committee that would defeat all other committees in pairwise majority voting contests, i.e. in each comparison this committee would be supported by more voters than its opponent. Suppose that the voter preferences over committees are induced by the respective misrepresentation values: the smaller the value the more preferable the committee. Focusing on three-member committees in Table 10.2, is the optimal BCD committee a Condorcet one?

10.4 Suggestions for Reading

The determination of the nearly optimal and/or nearly fully proportional committees has been shown to be computationally tractable by Skowron et al. (2015). Similar computational results pertaining to Condorcet committees – i.e. committees that are undefeated by any others in pairwise comparisons with a majority of individual votes – are presented by Darmann (2013). Methods based on dichotomous preferences and approval voting are discussed by Brams et al. (2006) and Kilgour (2010).

Answers to Selected Problems

1. It is possible and has, in fact, occurred a few times, most recently in the 2016 election where Donald J. Trump won, but his competitor Hillary Clinton received more popular votes.

2. The degree of misrepresentation for committee ABC is $5 + 6 + 7 = 18$. Any two-member committee that includes D or E can be improved by replacing this member by A, B or C thereby decreasing the degree of misrepresentation by 4, 6 or 7. Similarly, any one-member committee that includes D or E can be improved by replacing these by A, B or C. ABC cannot be improved upon by replacing any member with either D or E. Thus it is the optimal one under the modified count.
3. Yes, it is. For example, when compared with the committee ABC, each of the first four voters assigns ABC the misrepresentation value $1 + 2 = 3$, each of the next three voters assign ABC the misrepresentation value $4 + 2 = 6$ and each of the last two voters assigns ABC the misrepresentation value $3 + 2 = 5$. The corresponding values for BCD are $1 + 2 + 3 = 6$, $2 + 1 = 3$ and $2 + 1 = 3$. So, while four voters deem ABC preferable to BCD, five others have the opposite preference. Hence, BCD beats ABC by a majority. By studying all 10 committees of three, it turns out that BCD defeats all others and is thus the strong Condorcet committee.

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Chapter 11
Deliberation and Voting

Abstract The theory of voting often takes the agenda as exogenously given. The deliberative view of democratic decision making focuses on phases of decision process preceding the actual voting, occasionally even replacing the latter with deliberative processes. We discuss the plausibility of presuming that the best argument wins. We also take up issues pertaining to procedures that have to be resorted to when a consensus is not reached. We envisage a most useful role for the deliberative practices in agenda formation.

11.1 Voting With or Without Deliberation

In the preceding we have focused on the phase of decision making that immediately precedes the very act of choosing an alternative or candidate, viz. voting. There are, however, important stages in the decision process that precede the voting. Of particular importance is the stage where the voting alternatives are determined. After all, voting can determine only which alternative or candidate is best – in the sense intended by the voting procedure – of those that are being voted upon. The standard voting theory often glosses over the alternative formulation stage and restricts attention to the mapping from voter opinions regarding the *given* alternatives or candidates to the alternatives or candidates that are deemed best. And yet, the way decision alternatives are processed prior to voting can make an essential difference in the outcomes. In particular, by discussing the alternatives the voters may spot outcomes that benefit no one when compared with the *status quo*. Such collectively inferior outcomes sometimes exist and, more importantly, can actually ensue from some voting procedures unless special precautions are made. The possibility of such Pareto violation of the amendment (a.k.a. successive elimination) procedure illustrates this (see Table 11.1).

Suppose that the agenda of pairwise comparisons is: (i) B versus D, (ii) the winner of (i) versus A, and (iii) the winner of (ii) versus C. Suppose furthermore that each voter votes according to his preference in each pairwise comparison and that the winner is always the alternative receiving more votes than its contestant. Then B defeats D in ballot (i), A defeats B in (ii) and finally C beats A in (iii). Upon looking

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Table 11.1 Pairwise majority comparisons may lead to Pareto suboptimal outcomes

Voter 1	Voter 2	Voter 3
A	D	B
B	C	D
D	A	C
C	B	A

at Table 11.1 we see that the *C* is Pareto dominated by *D*. Hence, Pareto criterion is violated. In other words, if *D* is the *status quo* alternative, *all* voters are worse off as a result of voting.

11.2 The Role of Deliberation

The advocates of deliberative democracy stress the value of deliberation in improving the quality of collective decisions. This follows from free exchange of information, both factual and normative. The flow of factual information may improve the decisions by helping the participants to identify alternatives that are based on false assumptions, e.g. highways cannot be built through privately owned land without the permission of the land-owner or buying the land, physical punishments cannot legally be applied to school-children in order to root out bullying, military personnel may not be eligible for certain types of public offices, etc. In decision making involving technological projects, people cognizant of technology may be able to rule out certain alternatives as infeasible given our present-day knowledge. Ideally, the exchange of factual and normative information may result in a consensus. Should this happen, then no voting is needed at all.

More common are, however, situations where the deliberation does not lead to a consensus, but only modifies the original alternative set. Even so, the elimination of unrealistic alternatives and possible introduction of new realistic ones, are bound to improve the decision process. Still, the voting stage is required to yield the actual collective decision. The deliberation phase may alter the outcomes in other ways as well. To wit, after exchanging views of the alternatives at hand, the voters are probably more informed about each others' opinions (preferences) than at the outset. Given a known voting procedure, this may change their voting strategy. In particular, the voters may resort to sophisticated instead of sincere strategies. These necessarily exclude the possibility of Pareto dominated outcomes.

A more subtle argument for deliberative process is provided by List et al. (2013). If the main source of trouble in collective decision making comes from the cyclic majority preference relation, as is often argued, then some experimental evidence suggests that deliberation is prone to modify the preferences of the individuals towards single-peakedness thereby increasing the probability of Condorcet winners. If this finding is robust, then deliberation can reduce the apparent arbitrariness of voting outcomes. Especially, for those advocating Condorcet extension rules this experimental evidence is undoubtedly appealing.

11.3 Deliberation and Agenda Effects

The theory of collective decision making typically starts from a given set of alternatives. This glosses over a crucial determinant of the decision outcomes, viz. the process whereby the alternatives are formulated. Yet, the outcomes can be nothing else but subsets or ranking of the alternative set. If we widen our perspective to include also the alternative formulation stage, a host of negative findings are in front of us. In the following we mention just a few most important ones.

In a path-breaking article based on a pseudo-experiment Plott and Levine (1978) came to the following conclusion:

> Experimental results indicate that within a range of circumstances the agenda can indeed be used to influence the outcome of a committee decision.

The observation is based on packaging different options into bundles and manipulating voting order (agenda). Plott and Levine's target community was a private club of amateur pilots pondering upon the purchase of a new fleet of aircraft consisting of planes of various types. Given the preferences of the club members over various aircraft options, it turned out that basically any desired mix of airplanes could have been made the winner with a suitable packaging of options and the order of voting. A more general statement was later made by Saari (2001, p. 13):

> For a price, I will come to your organization to design your election procedure. You tell me who you want to win. After talking with the members of your organization to ascertain their preferences, I will construct a 'democratic voting procedure' which will ensure the victory of your candidate.

These observations suggest that the voting outcomes are even more agenda-dependent than the pioneering theorems of McKelvey (1979) imply. According to these, in the absence of a Condorcet winner or a majority undominated alternative, the pairwise majority comparisons do not in general guarantee even a rough similarity between voting outcomes and voter preferences. Instead, under sincere voting, the agenda builder can completely determine the voting outcome and yet the winner at each stage is determined by a majority of voters.

In the spirit of Levine, Plott and Saari, Marengo and Settepanella (2012) provide insights to packaging of issues into bundles and to the ensuing changes in social outcomes. In their model choices are made up of bundles of elements called *features*, i.e. $F = \{f_1, \ldots, f_n\}$. Each feature may take on one value out of a finite set of alternatives. If all features may take $m + 1$ values, there are $(m + 1)^n$ social outcomes, i.e. $n-$ tuples of feature values. An object scheme is a bundling of features into subsets (not necessarily distinct). As an example Marengo and Settepanella discuss a group of people considering how to spend an evening together. The features could be: where to go, when to go, how to go. The values, in turn, are: $\{restaurant, cinema\}$, $\{7PM, 8PM\}$, $\{car, walk\}$. A possible outcome would, then, be $(cinema, 7PM, car)$. If the agenda setter can bundle features any way he likes, the outcome can be far away from the individuals' desires. More specifically, Marengo and Settepanella show that it is always possible to manipulate the object scheme in such a way that the median voter theorem does not apply and the social choice may converge to social outcomes very distant from the median voter's preferred one.

The results referred to in this section stress the importance of the formation of the alternatives to be voted upon. It is in this stage that deliberation can play an important role or – to put it in a different way – where the lack of deliberation may undermine the experienced legitimacy of the voting outcomes, no matter how plausible the voting mechanism used in ballot aggregation.

11.4 Topics for Further Reflection

1. Construct a three-voter, three-alternative profile where the amendment procedure can result in each alternative as the winner depending on the agenda of pairwise comparisons.
2. Suppose that the successive procedure is used, i.e. each alternative is voted up or down by a majority vote at each stage of the procedure and winner of the final vote is the overall winner. Suppose moreover that every voter votes for the subset containing his/her first ranked alternative at every stage. Construct a profile and an agenda showing that the Condorcet winner is not elected.
3. What can be said about the election of Condorcet winners and losers under the amendment and successive procedures?

11.5 Suggestions for Reading

Miller's (1995) monograph covers much of what is known about agenda-based procedures. The art and science of packaging of issues is dealt with by Riker's (1982, 1986) books.

Answers to Selected Problems

1. Consider the Condorcet paradox profile:

Voter 1	Voter 2	Voters 3
A	B	C
B	C	A
C	A	B

   If one wants *A* to win, the agenda should be built so that in the first stage *B* and *C* are compared with each other and the winner takes on *A* in the second voting. If *B* is to be made the winner, the first vote should be between *A* and *C*, whereupon the winner faces *B* in the second vote. If *C* should win, then the first vote should compare *A* and *B* with the winner confronting *C*.
2. Consider the following profile:

4 voters	3 voters	2 voters
A	B	C
C	C	A
B	A	B

   Here *C* is the Condorcet winner. Suppose that the agenda puts *C* first to a up or down vote. Since it is the first ranked one by only two voters, it will be voted down. If *A* is the next in the agenda, it will be elected. If *B* is the next, then *A* is elected as well. In any event, the Condorcet winner will not be elected.
3. The Condorcet loser cannot be elected under the amendment procedure since in order to win it is required that the candidate defeats at least one other candidate, viz. the one it is confronted with in the final comparison. Since the Condorcet loser defeats no other alternative, it cannot win under the amendment procedure. The successive procedure cannot choose the Condorcet loser, either. If a Condorcet loser is subjected to an up or down vote, it will be defeated since it cannot be the first ranked candidate by a majority of voters in any subset of candidates (if it were, it would defeat the other remaining alternatives and hence would not be the Condorcet loser).

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Chapter 12
The Business Context

Abstract There is a distinction between using rules in the business context and in the political context. Furthermore, in the political context, another distinction is between using rules to select people and using rules to follow procedures. Usually an analyst chooses a Voting Procedure (VP), based on his/her knowledge with regard to the technical issues to do with the application. We argue that instead, DMs themselves should make the choices and that a DM's preferences should be considered in the context of the decision problem.

12.1 Introduction

Although one might think that rules have been designed for political elections rather than for business decisions, it can be observed that they are used quite often in business organizations in a group decision context. Voting procedures are very well-suited to tackling a specific range of business decision problems. For instance, decisions made by the Board of Directors in many organizations are reached by using VPs.

On the other hand, although rules can be applied in both contexts, choosing a rule for the business context needs to consider matters that are very different from those of elections for a political context.

Moreover, it is well known that there are differences of another kind when rules are used, namely, when the choice concerns choosing a person, on the one hand or a policy, on the other hand. Both kinds of choices may be found in the business context. However, choosing a policy happens more often. Furthermore, this kind of decision may be associated with choosing procedures or other similar decisions, such as an alternative course of action to be implemented in the business of the organization. For instance, choosing suppliers is a decision problem that occurs frequently. Another example is to choose projects from a list with several proposals.

Choosing people in business organizations is normally related to recruiting staff and to assigning people to new tasks with new functions, which is rather different from choosing the representatives of political parties or the candidates of other groups of people standing for elected positions.

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In the Business context, the issue of choosing a rule may take place in two situations: using a rule for either a specific decision problem or for every decision by a group of DMs. The latter usually happens at the highest level of the organization; for instance, at Board meetings of any business organization, for which norms of the organization (e.g.: explicit in bylaws) have to state which voting procedure should be applied. The specific decision problem is to do with each ordinary decision being made in conjunction with due regard to the several processes of the organization. For this situation, each kind of problem has its specific characteristics and may require different criteria to evaluate rules.

12.2 The Decision Process in the Business Context

Simon (1960) presents the three basic stages of a decision process. Several subsequent studies have added other stages. Most of these contributions come from information and decision systems (Bidgoli 1989; Sprague and Watson 1989; Davis and Olson 1985; Polmerol and Barba-Romero 2000).

The two additional stages come after the first three, making a total of five stages. According to Simon (1960), the initial stages are: Intelligence, Design and Choice. Stages 4 and 5 are Revising and Implementing the decision process.

In the intelligence stage, an organization and its environment are monitored in order to identify decision situations. This is not usual for most procedures of operational research and decision methods, although it is related to identifying a decision situation using the Value Focusing Thinking (VFT) approach as proposed by Keeney (1992). The vision for strategic management also incorporates this kind of approach, during which an organization and its environment are continuously monitored in order to obtain a diagnosis and to act proactively with a view to anticipating decision situations (de Almeida et al. 2015).

In most operational research techniques, it is assumed that a decision problem already exists, and the process starts with the second stage of Design by defining this problem (Ackoff and Sasieni 1968). In this stage the decision model is built, during which several ingredients of the decision model are dealt with, such as creating the set of alternatives, which are also evaluated in this stage. The MCDM/A method is chosen during this stage. In order to have the problem clearly defined, Problem Structuring Methods (PSM) may be applied (Eden 1988; Rosenhead and Mingers 2004; Eden and Ackermann 2004). Building the model includes establishing or estimating all the parameters of the mathematical model. With regard to the preference modeling to be done in this stage, the DM has a particular role in providing information.

The Choice stage is applied in order to evaluate the alternatives and produce a final recommendation. However, before presenting this recommendation to the DM, the fourth stage of Revising is conducted, in order to check for possible inconsistencies and to validate the model. This stage may incorporate a learning process being undertaken within the organization (Davis and Olson 1985). The recommendation is applied in the Implementation stage. There are several practical concerns to be considered in these two last stages (de Almeida et al. 2015), the most important of which are discussed below.

Throughout these stages, different actors play some kind of role in the process. There are a few possible issues to be considered regarding these actors and their role. Amongst them, we have already considered the decision maker (DM), who can be influenced by other actors, such as stakeholders. Stakeholders are affected by the implementation of an action chosen by the DM and for that reason they try to exert some influence on the DMs. An Analyst has the role of supporting the DM in all stages of the process, and does so by methodologically structuring the problem and building the model (Roy 1996; Belton and Stewart 2002; Figueira et al. 2005; Polmerol and Barba-Romero 2000).

12.3 Types of Aggregation of DMs' Preferences

The aggregation of DMs' preferences consists of reducing the set of each individual DM's preferences to a collective preference system for the whole group of DMs. With a group of DMs, the preference aggregation process is closely related to a few factors, such as the way in which the DMs interact, including their power relation system, the time they have available to spend on this process, whether they are available to interact simultaneously and the role of other actors in this process.

Regarding the power relation system amongst the DMs, one of them may be a supra-DM, who usually has a hierarchical position in the organization's structure that is higher than that of the other DMs. The supra-DM is in charge of making the decision on main issues, such as the decision process itself, global evaluations and evaluating the other DMs' choices. The supra-DM is called a 'benevolent dictator' by Keeney (1976), and acts in accordance with one of the two types of Group Decision process. The other type is called the 'participatory group problem', in which the group acts jointly in the process and each DM has the same power and hierarchical position. Regarding other actors, instead of an analyst, in some situations a role is played by a facilitator or a mediator. With a facilitator, the interaction between DMs may be a more detailed process, assuming that the DMs are available for this. These issues play a conclusive role in the kind of group decision (GD) process, when classifying the types of GD aggregation.

Regarding the way in which the preferences are combined in order to obtaining a collective preference, whether or not a supra-DM is present, the decision process can be implemented in two ways as shown in Fig. 12.1 (Nurmi 1981; Kim and Ahn 1999; Leyva-López and Fernández-González 2003; Dias and Clímaco 2005; de Almeida et al. 2015):

- Procedure 1—Aggregate DMs' initial preferences Pi.
- Procedure 2—Aggregate DMs' individual choices Ri.

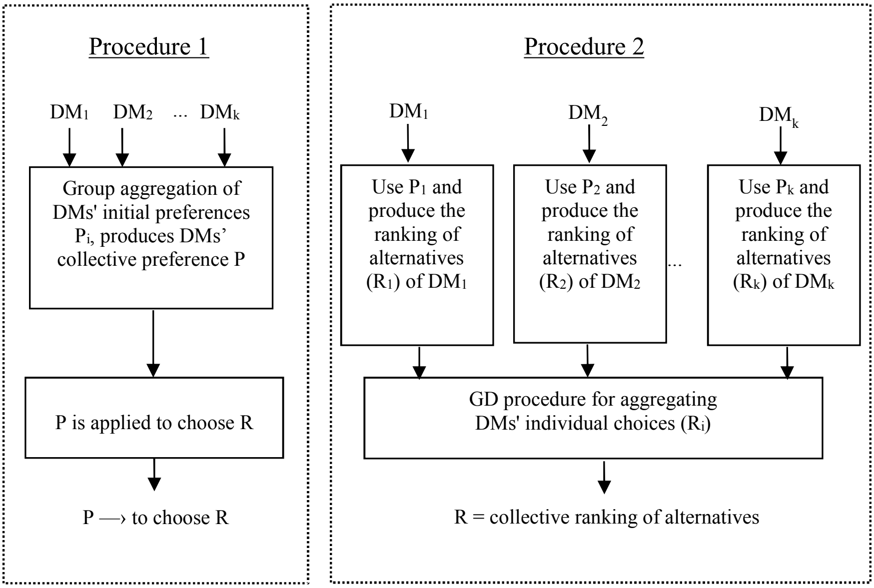

Fig. 12.1 Group decision aggregation processes

A distinction needs to be made between preferences and choices. Let us assume that each DM_i ($i = 1, 2, \ldots, k$), has an initial preference system (P_i) over the consequences (or outcomes) and could apply an MCDM/A method in order to obtain a separate ranking R_i of the alternatives (individual choices of DM_i), such as illustrated for procedure 2 in Fig. 12.1.

According to Nurmi (1981), with regard to preferences, given that there are individual preferences, aggregate these into collective preferences and then make the choice from this collective preference relation. As to choices, aggregate the individual preferences directly into collective choices (without the intermediary collective preference relation).

Therefore, in procedure 1 there is an integration of P_i, in order to produce a collective preference P, whereas in procedure 2, the process is completely separate for each DM.

In procedure 1, the group of DMs provides P_i in an integrated process, such that the aggregation of those preferences is intrinsically considered from the start. Then, a collective preference P is produced. To finalize, P is applied in order to make the final choices for the set of alternatives. These choices may be presented either with ordinal ranking only, or may include a cardinal score for each alternative. This depends on the method, which is jointly applied to all DMs. In this case, it is assumed that all DMs have the same criteria, although after applying these, they may have different evaluations for the intra-criterion and inter-criteria information. Usually the evaluation of intra-criteria is the same, and the differences in P_i concern the weights of the criteria.

For procedure 2, each DM_i provides R_i (ranking of alternatives for DM_i), which are the individual DMs' choices. The GD process consists of producing the final ranking of alternatives R. R_i may be obtained by completely different methods for each DM_i, and even if a different set of criteria is used, i.e. each DM may have a specific set of objectives. The only information that matters to the group decision aggregation is R_i. So, a voting procedure may be applied over R_i.

The latter is the focus of this text, since a VP is a natural method to be applied in order to combine R_i into R. Another step needs to be followed in this process: choosing the most suitable VP.

Typically, choosing a VP is a decision the analyst him/herself likes to make. In general, this choice is based on technical issues rather than the DMs' preference regarding how to tackle solving the final problem. Characteristics and properties of the VPs are considered. In other words, this part of the process appears simply as one of the technical issues to be considered during the process, since this step is not directly related to the final decision faced by DMs.

We argue that the DMs' preference regarding the final objective in this process should be taken into account. Therefore, they should be provided with methodological and technical support (de Almeida and Nurmi 2014, 2015).

12.4 Business Decision Process and Rule Choice

Often in business context, in the decision process DMs make their own ranking of alternatives, before a group aggregation procedure can be considered. Thus, the Business Decision Process can be divided into two specific decision processes:

- DPVP (decision process for choosing a voting procedure);
- DPBO (decision process for the business organization).

The DPVP is a first modeling step in the whole decision process, in which an MCDM method is applied. The DPBO is the subsequent step in the decision process. It focuses on the main concern of the business organization, in which the chosen VP is applied. Regarding the kind of support for these two processes, it should be noted that the DPVP is implemented using an MCDM model and the DPBO is conducted by means of a VP.

The DPVP is the main focus of this text and should use the framework that is presented in a later chapter. On the other hand, the main focus of the DPVP is the DPBO. For this reason a discussion on who should make the decision in the DPVP is worthwhile.

Usually an analyst chooses a VP and it is usually assumed that the DMs have agreed with the analyst´s choice of VP. We argue that this decision should be made not by the analyst but by the DMs themselves.

In such a case the DPVP is not strictly applied and the choice of a VP is made by taking some convenience for the modeling process into account. Although the analyst may consider many technical concerns regarding Social Choice Theory, typically, a structured process is not applied in order to make this choice. In other words, the choice of VP is dealt with as an additional technical issue in the whole decision process.

We argue it is important the DMs should act in the DPVP, since applying different VPs to the same set of alternatives ranked by individual DMs might lead to different results. An important issue is that the analyst's preferences (or technical predisposition) should prevail over the DMs' preference in the DPVP.

12.5 The Sequence in the Decision Process

Although the DMs supply their preference for the DPVP, such a process includes receiving the support of an analyst or facilitator, whose role is to support all DMs in the group decision process.

In the DPVP, the analyst supplies the DMs with the necessary information about the VPs. This includes listing the main VPs available and explaining their main characteristics, which would include their main properties and behavior regarding paradoxes.

There are two possible sequences for the decision process:

- DMs choose the VP before they rank the alternatives (of the DPBO);
- DMs choose the VP after ranking the alternatives (of the DPBO).

If the DMs have no knowledge about how other DMs have ranked the alternatives regarding the DPBO, then, the latter sequence would be fine. On the other hand, the former sequence could make some kind of manipulation possible i.e. the DMs might be tempted to adopt strategic choices for the ranking of alternatives in the DPBO. In the latter sequence, the DPBO is divided into two parts. The alternatives are ranked before the DPVP, as a preliminary part of the DPBO, which is finalized afterwards. However, in the first sequence, the DPBO is concluded at once, only after the DPVP has been conducted.

A bias in choosing a VP may happen if the VP is evaluated only after the data are known (the ranking of the alternatives). A DM may feel attracted to favor a VP that is not the one that is best suited to the DPBO, since this VP might suggest the alternative that the DM would like to choose. The analyst needs to be aware of these possibilities and be ready to deal with them, since this tendency may be present.

12.6 Topics for Further Reflection

As already mentioned an MCDM method is applied in the DPVP. Now, one could raise another relevant question:

How should the DMs interact in order to choose the VP in the DPVP? Or,

What is the DPVP group decision process like?

Also, one could imagine that another model should be built to aggregate the DMs' individual preferences with regard to the criteria for evaluating the VPs; alternatively a more complex negotiation process could be conducted with the DMs. This issue is discussed in the chapter related to the framework for the DPVP.

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Chapter 13
Overview of MCDM/A Methods

Abstract Most decision problems have multiple objectives. The basic ingredients of these problems need to be identified in order to build decision models. There are many MCDM/A (Multi-Criteria Decision Making/Aiding) methods and multi-objective approaches. This chapter places an emphasis on the MCDM/A methods, which are more closely related to rules for making choices. We mainly consider the pros and cons of compensatory and non-compensatory rationality for classifying MCDM/A methods.

13.1 Introduction

MCDM/A is an acronym that stands for a number of methods related to the decision process in multicriteria problems, and was formed by amalgamating the acronyms MCDM (Multi-Criteria Decision-Making) and MCDA (Multi-Criteria Decision-Aiding).

In a classical optimization problem, a maximization (or minimization) procedure is applied to a unique objective function, representing gains (or losses). A multicriteria problem consists of a situation, in which there is more than one objective (each objective being represented by one criterion) and in many situations, these objectives conflict with each other.

MCDM/A methods cope with problems for which there is more than one objective. These methods enable DMs to deal with these objectives simultaneously. The criteria (or attributes) are related to outcomes that may be obtained by choosing an alternative in the decision process. The criteria represent the objectives in the decision-making process.

This vision of dealing with several objectives at once was first put forward many centuries ago. For instance, an evaluation between two sets of criteria for choosing a course of action, was made around 300 B.C., by Socrates, as recorded by Plato in the Protagoras dialogue. The idea is about how to measure two types of criteria (pleasure and pain), as follows (de Almeida et al. 2015):

A. T. de Almeida et al., *Systems, Procedures and Voting Rules in Context*,
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> …What measure is there of the relations of pleasure to pain other than excess and defect, which means that they become greater and smaller, and more and fewer, and different in degree? "I should reply: And do they differ in anything but in pleasure and pain? There can be no other measure of them. And do you, like a skilful weigher, put into the balance the pleasures and the pains, and their nearness and distance, and weigh them, and then say which outweighs the other. If you weigh pleasures against pleasures, you of course take the more and greater; or if you weigh pains against pains, you take the fewer and the less; or if pleasures against pains, then you choose that course of action in which the painful is exceeded by the pleasant, whether the distant by the near or the near by the distant; and you avoid that course of action in which the pleasant is exceeded by the painful. Would you not admit, my friends, that this is true? I am confident that they (i.e. the sophists) cannot deny this.

This idea also appears in a text of 1722, by Benjamin Franklin, that deals with analyzing a specific kind of problem, with only one alternative (there are two options: implement it or don't do so), which was expressed in a letter proposing a decision procedure (Hammond et al. 1998; Figueira et al. 2005; de Almeida et al. 2015), as follows:

> In the affair of so much importance to you, wherein you ask my advice, …. […], my way is to divide half a sheet of paper by a line into two columns; writing over the one Pro, and over the other Con. […] When I have thus got them all together in one view, I endeavor to estimate their respective weights; and where I find two, one on each side, that seem equal, I strike them both out. If I find a reason pro equal to some two reasons con, I strike out the three. If I judge some two reasons con, equal to three reasons pro, I strike out the five; and thus proceeding I find at length where the balance lies; and if, after a day or two of further consideration, nothing new that is of importance occurs on either side, I come to a determination accordingly.

Perspectives and historical views for the MCDM/A may be found in several texts (Koksalan et al. 2011; Edwards et al. 2007).

The procedure most frequently applied for aggregating criteria is the additive model, also called the 'weighted sum' model, which is introduced below, in which the global value ($v(x_i)$) is considered for a consequence vector $x_i = (x_{i1}, x_{i2}, \ldots, x_{in})$, for the alternative i, which is the same as the global value $v(a_i)$ for alternative a_i, which has the consequence vector x_i.

$$v(x_i) = \sum_{j=1}^{n} k_j v_j(x_{ij}) \tag{13.1}$$

where:

x_{ij}	is the consequence or outcome of alternative i for criterion j.
$v_j(x_{ij})$	is the value of consequence for criterion j, for alternative i.
k_j	is the scale constant (weights) for attribute or criterion j ($k_j > 0$), which is usually normalized as follows:

$$\sum_{j=1}^{n} k_j = 1. \tag{13.2}$$

13.2 Basic Ingredients in a Multicriteria Problem

As to the modeling approach for building an MCDM/A model, some basic ingredients are subsequently highlighted that should be applied in the decision process for choosing a voting procedure (DPVP). One of them is the set of alternatives, which consists of the VPs to be evaluated for that particular decision process for the business organization (DPBO). In a business context, decision problems usually have a set of alternatives that is offered to the DM, which consists of a discrete set of elements a_i,. This set is represented by $A = \{a_1,\ a_2,\ a_3, \ldots, a_m\}$.

The set of alternatives is associated to the concept of problematic. This is related to the kind of analysis that is to be made of the set of alternatives and therefore, to the format of the recommendation to the DM. A few types of problematic are considered in the literature (Roy 1996; Belton and Stewart 2002), the most relevant two for this text being the problematic of choice and that of ranking. For the latter, the final result consists of ranking all the elements a_i in the set of alternatives.

In the choice problematic, the solution is a subset of chosen alternatives. It would be preferable to have only one alternative in this subset, which corresponds to optimization; a particular situation in this problematic. Nevertheless, it may happen that the procedure applied is not able to achieve optimization. Thus, in this subset, there is more than one alternative, which should be considered non-comparable with each other. Nevertheless, in the end, only one alternative is to be implemented.

Apart from the set of alternatives, another basic ingredient is the consequence x in the decision problem, which leads to a set of consequences X. This set X connects to concepts related to the family of criteria and matrix of consequences.

The set of consequences consists of the possible outcomes that the DM can obtain, after choosing an alternative. Consequences are directly linked with the objectives in the decision problem. A consequence is the result obtained by the DM after implementing a chosen alternative.

In a MCDM/A problem, there is a set of possible consequences for each of the multiple objectives. Therefore, a vector of consequences $x = (x_1,\ x_2, \ldots x_n)$ is considered, where x_j is the consequence for the criterion j.

Therefore, the alternatives in set A are evaluated by the consequences they can provide to the DM. In fact, the choice is made from among the consequences. These consequences are evaluated by the DM in order to make explicit what the DM's preference structure is over the consequence space. A decision model will recommend an alternative based on this preferential information over the set of consequences. Thus, for each alternative I, a possible consequence x_{ij} can be obtained for the criterion j.

This leads to compiling the matrix of consequences as illustrated in Table 13.1, in

Table 13.1 Consequence matrix

A	Criterion 1	Criterion 2	Criterion 3	…	Criterion j	…	Criterion n
A_1	x_{11}	x_{12}	x_{13}	…		…	x_{1n}
A_2	x_{21}	x_{22}	x_{23}	…		…	x_{2n}
…	…	…	…	…	…	…	…
a_i				…	x_{ij}	…	…
…	…	…	…	…	…	…	…
a_m	x_{m1}	x_{m2}	x_{m3}	…	…	…	x_{mn}

which there is a specific outcome x_{ij}, for each combination of criterion and alternative.

As shown in Table 13.1, since the value of each consequence $v_i(x_i)$ for a given alternative a_i can be obtained, then, the value of alternatives $v_i(a_i)$ can be found.

A dominance relation D between two alternatives a_k and b_l is defined as follows: a_k dominates a_l $(a_k D a_l)$, if $v_j(a_k) \geq v_j(a_l)$, for all $j = 1, 2, 3, \ldots, n$, and for at least one of the criteria j, the inequality is strict (>).

If the dominance relation applies between two alternatives, there is no need to use an MCDM/A method for aggregating all criteria j in order to compare them. Rarely can a solution be found only by applying the dominance relation to all alternatives in the set and therefore, generally speaking, an MCDM/A method is required.

In general, preference binary relations are applied in order to represent the DM's preferences in a preference modeling process. The following basic preference relations can be considered for the subsequent explanations:

- Indifference (*I*)—*zIy* means that the DM is indifferent between the two consequences *z* and *y*.
- Strict Preference (*P*)—*zPy* means that the DM clearly prefers *z* to *y*.
- Weak Preference (*Q*)—*zQy* means that *z* is at least as preferable as *y* to the DM. In other words, the DM can find that *zPy* or *zIy*.
- Incomparability (*J*)—*zJy* means that the DM is not able to compare the two elements.

13.3 Classifying MCDM/A Methods

MCDM/A methods may be classified in several ways, one of which is to do so according to the nature of the set of alternatives. Since the set of alternatives that is taken into account in the kind of problem analyzed here is a discrete set, only discrete methods are considered. This means that making use of multiobjective mathematical programing approaches is excluded.

One of the classifications for methods most found in the literature (Roy 1996; Vincke 1992; Belton and Stewart 2002; Pardalos et al. 1995; de Almeida et al. 2015) includes two kinds of methods, when only discrete methods are considered:

- Unique criterion of synthesis methods;
- Outranking methods.

These kinds of methods could be classified in another way, in accordance with their rationality, namely, as being either compensatory or non-compensatory (de Almeida et al. 2015). However, it is worth noting that 'unique criterion of synthesis methods' are compensatory, while 'outranking methods' are non-compensatory.

In the unique criterion of synthesis methods, the criteria are aggregated by an analytical model in such a way that a global score for a consequence or alternative is produced. These methods synthesize all the criteria in a unique criterion (global evaluation). They can deal with either deterministic consequences or probabilistic consequences. The former is the scope of Multi-Attribute Value Theory (MAVT) and the latter, Multi-Attribute Utility Theory (MAUT) (Keeney and Raiffa 1976).

The additive model for aggregating criteria, shown in (13.1), is the most commonly applied model for this kind of method within the scope of MAVT or MAUT (Keeney 1992). It is the aggregating model for many additive methods (Keeney and Raiffa 1976; Vincke 1992; Belton and Stewart 2002), especially in MAVT, examples include SMARTS, FITradeoff, MACBETH, AHP, TOPSIS, etc. Even swaps (Hammond et al. 1998).

These methods can be applied if the DM's preference is compatible with the preference structure (P, I), and they can produce a complete pre-order.

Outranking methods usually do not produce a unique criterion of synthesis. Therefore, these methods can produce recommendations with no scores for alternatives. These methods consider the incomparability relation and are compatible with the DM's preference structure (P, Q, I, J). In this case they can produce a partial pre-order. The main methods in this group (Vincke 1992; Belton and Stewart 2002; Polmerol and Barba-Romero 2000) are the ELECTRE (Roy 1996) and PROMETHEE (Brans and Vincke 1985) methods.

As already mentioned, these two kinds of methods differ according to the DM's rationality with regard to the compensation of criteria. These concepts may be rather important in choosing a MCDM/A method as shown in de Almeida et al. (2015). However, how best to analyze this is seldom discussed in the literature (de Almeida et al. 2015). Bouyssou (1986) and others (Munda 2008; Roy 1996; Vincke 1992) have commented on the concepts related to compensation and non-compensation.

The compensation concept between two criteria clearly includes the notion of a tradeoff between these two criteria (Bouyssou 1986). On the other hand, as to non-compensation, there is no tradeoff between the criteria. Additionally to outranking methods, a lexicographical procedure is non-compensatory and illustrates this absence of non-compensation between criteria.

A preference relation P, for comparing consequences x, y, z and w, is non-compensatory if the preference between them only depends on the subset of criteria in favor of x and y (Fishburn 1976). If the DM has a non-compensatory rationality, it does not matter what the level of the performance of x or y in each criterion is. The only information necessary is to know if one is higher or lower than the other, which is directly associated with the strict preference relation P, as follows.

Let $P(x, y) = \{j{:}x_j P_j y_j\}$. That is, $P(x, y)$ is the collection of criteria for which $x_j P_j y_j$. Then, for a non-compensatory rationality (Fishburn 1976):

$$\left\{\begin{matrix} P(x, y) = P(z, w) \\ P(y, x) = P(w, z) \end{matrix}\right\} \Rightarrow [xPy \Leftrightarrow zPw] \tag{13.3}$$

That is, in the decision matrix, the only information needed is whether $v_j(x_j) > v_i(y_j)$ or otherwise. This would mean that the $x_j P y_j$. or otherwise, respectively.

On the other hand, it is essential, for a compensatory relation P, to know the level of performance $(v_i(x_j))$ of x_j for criterion j. This is related to the process by which the DM on using a compensatory rationality evaluates how a disadvantage in one criterion may be compensated for by an advantage in another criterion.

The additive model in (13.1) illustrates this notion very well. In order to maintain the same global score in (13.1), if the performance in one of the criteria decreases, the performance in one of the other criteria should be increased. It is valuable to note at this moment what the meaning of the scale constants ($\mathrm{k_j}$) of criteria is. It is not about the degree of importance of the criteria, as one could intuitively think. The scale constants ($\mathrm{k_j}$) indicate in (13.1) by how much the performance of a criterion should increase in order to make the compensation needed to keep the same global score.

Therefore, it is easy to cope with situations in which compensatory rationality applies, when analyzing consequences with multiple criteria. However, for non-compensatory rationality this may be not easy at the beginning, although several real situations may illustrate the use of a non-compensatory rationality. Many of them are found in sports and some of them may be found in voting systems. Examples of these are given below.

Let us visualize a volley-ball game, in which a non-compensatory rationality is applied. That is, the number of sets a team has won indicates the final result. If the total number points were used to determine who has won, that would be a compensatory rationality, similar to the additive model (de Almeida et al. 2015).

In the volley-ball game a non-compensatory rationality is applied. The criteria are represented by the sets, each of which has the same weight. For instance, Table 13.2 shows the results of a volley game between teams A and B (de Almeida et al. 2015). Team A is considered the winner, since it wins three sets and team B wins only two sets. The number of points each team gets in each set is irrelevant. Whoever wins the set gets the whole value of the set. However, in the example, team A wins a total of only 93 points, while team B wins 104 points. If a compensatory rationality were applied, Team B would be the winner, since its total of points is greater than the total points of team A.

Munda (2008) calls attention to an evaluation of students on a course, considering grades in a scale from 0 to 10. This kind of evaluation usually is a compensatory

Table 13.2 A volley-ball game—compensatory or non-compensatory evaluation

Team	*A*	*B*	Set won by
Set 1	25	23	*A*
Set 2	25	20	*A*
Set 3	11	25	*B*
Set 4	17	25	*B*
Set 5	15	11	*A*
Total points	93	104	

procedure. For instance, a student could compensate a grade 4 received for mathematics, with a grade 10 in language, and thus pass the final evaluation. However, an evaluation system could use a non-compensatory rationality, if the system did not wish to allow this kind of compensation amongst different subjects. For instance, each student could be required to have a minimal performance in each subject.

Voting systems may be the source of interesting examples in this regard. A presidential election in the United States of America (USA) is a case in point (de Almeida 2015). In 2016, the USA presidential election gave an interesting result which illustrates this subject. In that system, the candidate has to get 'electoral votes', which are based on winning individual states. Each state has a certain number of 'electoral votes' that are assigned to the candidate who was chosen by the majority of voters in that state. No matter the number of voters in a given state or the winner's margin of victory in that state, whether this is by one vote or by hundreds of thousands of votes, the winning candidate receives all that state's 'electoral college votes'. In 2016, in order to win, a candidate needed 270 'electoral votes'. Candidate Trump got 306 of these votes, while Clinton got only 232. On the other hand, Trump obtained 46.09% of the voters in his favor, while Clinton had 48.18% which means that Clinton 'won' the popular vote by approximately 2.9 million votes. Trump won the election because this decision system uses a non-compensatory rationality. Otherwise, Clinton would have won the election, since she won the popular vote. This is similar to the example above about the volley-ball game, illustrated in Table 13.2.

In the US Presidential voting system, each state (representing a criterion) has a symbolic weight, which is represented by the 'electoral votes'. These 'electoral votes' are related to the number of senators and congress representatives each state has, which is also associated to the population of the state, with some exceptions. Thus, in order to get all the 'electoral votes' (weight) of one state (criterion), a candidate needs to win the majority of the votes cast by the electors registered in that state. For instance, the state of California has 55 'electoral votes'. With 60.4% of the votes, Clinton won all these 55 'electoral votes'. And the state of Florida has 29 'electoral votes'; all of them won by Trump with a small difference of votes in that state (49.1% of the votes cast were for Trump, against the 47.8% for Clinton).

Comparing this electoral system to a multicriteria problem, the states are equivalent to criteria and the outcome for each criterion is the number of votes obtained in that state. A group of states plays the same role as a subset of criteria in an outranking method (Vincke 1992), as a coalition in favor of one of the alternatives (candidates).

In order to win a candidate has to get the best coalition of criteria (states), with the greatest summation of criteria weights ('electoral votes').

It is worth remarking that the presidential election in the USA consists of e elections, in which $e =$ number of states. These e elections are combined with a non-compensatory rationality.

There now follows a brief overview of a few discrete MCDM/A methods. First, we describe some of the methods related to the additive aggregation of criteria, which are classified as unique criterion of synthesis methods. Then, we introduce some outranking methods, which are related to non-compensatory rationality.

13.4 Methods of the Type: Unique Criterion of Synthesis

The additive model is the procedure, within the group of compensatory methods, that is most often applied in order to aggregate criteria. It may be considered in two different contexts (Belton and Stewart 2002; Keeney and Raiffa 1976; de Almeida et al. 2015): Multi-Attribute Value Theory (MAVT), Multi-Attribute Utility Theory (MAUT). This aggregating model is also called the 'weighted sum' model and is shown in Eq. (13.1).

In MAVT, the consequences are deterministic; that is, certainty is assumed. So, the performances of the consequences are assumed to be known. In MAUT the consequences are assumed to be uncertain. Thus, one may either know the probabilities for performances of the consequences in each criterion or these probabilities may be unknown. The former characterize a decision under risk and the latter, decision under uncertainty. In this text an emphasis is given to the MAVT context, since it seems to be more related to analyzing the decision process for choosing rules.

A few properties are assumed for the additive model. A complete pre-order or a complete order is assumed in the DM's preference structure. That is, the DM should be able to compare and order all consequences. The property of transitivity is also assumed for this model. These two properties are assumed for aggregating procedures in this kind of MCDM/A method, the unique criterion of synthesis method. On the other hand, an outranking method, the other kind of MCDM/A method, should not be applied.

A relevant property of the additive model is the mutual preference independence condition amongst the criteria (Keeney and Raiffa 1976), which may not be followed in other aggregating procedures of the unique criterion of synthesis kind of methods. The preference independence between two criteria Y and Z occurs if and only if

$$(y', z')P(y'', z') \Leftrightarrow (y', z)P(y'', z), \text{ for all } z,\ y' \text{and } y''.$$

That is, the preference for the whole space of Y (the marginal value function for different levels of y, e.g., y' and y''), given a level of z (let us say $z = z'$), does not depend on z level. This means that (Vincke 1992) for four consequences (a, b, c and d) with these two criteria, Y and Z are preferentially independent if the following condition holds:

$$If \left\{ \begin{array}{c} v_y(a) = v_y(b) \\ v_y(c) = v_y(d) \\ v_z(a) = v_z(c), \\ v_z(b) = v_z(d) \end{array} \right\} then\, aPb \Rightarrow cPd \tag{13.4}$$

When building a decision model, these properties should be assessed in order to validate the use of such a model. On the other hand, it has been observed that the property of preference independence is not violated in many practical situations. Moreover, Keeney (1992) points out that dependence between criteria in the DM's preference may happen when a criterion is missing from the family of criteria.

Therefore, it is important to check the properties of this model. However, this is not the main issue when building additive decision models. The main concern for this model is related to the DM's preference modeling in order to specify the scale constants, k_i. There are two main issues here: the meaning of these scale constants and how to obtain them in a consistent way.

The meaning of the scale constants (or weights) is related to substitution rates between the criteria (Keeney and Raiffa 1976; Vincke 1992; Belton and Stewart 2002). This issue is well explored by by Keeney and Raiffa (1976) and Keeney (1992). A common mistake is to associate the meaning of a criterion weight with its degree of importance. Actually the name 'weight' may induce this misconception. Maybe the name of scale constants would be more appropriate (de Almeida et al. 2015). Keeney and Raiffa (1976) call attention to the possibility of a criterion having a scale constant larger than another and being of less importance. Also, changing the normalization procedure for consequences with a linear value function v(x) and using different scales, such as a ratio or an interval scale, implies that new values for the scale constant of criteria should be computed. Although this is necessary for the additive model, it is not needed for other methods

Furthermore, behavioral studies have shown the possibility of there being many inconsistences in the elicitation process with the DM in order to obtain the scale constant (Weber and Borcherding 1993). For this reason, although there are many MCDM/A methods based on the additive model in the literature, most of them differ from each other only in the elicitation procedures for obtaining the scale constants. These methods include: SMARTS (Simple Multi-Attribute Rating Technique with Swing) proposed by Edwards and Barron (1994); AHP (Analytic Hierarchy Process) (Saaty 1980); MACBETH Macbeth (Measuring Attractiveness by a Categorical Based Evaluation Technique) (Bana and Costa et al. 2005). Each of these methods is based on one of the basic elicitation procedures (for the scale constants). Amongst these procedures are the swing and the tradeoff procedures which are briefly described below and more fully in de Almeida et al. (2015).

Keeney and Raiffa (1976) presented the tradeoff procedure in detail and Weber and Borcherding (1993) considered this procedure as the one with the strongest theoretical foundation.

This is an algebraic procedure (Weber and Borcherding 1993), since the k_j are calculated from a simple system of equations, including Eq. (13.2). The other equations are based on a set of the DM's $n-1$ judgments over the consequence space. Since the scale constants k_j are not elicited directly from the DM, this procedure is also classified as an indirect procedure. Thus, the scale constants k_j are calculated using information given by the DM regarding the consequence space.

This information is obtained by asking the DM structured questions (Keeney and Raiffa 1976) which he/she answers, thereby identifying consequences which are preferentially indifferent to each other; i.e. trade-offs are identified. This indifference implies the need for an equation, since the value of these two consequences are equivalent.

Another elicitation procedure is called 'swing', which is used by many methods, such as the SMARTS method (Edwards and Barron 1994). Since the scale constants are based on direct information that the DM is asked for, this procedure is classified as being a direct one (Weber and Borcherding 1993). However, it should be noted that this direct information takes into account the range of the consequences, thus avoiding the usual mistake of sampling which is to regard the scale constants as being ratios that represent the degree of importance of criteria.

There is also a sequence of structured questions in this procedure (Edwards and Barron 1994). The first question considers that all criteria have the worst consequence, and the DM is asked to choose only one criterion and to improve its outcome from the worst to the best outcome; that is, to 'swing' from the worst to the best outcome. The chosen criterion should be that with the greatest value of k_j. Then, other similar choices are made in order to identify the ranking of k_j. In the next step of the procedure, the criterion with the largest k_j, is arbitrarily assigned a value of 100. This value acts as a reference for percentages, so that when points are assigned to the other criteria, they express percentages of k_j ranked first. In this case the value of k_j considers the range of each criterion. At the end, in order to produce the final scale constants, these percentages are normalized.

Possible inconsistencies of these elicitation procedures have been evaluated by behavioral studies. For instance, Borcherding et al. (1991) have reported that inconsistencies arise in 50 and 67% of cases, for swing and tradeoff procedures, respectively.

In the tradeoff procedure, the adjustment the DM has to make, in order to obtain two consequences that are preferentially indifferent, is considered to be a critical judgment in the tradeoff procedure and may easily lead to inconsistences. The FITradeoff (Flexible and Interactive Tradeoff) method is based on this procedure (de Almeida et al. 2016) and avoids these adjustments for indifferences by the DM, which ensures this procedure leads to more consistent results and yet it is based on the strongest theoretical foundation. In FITradeoff, the DM does not have to identify consequences with preferential indifferences. Instead, the DM compares consequences and has just to identify which one of them is preferable. This leads to inequalities that are applied as constraints in Linear Programing Problems which are structured to identify Potential Optimal Alternatives (POA). At each question answered, the FITradeoff calculates the current POA, with this partial information. By using these inequalities, the algebraic characteristic of the procedure is maintained. Amongst the many flexibilities of the model, the DM may skip some questions and yet, the method is able to carry on the process in order to identify the best alternative, according to the DM's preference. This method has a Decision Support System (DSS), available for free at www.fitradeoff.org. In order to increase the confidence of the results, the FITradeoff DSS has been improved with behavioral studies using Decision Neuroscience experiments (Roselli et al. 2019).

The use of methods with partial information (Weber, 1987) in the elicitation process, such as FITradeoff, may contribute to minimizing inconsistences (de Almeida et al. 2016). Other advantages of a process with partial information are that this avoids time-consuming and controversial processes (Kirkwood and Sarin 1985; Kirkwood and Corner 1993) and deals with the possibility of the DM being unable to respond specifically and precisely to tradeoff questions (Kirkwood and Sarin 1985).

13.5 Outranking Methods

These methods use a non-compensatory rationality and are completely different from the methods described in the previous section. For instance, in this method, a preference relation of incomparability is allowed in the DM's preference structure.

In outranking methods, the first two properties mentioned for the additive model are not assumed. Therefore, if the DM is not able to compare all consequences and order them, these methods may nevertheless be applied. Also, the transitivity property may not be followed. Consequently, these methods may be able to produce only partial pre-orders.

An important difference between outranking methods and those of the unique criterion of synthesis concerns the meaning given to the criteria weights. For the former, criteria weights mean the degree of importance of the criteria, which can be perceived when analyzing the mathematical structure of those methods and because of how the weights are used.

Amongst the methods in this group we briefly describe two of the most applied:

- ELECTRE (Elimination Et Choix Traduisant la Réalité)—(Roy 1996; Vincke 1992); and
- PROMETEE (Preference Ranking Organization Method for Enrichment Evaluation)—(Brans and Vincke 1985).

Pairwise comparison amongst the alternatives is a common feature of these methods. These pairwise comparisons are not made by the DM but rather the outranking relations between all pairs of alternatives in the set of alternatives are explored.

The notion that the meaning of weights is about the degree of importance of criteria is usually contextualized with a voting process and the weights are compared with votes (Roy 1996; Vincke 1992). The notion of coalition is an interesting feature in these methods. Let us look at this notion of coalition of criteria in order to compare two alternatives a and b. Consider two subsets of criteria G and H so as to compare them. Let us assume that the subset of criteria in G has weights that sum up to a greater value (and so are more important; or combine more votes) than the weights of criteria in the subset in H. If the following conditions hold (Vincke 1992): a is better than b in subset G, b is better than a in subset H, and a and b are indifferent for any other criteria, then: a is globally better than b. This means that the criteria in favor of a have a summation of weights that is greater than the weights for criteria in favor of b. In other words, a has a better coalition of criteria than b.

Two main steps characterize these methods (Roy 1996; Vincke 1992): Building the outranking relation and Exploiting this outranking relation.

The outranking relation is built by making the pairwise comparison for all the set of alternatives. Let us represent the outranking relation by S, and consider applying it to a pair of alternatives a and b. Then, aSb means that a outranks b. This indicates that a is at least as good as b.

These outranking relations are exploited by applying a procedure in order to find recommendations according the problematic in question.

In the ELECTRE methods, the outranking relation aSb, between two alternatives a and b, is based on concordance and discordance concepts, about which the DM gives preference information in the form of thresholds.

The family of ELECTRE methods includes several methods, which differs from the problematic and the kind of criteria (Roy 1996; Vincke 1992; Belton and Stewart 2002; Figueira et al. 2005). Two of these methods are of interest here, since they are related to the choice problematic: ELECTRE I (considering true criteria) and ELECTRE IS (considering pseudo-criteria). However, if the context requires a different problematic to be applied in order to analyze the VPs, then another method may be applied, such as the ELECTRE III for the ranking problematic.

In the ELECTRE methods, the outranking relation between two objects a and b (aSb) is established considering concordance and discordance concepts. The former concept indicates how much the coalition of criteria supports an outranking relation S between two alternatives. If the outranking relation is supported by the former, then, the discordance is applied in order to evaluate other issues and it may disagree with this outranking relation. Therefore, the following indices are applied in order to evaluate the outranking relation aSb: the concordance index $C(a, b)$ and the discordance index $D(a, b)$.

The concordance index $C(a, b)$ is given based on the summation of the weights of criteria in favor of a, as follows:

$$C(a,b) = \sum_{j:v_j(a) \geq v_j(b)} w_j$$

$$\text{with} \sum_j w_j = 1, \text{ for normalization of weights.} \tag{13.5}$$

where: w_j is the weight for criterion j; and $v_j(a)$ and $v_j(b)$ are the values, respectively of alternatives a and b, for criterion j.

Different discordance indices ($D(a, b)$) are proposed by (Roy 1996; Vincke 1992; Belton and Stewart 2002). Let us consider the following:

$$D(a, b) = \max\left(\frac{v_j(b) - v_j(a)}{\max[g_{vj}(c) - v_j(d)]}\right), \quad \forall \mathrm{j}|v_j(b) > v_j(a); \forall j, c, d. \tag{13.6}$$

The DM has to specify threshold levels for both indices; let us say, c' for concordance and d' for discordance. These threshold levels let the outranking relation *S* be built. Therefore *aSb* can be established as follows:

$$aSb \text{ if and only if } \begin{cases} C(a, b) \geq c' \\ D(a, b) \leq d' \end{cases} \tag{13.7}$$

It may happen that for a pair of alternatives, there is a simultaneous outranking, such that *aSb* and *bSa*. This represents a circuit in a graphical representation of these relations and for ELECTRE I, these alternatives are considered indifferent.

Once the outranking relations are built for all pairs of alternatives, by applying (13.7) with the parameters informed by the DM, then, the step of exploiting the outranking relation can be applied. In this step, a subset of alternatives, called the kernel is obtained in the ELECTRE I method. The kernel consists of the subset of alternatives that are not outranked by any other in the kernel. The kernel is the solution for the choice problematic. If the kernel has only one alternative, this problematic has the same result as an optimization. If the kernel has more than one alternative, this means that those alternatives in the kernel are incomparable.

The PROMETHEE family of methods is based on a valued outranking relation (Brans and Vincke 1985; Vincke 1992; Belton and Stewart 2002). In these methods, the information regarding concordance and discordance are not applied. Therefore, the DM has to provide weights of criteria and information on the indifference or preference thresholds regarding the evaluation of intra-criteria, if any of these thresholds are taken into account.

An outranking degree of *a* over *b,* which is denoted by $\pi(a, b)$, is obtained for each pair of alternatives *a* and *b*, in order to move forward to the step for building the outranking relation. $\pi(a, b)$ is obtained as follows:

$$\pi(a, b) = \sum_{j=1}^{n} w_j F_j(a, b) \tag{13.8}$$

The weights w_j are normalized, just as in the ELECTRE method. $F_j(a, b)$ is a function that informs the relation between the performance of outcomes for alternatives in criterion *j*. In the usual case, $F_j(a, b) = 1$, if $v_j(a) > v_j(b)$, which would contribute to *a* outranking *b*; otherwise, $F_j(a, b) = 0$. In the usual case, indifference or preference thresholds are not applied for criterion *j*. Therefore, the outranking degree, $\pi(a, b)$, for $F_j(a, b) = 1$ is the summation of the weights of those criteria, in the usual case, in which $v_j(a) > v_j(b)$.

Regarding other options for the function $F_j(a, b)$, the method has six different patterns, including the one explained above. In the other forms for $F_j(a, b)$, indifference or preference thresholds, or both, are considered. In these cases, $0 < F_j(a, b) > 1$, depending on the difference $[g_j(a) - g_j(b)]$. If this difference is in the range of these thresholds, then, a partial value of the weights is added to the outranking degree $\pi(a, b)$.

The information to be given by the DM regarding the evaluation of intra-criteria is the choice of the form for $F_j(a, b)$, and if applicable, the specification of indifference and preference parameters of thresholds, for each criterion j.

A matrix $\pi(a, b)$ is produced as result of this first step. In the next step, each alternative a is evaluated by exploiting the outranking relation, based on the outranking degree $\pi(a, b)$. The evaluation of these alternatives is based on the outgoing flow $\phi^+(a)$ and on the incoming flow $\phi^-(a)$, as follows:

$$\phi^+(a) = \frac{1}{n-1} \sum_{b \in A} \pi(a, b) \tag{13.9}$$

$$\phi^-(a) = \frac{1}{n-1} \sum_{b \in A} \pi(b, a) \tag{13.10}$$

Since n is the number of criteria, the division by $(n-1)$ in (13.9) and (13.10) implies there is a normalized scale between 0 and 1, for both flows.

The outgoing flow sums up the outranking degree of a over b. for all b, thereby indicating the advantage of the alternative a over all other alternatives (b) in the set of alternatives. The incoming flow sums up the outranking degree of b over a, for all b and thus represents the disadvantage of the alternative a compared with all other alternatives.

The combination of these two indices in (13.9) and (13.10), is applied in the PROMETHEE I method in order to compare all pairs of alternatives and to obtain preference relations (P), indifference (I) and incomparability (J) between them, and thus to produce two pre-orders. Then, a partial pre-order of all alternatives is obtained. (Brans and Vincke 1985; Belton and Stewart 2002).

The PROMETHEE II method uses another index for evaluating the alternatives, namely, the net flow $\phi(a)$ as follows:

$$\phi(a) = \phi^+(a) - \phi^-(a) \tag{13.11}$$

The liquid follow is given in a scale of -1 to 1 and the PROMETHEE II produces a complete pre-order of the set of alternatives.

13.6 Choosing an MCDM/A Method

Choosing a proper MCDM/A method for a particular decision problem is not a topic often found in the literature. On the other hand, this might be changing. The growing interest in using and developing hybrid methods as well as making adaptations in classical methods, has been noteworthy. This may be related to concerns with building methods that have more realistic assumptions, which would be more appropriate for the problem to be tackled. Some studies are concerned with choosing a method (Roy and Słowinski 2013; de Almeida et al. 2015).

There are a few issues to be considered when choosing an MCDM/A method. These include: the DM's preference structure; characteristics of the decision problem itself; contextual features related to the decision problem. The latter may include organizational aspects, and the time available for making the decision. Analysts should avoid using their own preference for choosing a method. This could raise important ethical considerations as discussed in de Almeida et al. (2015).

A framework for building an MCDM/A model, presented in de Almeida et al. (2015), includes how to choose a method. Detailed considerations are given in this regard, which is one of the twelve steps in the process of building a decision model. The main information to be considered is the DM's preference structure.

One of the items to be evaluated is how the DM's preferences fit in with regard to compensatory and non-compensatory rationality. Simon (1955) was one of the first to draw attention to this issue, and before the development of most MCDM/A methods. Bouyssou (1986) presents a discussion on the notion of compensation and non-compensation. Vincke (1992) remarks that choosing an MCDM/A method is equivalent to choosing the type of compensation between criteria. For instance, choosing an additive method is the same as choosing the tradeoff between performances on criteria. Roy and Słowinski (2013) have put forward the question "Is the compensation of bad performances on some criteria by good ones on other criteria acceptable?". This reveals their concern with this issue.

13.7 Challenges in MCDM/A Methods

Challenges in multicriteria decision methods have been pointed out that (de Almeida et al. 2018) considering the question in the previous section, which is related to the process of building decision models, since it includes choosing the most appropriate method. Although some frameworks may be found for building multicriteria models (Polmerol and Barba-Romero 2000; Belton and Stewart, 2002; de Almeida et al. 2015) this topic remains a challenging one.

The techniques that allow the use of partial information (or imprecise/incomplete information) for modeling DM preferences has been proved to be an enrichment for MCDM/A methods. First, surrogate weights has been applied (Edwards and Barron 1994) and still remains as a competitive approach (Morais et al. 2015; Danielson and Ekenberg 2017). There are methods using decision rules, simulation, or linear programing in order to reduce the space of criteria weights, while analyzing the set of alternatives (Weber 1987; de Almeida et al. 2016).

Multi-Criteria Group Decision Making (MCGDM) is another challenge related to the use of these methods. Since, for an individual DM the preferences modeling process is complex task in order to aggregate multiple objectives, adding to this the aggregation of multiple DMs is even more challenging.

All these mentioned questions are relevant issues to be improved in order to help organizations to find consistent decision models.

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Chapter 14
An MCDM/A Framework for Choosing Rules

Abstract Our focus is on the decision process. A Framework for the DPVP (decision process for choosing a voting procedure) is necessary in order to guide how best to aid DMs. It is assumed that DMs may evaluate the impact of VP (Voting Procedure) properties on their own business decision process. It is assumed that the DMs have agreed on some voting procedure. Choosing the most appropriate MCDM/A (Multi-Criteria Decision Making/Aiding) method is essential to ensure the quality of the decision process. When choosing an MCDM/A method, the DM's preferences should be taken into consideration. A check needs to be made on whether the DM uses compensatory or non-compensatory rationality.

14.1 Introduction

The decision context of a business organization is discussed and a framework is built to deal with the decision process of choosing a VP. This framework is included in an MCDM/A model that can aid this choice.

This framework considers preliminary ideas (de Almeida and Nurmi 2015; de Almeida and Nurmi 2014) for aiding the choice of a voting procedure (VP), by using an MCDM/A model and considering a business organization. As to implementing the framework, it is assumed that an analyst will give some methodological and technical support to the DM.

The framework guides how best to structure the elements to be considered in the process, which include: the MCDM/A method to be used, the criteria to be applied and what the outcomes of these voting procedures for these criteria are likely to be. These elements are not prior specified in a general way, but rather, they will be consolidated according to the business context under analysis.

In the DPVP, it is assumed that there is either: only one DM (a benevolent dictator or a supra-DM) or a group of DMs acting in agreement. In the latter, one of the DMs may be appointed to act on behalf of the others. Otherwise, a more complex group decision process would be conducted and the way in which the DMs interact in order to choose the VP in the DPVP may be an important issue. This situation is discussed in the last section of this chapter.

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14.2 The Framework for the DPVP

As explained in a previous chapter, the DPVP takes place in a phase at the beginning of the whole decision process and is when an MCDM/A method is used to choose a VP. This decision model is built for the particular characteristics of the business context and decision under analysis.

The decision model is built based on a framework presented in this chapter, which is founded on a more general procedure for building MCDM/A decision models (de Almeida et al. 2015), which has three phases: a preliminary phase, preference modeling phase, and a finalization phase.

In the first phase, which has five steps, the basic elements of the decision model e.g. the objectives, the criteria and the set of alternatives are established. The first step is used to identify the DM and other actors in the decision process and whether or not the situation is a group decision problem.

The phase for preference modeling has three steps and includes choosing the MCDM/A method. In the last phase, the alternatives are evaluated, a sensitivity analysis is made and a recommendation is put forward.

The framework for the DPVP is shown in Fig. 14.1. All the steps of the DPVP must be focused on the DPBO (Decision Process for the Business Organization), as it is this which determines the whole decision process.

In the first step, VPs, including those that are technically appropriate for the business decision process, are pre-selected and placed in a set. Most recognized VPs could be included in this step, while those excluded are mainly those that may be incompatible in some way with the decision process. For instance, a VP may require data to be input which it is not feasible to provide for the business decision process.

The second step consists of establishing the criteria, which are associated with the DM's objectives in choosing a VP, including the paradoxes and desirable properties of VPs. Then the consequence matrix and decision matrix are built, since the alternatives (VPs) and criteria are given. The following step consists of choosing the MCDM/A method to be applied for analyzing these VPs according to the criteria given. What MCDM/A method to choose is an important issue to be considered (Roy and Słowinski 2013; de Almeida et al. 2015). The finalization is obtained with the steps for parameterizing and applying the MCDM/A model.

The dashed frame in the middle of Fig. 14.1 shows who makes the decision in each step. Usually the DM gives preference information to be included within the model. The analyst has also to make technical choices; for instance when pre-selecting VPs. There are some steps in which instead of decisions, only technical actions are taken in the modeling process; for instance for building the consequence matrix.

The interactions between the analyst and the DM (or a group of DMs) are shown in Fig. 14.1. The dashed frame on the right side of Fig. 14.1 shows how the analyst supports the decision process. In most of the steps, the analyst conducts a structuring or modeling activity. Some of these activities involve interaction with the DM, e.g. establishing criteria or building the decision matrix.

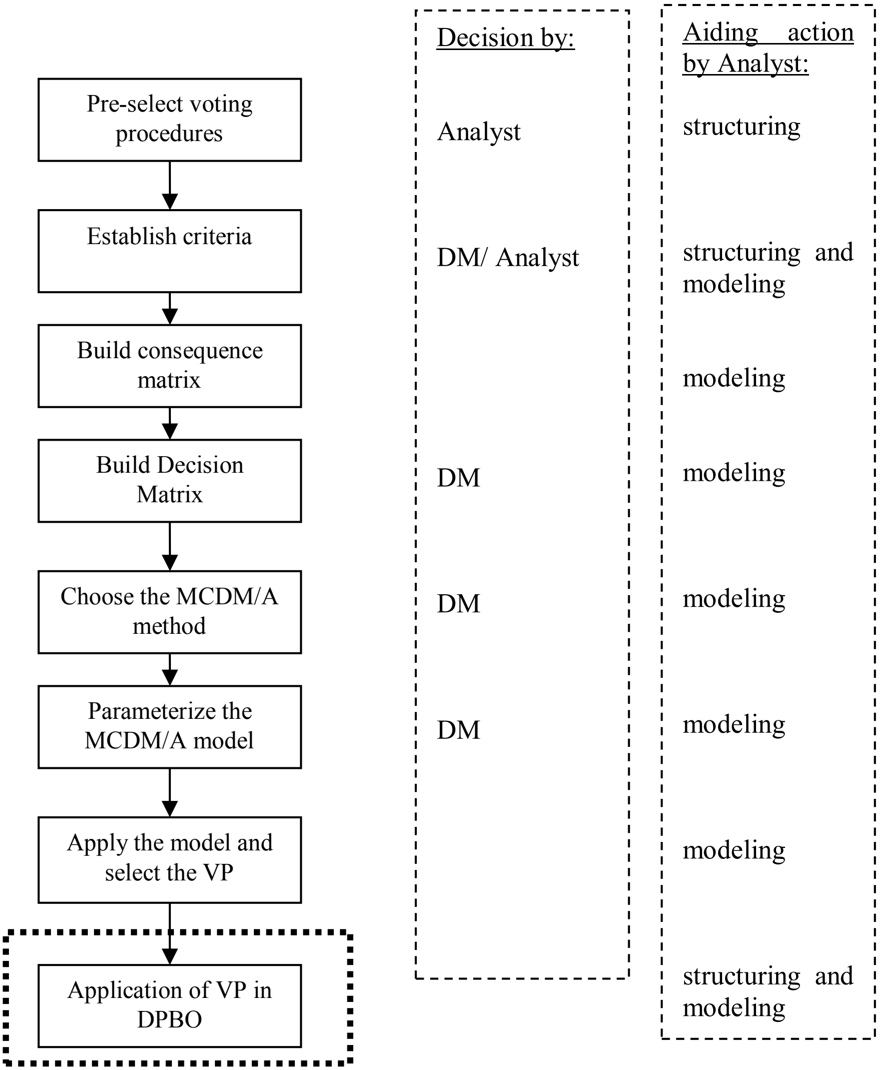

Fig. 14.1 Framework for DPVP—choosing the voting procedure

For the step of pre-selecting voting procedures, many of the considerations given in Part I and II of this book are applied. The main issue to emphasize at this moment is the importance not only of having regard to these but especially also of taking into account the business context of the decision to be made.

The following sections give a few details about other steps of the framework.

14.3 Criteria for Selecting a Voting Procedure

In the DPVP, there are two kinds of criteria to be considered, which are related to two main objectives directly associated with the context of the business decision problem:

- Maximizing the full use of VP's properties that are desirable and appropriate to the DPBO;
- Maximizing the matching between the nature of input required by the VP and its impact on the DPBO.

The latter objective is directly related to the DM's interaction with the DPBO and the way in which the DM supplies information to be processed in the VP. This objective produces a criterion called the 'Input criterion'.

The first objective concerns the characteristics of VPs, associated with their properties and paradoxes, that may have a relevant impact and the way they affect the DPBO. The property or the paradox of each VP may be considered a criterion and the consequence matrix indicates how this criterion affects each VP. This objective produces a criterion called the 'property criterion'.

14.3.1 Criteria Related to the Properties of VPs

The results obtained from Social Choice Theory have shown compatibility issues for choice desiderata. So, a set of intuitively plausible principles of choice has been considered. The incompatibility captured by Arrow's Impossibility Theorem (Arrow 1963) is one of the main issues to be considered in this matter of choosing a VP and becomes an important restriction on establishing preference profiles.

A few studies regarding the analysis of properties and paradoxes of most VPs are reported in the literature (Nurmi 1983, 1987, 2002; Felsenthal and Nurmi 2018). These VPs' features are relevant for choosing one of them for a specific decision problem. For this objective, the selection of a set of relevant properties of the preselected VPs may be a set of criteria to be considered. A set of these criteria is shown in Table 7.5. The set consists of the Condorcet winner criterion; the Condorcet loser criterion; the strong Condorcet criterion; monotonicity; Pareto; consistency; Chernoff property; independence of irrelevant alternatives; and invulnerability to the no-show paradox. A wider set of this kind of criteria is introduced and discussed in Sect. 7.3.

There are several criteria for comparing VPs, one of which is particularly important for the business context, since it can be associated with rationality, which is Pareto optimality. It is the collective rationality criterion. It states that an alternative y is not chosen, if each DM strictly prefers alternative x to alternative y. If a VP fails on Pareto optimality, this means that it is collectively irrational.

The Condorcet winner has usually been indicated for social choice rules as a reasonable desideratum. It dictates that an alternative that defeats all the others in pairwise majority comparisons should be chosen. On the other hand, commonly, a VP that satisfies the Condorcet winner criterion, does not satisfy the positional dominance criterion (Fishburn 1982), which is another acceptable criterion. It states that alternative x positionally dominates alternative y, if for each possible rank r, the number of DMs assigning x to rank r or higher is larger than the number of DMs assigning y to rank r or higher. The positional dominance criterion indicates that an alternative may not be chosen if it is positionally dominated by another alternative.

Many paradoxes of VPs have consequences that need to be evaluated with regard to their impact on the DPBO. However, when we try to avoid a paradoxical possibility, this leads to another kind of paradox. Therefore, some trade-offs can be made when dealing with paradoxes. On the other hand, given that the DM's preference in the decision process may lead to a different kind of rationality, in which non-compensatory rationality (de Almeida et al. 2015) takes place, in this kind of situation, other inter-criteria relations are considered, instead of tradeoff.

14.3.2 Criteria Related to Data Input of VP

These kinds of criteria seek to maximize the matching between the nature of input required by the VP and its impact on the DPBO. The input consists of the kind of information the DMs give about the alternatives in order to introduce a chosen VP. An input required by a VP could be, for instance, the ranking of all alternatives by each DM; or the pairwise comparison of all alternatives by each DM; or rating of all alternatives, in such a way that the DM could give a score (for instance, on a five-level scale of 1–5) for all alternatives.

Another classification of input could consider the amount of information. For instance, a VP could require full information; partial information; or the most relevant information (e.g., Approval Voting procedure).

An impact on the DPBO could be, for instance, the effort that the DMs require to make in order to produce the information required. Other aspects may affect the DPBO, such as the ease with which a particular kind of information can be provided. Therefore, the ease of making the input that a particular VP requires may be considered a criterion in this group.

The reliability of information may be also associated with the input required from DMs. For instance, it is well known that choosing alternatives from among a large number of alternatives is affected by the bounded rationality of humans (Simon 1955, 1960, 1982), particularly when the number of alternatives is greater than 7. In this case, it might be useful to consider the style of input: pairwise comparison or ranking, for instance. Another possibility is a grade based on the number of alternatives, for which an exponential function with a negative value could be applied. Also, problems of reliability may arise due to there being a large number (i.e. well over 7) of alternatives in the ranking. For pairwise comparisons, the number of options can be used as a parameter with a function of similar value.

In many circumstances in a business organization, there is a decision process for each DM, which requires another MCDM/A model to be applied to compare all alternatives. In general, this decision model is much more complex than the MCDM/A model for the DPVP. In this case, this kind of criterion may not be relevant in the DPVP, since the MCDM/A model for the DPBO will compare alternatives. The only issue is the compatibility of this information with that required by the pre-selected VPs. On the other hand, this criterion may be rather relevant with a high weight, for problems related to a group of DMs dealing with leisure choices (Naamani-Dery et al. 2014), in which case, the options are evaluated directly.

Of course, the analyst's convenience and personal objectives should not have any kind of influence on this decision process i.e. they should not be included in the set of criteria for comparing the VPs. In other words, only the DM's objectives connected to the business problem should be considered for the set of criteria.

For instance, an analyst may wish to reduce the computational complexity of the VP. However, this criterion should not be accounted for, since the analyst's preference should not be included in the model. On the other hand, if this kind of criterion has some influence on the DPBO, it could be considered. After all, the purpose of the DPVP is to improve the outcomes of the DPBO.

14.3.3 Weights for Criteria

The way in which the weights of criteria are established depends on the kind of MCMD/A method applied. In different methods, these weights have different meanings. For instance, in compensatory methods such as those with additive aggregation for criteria they mean scale constants and will change with the normalization procedure or the scale applied to the input data. For non-compensatory methods such as outranking methods, they mean degree of importance.

Therefore, the elicitation procedure for weights or preference modeling will be conducted in different ways according to the MCDM/A method applied. Also, using partial or incomplete information precise weights may not be needed (de Almeida et al. 2016). In that case, a concept of the space of weights may be applied thereby bringing more robustness to the process. Also, surrogate weights may be applied, when considering partial information (Edwards and Barron 1994).

14.4 The Consequence Matrix

Building the consequence matrix is straightforward and will produce information in the format of Table 14.1.

For the objective related to the nature of the input required by the VP, there are many possibilities for the format of data, and natural attribute (Keeney 1992) should be the first choice.

Table 14.1 Consequence matrix for discrete binary outcome

	Criteria								
Voting system	Condorcet winner	Condorcet loser	Strong condorcet	Monotonicity	Pareto	Consistency	Chernoff	Independence of IA	Invulnerability to the no-show paradox
Amendment	1	1	1	1	0	0	0	0	0
Copeland	1	1	1	1	1	0	0	0	0
Dodgson	1	0	1	0	1	0	0	0	0
Maxmin	1	0	1	1	1	0	0	0	0
Kemeny	1	1	1	1	1	0	0	0	0
Plurality	0	0	1	1	1	1	0	0	1
Borda	0	1	0	1	1	1	0	0	1
Approval	0	0	0	1	0	1	1	0	1
Black	1	1	1	1	1	0	0	0	0
Pl. runoff	0	1	1	0	1	0	0	0	0
Nanson	1	1	1	0	1	0	0	0	0
Hare	0	1	1	0	1	0	0	0	0

For the first objective, related to a VP's properties, there are two forms of doing this, namely:

- Discrete binary outcome.
- Continuous outcome.

The discrete binary outcome consists of the indication whether some property is violated by a VP. Table 14.1, which is based on Tables 7.4 and 7.5, is the consequence matrix of a type of discrete binary outcome. In that table, '1' indicates that the VP satisfies the property and '0' that it does not. This outcome in the consequence matrix is one of increasing preference; i.e. a score of 1 is preferred to a score of 0.

This kind of information imposes a great limitation on the modeling process, since the information provided is not so rich. That is, the consequence matrix informs only whether the property is satisfied or not. The continuous outcome supplies richer information.

The continuous outcome consists of providing information on how often the property is violated in that VP. If a criterion is not satisfied, we can consider how frequently a VP violates that criterion. This could be represented by the frequency of occurrence in a scale of 0–1. In this case, an indication of '1' in Table 14.1, would have a score of '0' in Table 14.2, and an indication of '0' in Table 14.1 would have a score between 0 and 1 in Table 14.2. This outcome in the consequence matrix is one of decreasing preference; i.e. a score of 0 is preferred to a score of 1. A score of 1 indicates that that property is always violated, while '0' indicates that it is always satisfied. Table 14.2 shows the consequence matrix for the continuous outcome.

In the continuous outcome more information is given, since it can represent that a drawback to a VP may not happen for some contexts of input data in a decision problem. In order to illustrate this situation, let us consider the property of the independence of irrelevant alternatives. Let us consider three different VPs: VP_1, VP_2 and VP_3 and let us make a few simplifications for the sake of clarifying how this kind of outcome may be evaluated. Assuming that for VP_1, the independence of irrelevant alternatives does not hold at all, then the outcome for $x_{18} = 1$, which means that for VP_1 this property is always violated. For VP_2, let us assume that the independence of irrelevant alternatives does hold in 50% of the cases, then $x_{28} = 0.5$. Finally, let us assume that for VP_3, the independence of irrelevant alternatives does hold for more than 90% of the cases, which is assumed to be a limit in this scale and represents a minimum frequency of occurrence, then $x_{38} = 0$. The marginal value function assigns the impact of these outcomes to the DPBO.

This kind of information is richer than that in Table 14.1 and can significantly improve the evaluation process and make it much more reliable. Information related to x_{ij} in Table 14.2 can be found in literature, although this is not available for all VP, considering all properties. Also this information depends on the number of DMs (voters) and the number of alternatives in the DPBO. Simulation studies on this matter might be required in order to complete information on those not found in the literature.

For illustrating this kind of information of x_{ij} in Tables 14.2 and 14.3 provides an example based on studies found in the literature, for the Condorcet winner criterion,

Table 14.2 Consequence matrix for the continuous outcome

Voting system	Criteria								
	Condorcet winner	Condorcet loser	Strong Condorcet	Monotonicity	Pareto	Consistency	Chernoff	Independence of IA	Invulnerability to the no-show paradox
Amendment	0	0	0	0	x_{ij}	x_{ij}	x_{ij}	x_{ij}	x_{ij}
Copeland	0	0	0	0	0	x_{ij}	x_{ij}	x_{ij}	x_{ij}
Dodgson	0	x_{ij}	0	x_{ij}	0	x_{ij}	x_{ij}	x_{ij}	x_{ij}
Maxmin	0	x_{ij}	0	0	0	x_{ij}	x_{ij}	x_{ij}	x_{ij}
Kemeny	0	0	0	0	0	x_{ij}	x_{ij}	x_{ij}	x_{ij}
Plurality	x_{ij}	x_{ij}	0	0	0	0	x_{ij}	x_{ij}	0
Borda	x_{ij}	0	x_{ij}	0	0	0	x_{ij}	x_{ij}	0
Approval	x_{ij}	x_{ij}	x_{ij}	0	x_{ij}	0	0	x_{ij}	0
Black	0	0	0	0	0	x_{ij}	x_{ij}	x_{ij}	x_{ij}
Pl. runoff	x_{ij}	0	0	x_{ij}	0	x_{ij}	x_{ij}	x_{ij}	x_{ij}
Nanson	0	0	0	x_{ij}	0	x_{ij}	x_{ij}	x_{ij}	x_{ij}
Hare	x_{ij}	0	0	x_{ij}	0	x_{ij}	v	x_{ij}	x_{ij}

Table 14.3 Consequence matrix for the continuous outcome with condorcet winner criterion for four VPs

	Criteria								
Voting system	Condorcet winner	Condorcet loser	Strong condorcet	Monotonicity	Pareto	Consistency	Chernoff	Independence of IA	Invulnerability to the no-show paradox
Amendment	0	0	0	0	x_{ij}	x_{ij}	x_{ij}	x_{ij}	x_{ij}
Copeland	0	0	0	0	0	x_{ij}	x_{ij}	x_{ij}	x_{ij}
Dodgson	0	x_{ij}	0	x_{ij}	0	x_{ij}	x_{ij}	x_{ij}	x_{ij}
Maxmin	0	x_{ij}	0	0	0	x_{ij}	x_{ij}	x_{ij}	x_{ij}
Kemeny	0	0	0	0	0	x_{ij}	x_{ij}	x_{ij}	x_{ij}
Plurality	0.21	x_{ij}	0	0	0	0	x_{ij}	x_{ij}	0
Borda	0.11	0	x_{ij}	0	0	0	x_{ij}	x_{ij}	0
Approval	x_{ij}	x_{ij}	x_{ij}	0	x_{ij}	0	0	x_{ij}	0
Black	0	0	0	0	0	x_{ij}	x_{ij}	x_{ij}	x_{ij}
Pl. runoff	0.23	0	0	x_{ij}	0	x_{ij}	x_{ij}	x_{ij}	x_{ij}
Nanson	0	0	0	x_{ij}	0	x_{ij}	x_{ij}	x_{ij}	x_{ij}
Hare	0.16	0	0	x_{ij}	0	x_{ij}	x_{ij}	x_{ij}	x_{ij}

considering 5 DMs (voters) and 7 alternatives (Nurmi 1988, 1992, 1995). Note that the Approval Voting procedure has not be considered in this particular example, since this information is not available in those references.

It might be the case that some information on x_{ij} may be neither available in the literature or there is not a possibility for making the simulations studies in order to found out the values for x_{ij}. In that case, there is a possibility of using Prior probabilities $\pi(x_{ij})$, which is obtained in the framework of Bayesian approach and consists of subjective probabilities, such as it is usually applied in Bayesian Decision Theory (Raiffa 1968; Berger 1985). Actually, $\pi(x_{ij})$ is prior probability density function over the possible 'state of nature' X_{ij} (see Berger 1985). These Prior probabilities $\pi(x_{ij})$, also known as degree of belief, are usually obtained from experts in the subject analyzed; that is the 'state of nature' in consideration.

In some situations, this prior probability might be considered more relevant than conducting simulations studies in order to obtain x_{ij}. In that case, the consequence is probabilistic and MAUT (Multi-Attribute Utility Theory) could be more appropriate than MCDM/A deterministic methods. This is more detailed in the subsequent section. However, it is also possible using an outranking method, when integrated with utility functions, in order to consider the probabilistic information, as it has been shown in de Almeida (2005) and Brito et al. (2010).

For illustrating the use of Prior probabilities $\pi(x_{ij})$, Table 14.4, adds this to Table 14.3, Condorcet winner criterion in Approval Voting procedure and also for all VPs regarding to the Independence of IA criterion. In this illustration, it can observed that the process can be conducted considering a mixing of different kind of information, such as the estimated value for x_{ij}, for some criteria and $\pi(x_{ij})$ for others.

There is an alternative for dealing with this possibility of not having either directly information about X_{ij}, or prior probability $\pi(x_{ij})$. This is the use of a simpler information from experts, which would be based on a score in a discrete scale of at least three levels (0, 1 or 2). Also, a score of five-level scale (0, 1, 2, 3, 4) could be applied, likewise the Likert scale (Likert 1932a, b). This score indicates the influence of that criterion on the VP. For the three-level scale, for instance, score '0' indicates that the VP satisfies the property and '2' that it either rarely satisfies the property or does not satisfies, and score '1' indicates that the VP may satisfy the property with a medium frequency comparatively to the score '2'.

For now, only technical information is provided in this step. The next step associates this information of consequence matrix, such as those of Tables 14.1, 14.2, 14.3 and 14.4, with its impact on the business process, in accordance with the DM's preference.

14.5 The Decision Matrix

The decision matrix provides preferential information over the outcomes in the consequence matrix. This preferential information is given by the DM, according to

Table 14.4 Consequence matrix for the continuous outcome including some prior probability density function

	Criteria								
Voting system	Condorcet winner	Condorcet loser	Strong condorcet	Monotonicity	Pareto	Consistency	Chernoff	Independence of IA	Invulnerability to the no-show paradox
Amendment	0	0	0	0	x_{ij}	x_{ij}	x_{ij}	$\pi(x_{ij})$	x_{ij}
Copeland	0	0	0	0	0	x_{ij}	x_{ij}	$\pi(x_{ij})$	x_{ij}
Dodgson	0	x_{ij}	0	x_{ij}	0	x_{ij}	x_{ij}	$\pi(x_{ij})$	x_{ij}
Maxmin	0	x_{ij}	0	0	0	x_{ij}	x_{ij}	$\pi(x_{ij})$	x_{ij}
Kemeny	0	0	0	0	0	x_{ij}	x_{ij}	$\pi(x_{ij})$	x_{ij}
Plurality	0.21	x_{ij}	0	0	0	0	x_{ij}	$\pi(x_{ij})$	0
Borda	0.11	0	x_{ij}	0	0	0	x_{ij}	$\pi(x_{ij})$	0
Approval	x_{ij}	x_{ij}	x_{ij}	0	x_{ij}	0	0	$\pi(x_{ij})$	0
Black	0	0	0	0	0	x_{ij}	x_{ij}	$\pi(x_{ij})$	x_{ij}
Pl. runoff	0.23	0	0	x_{ij}	0	x_{ij}	x_{ij}	$\pi(x_{ij})$	x_{ij}
Nanson	0	0	0	x_{ij}	0	x_{ij}	x_{ij}	$\pi(x_{ij})$	x_{ij}
Hare	0.16	0	0	x_{ij}	0	x_{ij}	x_{ij}	$\pi(x_{ij})$	x_{ij}

Table 14.5 Decision matrix

A	Criterion 1	Criterion 2	Criterion 3	…	Criterion j	…	Criterion n
a_1	$v(x_{11})$	$v(x_{12})$	$v(x_{13})$	…		…	$v(x_{1n})$
a_2	$v(x_{21})$	$v(x_{22})$	$v(x_{23})$	…		…	$v(x_{2n})$
…	…	…	…	…	…	…	…
a_i				…	$v(x_{ij})$	…	…
…	…	…	…	…	…	…	…
a_m	$v(x_{m1})$	$v(x_{m2})$	$v(x_{m3})$	…	…	…	$v(x_{mn})$

the framework shown in Fig. 14.1. The role of the analyst in this step is to aid the DM in the preference modeling process. At this point, the DM's preferences that are considered are those regarding the evaluation of the intra-criteria, and this leads to producing the marginal value $v(x_j)$ of the outcomes x_j related to criterion *j*. Table 14.5 shows how this kind of information is considered in an MCDM/A method.

Regarding the DM's preference, it may happen that a property is not satisfied and that this does not matter very much for a particular context decision. Additionally, it may happen that the value for the frequency of occurrence of violation is not a linear function. That is, the value of a frequency of 0.25 might not represent 50% of the value of a frequency of 0.50; i.e. either v(0.25) > 0.5 v(0.5) or v(0.25) < 0.5 v(0.5).

This depends on how the DM evaluates its impact in the business context. In the case illustrated above, the marginal value function would be non-linear. Actually, in most cases, it is expected that a linear function will be found (de Almeida et al. 2015).

For the discrete binary outcome, the following value function may be applied:

$$v_j(x_{ij}) = x_i \tag{14.1}$$

For the continuous outcome, which has an outcome of decreasing preference, such as that shown in Table 14.2 (and Table 14.3), the following value function may be applied, if it is linear:

$$v_j(x_{ij}) = 1 - x_i \tag{14.2}$$

Regarding a non-linear marginal value function, the literature offers many possible analytical forms (Keeney and Raiffa 1976; Belton and Stewart 2002).

Let us give an illustrative explanation of the continuous outcome, based on the example of the preceding section, for three different VPs: VP_1, VP_2 and VP_3. Applying Eq. (14.2) for the value function of x_i gives the outcome values for each VP for this particular criterion, as follows: $v(VP_1) = 0$, $v(VP_2) = 0.5$, and $v(VP_3) = 1$.

Therefore, using this value function, the DM assigns evaluations to these consequences x_i, in accordance with their impact on the DPBO. Of course, the impact on the DPBO could require a non-linear scale for the marginal value of x_i. For instance, considering a scale of 0 to 1, this would keep the values of extreme outcomes. So,

$v(VP_1) = 0$ and $v(VP_3) = 1$. As to VP_2, the value could be 0.5 different, either $v(VP_2) > 0.5$ or $v(VP_2) < 0.5$. If we assume that the harm caused to the DPBO by this drawback has an increasing rate with the outcome score, then $v(VP_1)$ could be higher than 0.5; for instance, $v(VP_2) = 0.7$. The question pointed out here is how often a drawback appears and to what extent this frequency affects each particular DPBO.

For the objective related to the nature of the input required by the VP, a non-linear value function may apply in many cases. Let us consider the reliability of information given by the DM. In this case, a non-linear value function may apply for the reliability criterion; e.g. a negative exponential function on the number of alternatives could represent this situation.

In order to build the decision consequence in Table 14.2, a discrete 5-level scale, such as the Likert (1932a, b) scale, could be used for subjective evaluations related to the marginal value function.

In the case of using prior probabilities $\pi(x_{ij})$, instead of a value function, a utility function is considered and the expected utility is associated with $\pi(x_{ij})$. Therefore, Eq. (14.3) is applied.

$$u_\pi\left(x_{ij}\right) = \int_0^1 \pi\left(x_{ij}\right) u\left(x_{ij}\right) \tag{14.3}$$

For a linear utility function over xij, the Eq. (14.2) can be applied in (14.3), becoming eq (14.4), as follows:

$$u_\pi\left(x_{ij}\right) = \int_0^1 \pi\left(x_{ij}\right)\left(1 - x_{ij}\right) \tag{14.4}$$

For an exponential utility function with parameter *a* (Keeney and Raiffa 1976; de Almeida 2005) over x_{ij}, the Eq. (14.4) becomes (14.5), as follows:

$$u_\pi\left(x_{ij}\right) = \int_0^1 \pi\left(x_{ij}\right) e^{-a x_{ij}} \tag{14.5}$$

The value of expected utilities $u(x_{ij})$ computed for a particular criteria *j* on a specific VP *i*, should be introduced in the Matrix of Table 14.4, for the case in which prior probabilities $\pi(x_{ij})$ are applied, replacing its equivalent value function $v(x_{ij})$ for the case of deterministic consequence.

For the case of the discrete scores applied instead of prior probabilities x_{ij}, the Eq. (14.6) should be applied

$$v_j\left(x_{ij}\right) = (y - x_i)/y \tag{14.6}$$

where y is the highest level in the scale. For the three-level scale, $y = 2$ and for the five-level scale, $y = 4$. The application of this three-level scale in Chap. 17 illustrates its use, with the Eq. (14.6).

14.6 Choice of MCDM/A Method for Comparing VP

After building the decision matrix, an MCDM/A method is applied in order to choose the most appropriate VP for the business decision in question. As shown in the framework of Fig. 14.1, a prior step still had to be followed. This step consists of choosing an MCDM/A method, which is one of the most important steps when building any MCDM/A model (Roy and Słowinski 2013; de Almeida et al. 2015).

The DM's preference is the main contribution to be considered when choosing the MCDM/A method. In this step the analyst should first evaluate the DM's rationality as to compensation amongst criteria, before the method itself is chosen (de Almeida et al. 2015). Then, an inter-criteria evaluation could be developed in order to proceed to the final preference modeling process, and establishing the criteria weights.

As described in the previous Chapter, there are two types of MCDM/A method for discrete action space, such as is the case of the DPVP. The main factor to be analyzed when choosing the MCDM/A method for the DPVP is the compensatory or non-compensatory rationality (de Almeida et al. 2015; Bouyssou 1986; Munda 2008). The question to put forward is: which fits the DM's preference structure for this particular decision problem and context? It should be evaluated if the DM is willing to make compensation amongst the VP's properties, for instance.

However, it should be noted that for a 'discrete binary outcome', it does not matter if the DM's rationality is compensatory or non-compensatory, from the point of view of choosing the method. Applying either method, the unique criterion of synthesis or outranking, would be analytically similar, at least for comparing a method with the additive model with outranking methods. For instance, considering the PROMETHEE method, for the discrete binary outcome, the result is similar to that from using additive models.

This can be easily verified by analyzing Eq. (13.1), which represents the additive model. For all criteria j to which $v_j(x_j) = 1$, the scale constants k_j of those criteria are summed up to the global value of the VP. Similarly the same happens for the PROMETHEE method, for the weights w_j of those criteria, which are going to be combined in (13.8). In this case, following the non-compensatory approach would be easier with regard to eliciting the weights of criteria.

On the other hand, if the consequence matrix is built with a 'continuous outcome', it makes a difference to the results if the DM's rationality is compensatory or non-compensatory. In this case, some reflections on the analysis of the most appropriate rationality to the DMs may be provided.

It does not seem to be natural to a DM to make compensation between the properties of two VPs. It seems more natural that a DM, when comparing two VPs, would be willing to check only which of them has a greater frequency in violating a particular property. It might not be relevant in most cases to consider precisely how frequently violations happened.

That is, it seems reasonable that a DM would compare two VPs for this kind of criterion by considering only which of them has a better performance rather than taking into account the extent of the difference between the two performances. In that case, the assumption of non-compensatory rationality seems to be reasonable and an outranking method could be applied in the DPVP. In this case an ordinal scale would be fine for the decision matrix in Table 14.4 and a linear marginal value function, such as in (14.1) could be applied. Then, the analyst could consider using either the ELECTRE or the PROMETHEE method.

Although the assumption above could be considered reasonable, this should be applied unconditionally. Actually, apart from the preceding considerations, the DM's preference must always be evaluated. It might happen that a DM requires compensation when comparing the properties of two VPs. That is, for comparisons between two VPs, a DM may be willing to take into account the magnitude of frequency in which a property is violated in each VP. In this case, a cardinal scale should be considered for Table 14.2.

If a compensatory rationality is found to be more appropriate, before choosing a method with the additive model, the properties of this model should be evaluated in order to confirm if it is actually adequate. For instance, the mutual preferential independence condition amongst the criteria should be checked. A simple evaluation could be to check the condition in (13.4) with the DM. Although the independence condition may be found in the most partial application, when a preferential dependence occurs between criteria, this model can produce undesirable results. There are some situations in which a preferential dependence may occur in the DM's preference structure. This usually happens when quality and quantity are the criteria to be considered (de Almeida 2013). For instance, in evaluating an Academic Institution, if the quantity and the quality of the degrees awarded are criteria in the evaluation, then, the additive model could make a compensation in such a way that an Institution of poor quality could compensate this and increase its global score in the model (13.1), because it has awarded a higher number of degrees.

Also, the preference structure (P, I) should be compatible with the DM's ability to express preferences. It might happen that the DM is not able to compare all consequences, in which case a preference structure (P, Q, I, J) should be considered.

A first thought in such a situation is to turn to outranking methods, since they first appeared with the possibility of approaching incomparability. However, this would not be a reason for using these methods. Actually, their ability to approach incomparability is an operational advantage in the process of preference modeling. If a compensatory rationality is required, these methods should be considered in the last case as an approximation.

When difficulties are found with regard to using a preference structure (P, I) in the preference modeling process, the additive model can still be considered, if a method with partial information is applied. For instance, the FITradeoff (Flexible and Interactive Tradeoff) method (de Almeida et al. 2016) could be applied in this case. Although this method uses the additive model in the MAVT scope, it is able to model preferences with partial information and has the flexibility of allowing the DM to refuse both to make some comparisons and to answer some typical questions of the elicitation process and yet, even so, a solution may be found. In general, these kinds of questions and other much more difficult ones are essential elements in the classical methods available.

14.7 Parameterization of the MCDM/A Model

Once the MCDM/A method is chosen, the parameterization of the model consists of specifying its parameters with the information collected in the preference modeling with the DM. There are a few methods described in Chap. 13, which also discusses the concerns with this process of making preference modeling in order to obtain parameters.

The main information required here are the scale constants or the weights of the criteria. Other parameters may be required depending on the method applied. In this case, the meaning of these parameters is the main concern, since they may have different meanings in unrelated methods. The main concern here is with the use of the concept of weights of criteria as indicating the degree of importance in additive models, since this is systematically applied in many situations.

For compensatory methods, such as those based on additive models, the elicitation of the scale constants are not applied in an appropriate way, since these parameters do not mean straightaway the degrees of importance of a criterion, as already explained. This is the main issue with regard to these methods.

On the other hand, for the non-compensatory methods, the weights do straightforwardly represent the relative degree of the importance of the criteria. So, when given by the DM, this kind of evaluation may be easier than was the case in the previous additive model. Let us consider two criteria in Table 7.4. For instance, the monotonicity could have a minor impact over the DPBO, compared with the Condorcet winner criterion. In this case, the monotonicity would receive a low weight from the DM, who could consider that the ‘Condorcet winner’ criterion is twice as important as the monotonicity, with a weight of 0.3. Therefore, the monotonicity criterion would have a weight of 0.15.

As to intra-criteria evaluation, different considerations are also applied. First, for compensatory methods a cardinal scale has to be considered for eliciting the marginal value function. On the other hand, for the non-compensatory methods an ordinal scale may be considered, which can be clearly seen on analyzing Eqs. (13.5) or (13.8), respectively, ELECTRE and PROMETHEE methods. Some exceptions may apply in some of these methods, such as for ELECTRE. The discordance indices may consider an interval scale so as to compute differences between the performance of two alternatives, if the analyst uses this kind of formulation. Regarding the PROMETHEE method, the ordinal scale is applied particularly for $F_j(a, b)$ when in Eq. (13.8). On the other hand, the preference flow consists of a cardinal scale (summation of the criteria weights) for scores of the alternatives. Even, if the usual criterion is not applied to the function $F_j(a, b)$, and indifference or preference thresholds are applied in these methods, the scale is still an ordinal scale, given the meaning of thresholds itself, contextualized for small amounts, with a high degree of approximation. Actually, the thresholds are applied as a benefit for the disadvantage of having less information than necessary as ordinal scales are used for (13.8).

14.8 Applying the Model and Selecting the VP

This step consists of applying the algorithm of the chosen method in order to select the most appropriate VP for the DPBO under analysis. Actually, for some methods the previous step of parameterization includes the process of obtaining the solution that will be recommended, as is the case in interactive methods (e.g., FITradeoff).

This step seems to be relatively simple. However, there is still a concern related to who is going to be the DM of the DPVP.

If there is a supra-DM in the process, this DM assumes the role of making the decision in the DPVP and the process is simpler. That is, the DPVP has an individual decision-making process. Generally, the supra-DM has a hierarchical position in the structure of the business organization that is above that of the other DMs. The concept of the supra-DM is similar to that of the 'benevolent dictator problem' (Keeney 1976).

On the other hand, if the group of DMs for the DPBO is going to play the role of making the decision in the DPVP, then this could bring some modeling difficulties that require additional discussion. In this case, the DPVP is a group decision-making process, in which the DMs act together such as in the DPBO. However, this can be also simplified by a discussion with the group of DMs by using problem structure methods (PSM), with the support of a facilitator (Eden 1988; Eden and Ackermann 2004).

In the case of a group decision process for the DPVP which deals with a more formal analytical approach, one may wonder if this could turn out to be an infinite regress, since a method should be selected in order to solve that second order group decision. Thus, another decision model would be necessary, and so forth. That is why a facilitator is needed in this process, as already mentioned, so that a supposed circle of numerous regresses could be broken.

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Chapter 15
Choosing a Voting Procedure for Assessing the Readiness of Technology for Generating Energy

Abstract This Chapter presents an illustration of the framework for choosing a voting procedure applied to the context of assessing the Readiness of Technology for Generating Energy. The illustration is based on Morais et al. (Math Probl Eng (Online), 1–11, 2015) and is used in order to prioritize technologies that are critical for power generation in the Energy Sector. This problem was tackled in response to a request from CGEE (*Centro de Gestão e Estudos Estratégicos*, in English, the Center for Management and Strategic Studies) which is a Center that offers support to decision-making processes related to topics in science, technology and innovation. It does so by undertaking research and conducting strategic assessment studies based on a wide-ranging collaboration with experts and institutions of the Brazilian System of Science, Technology and Innovation with a view to promoting collaboration between the education and business sectors.

15.1 Assessing the Readiness to Generate Energy

Technology Readiness or critical technology can be understood as an in-country domain of technology which will generate economic development and, as a result, the technology will no longer need to be supplied from outside that country (Melo et al. 2003). In this context, technology means forming a rationale for acquiring know-how and this is designed in response to new demands and social requirements, thus changing a whole set of values and attitudes and ends up being aggregated to the culture of an organization or country (Veraszto et al. 2008). Therefore, critical technology is a top priority when planning change within an organization or a country.

Some technologies can be selected as a priority or as being critical for an organization or a country, when related to a specific area of interest. In order to evaluate the strategic condition of technology, Technology Readiness Assessment (TRA) is applied. TRA uses methods and processes to evaluate the technology itself and by specific metrics verifies the status of its development, i.e., measures the maturity of the technology assessed (Schot and Rip 1997).

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When developing a new technology, an effective TRA should also incorporate some metrics that provide a consistent assessment of the “degree of risk” (Mankins 2009). An effective TRA includes the following main features:

- *Performance Objectives.* These include aspects of engineering and operational performance measures, with a view to ensuring that the performance objectives of the new technologies and/or the capabilities of the system are clearly understood.
- *Technology Readiness Level—TRL.* This concept was introduced by NASA (National Aeronautics and Space Administration), in the mid-1970s. It is a metric that evaluates the maturity level of a specific new technology more effectively. It consists of a 9-level scale: TRL1 is the lowest level of maturity and TRL9 is the highest. The TRL scale is simple and easy to operate and has been applied in many fields such as aeronautics, astronautics and energy resources (Wei-gang et al. 2013). However, this tool also contains some weaknesses, especially because it depends on qualitative assessment, which is derived from the professional knowledge of experts, whose assessment is prone to high subjectivity and low objectivity.
- *Degree of Difficulty of Research and Development.* During the formal TRA, it is important to develop a clear understanding of the barriers to be faced and the difficulties related to whether the new technologies can be successfully developed.

The TRA for energy consists of two phases: diagnosis and implementation (as shown in Fig. 15.1). First of all, a search for new technologies that could be developed is made in the diagnostic phase. Thereafter, using TRL metrics, the maturity level of that technology is analyzed. When a specific technology is chosen, then a new

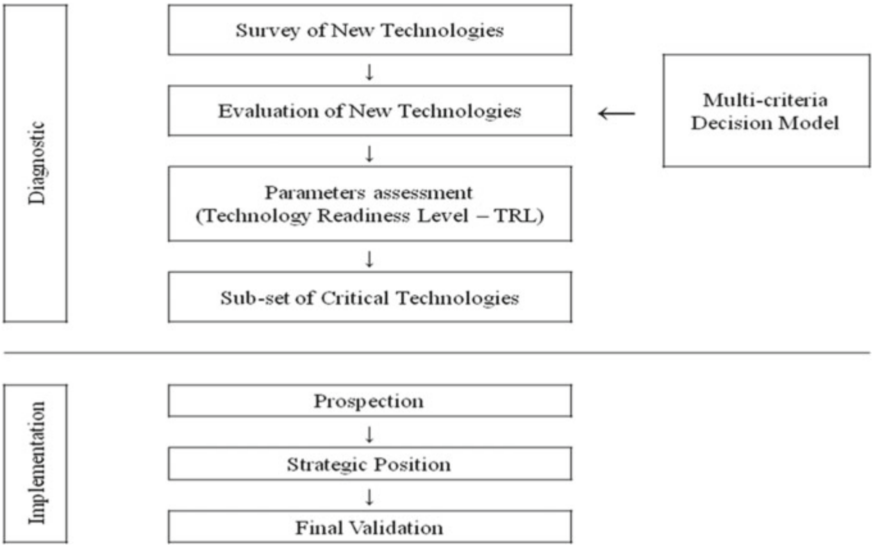


Fig. 15.1 Flowchart of the TRA and position of the multicriteria decision evaluation (Morais et al. 2015)

search on a subset of critical technologies is conducted and only after the results of this have been analyzed should the implementation phase begin.

Note that, based on these steps of the TRA, a multicriteria decision model can be applied in both phases, either in the diagnostic phase, when evaluating new technologies, or in the implementation phase, when evaluating the subset of critical technologies. Nevertheless, this study focuses on the first and most strategic part of the TRA of generating energy in order to aid how best to evaluate new technologies.

It is worth noting that some authors have conducted studies to contribute to TRA in order to improve the process for evaluating critical technologies: Chen et al. (2012), Wei-gang et al. (2013), Hoffmann et al. (2013), Li and Zhu (2011), Demirkiran (2012), Goetghebeur et al. (2012), Thokala and Duenas (2012). It is important to bear in mind that this kind of evaluation is complex and normally involves many actors in the process. However, none of these studies deal with aggregating the individual preferences of a group of decision-makers (DMs).

In this perspective, in order to take the DMs' multiple perspectives and objectives into account, this study aims to develop a group multicriteria decision model for the purpose of analyzing the strategic problem of evaluating technologies for the Brazilian energy matrix. It is worth mentioning that investments in this area are huge and an appropriate multicriteria model is necessary in order to ensure adequate efficiency in making a decision on which technology should be fostered. Also, an adequate voting process to aggregate the individual DMs' results is needed.

For this kind of strategic problem, the DMs should establish their objectives in order to analyze the alternatives (technologies). In this case, a multicriteria analysis can be a useful way to compare the technologies when using different criteria, which are important enough to disallow any kind of compensation. Therefore, this kind of decision problem has a non-compensatory rationality. However, for a non-compensatory rationality, DMs need to be able to give weights for criteria that represent their relative importance, and this task can be very hard for them.

Therefore, there are some cases in which the DM is neither able to provide such information nor feels comfortable about doing so but he/she may be able to rank the criteria by their importance. Having obtained such a ranking from the DMs, the use of surrogate weights can be considered. Therefore, it is proposed that surrogate weights could be used with the PROMETHEE method (Morais et al. 2015). The authors proposed a model for assessing the readiness of technology to generate energy a PROMETHEE-ROC method. De Almeida-Filho et al. (2018) argue that, according to their analysis, the surrogate weights approach that most faithfully represents a DM's value system is the ROC procedure.

In this Chapter, the real application conducted by Morais et al. (2015) regarding the strategic problem of TRA was adapted to incorporate the perspectives of different DMs and also, in order to illustrate the application of the Framework for choosing a Voting Procedure.

Figure 15.2 shows the group decision model to evaluate critical technology for generating energy and thereby to analyze which technology for power generation should be recommended. This model was developed in order to support DMs in choosing, with greater efficiency, the technology to be implemented in the sector.

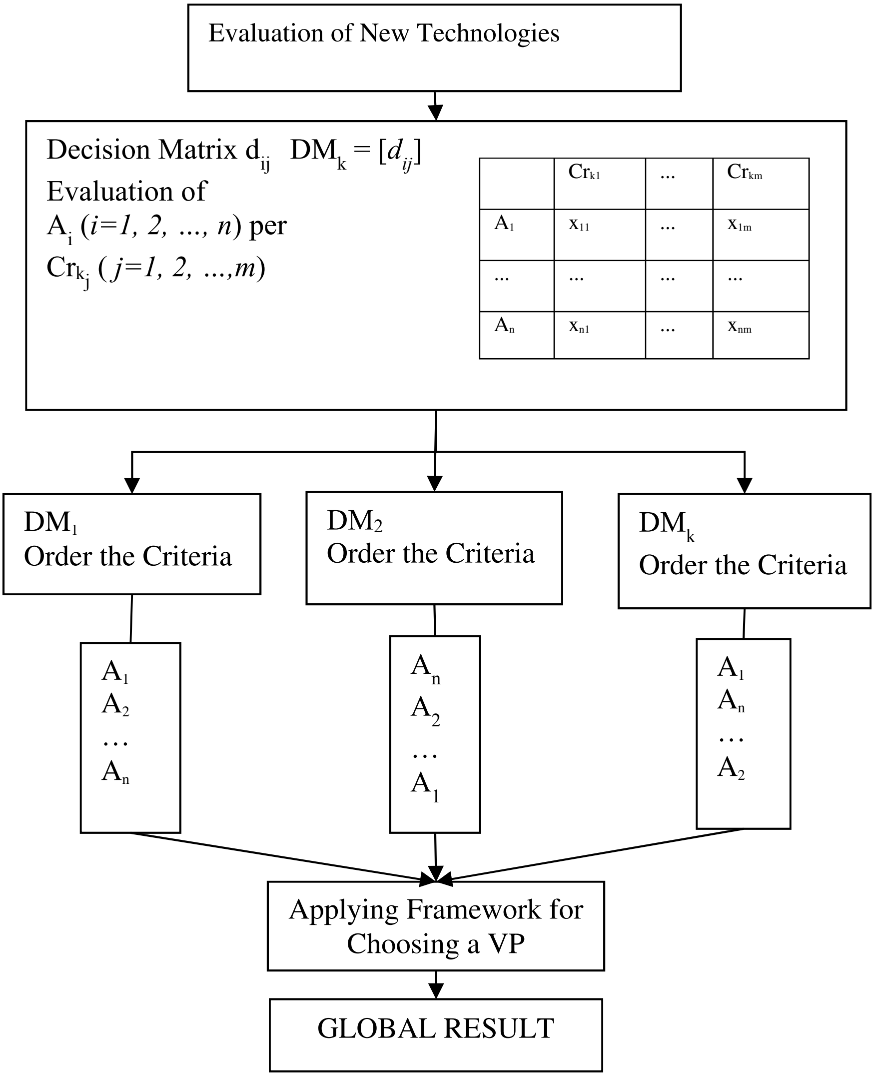


Fig. 15.2 Overview of the group decision model incorporating the framework for choosing a VP

The individual analyses take into account all the members' preferences and their perceptions about relative importance among criteria, i.e., only ordinal information. The individual evaluation explores the matrix of alternatives per criteria per DM and the individual ranks are obtained by applying the PROMETHEE-ROC method. To find the collective decision, the Framework for choosing a VP was taken into account Based on the chosen VP, the group decision-making process obtains the collective result by aggregating the results from the individual analysis.

The following sections present the application of the group decision model. First, the DMs' perspectives, their objectives and criteria are given, and the set of critical technologies to be considered as alternatives are established. On applying the PROMETHEE-ROC with each DM, the individual results are then found. Thereafter, the framework for choosing the VP that is appropriate for this problem is applied. Finally the global result is presented.

15.2 Structuring the Problem

The first step of the model consists of bringing together all members involved in the decision-making process. We call these members DMs. Thus, there is a group of DMs comprising k members, which is the set of $DM = \{DM1, DM2, \ldots, DMk\}$.

It is important to have a facilitator who will conduct the discussion process, but this person can be a member of the group, when it is not possible to have an external person. This member is called the Supra-decision-maker, i.e., the member of the group who has the most experience of and knowledge about the problem and, in conflict cases, he/she may have greater weight in the final decision and/or also establishes the DMs' priority as to calculating the final ranking. The problem of assessing the readiness of technology for generating power energy in Brazil considered there were three DMs in the process, namely, DM1, DM2 and DM3:

- DM1 is the representative concerned about environmental issues, e.g., the impacts on soil fertility, air, water, temperature and sound;
- DM2 is the representative concerned about social issues, e.g., local development, the quality of jobs generated and the know-how competence that needs to be developed;
- DM3 is the representative concerned about government issues, e.g., the development of national industry and the strategic impact on the energy matrix of the country.

15.2.1 Establishing Objectives and Criteria Based on Decision-Makers' Perspectives

The set of the objectives and criteria identified for the process of evaluating critical technologies is presented in Table 15.1 per DM. This table also shows the criteria codes, whether the interest is in minimizing or maximizing the criterion, and the unit or measurement scale for each criterion.

Table 15.2 presents the reference parameters for analyzing the critical technologies that are the units or measurement scales of the criteria. The details of these scales are shown below.

Table 15.1 Objectives and criteria

Decision-maker	Objectives	Criteria	Code	Min/Max	Unit/Scale
DM1	Environmental	Impact on soil fertility	Fert	Min	IS
		Impact on temperature	Temp	Min	IS
		Impact on sound	Soun	Min	IS
		Impact on water	Air	Min	IS
		Impact on air	Wat	Min	IS
DM2	Social	Impact on generating local or national employment	Emp	Max	IS
		Impact on the local development	Dev	Max	IS
		Impact on the quality of employment	QualEmp	Max	IS
	Knowledge	Impact on the synergy between boundaries of knowledge	Syn	Max	IS
		Impact on national competence (know-how)	Khow	Max	IS
	Economic	Period of time of interesting the market	PMar	Max	TS
		Impact of current demand in the domestic market	DDMar	Max	IS
		Impact of current demand in the global market	DGMar	Max	IS
		Impact on the growing of the national market	NaM	Max	IS
		Impact on the growing of the global market	GloM	Max	IS
DM3	Industrial	Impact on production capacity	Cap	Max	IS
		Impact of the availability of inputs	Inp	Max	IS
	Strategic	Condition for curtailing its development	Curt	Min	CS
		Impact on the energy matrix	Mtx	Max	IS

IS impact scale; *TS* time scale; *CS* curtailment condition scale

Table 15.2 Reference parameters for analyzing the critical technologies

Impact level	Value	
	Min	Max
a. Impact scale (IS) (Morais et al. 2015)		
No impact	5	1
Very low impact	4	2
Moderate impact	3	3
High impact	2	4
Very high impact	1	5
b. Time scale (TS) (Morais et al. 2015)		
Period of time		Value
Short term (up to 5 years)		1
Medium term (up to 15 years)		2
Long term (up to 30 years)		3
c. Curtailing condition scale (CS) (Morais et al. 2015)		
Curtailing condition		Value
The Energy Technology is not an important part of the development of another process		1
The Energy Technology is an important part of the development of another process		2

15.2.2 *Establishing the Set of Critical Technologies*

The process for selecting the set of alternatives was agreed to by all three DMs. They agreed to evaluate fourteen critical technologies, which are distributed in five technological areas. Table 15.3 shows the technological area and subarea, and the code of the critical technology.

15.3 Individual Results

Each DM has a decision matrix since each of them compares the alternatives from different perspectives. In order to evaluate the individual results, each DM considered in this study needed to establish the ranking of the criteria and computing their weights, based on ROC (Rank order centroid) weights (Vansnick 1986; Barron 1992). Table 15.4 shows the order and weights for the criteria per DM.

The next step is to build the consequence matrix, which evaluates the alternatives by criterion, using the scale shown in Table 15.2. The step of evaluating the critical technologies by criterion is supported by a decision support system and is illustrated in Figs. 15.3, 15.4 and 15.5.

Table 15.3 Set of alternatives

Technological area	Technological subarea	Critical technology	Code
Chemical	Physicochemical	Lithium-ion batteries	BIL
	Organic chemistry	Recycling	Rec
		Bioenergy	Bio
Optics	Photo-automation	Photosensors	FotS
	Photo-generation	Photo-voltaic panel	FotG
Telecommunications	Control and automation	Automation system	Aut
	Communication	Telecommunication systems using transmission power cables	Com
		Loading batteries by communication signals	Batt
Mechanics	Wind	Wind power	Wind
	Hydro	Hydro power	Hydr
		Small hydro power central unit	SHC
	Solar	Solar Energy	Solar
Electric	Battery	Advanced battery technologies	Acum
	System	Equipment and arrangements	EqAr

Source Morais et al. (2015)

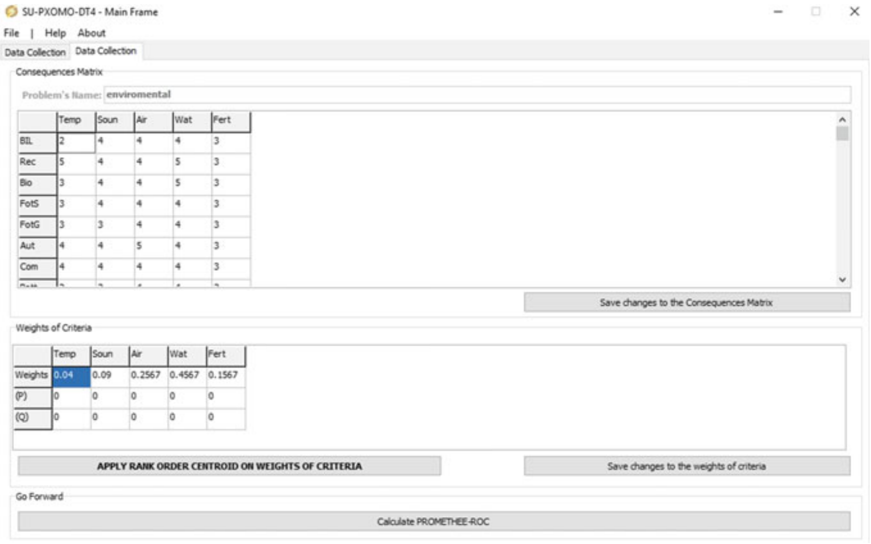
All DMs considered the Usual preference function for all criteria, which indicates that any difference between alternative performances represents a strict preference. The use of ROC weights minimizes the effort that a DM needs to make in the process for indicating the degree of importance of the criteria. Based on the consequence matrix and the value of the criteria, the performance of the alternatives can be evaluated by implementing a multicriteria method. Table 15.5 shows the individual results per DM regarding their preferences. The result is shown as the order of preference of the alternatives by DMs (1 being the most preferable and 14 the least preferable).

The critical technologies are evaluated based on PROMETHEE-ROC. The mathematical structure of the multicriteria method offers the first recommendation per DM and obtains the ranking of the alternatives.

In accordance with the results, the DMs do not agree with each other about which critical technology must be prioritized. Besides, note that there is a great divergence between DM 2 and DM3, since the first alternative of DM3 is the 11th alternative for DM2. Thus, the next step is to evaluate which Voting Procedure should be used in order to aggregate these individual results.

Table 15.4 Criteria order and weights by DMs

Decision-maker	Criteria code	Order of the criteria	ROC-weights
DM1	Fert	3	0.1567
	Temp	5	0.0400
	Soun	4	0.0900
	Air	2	0.2567
	Wat	1	0.4567
DM2	Emp	4	0.0900
	Dev	5	0.0400
	QualEmp	2	0.2567
	Syn	1	0.4567
	Khow	3	0.1567
	PMar	6	0.0606
	DDMar	5	0.0828
	DGMar	8	0.0262
	NaM	3	0.1477
	GloM	7	0.0421
DM3	Cap	4	0.1106
	Inp	9	0.0123
	Curt	2	0.2032
	Mtx	1	0.3143

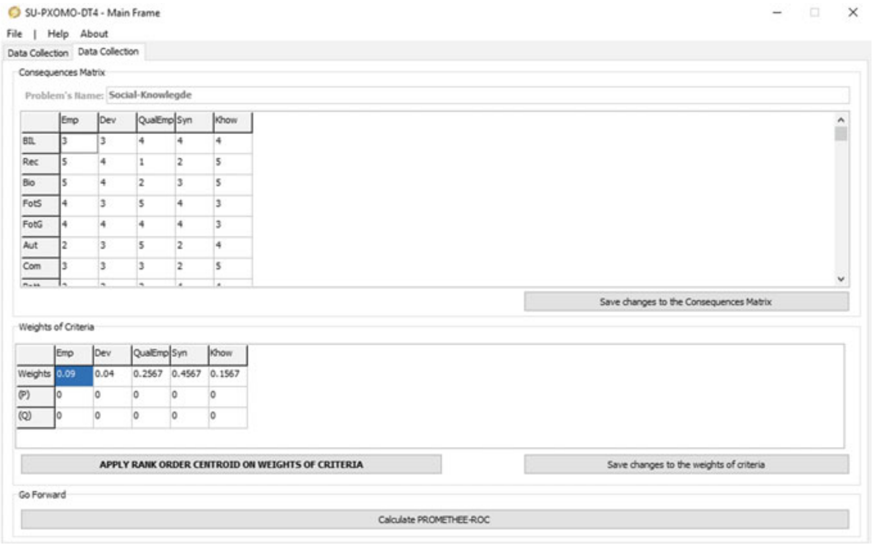


Problem's Name: enviromental

	Temp	Soun	Air	Wat	Fert
BIL	2	4	4	4	3
Rec	5	4	4	5	3
Bio	3	4	4	5	3
FotS	3	4	4	4	3
FotG	3	3	4	4	3
Aut	4	4	5	4	3
Com	4	4	4	4	3

	Temp	Soun	Air	Wat	Fert
Weights	0.04	0.09	0.2567	0.4567	0.1567
(P)	0	0	0	0	0
(Q)	0	0	0	0	0

Fig. 15.3 Establishing the consequence matrix and the weights of criteria by DM1

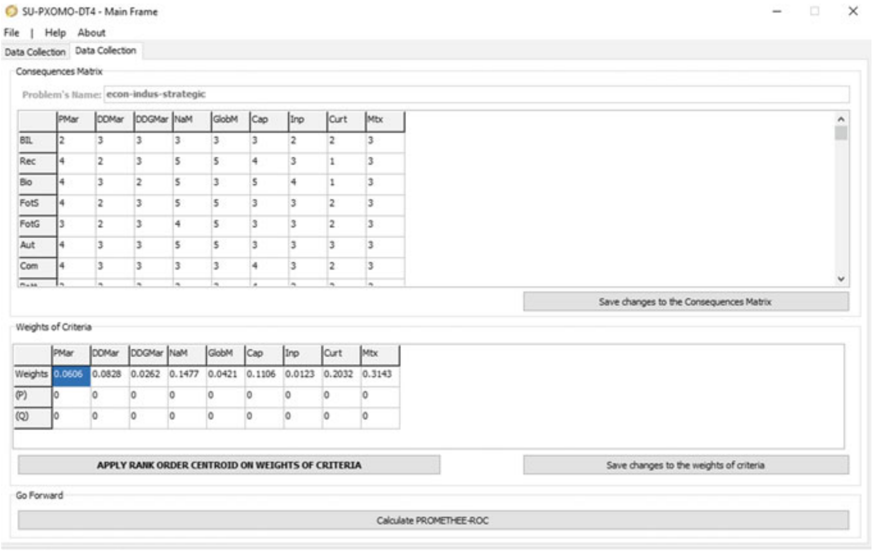


Problem's Name: Social-Knowlegde

	Emp	Dev	QualEmp	Syn	Khow
BIL	3	3	4	4	4
Rec	5	4	1	2	5
Bio	5	4	2	3	5
FotS	4	3	5	4	3
FotG	4	4	4	4	3
Aut	2	3	5	2	4
Com	3	3	3	2	5

	Emp	Dev	QualEmp	Syn	Khow
Weights	0.09	0.04	0.2567	0.4567	0.1567
(P)	0	0	0	0	0
(Q)	0	0	0	0	0

Fig. 15.4 Establishing the consequence matrix and the weights of criteria by DM2



Problem's Name: econ-indus-strategic

	PMar	DDMar	DDGMar	NaM	GlobM	Cap	Inp	Curt	Mtx
BIL	2	3	3	3	3	3	2	2	3
Rec	4	2	3	5	5	4	3	1	3
Bio	4	3	2	5	3	5	4	1	3
FotS	4	2	3	5	5	3	3	2	3
FotG	3	2	3	4	5	3	3	2	3
Aut	4	3	3	5	5	3	3	3	3
Com	4	3	3	3	3	4	3	2	3

	PMar	DDMar	DDGMar	NaM	GlobM	Cap	Inp	Curt	Mtx
Weights	0.0606	0.0828	0.0262	0.1477	0.0421	0.1106	0.0123	0.2032	0.3143
(P)	0	0	0	0	0	0	0	0	0
(Q)	0	0	0	0	0	0	0	0	0

Fig. 15.5 Establishing the consequence matrix and the weights of criteria by DM3

Table 15.5 Ranking of the alternatives by DMs

Alternatives	DM1	DM2	DM3
BIL	9	2	14
Rec	1	12	9
Bio	3	7	7
FotS	8	1	6
FotG	11	3	11
Aut	4	11	1
Com	7	13	8
Batt	13	4	13
Wind	10	6	2
Hydr	5	5	3
SHC	12	10	12
Solar	14	9	4
Acum	2	8	5
EqAr	6	14	10

15.4 Applying the Framework for Choosing a VP

In this session, the focus is on choosing which Voting Procedures (VPs) can be used to evaluate the specific problem of evaluating the Readiness of Technology for Generating Energy. The process is based on the Framework presented on Chap. 14 of this book. The main aspect of this kind of problem may be the type of input required by a VP. Based on the characteristics of this problem, the input was the VP rankings of alternatives, while the VPs that were considered adequate were: Amendment, Copeland, Dogson, Minmax, Borda, Nanson and Hare.

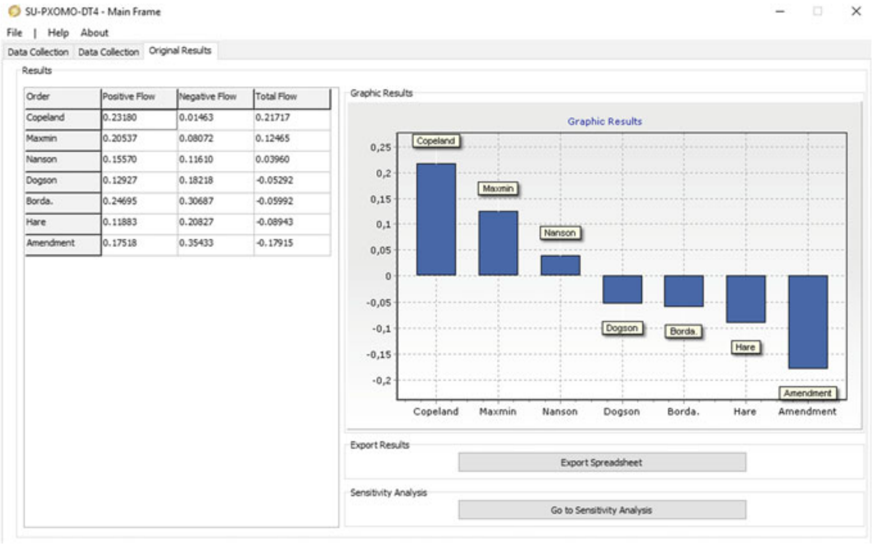
The Criteria used to evaluate the VPs is in accordance with the context of the problem of Readiness of Technology for Generating Energy, and with the characteristics of the VP and how they affect the problem. Criteria are generated which are associated with the properties and other characteristics, such as paradoxes that may be relevant for consideration when analysing a VP.

In order to facilitate the process, the same multicriteria process (PROMETHEE-ROC) was applied to evaluate the VP. The DMs involved in this process agreed with the order of the criteria to evaluate the VP. Table 15.6 shows the order and weights of these criteria.

For the process of choosing a VP, the Usual preference function was applied for all criteria, since it is a binary evaluation of the VPs. Besides, based on the consequence matrix (binary evaluation as shown in Table 14.1—Chap. 14), the value function as considered in Eq. (14.1) (Chap. 14), and the weights of the criteria, the VP appropriate for this problem can be found. Figure 15.6 presents this result.

Table 15.6 Order of criteria to evaluate VP

Criteria	Condorcet winner	Condorcet loser	Strong Condorcet	Mono-tonicity	Pareto	Consis-tency	Chernoff	Invulnerability to the no-show paradox
Order	4	5	2	3	1	7	8	6
ROC-Weights	0.1106	0.0793	0.2147	0.1522	0.3397	0.0335	0.0156	0.0543



Order	Positive Flow	Negative Flow	Total Flow
Copeland	0.23180	0.01463	0.21717
Maxmin	0.20537	0.08072	0.12465
Nanson	0.15570	0.11610	0.03960
Dogson	0.12927	0.18218	-0.05292
Borda.	0.24695	0.30687	-0.05992
Hare	0.11883	0.20827	-0.08943
Amendment	0.17518	0.35433	-0.17915

Fig. 15.6 Result of the VP chosen for the problem of evaluating the Readiness of Technology for Generating Energy

As can be observed, the Copeland method was identified as the most appropriate VP for the problem to aggregate the individual results from the DMs involved in the problem of evaluating the Readiness of Technology for Generating Energy.

15.5 Global Results

Since the Copeland method was considered the most appropriate voting procedure to conduct the aggregation of individual results for this problem. Table 15.7 shows the result of the aggregation of the alternatives using Copeland method.

In accordance with the results, the critical technology which must be prioritized in the first instance is Hydr, followed by Aut and Acum. The last position is taken by SHC and Batt. The reference to the name of the alternatives is shown in Table 15.3. This result reflects the aggregation of DMs' rankings after using the Copeland method.

Table 15.7 Global result by applying Copeland Method

Alternatives	BIL	REC	BIO	FotS	FotG	AUT	Com	Batt	Wind	Hydr	SHC	Solar	Acum	EqAr	Wins	Total
BIL	–	0	0	0	1	0	0	1	1	0	1	1	0	0	5	−3
REC	1	–	0	0	1	0	1	1	0	0	1	0	0	1	6	−1
BIO	1	1	–	0	1	1	1	1	0	0	1	1	0	1	9	5
FotS	1	1	1	–	1	0	1	1	1	0	1	1	0	1	10	7
FotG	0	0	0	0	–	0	0	1	0	0	1	1	0	0	3	−7
AUT	1	1	0	1	1	–	1	1	1	1	1	1	0	1	11	9
Com	1	0	0	0	1	0	–	1	0	0	1	0	0	1	5	−3
Batt	0	0	0	0	0	0	0	–	0	0	0	1	0	0	1	−11
Wind	0	1	1	0	1	0	1	1	–	0	1	1	1	1	9	5
Hydr	1	1	1	1	1	0	1	1	1	–	1	1	1	1	12	11
SHC	0	0	0	0	0	0	0	1	0	0	–	0	0	0	1	−11
Solar	0	1	0	0	0	0	1	0	0	0	1	–	0	1	4	−5
Acum	1	1	1	1	1	1	1	1	0	0	1	1	–	1	11	9
EqAr	1	0	0	0	1	0	0	1	0	0	1	0	0	–	4	−5
Loss	8	7	4	3	10	2	8	12	4	1	12	9	2	9		

15.6 Topics for Further Reflection

The problem presented used ROC weights to represent the importance of criteria in the decision problem, either during the individual phase or for the framework for choosing the VP. This model could be applied to other problems, since the DMs have a non-compensatory rationality and could not give complete information about the criteria, but they are able to give partial information about them.

If the DMs have a compensatory rationality, it is important to rank the criteria based on swing weights and also to use an additive multicriteria method.

In this Chapter, all DMs agreed with the order of criteria for choosing the VP. However, if this were not the case, a Supradecision-maker who is able to do this task should be appointed.

15.7 Suggestions for Reading

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Chapter 16
Choosing a Voting Procedure for a Water Resources Management Problem

Abstract The management of water resources involves multiple decision-makers, each with their own perspectives on the way the decision problem should be tackled. This Chapter presents an illustration of the framework for choosing a VP for a water resources management problem. The application is based on the Morais and de Almeida (Omega (Oxford) 40:42–52, 2012) group decision model to support the choice of an alternative to stem and reverse the degradation of the hydrographic basin of the Jaboatão River, Pernambuco-Brazil.

16.1 A Water Resources Management Problem

Many decisions on water resources management in Brazil are made by hydrographic basin committees, which were instituted by the Brazilian National Policy on Water Resources (Ministry of the Environment-MMA 2006). The responsibility of hydrographic basin committees is to make the decision process decentralized and participatory by involving civil society, public sector authorities and users of water resources.

However, it is not a simple task to plan activities in these committees, since their members must make decisions on complex problems that consider multiple conflicting criteria (i.e. economic, technical, social and environmental dimensions). Various models were developed to support water resources management decision making using multicriteria analysis (Raju et al. 2000; Hajkowicz and Collins 2007; Morais and de Almeida 2007; Morais et al. 2010; Silva et al. 2010; Morais and de Almeida 2010; Mutikanga et al. 2011; Roozbahani et al. 2012; Trojan and Morais 2012a, b; Markovic 2012; Coelho et al. 2012; Fontana and Morais 2013, De Almeida-Filho et al. 2017; Gonçalo and Morais 2018).

Another important issue in this kind of problem is that committee members are usually able to spend only a limited amount of their time on water resources management activities, since they are typically also engaged on other priority activities (Silva et al. 2010). Therefore, it is very difficult to schedule meetings to make decisions because the actors involved have other commitments. These meetings should be held once per month but, given that members have other priority commitments,

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there are months when the meeting of the hydrographic basin committee to discuss water resources management problems cannot be held.

When the meetings do occur, the members of this committee use the plurality method to reach a collective preference. Each member indicates a single alternative or abstains, so, the alternative with the highest number of votes is the final decision. Therefore, this decision may not correspond to the interests of the majority, i.e., sometimes, the alternative chosen is the worst option for many members involved in the decision process.

Given the complexity of this decision-making, it is important to have a group decision support method to guarantee transparency, swiftness and, especially, a structured analysis of the problem, which incorporates the points of view of all committee members.

In that perspective, it is presented a group decision-making model for analyzing the alternatives to stem and reverse the degradation of the Jaboatão River in the context of hydrographic basin committee. The hydrographic basin of the Jaboatão River (in the state of Pernambuco, Brazil) extends over parts of the townships of Cabo de Santo Agostinho, Jaboatão dos Guararapes, Moreno, São Lourenço da Mata, Vitória de Santo Antão and parts of the city of Recife and has a drainage area of 426.70 km^2. This basin forms part of the Eastern region of the Brazilian Northeast Atlantic, which is the region in Brazil with the second highest population density (about 80 inhabitants/km^2) and it is this which emphasizes the importance of the hydrographic basin in the region. Besides, the interior of the region experiences periods of drought and/or low rainfall. Therefore, the supply of water becomes very critical due to the intermittent nature of the flow of water in the watercourses.

The main problem of this region is environmental, social and economic degradation resulting from the uncontrolled use of soil and water throughout the hydrographic basin. Table 16.1 shows the sources of degradation of the hydrographic basin of the Jaboatão River, defines their degradation status and the areas in which degradation is at a critical level.

Figure 16.1 illustrates the decision model which considers the effective participation of all members involved of the hydrographic basin committee, thereby obtaining individual rankings of alternatives with the aid of a multicriteria method. Thereafter, the framework for choosing a voting procedure (VP) is applied in order to identify the most appropriate VP to aggregate the individual rankings. The final group decision result is the selection of an alternative, which represents the preference of the committee, and which takes into consideration the points of view and interests of the different sectors/entities involved.

This model can increase the transparency of the decision process, thus reducing the possibilities of conflicts involving the use of the hydrographic basin. The sections that follow present the application of the model that seeks to support the group representing the hydrographic basin of Jaboatão River. The aim is to stem and reverse the degradation of the river.

Table 16.1 Characterization of the hydrographic basin

Source of degradation	Degradation status	Critic areas
Public actions	High levels of thermotolerant coliforms and phosphorous in the Jaboatão River and its tributaries are evidence of domestic sewage entering the hydrographic basin	Urban areas of Jaboatão dos Guararapes
	The following solid residues are found in the hydrographic basin: pieces of fishing line and netting, rope for tying up boats, plastic bags, drink containers, foam packaging for food and drinks, containers of lubricating oil	Urban areas, fishing colonies, and areas where the springs are used for recreational purposes
Agro-industrial	Irrigation using the main liquid residues from sugar-cane	Irrigated areas
Industrial	Untreated industrial emissions	Township of Jaboatão dos Guararapes
Agricultural	About 30 principal activities are involved in farming practices in the region served by the river	Township of Vitória de Santo Antão

Source Morais and de Almeida (2012)

16.2 Structuring the Problem

In order to support the choice of an alternative to mitigate the degradation of the hydrographic basin of the Jaboatão River, Pernambuco-Brazil, first of all, the decision-makers (DMs), the alternatives for this problem and the set of criteria to evaluate the alternatives must be identified. This application is based on Morais and de Almeida (2012).

16.2.1 Identifying the Decision-Makers

According to the National Policy for Water Resources, the DMs are the participants that represent public sector bodies, civil society and users of water resources (industries, agro-industries, water treatment and supply companies). For this problem, only one member from each sector/entity was considered, in order to avoid making the group too large. Table 16.2 shows what the composition of the group was.

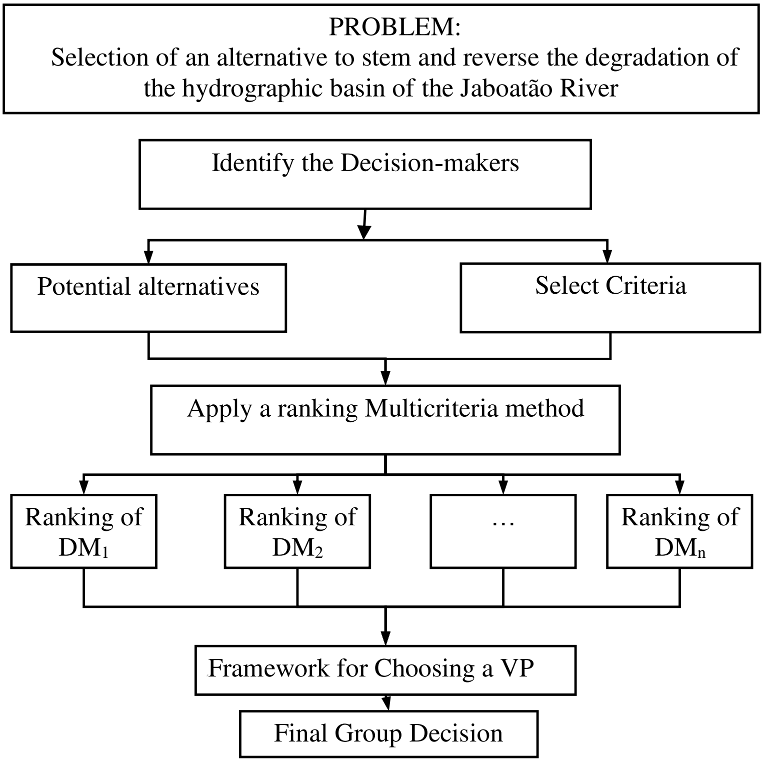


Fig. 16.1 Flowchart of the application in a water resources context (adapted from Morais and de Almeida 2012)

Table 16.2 Decision makers

Representation	Sector/Entity	Quantity
Water resources users	DM1 Industries	01
	DM2 Agro-industries	01
	DM3 Water treatment and supply company	01
Public sector	DM4 Union, State or City	01
Civil society	DM5 Universities or social organizations	01

Source Morais and de Almeida (2012)

16.2.2 Establishing the Set of Potential Alternatives

Based on the information about the status and main sources of degradation of the hydrographic basin presented in Table 16.1, a technical and specific study was conducted in order to formulate a set of alternatives to stem and reverse the degradation identified for the Jaboatão River basin. Some alternatives were also included based on a search for similar alternatives that have already been used in other basins to tackle degradation problems caused by sources of degradation of the same kind.

In order to establish the alternatives to mitigate the main degradation problem, the committee had an open discussion, focused on information about the sources of degradation and their status, to avoid considering alternatives based on specific and particular interests. For each alternative proposed, the DMs gave a technical explanation of how the action will contribute to mitigating the degradation problem detected. If someone disagrees with the arguments, he/she must explain why this is so and must emphasize their negative aspects against the positive ones. This exercise is often very beneficial for the group learning process.

Table 16.3 presents and describes the alternatives that were identified as being able to mitigate the degradation of the Jaboatão River. All DMs agreed that the alternatives identified were possible actions for reducing the degradation in the Jaboatão River Basin.

Table 16.3 Set of alternatives to mitigate degradation in the Jaboatão River Basin

Code	Description
A1	Secondary sewage treatment in Jaboatão dos Guararapes, which requires industrial waste to be pre-treated according to the standards laid down
A2	Educational campaigns in the townships within the hydrographic basin (with the exception of Recife)
A3	A campaign with industry to minimize the quantity of water used in production processes by offering monetary incentives for those industries that show positive results
A4	Maintenance of industrial facilities to prevent the water used for refrigeration from being contaminated by waste from industrial processes
A5	To institute policies for controlling the development of new businesses and/or expansion of current ones to avoid worsening industrial pollution
A6	Development of a plan of sustainable agriculture specific to the rural producers of Vitória de Santo Antão which focuses on soil and water conservation for the hydrographic basin of the Rio Jaboatão
A7	Recovery of native vegetation along the banks of the Jaboatão river
A8	Improving the collection of waste material all along the river, such as providing for the periodic removal of trash
A9	Recovery of the natural aquatic ecosystem
A10	Treatment of the Erosion Points in order to contribute to reducing the silting-up of the rivers and of the rainfall drainage network
A11	Restoring the biodiversity of the fauna in the basin
A12	Development of sustainable tourist activities along the Jaboatão river

Source Morais and de Almeida (2012)

16.2.3 *Selecting Criteria for Evaluation*

In order to evaluate the alternatives, five criteria were considered with regard to the economic, financial, social and environmental dimensions. These criteria consider the issues addressed by the members of the group during committees' meetings, and include the status of the degradation, its scope (point to the source or covers diffuse sources) and the urgency of implementing actions. Table 16.4 shows the code, name, description and the scale of each criterion.

Table 16.4 Criteria and their respective descriptions

Code	Criteria	Description	Scale
C1	Investment value	This is the monetary value for implementing action	Brazilian currency (Reais) and should be based on estimates of the State company responsible for water supply and sanitation. A smaller value is preferable to a higher value
C2	Maintenance costs	This is the monetary value to maintain the action in annual operation	Brazilian currency (Reais) and should be based on estimates of the State company responsible for water supply and sanitation. A smaller value is preferable to a higher value
C3	Dependence on third-parties	This is the action dependency, which does not consider the involvement and participation of others (society). The involvement of society diminishes the effectiveness of actions	Ordinal scale (very low, low, regular, high, very high). A lower value is preferable to a smaller value
C4	Industrial impacts	Corresponds to the negative impacts that the action will cause on industrial activities from the operational, economic or legal points of view	Ordinal scale (very low, low, regular, high, very high). A lower value is preferable to a smaller value
C5	Agricultural impacts	Corresponds to the negative impacts that the action will cause on agricultural activities	Ordinal scale (very low, low, regular, high, very high). A lower value is preferable to a smaller value

16.3 Individual Results

The group model aims to support DMs in ranking the alternatives while taking any aspects of the criteria into account. A multicriteria method was applied with each DM separately to obtain the individual rankings. It was considered that the DM's preference is directly influenced by the preference of the sector/entity that he or she represents.

The use of a multicriteria method for ranking is helpful due to the difficulty that DMs have in ranking the alternatives while thinking about the economic, environmental and social dimensions without a decision support method and to avoid the problem of manipulating the preference order (a common problem when a voting procedure is used). The choice of the multicriteria method depends on the context and characteristics of the problem analyzed and the DMs' rationality.

For this application, the PROMETHEE II method (Brans and Vincke 1985; Brans et al. 1986), which is appropriate for the ranking problematic, was used to prioritize the individual alternatives. Each DM evaluated the relative importance of the criteria and then attributed corresponding weights to each criterion and the preference functions for each criterion. It is easy to understand the concepts of this method and its inherent parameters, which makes preference modeling simpler and more efficient.

These weights which are defined by the DMs are non-negative numbers, independent of the measurement units of the criteria, whereby the higher value, the more important the criterion. The data should be normalized by dividing each weight by the total of all the weights attributed by a given DM. The sum of the normalized weights is equal to 1.

Since the PROMETHEE method suggests six types of preference functions (Brans et al. 1986), each DM can choose a different preference function per criterion. However, in this application, after discussions, the DMs decided that the preference functions for each criterion were to be chosen globally, that is, the same preference function and the parameters p and q would be the same to represent all DMs.

As criteria C1 and C2 are the measurable ones, the DMs' preference function is the V-shape criterion (or Linear Preference), where the preference for one alternative in relation to other increases linearly with the difference in performance between them, based on a preference threshold (p).

As the other criteria, C3, C4, and C5, are the subjective ones, they are evaluated on a verbal scale. The DMs' preference function is the Usual criterion, which seems to be the most appropriate one when subjective performances are evaluated. This preference function considers that if the performance of one alternative is slightly higher than the performance of another, then the former is entirely preferable.

Table 16.5 shows the normalized criteria weights attributed by each DM and the preference functions chosen per criterion with its respective parameters.

Before applying the multicriteria method, it is important to note that each DM can individually evaluate the alternatives by criteria, in the case of the subjective criteria. In this application, the evaluation was performed in an open discussion among DMs. The idea is to analyze each DM's assessment in order to increase understanding of the criteria, alternatives and scales in order to generate more realistic estimates of performance. So, the consequence matrix was the same for all DMs.

From the information collected (criteria weights per DM, judgments of the alternatives, preference functions and their respective parameters), the PROMETHEE II method was applied to obtain the ranking of the alternatives per DM. Table 16.6 presents these individual rankings.

Table 16.5 Criteria and their respective descriptions

Decision makers	Criteria				
	C1	C2	C3	C4	C5
DM1	0.46	0.36	0.06	0.06	0.06
DM2	0.30	0.30	0.10	0.27	0.03
DM3	0.38	0.28	0.14	0.10	0.10
DM4	0.19	0.22	0.29	0.22	0.08
DM5	0.20	0.10	0.10	0.20	0.40
Preference Functions	V-shape criterion	V-shape criterion	Usual criterion	Usual criterion	Usual criterion
Parameter p	100,000	50,000	–	–	–

Source Morais and de Almeida (2012)

Table 16.6 Individual rankings per decision-maker

Ranking	DM1	DM2	DM3	DM4	DM5
1	A5	A2	A5	A9	A9
2	A2	A5	A4	A6	A4
3	A10	A6	A3	A4	A3
4	A4	A4	A2	A2	A6
5	A6	A9	A10	A3	A12
6	A3	A10	A9	A11	A10
7	A9	A3	A6	A7	A8
8	A7	A7	A7	A8	A2
9	A11	A12	A11	A10	A5
10	A12	A8	A8	A5	A1
11	A8	A11	A12	A12	A7
12	A1	A1	A1	A1	A11

16.4 Applying a Framework for Choosing a VP

In this stage of the model, the framework for choosing a voting procedure (VP) is used, as presented in Chap. 14 of this book, in order to aggregate the individual rankings and select an alternative to mitigate the degradation of the Jaboatão River Basin.

First, it is important to analyze which VPs are appropriate for this problem, which has rankings as input, but only one alternative is needed as an output. Based on this perspective, either a VP that results in rankings or a VP that results in a single alternative can be considered. Therefore, the VPs that were considered for evaluation were: Amendment, Copeland, Dogson, Minmax, Kemeny, Plurality, Borda, Approval, Black, Pl. runoff, Nanson and Hare.

Voting proprieties, in terms of which the goodness of the procedures is assessed (Nurmi 1983), were considered as criteria to evaluate the VP. In this application we used the same set of proprieties that are presented in Table 7.4. Thus, the consequence matrix of the VPs versus their proprieties is based on a discrete binary outcome (see Table 14.1, Chap. 14). Value function considered is Eq. (14.1) in Chap. 14.

For this problem, there is a concern related to which DM will be given the preferences in order to evaluate the VPs. For this case, all DMs agreed that DM 5 (the representative from Universities and social organizations) should assume the role of making the decision, thus acting as a Supra-Decision-Maker. They argued that DM5 understands the voting proprieties better than they did, and therefore he will be better at evaluating their relative importance.

On the other hand, DM5 had difficulty in expressing his preference regarding relative importance among the criteria. So, he required the support of the "Simos' revised Procedure" by Figueira and Roy (2002). The aim of this procedure is to elicit the weights of the different criteria and it does so by using two sets of cards, thus facilitating the assessment of criteria. As it is a relatively simple technique, it can be learned inductively (Figueira and Roy 2002).

Under this process, the DM is given two sets of cards: in one set, each card has the name and description of a criterion, and the other set consists of blank cards. The DM takes the set of named cards (in effect, the criteria) and orders them in ascending degree of importance. If the DM has an equal preference for two cards, he/she should put them together, in pairs

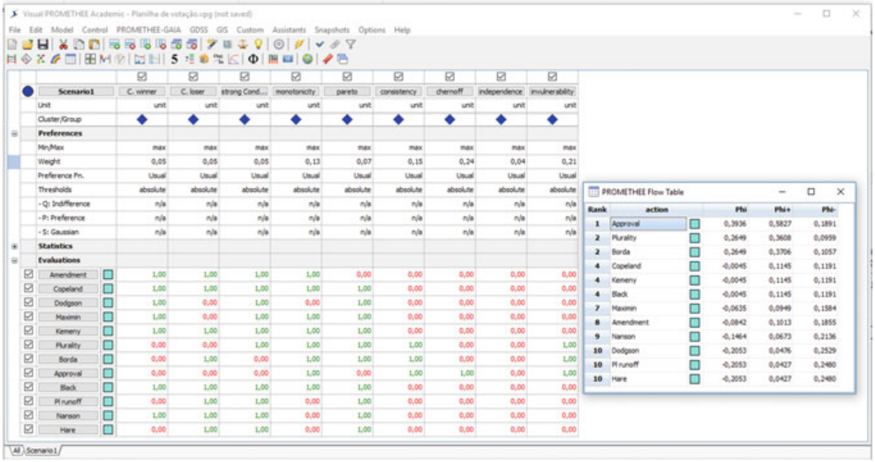
After the DM has ranked the named cards, the DM should think about the importance of a named card (criterion) relative to its immediate neighbors, and to express the degree of difference in importance between them, he should place one or more blank cards between the pairs of named cards.

Subsequently, an algorithm is set which will be used to calculate non-normalized and normalized weights. The SRF 2.2 (Simon Roy Figueira) software, developed by Lamsade (Paris-Dauphine University, Paris, France), is recommended to support this process (Figueira and Roy 2002). Table 16.7 shows the result of the weights by using SFR.

Using these weights for the VP proprieties, the PROMETHEE method is applied to evaluate the decision matrix [considering as value function Eq. (14.1), Chap. 14], based on the discrete binary outcome (see Table 14.1, Chap. 14). Figure 16.2 shows the result.

Table 16.7 Weight for VP proprieties given by DM5

Criteria	Order (ascending)	White cards	Normalized weights	Weights
h. Independence of irrelevant alternatives	1		3.5	0.035
		0		
a. Condorcert winner	2		5.4	0.054
b. Condorcert loser	2		5.4	0.054
c. Strong Condorcet	2		5.4	0.054
		0		
e. Pareto	3		7.3	0.073
		2		
d. Monotonicity	4		13	0.130
		0		
f. Consistency	5		14.9	0.149
		2		
i. Invulnerability	6		20.6	0.206
		1		
g. Chernoff	7		24.5	0.245



Scenario1	C. winner	C. loser	strong Cond...	monotonicity	pareto	consistency	chernoff	independence	invulnerability
Unit	unit	unit	unit	unit	unit	unit	unit	unit	unit
Cluster/Group	◆	◆	◆	◆	◆	◆	◆	◆	◆
Preferences									
Min/Max	max	max	max	max	max	max	max	max	max
Weight	0,05	0,05	0,05	0,13	0,07	0,15	0,24	0,04	0,21
Preference Fn.	Usual	Usual	Usual	Usual	Usual	Usual	Usual	Usual	Usual
Thresholds	absolute	absolute	absolute	absolute	absolute	absolute	absolute	absolute	absolute
- Q: Indifference	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
- P: Preference	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
- S: Gaussian	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
Statistics									
Evaluations									
Amendment	1,00	1,00	1,00	1,00	0,00	0,00	0,00	0,00	0,00
Copeland	1,00	1,00	1,00	1,00	1,00	0,00	0,00	0,00	0,00
Dodgson	1,00	0,00	1,00	0,00	1,00	0,00	0,00	0,00	0,00
Maximin	1,00	0,00	1,00	1,00	1,00	0,00	0,00	0,00	0,00
Kemeny	1,00	1,00	1,00	1,00	1,00	0,00	0,00	0,00	0,00
Plurality	0,00	0,00	1,00	1,00	1,00	1,00	0,00	0,00	1,00
Borda	0,00	1,00	0,00	1,00	1,00	1,00	0,00	0,00	1,00
Approval	0,00	0,00	0,00	1,00	0,00	1,00	1,00	0,00	1,00
Black	1,00	1,00	1,00	1,00	1,00	0,00	0,00	0,00	0,00
Pl runoff	0,00	1,00	1,00	0,00	1,00	0,00	0,00	0,00	0,00
Nanson	1,00	1,00	1,00	0,00	1,00	0,00	0,00	0,00	0,00
Hare	0,00	1,00	1,00	0,00	1,00	0,00	0,00	0,00	0,00

PROMETHEE Flow Table

Rank	action	Phi	Phi+	Phi-
1	Approval	0,3936	0,5827	0,1891
2	Plurality	0,2649	0,3608	0,0959
2	Borda	0,2649	0,3706	0,1057
4	Copeland	-0,0045	0,1145	0,1191
4	Kemeny	-0,0045	0,1145	0,1191
4	Black	-0,0045	0,1145	0,1191
7	Maximin	-0,0635	0,0949	0,1584
8	Amendment	-0,0842	0,1013	0,1855
9	Nanson	-0,1464	0,0673	0,2136
10	Dodgson	-0,2053	0,0476	0,2529
10	Pl runoff	-0,2053	0,0427	0,2480
10	Hare	-0,2053	0,0427	0,2480

Fig. 16.2 Result of the VP chosen for the problem of water resources management

As can be observed, the Approval voting method was identified as the most appropriate VP for the problem to aggregate the individual results of the DMs involved in the decision about choosing the alternative to mitigate the degradation of the Jaboatão River Basin.

16.5 Global Result

Table 16.8 shows the results after applying the Approval Voting (AV), a voting procedure in which DMs may vote for as many candidates as they wish. The AV rule selects the candidate receiving the maximum number of votes or “approvals”.

As can be observed, DM1, DM3 and DM5 decided to approve the first three alternatives in their ranking, while DM2 and DM4 approved the first two alternatives. In accordance with the results, the winner alternative is A5 with three votes in its favor. This alternative is related to instituting policies for controlling the development of new businesses and/or the expansion of current ones to avoid worsening industrial pollution.

Table 16.8 Result of approval voting

Alternatives	DM1	DM2	DM3	DM4	DM5	Total
A1						0
A2	X	X				2
A3			X		X	2
A4			X		X	2
A5	X	X	X			3
A6				X	X	2
A7						0
A8						0
A9				X	X	2
A10	X					1
A11						0
A12						0

16.6 Topics for Further Reflection

The group decision model for water resources management was applied for the specific problem of choosing an alternative to mitigate the degradation in the Jaboatão River Basin. This application serves to illustrate the framework for choosing the VP for a water resources problem. In this case, a Supra-decision-maker was used to evaluate the voting proprieties. This Supra-DM was a member of the group. The other DMs considered that he understood the properties of these consequences better than they did. The “playing cards” method, also known as the Simos’ revised Procedure was used in order to obtain the weights of the proprieties of the VP (criteria).

The discrete binary outcome was used in the decision matrix for the VPs, and the multicriteria method that was applied to evaluate the VPs was Promethee. The same method was applied in order to aid the DMs to rank the alternatives.

In this problem, only five DMs took part in the process. However, a larger number of members can do so.

This proposal considers that each DM interprets a given situation differently and can generate different results (based on their individual way of thinking), even although they evaluate the same alternatives.

The use of the framework for choosing the VP was helpful and makes the process more transparent and acceptable. Note, however, that there are other methods for making social choices other than voting procedures and there are different VPs that can be used, which may generate different results.

16.7 Suggestions for Reading

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Chapter 17
Choosing a Voting Procedure to Identify Technology for Generating Renewable Electric Power

Abstract Among other worldwide concerns is that of choosing the technology for generating electric power that should comprise the electricity matrix of a country. In this kind of decision process, multiple actors are involved, and they need to consider not just the financial dimension but also the technical, socio-economic and environmental dimensions. This Chapter presents an illustration of the framework for choosing a VP to aggregate information from the profile of the various Decision-Makers involved in this process. This illustration is based on Kang et al. (2018) and Soares et al. (working paper) which presented how a decision model using the FITradeoff method was applied to aid a decision on identifying technology to generate electric power for the Brazilian electricity matrix.

17.1 Generating Renewable Electric Power

When electric power is generated centrally and the demand for electricity rises, an increase in generation occurs until capacity is reached. When capacity is exceeded, new generation units are created, thereby increasing the costs of transporting and distributing energy. As an alternative to such traditional systems for generating energy, Alanne and Saari (2006) argued that distributed energy generation systems offer an alternative that is more efficient, reliable and environmentally friendly.

This new trend of distributed energy generation means that energy conversion units are situated close to the consumers of energy, and large units are replaced with smaller ones. Besides, distributing the generation of energy is well adapted to regions that suffer from the supply of low-quality energy, such as rural regions, since this form of generation is relatively easy to develop locally and is cost-effective compared to other solutions for generating energy (Irena 2016).

In the context of distributed energy generation, it is important to periodically evaluate the most suitable solution for a country due to changes that may have occurred in different dimensions. In emerging countries, particularly those that are dependent on oil, it is essential to diversify energy sources in order to guarantee the supply of energy, to create jobs and to develop sustainable energy (Al Garni et al. 2016).

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The concept of sustainability, in general, means that scarce resources and economic opportunities regarding society and the environment should be distributed fairly (www.sustainablemeasures.com), and should take account not only of society's well-being today, but also in the future, as it is known that the resources consumed will be different in the future (WCED 1987). Based on the need for sustainable development, making use of renewable energy sources emerges as a good option.

According to De Melo et al. (2016), energy is said to be renewable when it is generated by using natural resources. Such sources of energy are continually replenished by nature and derived from the sun, wind, hydropower, the photosynthetic energy stored in biomass or from other natural movements in and mechanisms of the environment (such as geothermal and tidal energy) (Ellabban et al. 2014). Renewable energy technologies turn these natural energy sources into usable forms of energy, namely electricity, heat, and fuels.

Therefore, renewable energy meets the dual goals of reducing greenhouse gas emissions, thereby limiting future extreme weather and climate impacts, and ensuring the reliable, timely, and cost-efficient delivery of energy (Ellabban et al. 2014). Although these sources enhance the economy of a country (da Silva et al. 2016; Pohekar and Ramachandran 2004), renewable energy technologies are more expensive than conventional ones (Balezentis and Streimikiene 2017).

In Brazil, most electricity is generated by hydropower (de Melo et al. 2016; Aquila et al. 2016). The main technologies that generate electric power and comprise the electricity matrix of Brazil are shown in Table 17.1.

The predominance of hydroelectric sources for power generation in Brazil can be explained by Brazil's topography (Aquila et al. 2016). Nevertheless, since this kind of generation is dependent on hydrological conditions (da Silva et al. 2016) and has significant socio-environmental impacts, it is prudent to evaluate other sources of power generation that would form ideal energy policies for Brazil, especially of renewable energy sources (Strantzali and Arovossis 2016) to ensure that this kind of generation makes up a high share of the total resources in Brazil's electricity matrix (da Silva et al. 2016).

Thus, making decisions in this context is of high complexity. Multiple factors should be considered when deciding on how best to generate energy. This is not

Table 17.1 Brazilian electricity matrix as at 2015

Source	Percentage (%)
Hydropower	64
Natural Gas	13
Biomass	8
Petroleum	5
Coal	4
Wind	4
Nuclear	2

Adapted from Kang et al. (2018)

only related to energy production and consumption but is also associated with social, economic and environmental aspects (Zografidou et al. 2016). From this perspective, multiple actors are involved in this kind of decision process and they have the complex task of considering all these aspects, and thus to ensure a balance of sources or to make a tradeoff between them (Balezentis and Streimikiene 2017). Therefore, the impact of this decision process affects not only a region or a country, but it is a worldwide concern (Al Garni et al. 2016).

Nonetheless, another important characteristic of this type of problem is that the set of alternative solutions depends on the values and desires of the actors involved in the decision process. In the energy sector, there are a large number of actors, each of whom brings different perspectives on and a different set of values regarding power generation.

As can be observed, this kind of decision-making cannot be treated as an optimization problem that can use a single dimension (commonly the economic one). Thus, in order to analyze the problem as a complex system, the most appropriate approach for considering all the conflicting dimensions appears to be one that uses multicriteria decision-making/aiding (MCDM/A) methods (Zhang et al. 2015).

Given the need to diversify Brazil's electricity matrix by investing in technologies that complement hydroelectric generation, and taking into account the multiple aspects that need to be considered when making such decisions, Kang et al. (2018) proposed a MCDM/A model to evaluate different electrical energy technologies, both renewable and non-renewable ones, that comprise Brazil's current electricity matrix under (financial, technical, environmental and socio-economic) dimensions of sustainability.

In this Chapter we use this model as an illustration of applying the framework for choosing a VP. However, we focus only on renewable sources of energy, based on the working paper of Soares et al. (w.p.).

The MCDM/A model proposed by Kang et al. (2018) focused on situations where there is not enough data regarding the parameters related to some criteria that are important for the decision context or where the available information is incomplete. This is a very relevant aspect in the area of renewable technologies for distributed electric power generation. Taking this perspective, they proposed applying the Flexible and Interactive Tradeoff (FITradeoff) method (de Almeida et al. 2016). This method requires less cognitive effort from the decision-maker (DM) when eliciting his/her preferences, since it is based on incomplete (or partial) information. The FITradeoff DSS (Decision Support System) can be downloaded on request at http://fitradeoff.org/.

The dynamic procedure to build this MCDM/A model followed the de Almeida et al. (2015), framework, which consists of three main phases subdivided into twelve steps, within a flexible sequence, where the DM can go back to previous steps when necessary, thereby enhancing learning and generating insights during the process.

In the first phase of the model, the preliminary information is defined, such as identifying the actors of the decision process (henceforth called DMs), their objectives and the related set of criteria, and the viable alternatives. In the second phase, which considers the characteristics of the problem and the DM's preference structure, an MCDM/A method is chosen and applied (de Almeida et al. 2015), in this case, the FITradeoff method. Finally, in the third phase, the alternatives are evaluated, and a sensitivity analysis is conducted.

For this Chapter, a fourth and fifth phase were added. These phases deal with applying the framework for choosing a Voting Procedure (VP) and the global result, respectively. In order to choose the VP, it is important to evaluate the properties of the desired VP and also which VP is appropriate for this decision context. Once the VP is chosen, then it is applied using the ranking obtained from the DMs during the first three phases. Figure 17.1 shows the flowchart of the model for selecting the most appropriate form of renewable electric power generation in Brazil.

17.2 Structuring the Problem

In order to support the analysis of technology for renewable distributed electric power generation, first of all, what must be done is to identify who the DMs are, what the alternatives for this problem are and what the set of criteria to evaluate the alternatives should be. This application is based on Kang et al. (2018) and Soares et al. (w.p.), considering Brazil's electricity matrix.

17.2.1 Identifying the Decision-Makers

Many actors or pressure groups can be involved in this problem of looking for renewable technologies to generate electric power in Brazil. Each of them has their own perspectives and different value structures. For instance, technical and financial aspects may be emphasized by a utility company, which is interested in the performance of a plant and a return on capital. On the other hand, the community is interested in social and environmental impacts. Consequently, conflicts may exist and what is preferred by one group may not be by another (Stein 2013).

In this chapter, it was considered that there were four decision-makers (DMs), whom Kang et al. (2018) call different decision profiles. Table 17.2 shows the concerns of these DMs and their codes.

17.2.1.1 Decision Profile A: Energy Production

This DM is primarily concerned with the operational performance of the renewable electric power generation plant. The technical dimension is his/her focus. This profile is especially interested in the efficiency of generation, the capacity factor and controllability.

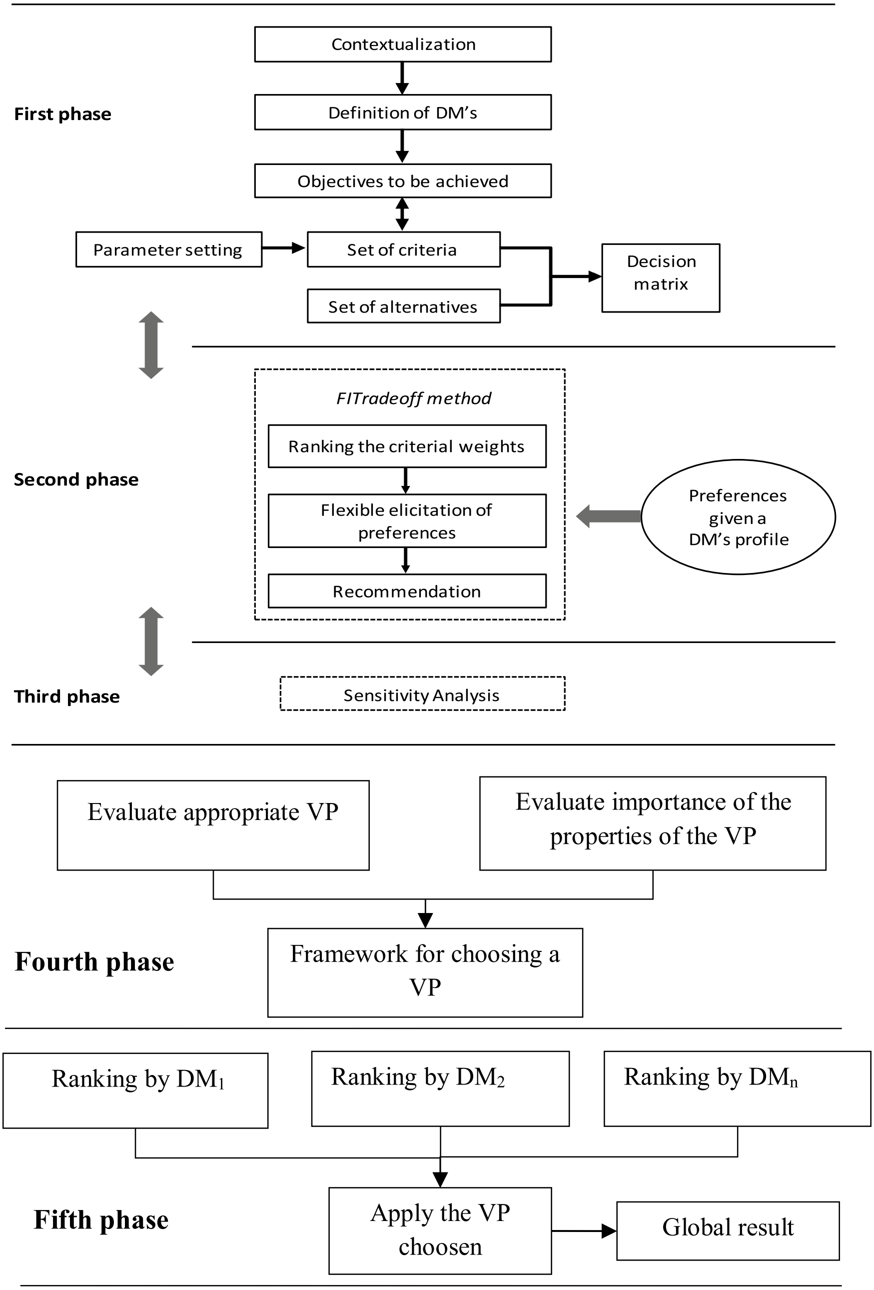

Fig. 17.1 Flowchart of the proposed MCDM/A model (adapted from Kang et al. 2018)

Table 17.2 Decision-makers considered in this problem

Decision-maker	Code	Concerns
Profile A	DM $_A$	Energy production
Profile B	DM $_B$	Return on investment
Profile C	DM $_C$	Environmental impact
Profile D	DM $_D$	Job creation

17.2.1.2 Decision Profile B: Return on Investment

This DM is concerned with the financial performance of the renewable electric power generation plant. The electric power technologies are evaluated from a financially-oriented perspective. The DM would prioritize the cost of investment and the average cost of operation and maintenance costs.

17.2.1.3 Decision Profile C: Environmental Impact

This DM is concerned with the environmental impacts and their interference in people's lives, and therefore seeks clean, renewable and non-polluting forms of energy.

17.2.1.4 Decision Profile D: Job Creation

This DM is concerned with the socioeconomic and political impact and creating jobs by setting up a renewable electric power generation plant. The number of jobs created is evaluated in the construction and installation phases, in the manufacturing phase, and during the operation and maintenance of the system.

17.2.2 Establishing the Set of Potential Alternatives

The set of potential alternatives, i.e., the set of viable alternatives, consists of four renewable electric power generation technologies that comprise Brazil's electricity matrix (Tolmasquim 2016). These alternatives are the technologies defined by ANEEL Normative Resolution No. 482/687 (ANEEL 2014, 2015). Table 17.3 shows the alternatives considered and their respective codes.

Table 17.3 Set of alternatives

Renewable electric power generation technology	Code
Wind power	WP
Solar photovoltaic	SPV
Small hydroelectric power plant	SHP
Biomass	Biofuels

According to (Ellabban et al. 2014), wind power results from using wind turbines to convert the energy from wind into electricity, using windmills for mechanical power, using wind pumps for pumping water or for drainage, or using sails to propel ships. Generating electricity from the wind requires that the kinetic energy of moving air be converted to mechanical and then to electric energy, thus challenging the industry to design cost effective wind turbines and power plants to perform this conversion. At the beginning of the 20th century, the first wind turbines for electricity generation were developed, and this technology has gradually improved since the early 1970s. Nowadays, wind energy has re-emerged as one of the most important sustainable energy resources.

A solar photovoltaic (PV) system is a semiconductor device (PV cell) that converts solar energy into direct-current electricity. PV cells are interconnected to form a PV module, typically up to 50 to 200 W. The PV modules, combined with a set of additional application-dependent system components (e.g., inverters, batteries, electrical components, and mounting systems), form a PV system. PV systems are highly modular, i.e., modules can be linked together to provide power ranging from a few watts to tens of megawatts (Ellabban et al. 2014).

Hydropower is a power derived from harnessing the energy of moving water. Flowing water creates energy that can be captured and converted into electricity by using turbines. The most prevalent form of hydropower is associated with dams. On the other hand, a small hydroelectric power plant (SHP) can be created by developing hydroelectric power on a scale suitable for a local community and industry, or to contribute to distributed generation in a regional electricity matrix.

Biomass energy is the term used for all organic material originating from plants, trees and crops, and is essentially about collecting and storing solar energy as a result of photosynthesis. Biomass energy (bioenergy) is the conversion of biomass into useful forms of energy such as heat, electricity and liquid fuels (biofuels) (Ellabban et al. 2014).

While these alternatives are different sources of renewable energy, it should be noted that each source of renewable energy has its advantages, disadvantages and these include there being some negative impacts on the environment, as shown in Table 17.4.

17.2.3 Selecting Criteria for Evaluation

The selection of the criteria was based on four decision-makers' profiles, henceforth called sustainability dimensions: financial, technical, environmental and socio-economic. Table 17.5 shows the relationship between these profiles and the dimensions considered.

Each dimension represents a group of criteria. Table 17.6 shows these criteria and their respective parameters. Such parameters are fundamental for the model, since they represent the consequence that can be obtained for each alternative, considering a deterministic problem. As to the financial dimension, two natural aspects were

Table 17.4 Advantages, disadvantages and negative impacts on the environment of the renewable energy resources considered

Source	Advantages	Disadvantages	Potential negative impacts on the environment
WP	– Is a free source of energy – Produces no water or air pollution – Wind farms are relatively inexpensive to build – Land around wind farms can have other uses	– Requires constant and significant amounts of wind – Wind farms require significant amounts of land – Can have a significant visual impact on landscapes – Need better ways to store energy	– Noises in the area, landscape change, soil erosion, the blades of the turbines kill birds
SPV	– Potentially infinite energy supply – Causes no air or water pollution	– May not be cost effective – Storage and backup are necessary – Reliability depends on availability of sunlight	– Soil erosion, landscape change, hazardous waste
SHP	– Abundant, clean, and safe – Easily stored in reservoirs – Relatively inexpensive way to produce electricity – Offers recreational benefits like boating, fishing, etc.	– Can cause the flooding of surrounding communities and landscapes – Dams have major ecological impacts on local hydrology – Can be used only where there is a water supply	– Change in local eco-systems, change in weather conditions, social and cultural impacts
Biofuel	– Abundant and renewable – Can be used to burn waste products	– Burning biomass can result in air pollution – May not be cost effective	– May not be natural CO_2, may release global warming gases like methane during the production of biofuels, landscape change, deterioration of soil productivity, hazardous waste

Adapted from Ellabban et al. (2014)

considered: the investment cost and the operational and maintenance costs. For the technical dimension, four criteria related to operational performance and efficiency were considered: the efficiency of generation, the capacity factor, maintenance and the controllability of input. The environmental dimension is concerned with evaluating the emission of CO_2, land occupation, safety and social welfare. Finally, what is

Table 17.5 Decision profile versus dimensions of sustainability

Decision-makers	Concerns	Dimension	Relates to	Objectives
Profile A	Energy production	Technical	Technical aspects of a technology that influences the generation of energy	Maximize operational performance and efficiency of the production process
Profile B	Return on investment	Financial	Costs related to investing in technology for generating electricity	Minimize costs
Profile C	Environmental impact	Environmental	Impact that a technology has on the environment	Minimize negative impacts on the environment and the well-being of the population
Profile D	Job creation	Socio-economic	Socio-economic impact caused by implementing a technology	Maximize the socio-economic impact and the financial return

evaluated for the socioeconomic dimension is the lifespan, secondary gains, jobs created in the construction and installation phase, and jobs created in the manufacturing phase and during operation and maintenance.

Regarding the financial dimension, it is very objective and in order to parameterize its criteria, it is necessary to define the desired application, as to the location, and to consider the energy potential and consequent choice of the energy generating devices. In this case, for the investment cost criterion, the data were obtained from the literature review (Skystream 2018; ENERGIA 2018; Solar 2018; BGS 2018; Branco 2018). Moreover, for O&M, the fixed costs related to operating and maintaining the electrical power generation plant were considered (Tolmasquim 2016).

As to the technical dimension, two criteria are natural ones, namely Generation Efficiency and Capacity Factor. These are measured in percentage terms (%), with values for each technology being well established in the literature (Evans 2010; Tidball 2010; EIA 2013). The other criterion is related to the maintenance of the electricity generation system. It is important to notice that this criterion is fundamental for choosing a technology. However, as yet no data for distributed production have been established. In order to evaluate the maintenance needed for distributed renewable electricity generation technologies, Komor and Molnar (2015) presented a simplified Likert scale that uses generalist parameters (high, low or medium). In this

Table 17.6 Set of criteria

Code	Criteria	Definition	Unit
C1	Investment cost	Comprises the costs related to build and install a power generation plant	US$/kW
C2	O&M	Considers the costs related to operating and maintaining the electric power generation plant	US$/MWh
C3	Generation efficiency	Considers the conversion in electric energy capacity by each generation technology, i.e., establishes the relationship between electricity generated by the plant and the energy provided by the source	%
C4	Capacity factor	This refers to the time period in which the plant is actively generating electricity. Natural conditions that occur in places where the plants are located and scheduled stops for repairs and maintenance have to be considered	%
C5	Maintenance	This considers the facility/simplicity of carrying out maintenance on the generation devices	–
C6	Input controllability	The possibility of controlling both the availability of the source that generates power and storing this power	
C7	CO_2 emission_t	CO_2 is one of the gases that contribute to the greenhouse effect and that can be emitted as a result of the production process of generating electric power	gCO_2EQ/kWh
C8	Land occupation	This is a measure of the area available for a technology to work in	m^2/MWh
C9	Safety	This considers the degree of possibility of accidents occurring that are inherent to each power generation system	–
C10	Social welfare	This considers the impact of technologies on people's lives and well-being	–
C11	Lifespan	Length of time, in years, in which the plant can generate electricity in a sustainable way	Years
C12	Secondary gain	This considers what value-added by-products there may be as a consequence of generating energy	Years
C13	Jobs in the construction and installation phase	This considers how many jobs will be generated while devices and equipment of the power generation plant are being manufactured	Jobs/year/MW
C14	Jobs in the manufacturing phase	This considers the jobs generated when building the infrastructure and installing the devices and equipment of the electric power generation plant	Jobs/year/MW
C15	Jobs during operation and maintenance	This considers the jobs generated when operating and maintaining the devices and equipment of the electric power generation plant	Jobs/year/MWh

Adapted from Soares et al. (w.p.)

application, three basic aspects were used for this evaluation of maintenance: lubrication needs, availability of spare parts and the need for specialized labor. Table 17.7 analyzes the maintenance criterion.

Considering these aspects and influences, a five-point Likert scale was used to determine a qualitative evaluation of this criterion, as shown in Table 17.8.

The last criterion of the Technical dimension is Input Controllability, which considers if it is possible to control the availability of the power source for generation, and of the storage of power. Table 17.9 shows its binary evaluation.

As to the environmental dimension, two aspects were considered: the emission of CO_2, as a greenhouse gas (GHG), and the external costs generated when producing electrical energy, such as land occupation, safety and social welfare.

Regarding the emission of CO_2, according to (Weisser 2007), all energy systems emit greenhouse gases (GHG) and therefore contribute to anthropogenic climate change. In the case of renewable energy technologies, the majority of GHG emissions typically occur as a result of producing and constructing the technology and/or its supporting infrastructure, although, for biomass systems, depending on the choice of biomass fuel, most emissions can arise during the fuel-cycle. With regard to GHG emissions from different energy technologies, Daniel Weisser (2007) conducted an interesting study. This compared and analyzed the results of the GHG emission life-cycle and reviewed and summarized this kind of emission for the renewable energy technologies. Moreover, the National Renewable Energy Laboratory (NREL) (Edenhofer 2011) conducted a similar review, by building a database to assess GHG in the life cycle of electricity. The NREL data were used in this application, and are within the range obtained in the studies of Weisser (2007).

Table 17.7 Maintenance criterion and its aspects and influences

Maintenance aspects	Possibilities
Availability of spare parts	– High availability – Low availability
Lubrication needs	– Needs lubrification – No need for lubrification
Need for specialized labor	– High complexity – Low complexity

Table 17.8 Maintenance criterion scale of evaluation

Description	Level
Low availability, lubrication and high complexity	1
High availability, lubrication and low complexity	2
Low availability, no lubrication and high complexity	3
Low availability, no lubrication and low complexity	4
High availability, no lubrication and low complexity	5

Adapted from Soares et al. (2018, w.p)

Table 17.9 Input controllability scale of evaluation

Description	Level
Non-controllable technology	0
Controllable technology	1

For the external costs, the environmental impact of a power generation plant on human populations and natural systems was considered. Such impacts should be measured considering not only their operation, but also all stages of the technology's life cycle. However, few studies about this issue have been conducted and there is very little information in Brazil. In fact, the only one available has no technical proof. Therefore, because of the generality of external cost data—since they do not consider the specificities for the Brazilian case, three representative criteria were proposed for the concept of external costs (considering their negative nature): land occupation, safety and social welfare.

- Land occupation: this considers the amount of area needed, directly and indirectly, for a technology to work. Neither how the land is used nor for how long it is used, nor if the technology damages the site are observed (Evans 2010). As to the generation of renewable energy, wind and solar photovoltaic typically use little space directly, although what is required is to disperse these technologies over large areas (Fritsche 2017). Other simultaneous uses of the land are often allowed, such as grazing and even arable farming, possible under or on wind and photovoltaic farms. In this application, due to its distributed characteristics, space is saved by considering only placing photovoltaic panels directly on the roof of buildings. As to hydropower, the use of land is more limited, since flooded areas preclude other uses of land (except recreation/fishing) and can create barriers to the migration of aquatic life. Nevertheless, for the SHPs, this application considers the solution to be to use shallow water as the source from which to derive the energy to drive turbines which avoids generating a flooded area. As to biofuel, the land occupation is close to zero, because this fuel is a by-product, since bioenergy can be obtained simultaneously from the same land with other products, for example, milk and beef, pork or poultry meat (Rafaj and Kypreos 2007). Other data on land use for the generation of electrical energy from renewable sources can be found in Evans (2010) and Fritsche (2017).
- Safety: this concerns the risk of accidents to the electric energy generation devices, considering the types of elements that they consist of and the different features of the technologies that generate energy. Three aspects of safety involving energy control are considered: kinetic energy (moving parts in relative motion), inertia energy (size and weight of components) and energy potential (height of the installation). For the safety criterion, a seven-point Likert scale of values was established, as shown in Table 17.10.
- Social welfare: considers the impact of each generation technology on people's lives. For the social welfare criterion, a four-point Likert scale was drawn up to conduct a qualitative evaluation. Table 17.11 shows the levels defined for the

Table 17.10 Safety evaluation scale

Description	Level
There are elements with low weight without relative movement and situated at a low height	1
There are elements with high weight, without relative movement and situated at a low height	2
There are elements with low weight, with relative movement and situated at a low height	3
There are elements with high weight, with relative movement and situated at a low height	4
There are elements with high weight, without relative movement and situated at a great height	5
There are elements with low weight, with relative movement and situated at a great height	6
There are elements with high weight, with relative movement and situated at a great height	7

Adapted from Soares et al. (2018, w.p)

Table 17.11 Social welfare scale of evaluation

Description	Level
No sound impact, no visual impact, no risk to animals, no direct risk to human beings	1
Low sound impact, no visual impact, no risk to animals, no direct risk to human beings	2
Low sound impact, low visual impact, no risk to animals, low risk to human beings	3
With sound impact, with visual impact, with risk to animals, with direct risk to human beings	4

Adapted from Soares et al. (2018, w.p)

consequences for this criterion, based on the impact of sound, the visual impact, the risk to animals and the risk to human beings.

Regarding the socioeconomic dimension, five criteria were considered to evaluate its impact: lifespan, secondary gain and the capacity to generate jobs in the different phases, including design, construction, operation and maintenance.

- Lifespan: This considers values available in the literature (Tolmasquim 2016) such as the service life based on the operating life of the devices and equipment of the energy generation plant.
- Secondary gain: considers the opportunity of obtaining a by-product with added economic value because of the generation of electric energy. Table 17.12 presents the evaluation scale for this criterion.
- Jobs: Those that are considered are the ones created when the devices are being constructed and installed; when devices and equipment of the electrical energy generation plant are being manufactured; and the ones generated during the operation and maintenance of these devices and equipment (Wei 2010). Due to the

Table 17.12 Evaluation scale for secondary gains when energy is generated

Description	Level
The technology does not generate any by-products	0
The technology generates a by-product	1

lack of data for the region of small scale electrical energy generation, data were based on a Greenpeace study (Greenpeace 2013), which compares the different electricity generation technologies associated with the capacity to generate jobs in Brazil.

17.3 Individual Results

The application of the model was developed in a case study carried out in a rural southeast region of Brazil in the State of São Paulo, chosen due to the availability of the data on the generation technologies to be analyzed. It corresponds to the area of the Mogiguaçu River Basin.

For each decision profile, the FITradeoff elicitation process was performed based on data from the decision matrix (Table 17.13), which therefore simulated the specific interests of different pressure groups regarding the problem.

Moreover, a different structure of preferences was assumed when ranking the criteria weights and expressing preferences. Then, the FITradeoff elicitation process was performed with each decision profile (here understood as a group of decision-makers) based on data from the decision matrix (Table 17.13). These decision profiles simulated specific interests of different pressure groups regarding renewable electric power generation. This led to different results. Table 17.14 shows the final rankings per decision profile, where w_j corresponds to the weight of a criterion c_j.

Table 17.15 presents the results found by FITradeoff for each decision profile. For each solution, there is an associated space of weights in which each criterion weight is limited by a minimum and a maximum value. This weight space was narrowed as more information, in the form of preference statements, was obtained from the DMs' responses. Column "Number of Questions" in Table 17.15 shows how many questions were answered, i.e., how many preference statements were given.

When analyzing the results for the four groups, SHP is considered the best option for two groups (A and B), but it is considered the worst for group D. While the Biofuel option is the best for group C and D, it is never considered as the worst alternative.

Table 17.13 Matrix of consequences for choosing power generation technology

Electric Power Generation Technology	Criteria														
	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15
	US$1.000/kW	US$/kW/ano	%	%	–	–	gCO_2EQ/kWh	m^2/MW_h	–	–	Year	–	Jobs/year/MW	Jobs/year/MW	Jobs/year/MW
Wind	3.23	85	35	43	1	0	12	1	7	4	20	0	7.7	3.3	0.6
Solar (FVT)	2.34	19	20	24	5	0	46	10	1	3	25	0	10.9	6.9	0,3
Hydropower	1.26	13	90	55	3	1	4	10	4	1	20	0	6	1.5	0.6
Biomass	1.119	22	25.3	60	2	1	18	0	3	2	20	1	14	2.9	1.5

Adapted from Soares et al. (2018, w.p)

Table 17.14 Ranking of criteria per decision profile

Group of DMs	Ranking of criteria
Profile A: Energy production	$w_3 > w_4 > w_6 > w_5 > w_7 > w_8 > w_9 > w_{10} > w_{11} > w_{12} > w_{13} > w_{14} > w_{15} > w_1 > w_2$
Profile B: Return on investment	$w_1 > w_2 > w_3 > w_4 > w_5 > w_6 > w_{11} > w_{12} > w_{15} > w_{14} > w_{13} > w_8 > w_9 > w_{10} > w_7$
Profile C: Environmental impact	$w_7 > w_{10} > w_8 > w_9 > w_{12} > w_{11} > w_{15} > w_{13} > w_{14} > w_3 > w_4 > w_5 > w_6 > w_1 > w_2$
Profile D: Job creation	$w_{15} > w_{14} > w_{13} > w_{12} > w_8 > w_{11} > w_3 > w_4 > w_5 > w_6 > w_1 > w_2 > w_7 > w_9 > w_{10}$

Table 17.15 Results for the group decision profiles

Ranking	Profile A: Energy production	Profile B: Return on investment	Profile C: Environmental impact	Profile D: Job creation
1	SHP	SHP	Biofuel	Biofuel
2	WP	Biofuel	SHP	WP
3	Biofuel	SPV	WP	SPV
4	SPV	WP	SPV	SHP
Number of questions answered	21	14	23	45

17.4 Applying the Framework for Choosing a VP

Since the decision profiles found a different ranking of the alternatives, in this stage of the model, the framework for choosing a voting procedure (VP) is used to aggregate the results of the decision profiles in order to find a global result which will be the best alternative for renewable power technology for a Brazilian region.

The characteristics of this problem reveals that there is a need for a voting procedure that deals with rankings, since the problem evaluated has only four alternatives and it is important to analyze how the decision profiles classified them. Thus, the VPs considered for this evaluation were: Copeland, Borda, Black, Nanson and Hare.

Another important aspect to consider is the voting proprieties to evaluate the VP. The proprieties analyzed in this application were: Condorcet winner; Strong Condorcet; Monotonicity; Consistency and Invulnerability to the no-show paradox. The proprieties of Condorcet loser and Pareto were not considered since all VPs analyzed satisfy these conditions. Similarly, the Chernoff and Independence of irrelevant alternatives were not considered since none of the VPs analyzed satisfies these conditions.

For this analysis, it will be present two ways of consequence matrix: binary outcome and discrete score.

17.4.1 Using the Consequence Matrix of Binary Outcome

The consequence matrix of the VPs and their proprieties based on a binary outcome (Chap. 14), is as shown in Table 17.16. In this table, “1” indicates that the VP satisfies the property and “0” that it does not. The value function is in Eq. 14.1 (Chap. 14). Also, Table 17.16 gives the weights of the five voting proprieties considered, where the DMs agreed about the weights considered.

Table 17.17 presents the results after applying the PROMETHEE II method to evaluate the decision matrix, using the usual preference function.

As can be observed, the result for the PROMETHEE II method is equivalent to that of the additive model, when using this binary outcome matrix (see Table 17.18).

Thus, the Borda voting procedure was identified as the most appropriate to aggregate the decision profile to find an alternative renewable power generation technology for a Brazilian region.

Table 17.16 Matrix of consequence of the VP considered

Voting system	Criteria/weights				
	Condorcet winner	Strong condorcet	Monotonicity	Consistency	Invulnerability to the no-show paradox
	0.1	0.1	0.2	0.35	0.25
Copeland	1	1	1	0	0
Borda	0	0	1	1	1
Black	1	1	1	0	0
Nanson	1	1	0	0	0
Hare	0	1	0	0	0

Table 17.17 Results after applying the PROMETHEE II method

Rank	VP	Phi	Phi+	Phi−
1	Borda	0.525	0.7	0.175
2	Copeland	0.025	0.175	0.15
2	Black	0.025	0.175	0.15
4	Nanson	−0.225	0.075	0.3
5	Hare	−0.35	0.025	0.375

Table 17.18 Results after applying the additive method

Voting system	Criteria/weights					Result	Rank
	Condorcet winner	Strong con-dorcet	Monotonicity	Consistency	Invulnerability to the no-show paradox		
	0.1	0.1	0.2	0.35	0.25		
Copeland	1	1	1	0	0	0.40	2
Borda	0	0	1	1	1	0.80	1
Black	1	1	1	0	0	0.40	2
Nanson	1	1	0	0	0	0.20	4
Hare	0	1	0	0	0	0.10	5

17.4.2 Using the Consequence Matrix of with Discrete Score

The consequence matrix of the VPs and their proprieties can also be evaluated by using a discrete score of three levels (0, 1, 2), instead the binary outcome. This score is elicited from an expert indicating the influence of that criterion on the VP. Table 17.19 shows what the score represents for the VP considered.

Considering these scores, it is obtained the following consequence matrix (Table 17.20), for the VP considered for this problem.

Table 17.19 Discrete score of the VP considered

Score	Description
0	It indicates that the VP satisfies the property
1	It indicates that the VP may satisfy with a medium frequency the property
2	It indicates that the VP does not satisfy the property

Table 17.20 Matrix of consequence of the VP with discrete score

Voting system	Criteria/weights				
	Condorcet winner	Strong condorcet	Monotonicity	Consistency	Invulnerability to the no-show paradox
	0.1	0.1	0.2	0.35	0.25
Copeland	0	0	0	1	1
Borda	1	1	0	0	0
Black	0	0	0	1	2
Nanson	0	0	1	1	2
Hare	1	0	1	2	1

Table 17.21 Results after applying the additive method for discrete score

Voting system	Criteria/weights					Result	Rank
	Condorcet winner	Strong condorcet	Monotonicity	Consistency	Invulnerability to the no-show paradox		
	0.1	0.1	0.2	0.35	0.25		
Copeland	1	1	1	0.5	0.5	0.70	2
Borda	0.5	0.5	1	1	1	0.90	1
Black	1	1	1	0.5	0	0.58	3
Nanson	1	1	0.5	0.5	0	0.48	4
Hare	0.5	1	0.5	0	0.5	0.38	5

At this point, the score has an outcome of decreasing preference, this leads to producing the marginal value $v(x_j)$ of the outcomes x_j related to criterion j. The following value function (see Chap. 14; Eq. 14.6) may be applied:

$$\mathrm{v_j(x_{ij}) = (y - x_i)/y}$$

Where y is the highest level in the scale (for this case of three-level scale, y = 2).

Using the additive model, Table 17.21 shows the result and respectable rank.

As can be observed, also the Borda voting procedure was identified as the most appropriate to aggregate the decision process. So, for this case, the result using the discrete score is the same as using the binary outcome. However, it is possible to have a complete order. No ties were found between Copeland and Black VP.

17.5 Global Result

In order to find the global result, the Borda count is applied to the data presented in Table 17.15. Thus, the Borda voting procedure was identified as the most appropriate for aggregating the decision profile to find an alternative renewable power generation technology for a Brazilian region (Table 17.22).

Based on the ranking obtained by the Borda count, the Biofuel was the first alternative, followed by Small Hydropower, Wind power and finally Solar Photovoltaic.

17.6 Topics for Further Reflection

The results obtained by using the decision model based on the FITradeoff method applied to different decision profiles and then aggregated by a Voting procedure,

Table 17.22 Results for the group decision profiles

Alternatives	Points				
	Profile A:	Profile B:	Profile C:	Profile D:	Results
SHP	4	4	3	1	12
WP	3	1	2	3	9
Biofuel	2	3	4	4	13
SPV	1	2	1	2	6

shows the model has potential to assist a group of decision-makers to tackle complex problems related to energy planning.

17.7 Suggestions for Reading

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Chapter 18
Choosing a Voting Procedure for a Group Decision Support System (GRUS)

Abstract Group Decision Support Systems (GDSS) are tools that are being increasingly used in group decision-making processes. In this context, GRUS (GRoUp Support System) is a web-based system to support group decision processes which consider the individual preferences of different actors involved in the same problem. The system supports a multicriteria approach for solving the problem. One of the ways to aggregate individual preferences is by using a voting procedure. This Chapter presents how the framework for choosing a Voting Procedure can be implemented in this GRUS System in order to facilitate this process. Two different situations for applying the framework are considered. In the first one, the users evaluate the problem and apply the framework for choosing a voting procedure. In the second situation, the result of the framework applied with an expert is presented for the users as a generic voting procedure to aggregate the individual rankings of the decision-makers.

18.1 GRoUp Support System (GRUS)

In many organizations, where collective decisions should be made, it is common to have conflict situations due to decision-makers (DMs) having different points of view and interests from each other. Furthermore, many managers spend their productive time (between 25 and 80%) in meetings at which decisions are made, but approximately 50% of this time is wasted as a result of information being lost or distorted (Dufner et al. 1995). Therefore, in order to reduce such losses and raise the productivity of managers, several Group Decision Support Systems (GDSS) have been proposed in the literature (Colson 2000; Damart et al. 2007; Adla et al. 2011; Lolli et al. 2015).

GDSS are often built based on computer platforms with a formal framework that uses a multi-criteria approach to help DMs express and evaluate their preferences and the parameters that will be used. Thus, Zaraté et al. (2016) built a GDSS on a web-based platform, called GRoUp Support System (GRUS), which is free and available upon request at http://www.irit.fr/GRUS.

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Advances in Group Decision and Negotiation 9,
https://doi.org/10.1007/978-3-030-30955-8_18

GRUS presents services commonly available in GDSS, such as the definition/design of a static or dynamic group decision process, the management of collaborative tools (add, modify, delete), and the management of automatic reporting such as PDF files (Zaraté et al. 2016). This system aids the conduct of meetings which may be synchronous or asynchronous, and distributed or face-to-face. The users of such a system are DMs and the facilitator. It is modularized to allow the facilitator to build a structure that best fits the problem. The facilitator is also responsible for managing the process of how the DMs interact with each other. The DMs should describe their points of view and ideas, whether anonymously or not, in the step called brainstorming as to the electronic interaction. They suggest the criteria and alternatives related to the problem to be solved, and then give their assessment of each alternative on each criterion, thereby generating a consequence matrix.

For the evaluations, the DMs indicate their preferred weights for the criteria, and enter a suitability equation function, thereby defining their interpretation of each criterion. In order to calculate the score of each alternative, two aggregation techniques are implemented in the GRUS: The Simple Additive Weight (SAW) (Keeney and Raiffa 1976) and the Choquet Integral (Ebadi et al. 2010).

The final decision must be managed by a facilitator who does so with a consensus process that is conducted in a face-to-face group meeting. Sometimes this process requires DMs to change their positions with regard to how to solve the problem until a potential compromise is found. This is usually time-consuming especially when the DMs have different objectives regarding the same problem. Therefore, the concern that is raised here is how to deal with this process when the DMs have different objectives.

De Almeida et al. (2015) noted that when DMs have divergent opinions regarding the objectives, it is necessary to work with their individual rankings of the alternatives and aggregate them in order to reach final choices that they can agree to. One way to deal with this type of aggregation is to use a voting procedure (VP). In this case, it is usually the facilitator who is responsible for choosing a VP compatible with the DMs´ needs so as to reach a group decision.

Numerous VPs have been studied over the years that have been applied in different situations. A comparative analysis of some of these VPs is given in Nurmi (1999), who showed that each procedure is associated with advantages and disadvantages and seeks to avoid different voting paradoxes. Nevertheless, the definition of the best VP usually depends on the properties of each procedure, which have been discussed over the years in the literature (Nurmi 2015) besides which many authors have compared VPs by considering their properties (Nurmi 1983, 2004; Fishburn and Gehrlein 1982; Lepelley and Valognes 1999; Kim and Roush 1996; Kim et al. 2002).

Bearing this in mind, according to de Almeida and Nurmi (2015), in specific situations, the facilitator is perhaps not the best person to conduct this task of choosing a VP, since he/she will not deal with the consequences of the social choice. Thus, the framework for choosing a VP that is applied here to aid this choice allows the DMs to have their preferences considered in this analysis.

The main idea of this framework is to consider a decision matrix where the VPs are the alternatives which are evaluated regarding some criteria (that are voting properties, which are characterized by the ability of a procedure to overcome voting paradoxes, and are related to the context of the problem, by considering how easily this matrix can be applied). A multicriteria approach is used to evaluate this decision matrix, which considers the characteristics of the methods and the problem itself (de Almeida et al. 2015).

In this Chapter, two situations for applying this framework for choosing a VP are considered:

Situation 1: In order to aid the users choosing a VP. In this case, the framework for choosing a VP is implemented in the GRUS and it is applied when the users are willing to choose a VP so as to make an in-depth evaluation of the characteristics of the problem and the advantages and disadvantages of the voting procedures for the specific case.

Situation 2: In order to indicate generically a voting procedure for the GDSS. In this case, the users are not willing to choose a specific VP, or do not have enough information regarding voting properties. Thus, the framework for choosing a VP is applied with an expert in voting rules who will make a holistic evaluation of the properties, yet thinking about the information that the GRUS is providing as individual rankings. The VP chosen by this expert will be implemented in the GDSS as a suggestion for an aggregation procedure that will result in a final recommendation being made.

Figure 18.1 shows the flowchart for applying the framework for choosing a VP to be included in the GRUS.

18.2 Structuring the Problem

The problem will be structured predominantly in the same way for both situations. In order to evaluate the voting procedures, the voting properties will be considered as criteria, which are presented as follows (Palha et al. 2017; Nurmi 1999; Arrow 1963; Felsenthal and Nurmi 2018):

- Condorcet winner: evaluates if the procedure chooses a Condorcet winner when there is one, i.e., the alternative which defeats all alternatives in pairwise comparisons.
- Condorcet loser: evaluates if the procedure does not choose a Condorcet loser when there is one, i.e., the alternative which is defeated by all other alternatives in pairwise comparisons.
- Strong Condorcet: evaluates if the procedure ends up with a strong Condorcet winner when there is one, i.e., the alternative which is ranked first by most individuals.
- Monotonicity: evaluates if the procedure displays monotonicity, i.e., “if an alternative y wins in a given profile P when a certain VP is being applied, it should also win in the profile P’ obtained from P by placing y higher in some individuals’ preference rankings” (Nurmi 1999). This means that additional support cannot transform a winning alternative into a non-winning alternative.

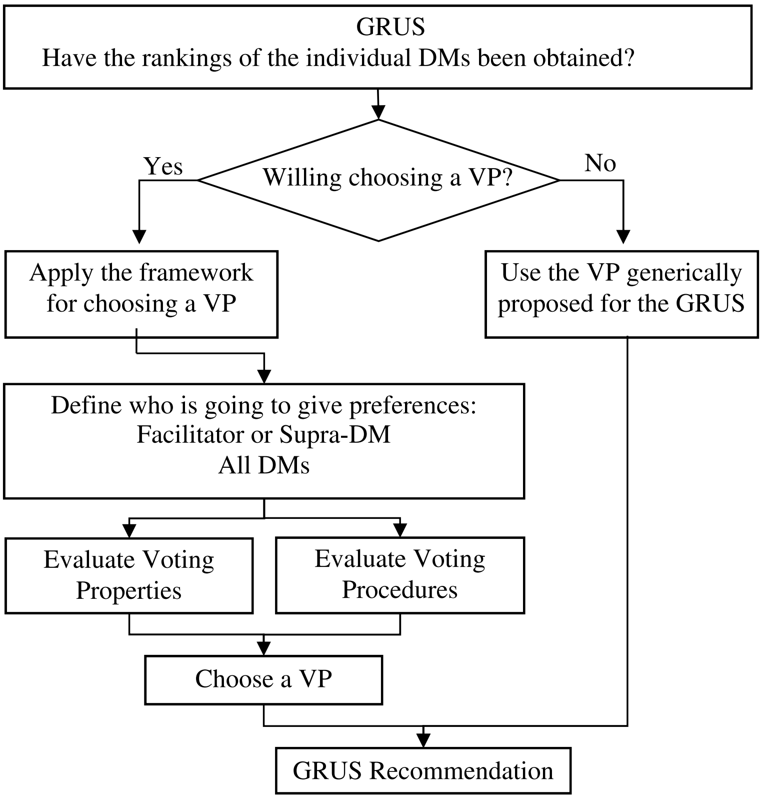

Fig. 18.1 Flowchart of the GRUS incorporating the framework for choosing a VP

- Pareto: evaluates if the procedure has a collective rationality, i.e., whenever all individuals strictly prefer x to y, then y is not chosen.
- Consistency: evaluates if the procedure satisfies the condition of the invariance of the set chosen when different decision-making groups are gathered together to make social choices. Suppose a group is divided into two groups and applies a voting procedure, which results in the same alternative being chosen by both subgroups. Then the procedure is consistent if the same alternative is chosen when the procedure is applied to the group as a whole.
- Chernoff: evaluates if the procedure presents the Chernoff property, i.e., if an alternative is a winner in a set of alternatives, it must be the winner in every subset of these alternatives.
- Independence of irrelevant alternatives: evaluates if the procedure satisfies this property, i.e., a procedure satisfies the condition of the independence of irrelevant alternatives, if, whenever two profiles have identical rankings over a pair of alternatives, the collective ranking over these two alternatives is the same in the two profiles, regardless of the rankings over the other pairs.

- Invulnerability of the no-show paradox: evaluates if the procedure satisfies this property, i.e., if a DM may achieve a better result by not voting, thus prompting him/her to manipulate the voting result by abstaining.

Many authors consider that the voting procedures can be evaluated regarding these criteria (or characteristics) by a binary evaluation, so a procedure either satisfies or does not satisfies the property (de Almeida and Nurmi 2015). Whenever the procedure satisfies the property sought, it will be represented by 1 (one), and when it does not, the representation is 0 (zero).

Note that consideration was not given to any criterion related to the context of the problem since any of the voting procedures would then receive the same input and would give the same output to the DMs. Moreover, the difficulties related to creating the algorithm within the system and to executing it in the GRUS system were not evaluated. Therefore, only the voting properties were considered as criteria.

The subset of voting procedures considered in this analysis were: Amendment, Copeland, Dodgson, Maxmin, Kemeny, Plurality, Borda, Approval Voting, Black, Plurality runoff, Nanson and Hare (Borda 1781; Brams and Fishburn 1978, Nanson 1883; Nurmi 1987, 1999; Saari and Merlin 2000; Felsenthal and Nurmi 2018). There are other methods available but these were not considered here. These include those that consider partial information (Cullinan et al. 2014; Ackerman et al. 2013) and the quartiles method (Morais and de Almeida 2012; de Almeida-Filho et al. 2017).

The binary evaluation of the VP considered regarding the voting properties is shown in Table 18.1. Note that the criterion of the independence of irrelevant alternatives has been excluded from the table since none of the voting procedures considered satisfies this property, so it does not make sense to consider this criterion in this case. It is important to have a procedure that is independent of irrelevant alternatives, but none of the voting procedures considered are, which leads to these criteria being excluded from the analysis.

As to a non-compensatory rationality to evaluate this set of VPs, a non-compensatory multicriteria method should be selected. In order to establish the relative importance of the weights of the criteria, this can be evaluated by considering a five-level scale as presented in Table 18.2.

As can be observed, the verbal scale was converted to a numeric scale. It is worth noting that this parametrization begins with 0.20. Since the value of 0 meant that the criterion had no relevance at all for the user, it was not considered in the analysis.

The user should use this scale to evaluate each criterion and, after this step has ended, the values should be normalized by considering the scaling process presented in the Equation (Palha et al. 2017):

$$\pi_i' = \frac{\pi_i}{\sum_j \pi_j}$$

where: π'_i is the value of the scaled weight of criterion i.

π_i is the value of the weight of criterion i on the five point scale.

$\Sigma_j \pi_j$ is the sum of the weights of all criteria.

Table 18.1 Consequence matrix for discrete binary outcome

	Criteria							
Voting procedure	Condorcet winner	Condorcet loser	Strong condorcet	Mono-tonicity	Pareto	Consistency	Chernoff	Invulnerability to the no-show paradox
Amendment	1	1	1	1	0	0	0	0
Copeland	1	1	1	1	1	0	0	0
Dogson	1	0	1	0	1	0	0	0
Maxmin	1	0	1	1	1	0	0	0
Kemeny	1	1	1	1	1	0	0	0
Plurality	0	0	1	1	1	1	0	1
Borda	0	1	0	1	1	1	0	1
Approval	0	0	0	1	0	1	1	1
Black	1	1	1	1	1	0	0	0
Pl. runoff	0	1	1	0	1	0	0	0
Nanson	1	1	1	0	1	0	0	0
Hare	0	1	1	0	1	0	0	0

Table 18.2 Notation scale for voting properties

Verbal scale	Notation	Numerical scale	Description
Very unimportant	VU	0.20	In this context, the criteria do not add any important feature to the problem
Not important	NI	0.40	In this context, the criteria do not add more than two important features to the problem
So-so	SS	0.60	In this context, the user is indifferent to the features added by the criteria
Important	I	0.80	In this context, the criteria add at least one important feature to the problem
Very important	VI	1.00	In this context, the criteria add more than two important features to the problem

Adapted from Palha et al. (2017)

18.2.1 Situation 1

In this situation, the users of the GRUS are willing to apply the framework to choose a Voting Procedure, since they have knowledge about the criteria considered to evaluate the VPs. Thus, in order to evaluate the preferences regarding the voting proprieties, it is necessary to establish who will define the required parameters of the multicriteria approach. For this task, three possibilities are considered:

- The facilitator will give his/her preferences, thereby allowing him/her to decide which VP would be best suited for the problem to be solved.
- The Supra-Decision-Maker will give his/her preferences if the problem has one and he/she would like to express his/her opinion instead of leaving the facilitator to do so.
- The DMs give their preferences by achieving an agreement as to the voting properties.

Although there are three possibilities to consider who will give the preference parameters regarding the voting properties, once the framework for choosing a VP is established, it will run in the same way, independently.

Thus, this situation will be illustrated based on the application presented by Palha et al. (2017), where one of the authors plays the role of the facilitator.

The facilitator considered using all the voting procedures available in the GRUS system, which were: Amendment, Copeland, Dodgson, Maxmin, Kemeny, Plurality, Borda, Approval Voting, Black, Plurality runoff, Nanson and Hare.

Therefore, her preferences were elicited by an interview regarding the voting procedure to be analyzed in order to aggregate the rankings of the group members. Thus, the facilitator expressed her preferences regarding the voting properties in accordance with Table 18.2. Table 18.3 presents the facilitator's preferences and the respective scaled weights.

Table 18.3 Preferences of the facilitator using the scale to evaluate the voting properties

Voting proprieties	Verbal scale	Numerical scale	Scaled weights
Condorcet winner	I	0.80	0.148
Condorcet loser	I	0.80	0.148
Strong Condorcet	VI	1.00	0.185
Monotonicity	I	0.80	0.148
Pareto	VI	1.00	0.185
Consistency	NI	0.40	0.074
Chernoff	VU	0.20	0.038
Invulnerability to the no-show paradox	NI	0.40	0.074
Total		5.40	1.00

As can be observed in Table 18.3, the criteria considered VI (Very Important) were Strong Condorcet and Pareto. The facilitator argued that the solution must be in the set of non-dominated alternatives and also Pareto-optimal.

The criteria considered I (Important) were Condorcet winner, Condorcet loser and Monotonicity. The facilitator stated that the procedure should be reliable, and it is important to guarantee that the best alternative in a pairwise comparison will be the Condorcet winner and the worst will not, if there these alternatives. Moreover, additional support should not lead a winning alternative to become a non-winning one.

The criteria of consistency and invulnerability were evaluated as NI (Not Important), since the analysis will hardly ever be made considering subsets of DMs, and the DMs will not be able to manipulate the analysis at this point.

Finally, the Chernoff criterion was considered VU (Very Unimportant) because it is unlikely that the group will decide to visualize a subset of alternatives during the analysis.

The scaled weights were calculated by normalization i.e., by dividing the nominal weight by the sum of all criteria (total). For example, the Condorcet winner has a nominal weight of π a $= 0.80$, being the sum of all criteria Σj π j $= 5.4$, thus, π' a $= \pi\text{a}/\Sigma\text{j}\,\pi\text{j} = 0.8/5.4 = 0.148$. The same calculations were used in all criteria and the results are presented in Table 18.3.

With the consequence matrix (as presented in Table 14.1—Chap. 14) and the weights, the analysis was conducted by applying ELECTRE III (Roy and Bouyssou 1993; Roy and Słowiński 2013). This multicriteria outranking method is based on comparisons between alternatives. It aims to eliminate the least advantageous and to indicate the most preferred action as determined by most of the criteria (Roy 1996). This method introduces concepts of preference p_j and indifference q_j to each criterion g_j. Consequently, the DM should establish a range of values in which one action is strictly preferable to another, and a range in which one action is indifferent.

Then, the facilitator considered three concordance indices (Figueira et al. 2005): s1 $= 0.9$, s2 $= 0.85$ and s3 $= 0.8$. The objective of applying different concordance indices was to verify if the kernel (Roy and Bouyssou 1993) would be altered if the strength of the concordance coalition is increased. Since the values were only binary, i.e., all the differences between evaluations are 0 or 1, discordance indices were not considered. Thus, the result indicates three voting procedures in the kernel (Copeland; Kemeny and Black). Figure 18.2 presents the result and the relationship between all alternatives.

In order to compare this result with other outranking multicriteria methods, and also to verify if changing the method would modify the result, PROMETHEE I (Brans et al. 1986) was also applied. This method provides a partial ranking based on pairwise comparisons. It considers six preference functions to evaluate criteria, and does so by considering the deviation between the evaluations of two alternatives on a particular criterion. For small deviations, the DM will allocate a small preference to the best alternative and even possibly no preference, if the DM considers that this deviation is

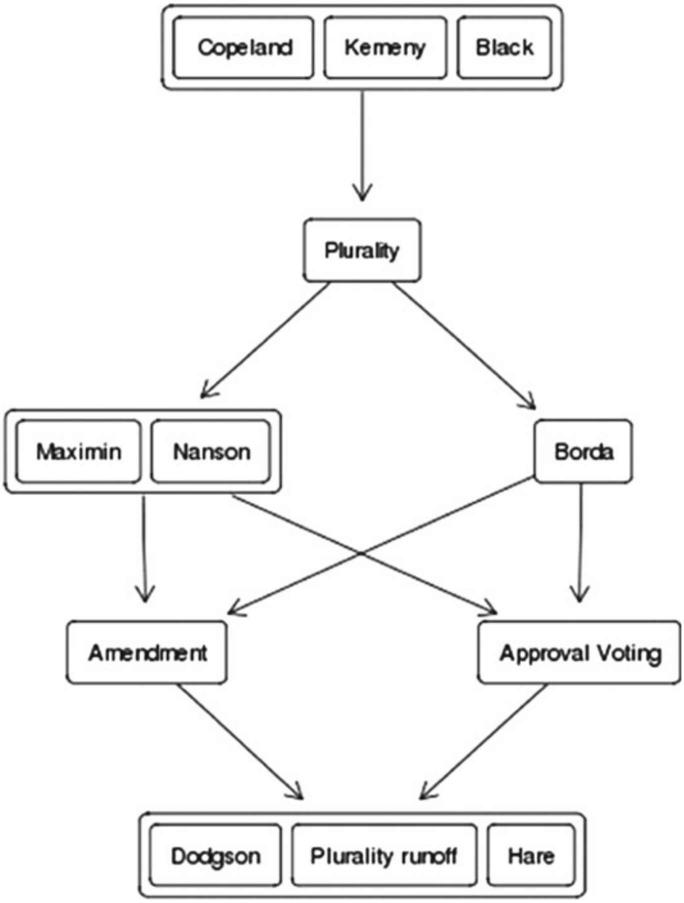

Fig. 18.2 Result of ELECTRE III. *Source* Palha et al. (2017)

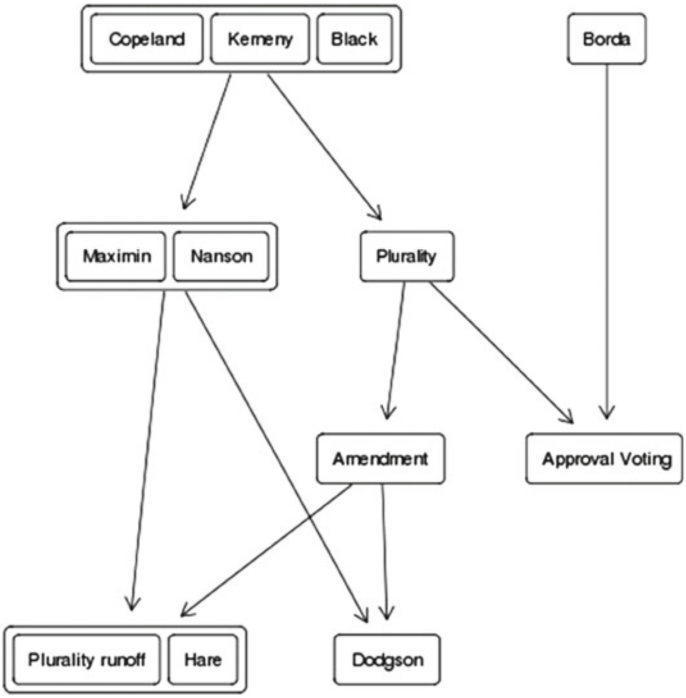

Fig. 18.3 Result of PROMETHEE I. *Source* Palha et al. (2017)

negligible. The larger the deviation, the stronger the preference. For this case, where only binary performance was considered, the usual preference function was used. This function means that any difference between the performance of alternatives will be strictly preferred. Figure 18.3 shows the result of applying PROMETHEE I.

As can be seen, the result found by applying PROMETHEE I was like that found from ELECTRE III, and the voting procedures Copeland, Kemeny and Black were, once again, found to have no differences. In fact, this outcome was expected since the evaluation of these alternatives was the same in all criteria. The difference when applying PROMETHEE I that can be highlighted is that Borda was not comparable to these three procedures. Thus, its position changed from third to first. Besides, Plurality remained in second position in both methods. However, while Maximin and Nanson were placed third by ELECTRE III, using PROMETHEE I, they were placed second but were not comparable with Plurality.

To sum up, on applying the two multicriteria methods, the voting procedures Copeland, Kemeny and Black are presented as a tie, and PROMETHEE I also presented Borda's rule as not being comparable with these voting procedures.

This performance links the problem to another concern which is: how should the voting procedure be chosen when the result of the framework for choosing a VP is a tie? This situation might occur because some criteria are missing. Thus,

Table 18.4 The expert's preferences on using the Scale to evaluate the voting properties

Voting proprieties	Verbal scale	Numerical scale	Scaled weights
Condorcet winner	I	0.80	0.133
Condorcet loser	I	0.80	0.133
Strong Condorcet	VI	1.00	0.167
Monotonicity	SS	0.60	0.100
Pareto	VI	1.00	0.167
Consistency	VI	1.00	0.167
Invulnerability to the no-show paradox	I	0.80	0.133
Total		6.00	1.00

by considering other criteria, the tie between these procedures could be broken. Other voting properties could be considered e.g., the possibility of adapting the procedures to a partial information environment. Or even using the three procedures and discussing the results that these achieve.

It is worth noting that Copeland, Kemeny and Black are all distance-based procedures, and although they have the same type of input information, they have different algorithms that can provide DMs with a final ranking of alternatives.

18.2.2 Situation 2

In this situation, the users of the GRUS are not willing, or do not have enough information, to decide which voting procedure best fits the problem. They would simply like to know the final recommendation since they have already given their preferences regarding the alternatives and the criteria of the problem studied. Therefore, they have the individual DMs' rankings.

For this situation, the framework for choosing a Voting Procedure was applied with an expert, who in this case was one of the authors of this book who played the role of the expert, in order to make a generic recommendation for a voting procedure for the GRUS.

The expert of voting rules made a holistic evaluation of the properties, while taking into account the information that the GRUS provided as individual rankings.

Based on that perspective, the subset of voting procedures that the expert considered was: Copeland, Dodgson, Maxmin, Borda, Nanson and Hare.

Considering these voting procedures, it makes no sense to evaluate the voting properties of Chernoff and of the Independence of irrelevant alternatives, since all VPs considered fail in these criteria. The consequence matrix (VP x properties) considered for discrete binary outcome is presented in Table 14.1 (Chap. 14). And the decision matrix uses the value function in Eq. 14.1 (Chap. 14).

Table 18.4 shows the expert's preferences on using the Scale for evaluating the

Table 18.5 Result of applying the PROMETHEE II method

Rank	VP	Phi	Phi+	Phi−
1	Borda	0.1398	0.4132	0.2734
2	Copeland	0.1398	0.1998	0.06
3	Nanson	0.0198	0.1398	0.12
4	Maximin	−0.0198	0.1466	0.1664
5	Dodgson	−0.1398	0.0866	0.2264
6	Hare	−0.1398	0.0866	0.2264

voting properties, as proposed in Table 18.2.

After applying the PROMETHEE II method, which is similar to an additive method in this case of using the binary performance of the alternatives, the result achieved is shown in Table 18.5.

The Borda procedure, the VP chosen by this expert, could be implemented in the GDSS as a suggestion for an aggregation procedure to give a final recommendation.

18.3 Topics for Further Reflection

This chapter presented how the framework for choosing a voting procedure could be implemented in the GRUS GDSS when the group does not wish to reach a consensual decision. In this case, the group can proceed to use the framework itself or the group can use a generic voting procedure chosen by an expert using the framework.

It is important to note that applying the framework avoids it being manipulated on behalf of one or more parties, even when it is applied considering the facilitator's preferences.

18.4 Suggestions for Reading

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Chapter 19
When Does a Given Procedure Work Best?

Abstract By way of summarizing our account on collective decision procedures, we shall provide a brief exposition of those methods that have been discussed in earlier chapters from the viewpoint of their applicability. The question we seek to answer is: what are the circumstances under which each system works best? Since each system has specific proprieties, it is important to understand them in order to choose a Voting Procedure (VP) to evaluate a decision problem. The framework for choosing a voting procedure can guide the Analysts and Decision-Makers with regard to this issue and therefore it evaluates the impact of VP properties on their own business decision process.

19.1 Introduction

As was pointed out in the preceding chapters, every system is based on some intuition regarding the notion of the 'collectively best' candidate or alternative. This intuition may, however, be incompatible with many others and, hence, the overall attractiveness of each procedure has to be based on a balance of desirable and undesirable properties of procedures in the intended context of application. For example, systems that aim at making each participant reasonably satisfied with collective outcomes are likely to differ from those seeking to maximize the benefit for a majority of participants. Our concluding discussion is partially based on system classification presented more than thirty years ago (Nurmi 1983). It divides Voting Procedures (VP) into:

1. binary systems
2. positional systems
3. hybrid systems
4. systems based on non-ordinal input.

The first of these classes consists of those procedures which are typically defined by means of pairwise comparisons and where the winner is determined as a result of those comparisons. The systems in the second class single out winners or determine collective preference ranking based on the positions that each candidate or alternative

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occupies in the individuals' preference rankings. The third class consists of multi-stage systems where the selection of the winner may be preceded by several rounds of computation (typically eliminating alternatives on the way). All three of these classes can be implemented based on preference profiles submitted by the voters.[1] The systems in the fourth class require information other than individual preference rankings to determine the collective outcomes. These include systems that enable the individuals to signal their degree or intensity of preference for alternatives *vis-à-vis* each other, or to classify them into acceptability classes. In what follows we shall briefly comment on the properties of the systems described above from the view point of business decision making. So, our perspective here is different from political decision-making. Rather, we stress the properties of systems that are conducive to team work, consensus building and making full use of information.

19.2 Binary Systems

Since these systems are based on pairwise contests, the best performance of any alternative would seem to be that it defeats all its contestants. If the majority of votes defines the winner in each comparison, this would amount to electing the Condorcet winner whenever such an alternative exists.[2] Perhaps the mostly widely applied of these systems is the amendment (or successive elimination) system which introduces an additional factor to the determinants of the voting outcome: the agenda of pairwise comparisons. Also the other major parliamentary voting procedure, the successive one, is based on an agenda. It is difficult to recommend these systems for use in business contexts because of their obvious bias stemming from agenda control. Both of these also suffer from not separating cases where the outcome is the Condorcet winner from those where this just happens to be one outcome in a collective majority cycle. In decision settings where it is important to avoid outcomes that are deemed most undesirable by the individuals (e.g. in recommendation settings), the sequential voting by veto has much to recommend it. If all individuals may eliminate their worst-regarded alternative (of those available when their turn arrives), it is clear that the procedure guarantees a modicum of satisfaction to all parties involved. Since obviously individuals are treated differently depending on their position in the

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[1] There are elimination systems that can be—and often are—implemented through several rounds of voting, e.g. the Finnish and French presidential election. This makes it possible for the voters to change or to work out their preferences gradually upon finding which alternatives are available at each stage. Thereby strategic elements enter the analysis. Since we are primarily interested in the properties of systems as methods of aggregating expressed preferences, we assume that the voters's preferences do not change over time in the balloting process.

[2] There are, however, binary systems, notably the Borda count, that *can be* (although typically aren't) implemented via pairwise comparisons which do not always result in an eventual Condorcet winner. In these, the notion of pairwise winning is not the sole determinant of the winner. These systems are not discussed in the present section.

sequence of eliminations, it is important that in repetitive decision settings, the order of voting is determined by a random device.

Eventual cycles in outcomes are clearly discernible when the Copeland, Dodgson, Kemeny, max-min, Schwartz or Young procedures are used. Their major flaw is their vulnerability to various kinds of monotonicity failures (see Felsenthal and Nurmi (2017) for a comprehensive discussion) in variable electorates. In terms of this consideration, the max-min and Young procedures have a clear advantage over the others. This edge comes, however, with a high price, viz. these two procedures may lead to choosing a Condorcet loser. Nonetheless, the max-min procedure is particularly attractive in business contexts since it does not have a clear majoritarian flavor, i.e. it does not clearly divide the individuals into a majority of winners and a minority of losers. Instead, it results in alternatives that have a reasonably strong showing against even their toughest competing alternative. Thus, the max-min method has some consensual features that are important in organizations that are certain to make a sequence of decisions over time.

19.3 Positional Procedures

Of positional systems, the Borda count seems superior to plurality and vote-for-*k* procedures. It fully utilizes the available ordinal information about individual preferences, avoids choosing eventual Condorcet losers, satisfies all conceivable monotonicity criteria in fixed and variable electorates and always leads to Pareto-optimal outcomes. Its instability under expansion or restriction of the set of alternatives has been shown by Saari to be no more dramatic—in fact it is less so—than that of other positional systems (Saari 2001). The Borda count is strategically manipulable, but basically so too are all systems. If the individual misrepresentation of preferences is deemed of particular concern, the sequential elimination procedure based on Borda scores, the Nanson method, seems worth recommending, although it belongs to hybrid systems.

19.4 Hybrid Systems

Hybrid systems consist of several stages of computation, in some cases even several rounds and different kinds of balloting. Typically, these aspects are bound to complicate efforts to benefit from strategic misrepresentation of preferences. This is not say that these systems are strategy-proof. In fact, some—notably plurality runoff—require a fairly limited amount of information about preference profiles to make misrepresentation efforts worthwhile. On the whole, however, using these systems makes it plausible to look at other than strategic properties when trying to determine the most appropriate system. Those advocating the Condorcet winner selection no doubt favor Nanson's procedure which—as was pointed above—also has the advantage of requiring a good deal of information about preference profiles in order to make successful strategic behavior feasible. In business contexts, Coombs's system—which is not a Condorcet extension—has the distinctive advantage of excluding alternatives that are regarded as being the worst ones by large groups of individuals. This feature would seem to make it appropriate in recommendation systems, especially if several consecutive choices are to be made by the same group.

19.5 Systems with Non-ordinal Input

Range voting, approval voting and majority judgment are systems that rely on input that differs from the standard (complete and transitive preference relation) assumptions. Using the assessment criteria applied in rank-aggregation contexts may, therefore, be questioned. For example, if we are given individual cardinal utility assignment over alternatives, it may be argued that the Condorcet winner or loser concepts lose their significance as we can now strive for a maximal utility sum instead of pairwise utility comparison winners. This striving is, however, open to criticism stemming from those views that stress the justice or fairness of collective outcomes instead of maximizing the utility sum associated with outcomes. Both range voting and majority judgment are based on a single indicator of individual value distributions: the former focuses on the mean value and the latter on the median value of the distribution. The choice between justice-driven and utility-maximizing methods is fundamental and cannot be resolved *in abstracto*. It is important to observe, though, that both majority judgment and range voting are maximizing methods. If all individuals have dichotomous preferences—acceptable vs. non-acceptable alternatives—the approval voting would seem a natural choice method. More sophisticated aggregation methods enter the picture when the more graded preferences are at stake.

19.6 Applying the Framework for Choosing a Voting Procedure

As can be observed, there are different properties and characteristics that each voting procedure has, some of which are acceptable or desirable in some decision problem contexts and not in others. Given this situation, the framework was built to deal with choosing a Voting Procedure (VP), by using a multicriteria decision-making/aiding (MCDM/A) model, as explained in Chap. 13 when discussing MCDM/A methods (Pardalos et al. 1995; Roy 1996; Vincke 1992; Polmerol and Barba-Romero 2000; Belton and Stewart 2002; Figueira et al. 2005; Munda 2008; de Almeida et al. 2015).

It is relevant to mention that this framework considers the specificity of the application (the organizational context), in which the decision process is conducted. Also, the issue of choosing a method is considered (Polmerol and Barba-Romero 2000;

Belton and Stewart 2002; Roy and Słowinski 2013; de Almeida 2013, de Almeida et al. 2015).

There are some important issues that the framework (see Chap. 14) deals with so as to guide the better structuring of the elements to be considered in the process:

1. The multicriteria method to be used
2. The criteria to be applied
3. What the outcomes of the VP for the criteria are likely to be.

The first issue is essential to ensure the quality of the decision process. An analyst should aid this process to determine the MCDM/A method to be used based on the preferences of the Decision-Maker (DM). The method is chosen whether the DM uses compensatory or non-compensatory rationality, since this will also imply the way in which the weights of criteria are established. Apropos, it is important to notice here that the framework assumes that the DM can be either one person (a benevolent dictator or a supra-DM) or a group of DMs acting in agreement.

Regarding the second issue, the framework considers that there are two kinds of criteria to be considered. One set is related to the properties of VPs (and aims to maximize the full use of a VP's properties which are desirable and appropriate for the organizational problem) and the second set is related to the data input to VP (the aim of which is to maximize the match between the nature of input required by the VP and its impact on the organizational problem).

The third but not least important issue concerns the information on the consequence matrix related to a VP's properties. There are two ways of doing this: Discrete binary outcome, which indicates whether some property is violated by a VP; or, Continuous outcome, which supplies richer information, and thus indicates how often the property is violated in that VP, for which a 0 to 1 scale is used to show the frequency of occurrence.

It is worth emphasizing that for a 'discrete binary outcome', the DM's rationality (compensatory or non-compensatory) does not matter with regard to choosing an MCDM/A method, since the result would be analytically similar.

To sum up, applying the framework helps to decide which VP best fits the problem and the DMs' preferences, thereby making it much more difficult for one or more parties to manipulate it.

Future work remains to be done with regard to investigating the frequency of violations of properties by VP in order to use 'Continuous outcome'. There are already some studies in the literature about this, but many more are expected. Note that using 'continuous outcome' considering the DM's rationality (compensatory or non-compensatory) would have on impact on guiding which MCDM/A method to use.

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