Chapter 22
MIMO Input Derivations, Optimizing Input Force Against Output Accuracy

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Abstract Multi-Input-Multi-Output (MIMO) vibration testing is considered more representative of the true loading environment (flight or wind induced vibration) where the inputs are not through a single point. The derivation of N inputs for testing typically involves matching the response at M locations (outputs). This involves inversion of a N × M Transfer Functions (TRF) matrix corresponding to the N input and M output locations. The matrix inversion is affected by both mathematical and physical parameters (ill-conditioned matrix, structural modes, signal noise).

Tikhonov regularization is commonly used in inverting an ill-conditioned N × M matrix. A low value of the Tikhonov regularization parameter minimizes the distortion of the original equations while a higher value can minimize error. In practice this introduces an interesting dilemma where obtaining realistic input loads and maintaining accuracy of output are often pitted against each other. A study was conducted using data synthesized from a simply-supported plate structure with known vibration modes with added noise at outputs. The objective of the study was to understand how noise or errors in the output and the Transfer function affect the input. This leads to a more judicious choice of the Tikhonov parameter that can achieve a balance between reducing input loads while preserving desired accuracy of output vibration.

Keywords Vibration testing · Multiple input · MIMO

Introduction

The vibration of a simply supported plate (1.0 × 1.5 m, 10 cm thick) under multiple input and response locations was studied. Force input was applied at various locations on the grid shown in Fig. 22.1 (the grid lines are equally spaced from 0.5 to 0.95 × length in both x and y axes). Response is also calculated at the same locations. For this specific study the two inputs were applied at locations 0.25 and 0.45 × length (circles in Fig. 22.1). Response was monitored at 4 locations, 0.25, 0.45, 0.65, and 0.85× length (circles and squares in Fig. 22.1). The response and the input are related by the plate response (Transfer Function TRF) as shown in Eq. (22.1). Input can be derived from response using Eq. (22.2) where TRF^{-1} is the pseudo-inverse of the rectangular TRF matrix. To invert an ill-conditioned matrix, the Tikhonov regularization parameter is used, which prevents the very low values of TRF from resulting in distorted (nearly singular) values in TRF^{-1}.

$$\left[TRF\right]_{4\times 2} \times \left[Input\right]_{2\times 1} = \left[Response\right]_{4\times 1} \tag{22.1}$$

$$\left[TRF\right]^{-1}{}_{2\times 4} \times \left[Response\right]_{4\times 1} = \left[Input\right]_{2\times 1} \tag{22.2}$$

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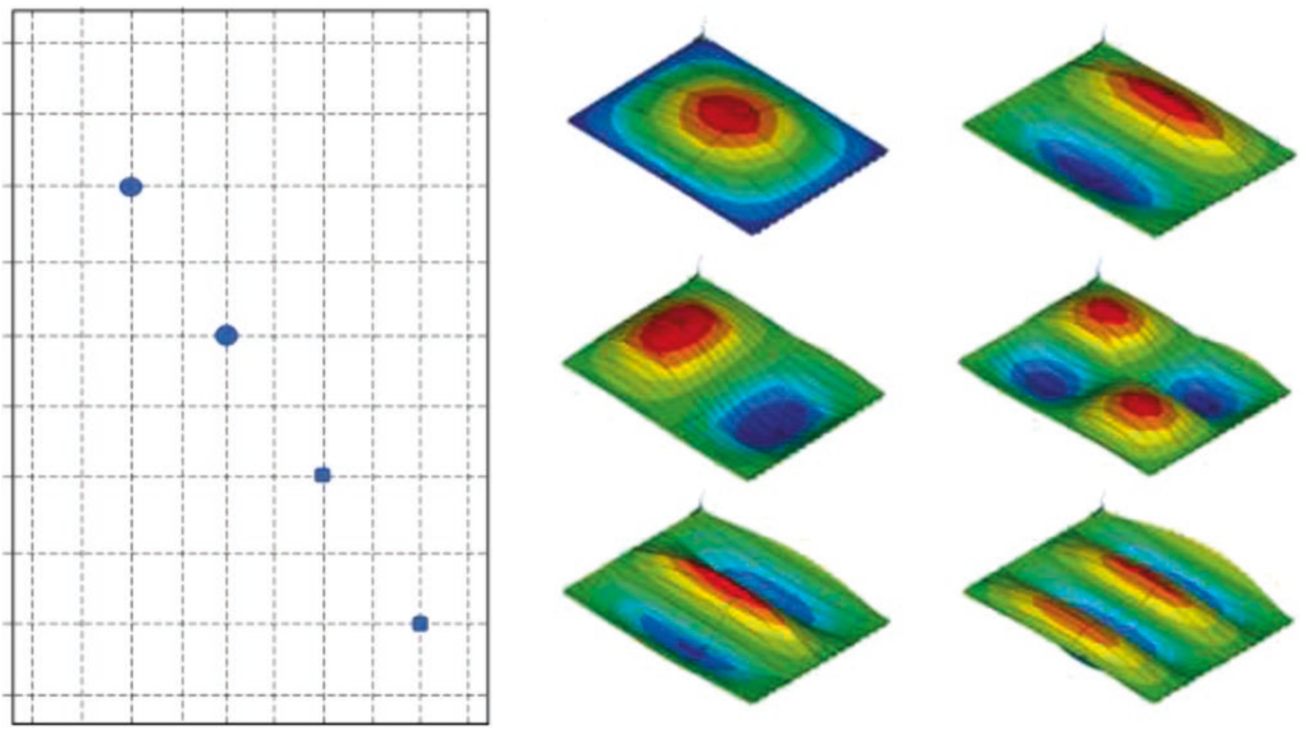

Fig. 22.1 Simply-supported plate with input and output locations and mode shapes

Table 22.1 Modes below 300 Hz (numbers refer to values of r_1 and r_2 in equation above)

Modes	1	2	3	4	5
1	34.04	65.46	117.82	191.14	285.39
2	104.73	136.15	188.52	261.83	
3	222.56	253.97			

The frequency domain Response of a simply supported rectangular plate to an input force was determined based on analytical solution provided Fahy and Gardonio [1]. Transverse displacement $d(\omega)$ at position x_2, y_2 due to a force $F(\omega)$ at location x_1, y_1 is given by the expression:

$$d(\omega) = \sum_{r=1}^{\infty} \frac{\phi(x_2,y_2)\phi(x_1,y_1)}{M_r\left[w_r^2(1+j\eta)\right]-\omega^2} F(\omega) \tag{22.3}$$

$w_r = \sqrt{\frac{D}{m}}\left[\left(\frac{r_1\pi}{l_x}\right)^2 + \left(\frac{r_2\pi}{l_y}\right)^2\right]$ represents the modal frequencies of the plate $\phi(x,y) = 2\sin\left(\frac{r_1\pi x}{l_x}\right)\sin\left(\frac{r_2\pi y}{l_{xy}}\right)$ Represents a modal shape function.

To obtain velocity instead of displacement the equation is multiplied by $j\omega$; to get acceleration $a(\omega)$ it is multiplied by $(-\omega^2)$.

$D = Eh^3/(12(1-\nu^2))$, is plate stiffness, E, h and ν are elastic modulus, plate thickness and poisson's ratio. m is the mass per unit area of the plate, l_x and l_y are plate dimensions. r_1 and r_2 are integers representing mode numbers. M_r is the total mass of the plate. η is damping coefficient. Properties of Aluminum were used ($E = 70$ GPa, $\nu = 0.2$). Table 22.1 shows the modes <300 Hz.

The following steps were used in these analyses:

- Frequency domain Transfer function (TRF) relating input force to response acceleration was generated using the equation 3 above for all 100 points of the grid shown in Fig. 22.1 (only a few of these were used). This was done for frequency values 1–8192 radians/s.

- Frequency domain Input force was generated using random phase (uniformly distributed between ±180°) and constant amplitude = 100 N. Input force values for the first 100 low frequencies were set to zero such that the length of data was statistically representative. Input force values at high frequencies were also set 0 to satisfy Nyquist criterion.
- The TRF and Input were used to calculate the response as per Eq. (22.1).
- Response was polluted with increasing amounts of white noise at each frequency and the input was recalculated based on Eq. (22.2).
- Error in the new input was determined compared to the original input.
- New Response was calculated using the new input (based on TRF inversion)
- Error in response was calculated relative to the original response.

Results

Figure 22.2 shows that Input error (=True Input/MIMO-based input) is correlated with condition # of the TRF; ill-conditioning of the matrix leads to higher error. However, numerical analysis involved in inverting a 4 × 2 matrix results in negilible error (note scale 5×10^{-14} = in Fig. 22.2). It is also worth noting that the relatvely higher errors for Input 2 are at frequencies where the input 2 has very little contribution (at location 0.45 × length it is close to the node at the center of the plate).

Figure 22.3 shows the effect of adding a noise of amplitude 1.0 and uniformly distributed random phase (±180°) to the response at each frequency. The right side of Fig. 22.3 shows that the noise is very low in comparison to the amplitudes of the response (peak response is a 1000), and the response with and without noise is visually indistinguishable. The input error is now considerable. At frequencies of 65, 136 and 254 Hz where Input 2 is near a node it has considerably more error than Input 1. Input 2 has small error at 118, 223, 285 Hz because it is no longer at a node and can contribute to those modes as well as Input 1 can. Also recognizable is the gradual increase in Input error below the 1st mode of 34 Hz, which can be attributed to the lack of motion in the plate.

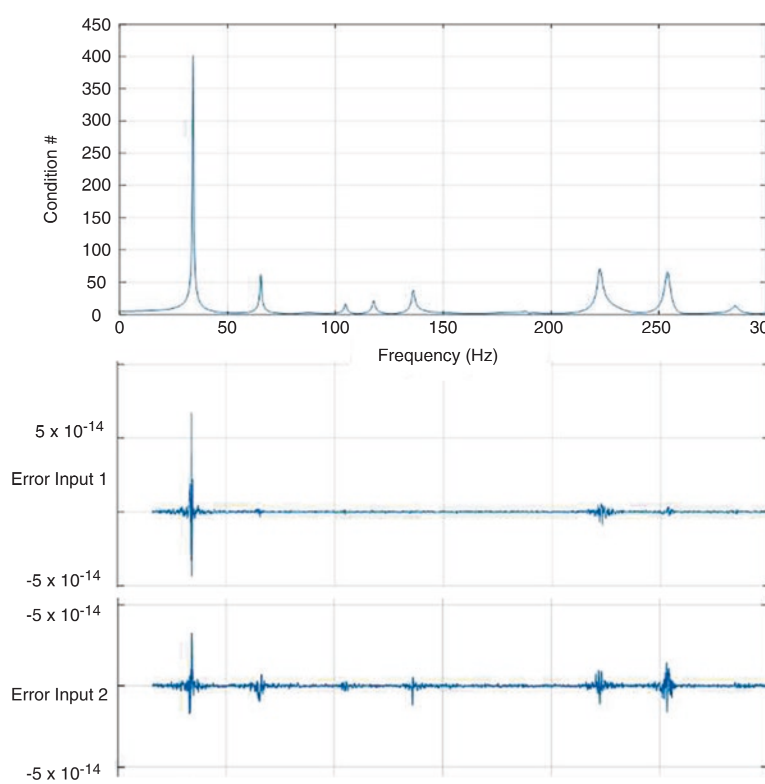

Fig. 22.2 Condition # vs. input error with zero noise

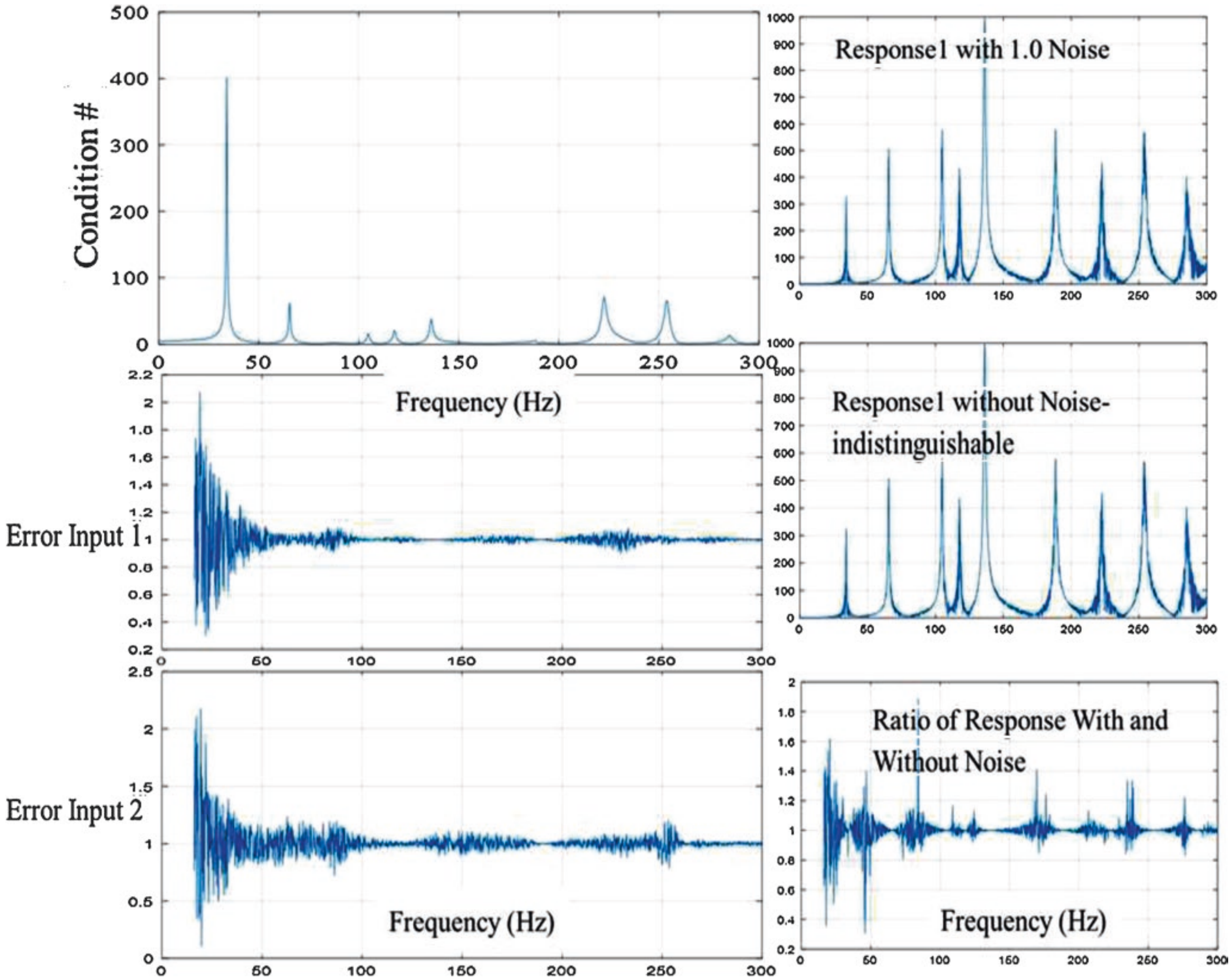

Fig. 22.3 Condition # vs. input error with noise = 1.0

Figure 22.4 shows results with the noise level increased to 10.0 at each frequency. The resulting error in the input is now greater in roughly the same proportion (≈×10 greater than that in Fig. 22.3). The bottom-right plot shows the input error when the tolerance value in the MatLab pseudo-inverse function '*pinv*' is change from 0 to 0.1. This is known as the regulariation parameter and referred to in this paper as *tol*. The two plots on the left (in Fig. 22.4) are the same data with the lower figure with a vertical scale matching the one on the lower right for comparison. The input error is significantly lower at and below the 1st mode of 34 Hz. This is because the lower singular values in TRF is set = 0.1 removing the inversion inaccuracies caused by extremely low singular values.

Additional insight can be gained by examining the relationship between the results of the Singular-Value-Decompostion (SVD) of the Transfer Function Matrix (TRF) at each frequency. The high and low singular values are shown in the top right plot of Fig. 22.4 (note long-normal of svd is presented instead of svd to better visualize the effect of singular values and *tol* on the error). The selection of the tolerance value (*tol*) results in selective minimization of input error. So, when tol is increased to 0.165 more of the input errors are reduced as seen in the plot in the right-center. The dotted horizontal lines represent the 2 *tol* values relative to the singular values. The circles show the corresponding decrese in the error in Input 2 where the lower svd values are below (ln 0.165 = −1.8). Likewise the two dotted lines below 100 Hz show where the lower svd values are below (ln 0.1 = −2.3) and the corresponding decrease in Input 2 error in those regions.

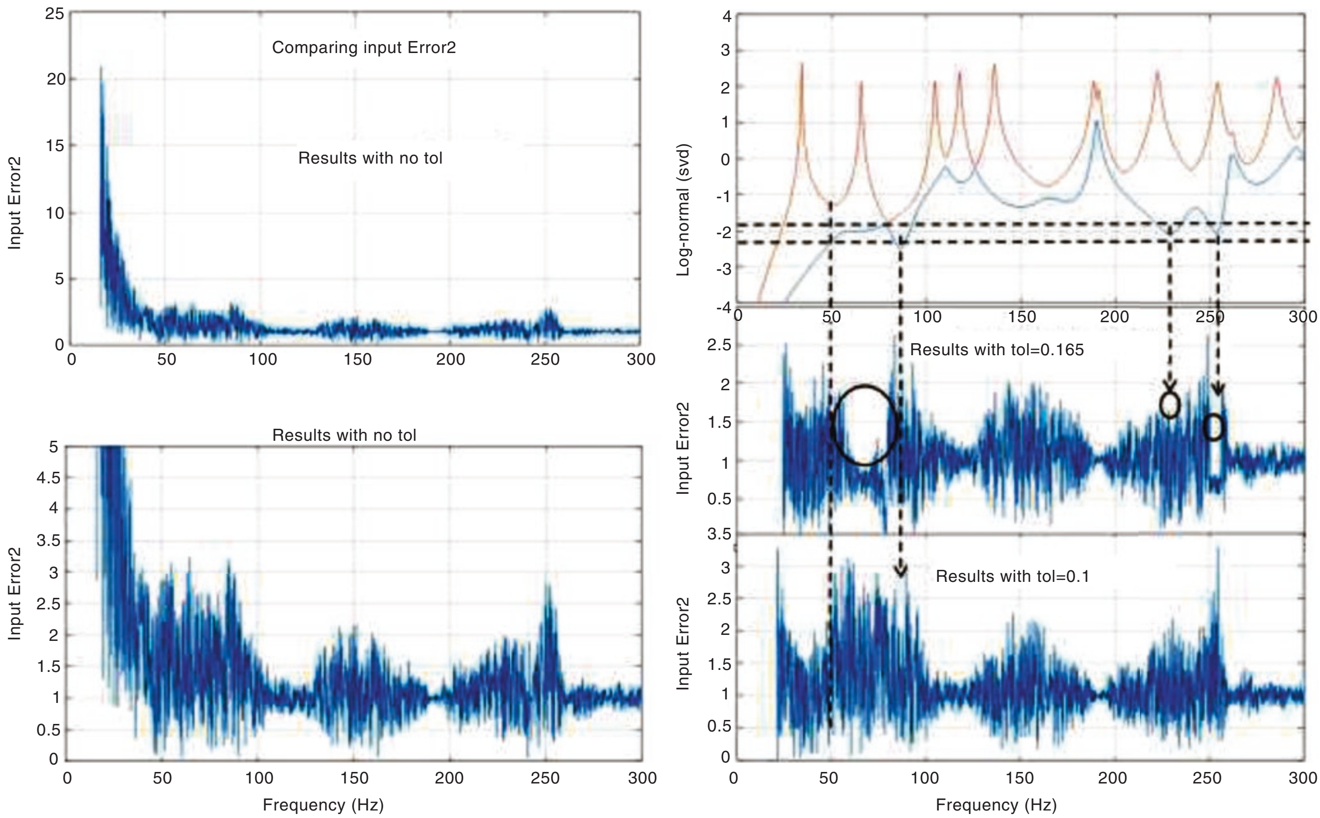

Fig. 22.4 Input error with noise = 10 and effect of regularization parameter

Conclusions

- Large condition #s associated with resonant modes is the first source of error in MIMO inversion. In the absense of other sources of noise this error is negligible.
- When response is polluted with white noise the input error is in proportion to the relative error in the response (effect of white noise to the response is low at and near resonances).
- Input error at specific frequencies is higher when the input is near a node for that frequency.
- A 10× increase in the noise results in a corresponding 10× increase in the input error.
- Choosing the regularization parameter based on the lower value from singular-value-decomposition of the complex TRF matrix can be used to decrease input error.

Reference

1. F. Fahy, P. Gardonio, Section 2.4: mobility and impedance functions of thin uniform flat plates, in *Sound and Structural Vibration*, 2nd edn. (Elsevier, Amsterdam, 2007). ISBN# 0-12-373633-1