Chapter 2
Walkthrough and History of the Virtual Fields Method

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Abstract This paper presents the history of the Virtual Fields Method, an identification method developed 30 years ago in order to identify constitutive parameters from full-field measurements. This method was initially proposed in the case of bending plate problems, but it has progressively been adapted to many other testing configurations. In this paper we look back at the first developments of the method and highlight the main problems that were progressively tackled, especially in terms of increasing complexity of the constitutive equations that were considered, and improvement of the identification technique itself to make it more efficient.

Keywords Full-field measurement · Identification · Material characterization · Principle of virtual work · Virtual fields method

Introduction

The Virtual Fields Method is an identification method, which has been developed in order to identify parameters from fields of displacement/strain measurements. It is based on the principle of virtual work. Its main feature is that the resolution of the direct problem, namely numerically calculating the distribution of the displacement and strain fields in the specimen under test, is not required during the identification process. Originally developed during the late 1980s to characterize the elastic properties of composite materials, this method has since then been progressively improved and used in various cases of identification problems. The objective of this paper is to recall the origins of this method and to set out the main milestones in its development, with a special emphasis on the different hurdles that were progressively overcome.

The First Beginnings

The starting point is the PhD work of the author [1] performed during the late 1980s under the supervision of late Pr. Alain Vautrin. The objective of this work was to characterize the six independent bending rigidities of a thin composite (and thus anisotropic) plate by performing only one test. Classic material characterization procedures such as the tensile or the three-point bending tests are based on configurations for which closed-form solutions for the displacement/strain/stress distributions as functions of the constitutive parameters are available. In this case, three stiffnesses at most are involved in the response of the specimen, so no more than three parameters can be measured at the same time. It means that one test is not sufficient with this type of approach when anisotropic materials are to be characterized. On the contrary, if the test is performed on one plate in which heterogeneous strain/curvature fields occur, a greater number of parameters are involved in the response, and these parameters are therefore potentially identifiable. The underlying benefit of this type of approach is that the number of tests is reduced, the ultimate case being one test only. This also paves the way for similar, but more complicated situations for which heterogeneous strain fields are unavoidable because they automatically take place in the specimen. This situation arises for instance in heterogeneous materials, when the strain field fluctuates from one constituent to another because of local changes of stiffness, or in materials with gradient properties such as solders or welding seams, where the local mechanical properties continuously spatially change.

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With the type of test described above, the main problem is that displacement/strain/stress distributions are generally not modeled by a closed-form solution. The challenge to be faced for solving this problem is twofold. The first challenge is of a practical nature since the displacement and strain components shall be measured at a sufficient number of points to reflect the actual heterogeneous response of the specimen. Ideally, full-field measurements shall be used to perform such measurements. In [1], this first problem was resolved by using deflectometry, a full-field measurement technique based on the observation of a regular pattern by reflection over the surface of the specimen [2]. The second problem is that some suitable identification procedure shall be used in order to link the measurements and the sought parameters in order to eventually retrieve the latter. The natural reflex for solving this problem is to rely on the finite element method. Indeed this method provides the displacement/strain field over the surface of the specimen, assuming that the geometry of the specimen, the boundary conditions and the material properties are correctly modeled. The latter being the unknowns for this identification problem, a cost function can be iteratively minimized with respect to these parameters. This technique, often referred to as Finite Element Model Updating (FEMU), has been proposed first in [3] for composite materials.

In [1], the main motivation for finding another route that FEMU is that iterative calculations shall be performed with FEMU. An interesting but relatively unnoticed solution has been proposed in [4] to overcome this drawback in the case of plate bending problem. It consists in considering three different bending tests performed on the same plate (only the location of the supports and applied loads changes from configuration to another), and writing the Maxwell-Betti reciprocal work theorem with this set of tests. When considering the identification problem at hand and assuming that full-field measurements are available over the surface of the specimen for each test, applying the Maxwell-Betti theorem leads to a system of linear equations where the bending rigidities of the plate specimen are the unknowns. This linear system is invertible if the three tests are different, which means that compared to FEMU, we have a direct identification technique since no iterative calculation is required in the case of elasticity. This approach has however two major drawbacks. The first one is that several tests are necessary, which is a strong limitation compared to FEMU where only one test is sufficient. The other one is that two experimental fields are involved in each equation. Measurements being noisy, this causes the resulting system of linear equations to be relatively sensitive to noise.

In [1], it is proposed to stick with the idea of considering scalar quantities such as works, but virtual works, by applying the principle of virtual work instead of the Maxwell-Betti reciprocal work theorem. The principle of virtual work can be written as follows:

$$\underbrace{-\int_V \underline{\underline{\sigma}} : \underline{\underline{\varepsilon}}^* \, dV}_{W^*_{\text{int}}} + \underbrace{\int_S \left(\underline{\underline{\sigma}}\,\underline{n}\right) . \underline{u}^* \, dS}_{W^*_{ext}} = \underbrace{\rho \int_V \underline{\gamma} . \underline{u}^* \, dV}_{W^*_{acc}} , \forall \text{K.A.} \underline{u}^* \quad (2.1)$$

where σ is the actual stress field in the specimen, u^*/ε^{**} a kinematically admissible (K.A.) displacement/strain field, n a unit vector perpendicular to the external surface, V the volume of the specimen and S its external surface. Compared to the procedure described above, employing the principle of virtual work leads to two main differences. The first one is that each kinematically admissible virtual field provides a linear equation where the stiffnesses are the unknowns. Since there is an infinite number of K.A. virtual fields, there is also potentially an infinite number of linear equations. In Eq. (2.1), combining these different virtual displacement/strain fields with actual heterogeneous strain fields involving all the unknown parameters leads to a system of linear equations, which provides these unknowns after inversion. One test only instead of several ones is therefore sufficient to identify the unknowns, provided that this test leads to heterogeneous strain fields and that at least as many independent virtual fields as unknown parameters can be defined to get an invertible linear system. The second difference is that the virtual fields can be defined by close-form equations, so these virtual fields are by definition not noisy. Hence, only one measurement field is affected by noise in the principle of virtual work instead of two in the Maxwell-Betti theorem. In the resulting linear equations, the coefficients by which the unknown stiffnesses are multiplied can be regarded are weighted averages of the actual strain measured over the whole specimen, the weights being the virtual strains. This contributes to average out the noise, and thus to regularize the identification problem.

The main results obtained in [1] were published in three different papers, where the principle of the method [5], numerical simulations [6] and experimental results [7] were presented in turn. Compared to FEMU, it can be said that the present approach is direct and not iterative in the case of linear elasticity. Full-fields measurements are however necessary, which is not the case with FEMU since only a limited number of measurements can be processed in this case. Another point is that volume integrals are involved in Eq. (2.1) but measurements are generally obtained only over the external surface. It means that some suitable assumptions are necessary to deduce the actual strain field from the measurements over the surface. This is possible when specimens are thin, which is generally the case.

From Bending Plate Problems to Other Applications

The principle of virtual work being the weak form of equilibrium, it is of general purposes and can therefore be used in other situations than plates subjected to bending tests, as mentioned in [1]. In the early 1990s, fruitful discussions with Fabrice Pierron, who was at that time a PhD student, led to a longstanding collaboration during which various applications of this identification method, as well as its improvement, were investigated. Two main questions were progressively addressed: *i*- how to use this approach in cases different from plate bending, and *ii*- how to define optimal virtual fields, bearing in mind that having potentially an infinite number of K.A. virtual fields means that some combinations are inevitably better than others.

From Elasticity to more Complex Constitutive Equations

Concerning the first point, the whole set of in-plane stiffnesses of a composite plate was first measured in [8]. Vibrating composite plates were then investigated in [9], paving the way for the identification of complex moduli of polymers or composites [10]. In the meantime, realizing that this procedure was general and could be applied in various situations, it was decided to call it the *Virtual Fields Method* (VFM). This name was employed for the first time in 2000 [11]. Non-linear constitutive equations were also progressively considered. A simple damage model for composites was characterized in [12], but the parameters governing the model were still obtained directly, by inverting a system of linear equations. An important step was to apply the VFM in the case of plasticity [13]. Since the constitutive equations are by essence non-linear in this case, the idea was to minimize a residual defined by the squared left-hand side of Eq. (2.1) (the third integral is null in this case) fed by one or several virtual fields chosen by the user. Minimizing this cost function with respect to the sought parameters leads to an iterative identification procedure. This obviously requires more computational resources than in the linear case, but less than FEMU. Indeed the forward problem, namely finding the displacement/strain fields from a set of parameters chosen a priori, does not have to be solved with VFM. Large strains constitute another cause of non-linearity. This problem has been addressed for the first time in [14], where the parameters governing the hyperelastic response of rubber were identified. Various other types of materials such as wood [15], foams [16] or biological tissues [17] were studied with the VFM during the last few years. The interested reader is referred to [18] for a complete overview.

Virtual Fields Are Smart Filters

Concerning the use of optimal virtual fields in the identification procedure, we can summarize the conclusions of several studies on this particular point by saying that virtual fields can be regarded as smart filters, in the sense that they enhance or minimize, even nullify, the contribution of zones or parameters in Eq. (2.1) above. They can even minimize the effect of noise in the measurements on the identified parameters. Virtual fields, which can either be defined piecewisely [19] or with functions continuously defined over the whole domain, can therefore be tailored in order to resolve given problems in an optimal way. Some examples are listed below.

Optimizing the Virtual Fields for a Robust Identification

Measurements are affected by noise, so it is important to try to minimize the propagation of this noise through the identification process. First attempts were proposed in [20] in the case of elasticity, where virtual fields leading to the independent identification of the parameters were proposed. More interestingly, a procedure minimizing the negative effect of noise on the final results is described in [21]. The procedure is iterative, but it is shown that with the fixed-point algorithm used to retrieve the unknowns, only one iteration is generally sufficient to reach converge. In the same spirit, optimized virtual fields were also recently proposed in the case of plasticity [22]. These fields maximize the sensitivity of the sought parameters in the cost function.

Mastering the Boundary Conditions

In Eq. (2.1), it can be seen that the force distribution applied onto the boundary of the specimen is involved in the second integral. However, the precise distribution remains generally unknown whereas the resulting force can be measured, at least at some points. The virtual fields can therefore be adjusted in such a way that only the resulting force virtually works, as explained in [18], thus avoiding any measurement of, or assumption on the force distribution.

Considering the Specimen Itself as a Load Cell

In dynamic testing, for instance with Hopkinson's bars, measuring the dynamic force applied to the specimen is challenging because of the transient nature of this force. In [23], it is proposed to remove the need for impact force measurement by processing full-field strain and acceleration measurements with Eq. (2.1). In this case, the virtual fields are chosen in such a way that the second integral in Eq. (2.1) vanishes, for instance by imposing a null virtual field along the boundary of the gauge section. As a result, the first integral in Eq. (2.1), namely the internal virtual work, is balanced with the integral in the right-hand side of Eq. (2.1) only. This can be interpreted by the fact that the specimen itself plays the role of a load cell thanks to a suitable choice of the virtual fields.

Making the Characterization of Heterogeneous Materials Easier

Heterogeneous materials feature spatially-changing stiffnesses. Identifying the parameters that govern the models describing these spatially-changing stiffnesses is the aim of various papers relying on the VFM. In [24], it is shown that it is possible to detect local stiffness fluctuations in composite plates (for instance caused by a local damage) by relying on piecewise virtual fields. The quality of the results however strongly depends of the choice of the mesh used to define the virtual fields. In [25], it is proposed to describe with a Fourier basis both the virtual fields and the model used to describe the spatially changing stiffnesses. This simplifies the calculations thanks to the orthogonality of the basis functions, and eventually leads to a very fine description of the spatially changing stiffnesses in heterogeneous materials, as illustrated in [25] with paper.

Measuring the Force Distribution Applied on the Specimen

If the material properties are known, it is possible to focus on the second integral of Eq. (2.1) in order to characterize the load distribution applied on the specimen. This type of problem is especially relevant in vibroacoustics. In [26] for instance, the authors use the VFM in order to measure the load time history in addition to magnitude and location.

Current State of Play

The VFM is spreading slowly but surely in the experimental mechanics community, as illustrated in Fig. 2.1, where the growing number of citations of the VFM in scientific publications is shown. The main reason given by the users for choosing this technique instead of others is the computation time, as in [27] for instance where FEMU and VFM are compared. A commercial code now includes the VFM as an add-on of a Digital Image Correlation program (https://www.matchid.eu), which illustrates the versatility of this method.

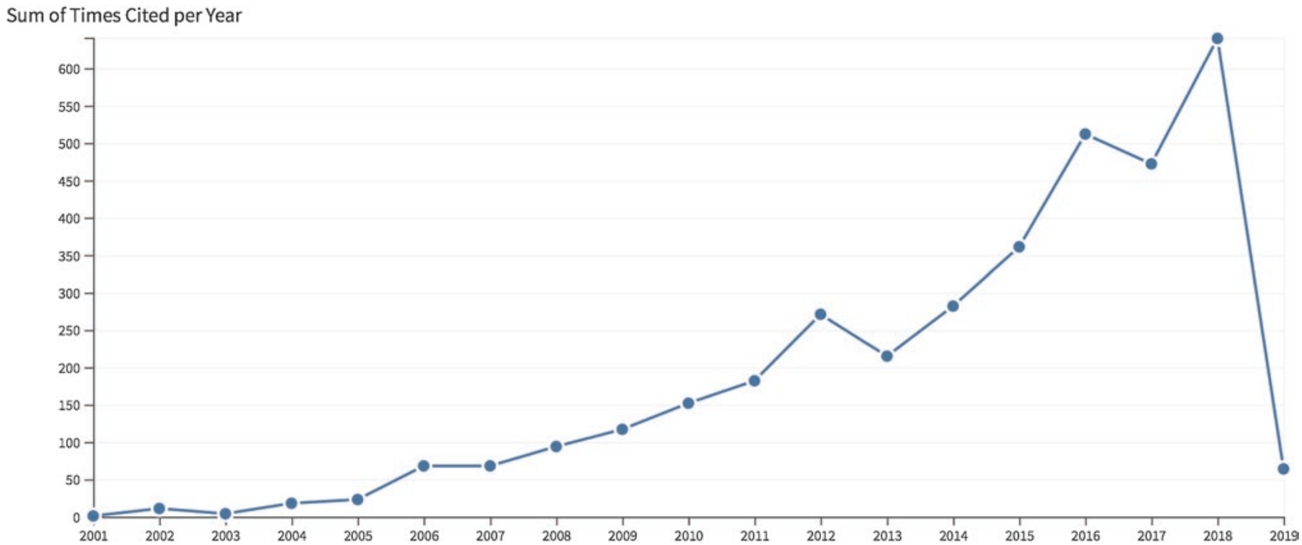

Fig. 2.1 Number of citations in scientific publications related to the virtual fields method (Web of Science, February 2019)

Conclusion

A technique initially designed to identify bending stiffnesses of composite plates by using full-field measurements has progressively been developed to become a general and versatile method. It has been applied to an increasing number of identification problems. Various types of constitutive equations and materials were studied in the recent past, and depending on the type of problem to be resolved, different strategies were proposed for the best choice of the virtual fields. The effectiveness of the VFM also depends on the geometry of the specimen as well as on the nature of the loading since these parameters directly influence the heterogeneous nature of the actual strain fields. This problem has been underexplored so far, and this surely constitutes a promising route for improving the quality of material characterization with this type of approach based on the processing of heterogeneous strain fields.

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