Modeling the Behaviour of Economic Agents as a Response to Information on Tax Audits

Suriya Kumacheva[1(✉)], Elena Gubar[1], Ekaterina Zhitkova[1], and Galina Tomilina[2]

[1] Faculty of Applied Mathematics and Control Processes, St. Petersburg State University, Universitetskii prospekt 35, Petergof, Saint-Petersburg 198504, Russia
{s.kumacheva,e.gubar,e.zhitkova}@spbu.ru

[2] EPAM Systems, Inc., Pevchesky per., 12, Saint-Petersburg 197101, Russia
g.tomilina@yandex.ru
http://www.spbu.ru

Abstract. Information is a strategic tool in many areas of the economics, business and social processes. Particularly, information can be considered as a control action in the problem of tax control. Following recent studies, we consider the auditing probabilities, which guarantee the absence of tax deviations, as the optimal strategies of the tax authority, however an employment of this strategy is unprofitable for tax authority and thus practically impossible. Therefore the dissemination of information on future tax audits over the taxable population can be offered as one of specific methods to stimulate people to pay taxes. Previously, it has been proofed that the process of information spreading resembles evolutionary dynamics in nature. Therefore, we consider a set of taxpayers as a population of interacted economic agents which intercommunicate with information/rumors. We design the propagation process as an imitation evolutionary dynamics over the structured population. We assume that it is more natural that agents will spread information or rumors over their own contact network, including neighbors and colleagues instead of randomly chosen agents. Thus in current study we investigate the imitation dynamics over the networks with different topology. We consider series of experiments where information spreads in the network of taxpayers with different topology and different modifications of bimatrix games to construct evolutionary dynamics describing the changes of agents behaviour. Numerical simulations give visualization of the information dissemination process over the different variant of the networks, imitation protocols and players payoffs. The results of simulations confirmed the influence of information on the final distribution of tax payments among the population with different levels of the risk propensity.

Keywords: Tax evasions · Tax control · Information dissemination · Evolutionary game · Networks

N. Agarwal et al. (Eds.): MSBC 2019, CCIS 1079, pp. 96–111, 2019.
https://doi.org/10.1007/978-3-030-29862-3_8

1 Introduction

In the modern world, an access to information helps people to make a decision about her/his strategy and choose an action which leads to success. The current study investigates such important problem as the impact of the information on possible tax audits on individuals' decisions to evade taxation or not [1,2]. Earlier in [8–10,16] based on the static model [4] we have constructed a dynamic system to describe the process of dissemination of information over the network of taxpayers. In the current study we use this system as the basic model to present the problem of taxation in a large but finite population of taxpayers.

In a series of papers on tax modeling [4,5,23], the conclusion about optimal tax control strategies was formulated as a "threshold rule". Numerical implementation of such strategies is an expensive procedure. In practice, it can be considered as unattainable due to the limited state budget. Therefore the tax authority needs to look for additional ways of encouraging taxpayers to fair tax payments. Among such incentives the dissemination of information about future audits can be considered as a useful tool. For example, an information can be similar to that the real probability is at least equal to the optimal probability value according to the mentioned "threshold rule", and be spread among the taxpayers, further mentioned as economic agents.

During the past decades different models for the propagation viruses and information in networks have been developed. One of the first papers, applying epidemic processes to the spreading of the rumors, ideas and information, is [7]. In many sources such as [12,17] information is considered as "infection of the mind" and its spreading can be formulated as epidemic or evolutionary process.

In the current paper we formulate a model of spreading information as an evolutionary network game to analyze how the dissemination of information affects on the decision of agents to evade taxation or not. Additionally, not only information, but also many other factors such as topology of the contact network, conflicts between agents etc. can also influence this decision.

In contrast to previous studies [20,24], where exampled agents were chosen at random, in the presented work construction of evolutionary dynamics of imitation is based on usage of special algorithms of choosing neighbors. Moreover, bimatrix games with different structure underlie the definition of the dynamics. Along with models built for risk-neutral agents [8–10,14], in the current study we also consider population, where agents with a different propensity to risk can change their actions, depending on external circumstances, as it was done in [15]. Due to this fact we design various scenarios of taxpayers' behaviour.

As we have showed in the previous researches [8,9,16] it is natural to represent a population of taxpayers as a connected network, because usually people prefer to share information with their family, neighbors, colleagues and friends. Following this assumptions we run the series of numerical experiments over the networks with different topology.

The paper is organized as follows. In Sect. 1, we present the overview of the basic and dynamic models. Section 2 presents the models of behaviour of risk-neutral agents in Subsect. 2.1 and the reaction of the system in the model for agents with different risk propensity in Subsect. 2.2 . In Sect. 3, we present the network model of annual process of tax audit. In Sect. 4, we present numerical simulation to illustrate the results. The paper is concluded in Sect. 5.

2 Aggregated System Costs in the Basic Models

In this paragraph we will obtain aggregated system costs for the cases of the model considered as a basis for dynamic processes of the information spreading.

2.1 Model with Risk-Neutral Agents

This section presents the problem of tax control based on the static model [4]. Due to the mentioned work we consider a large but finite population of n taxpayers, where ξ is true income, η is declared income, $\eta \leq \xi$. In the current model, $\xi, \eta \in \{L, H\}$, where $0 < L < H$, similar to [3]. Therefore here we obtain two groups of population: n_H and n_L, where

$$n_L + n_H = n.$$

Let P_L be the probability of random audit of agents, who declared income $\eta = L$. In the case when the tax arrears is revealed, the evader should pay $(\theta+\pi)(\xi-\eta)$, where constants θ and π are tax and penalty rates correspondingly.

For every profile of tax paying we obtain three possible taxpayers' profit functions:

$$u\left(L(L)\right) = (1-\theta)\cdot L; \tag{1}$$

$$u\left(H(H)\right) = (1-\theta)\cdot H; \tag{2}$$

$$u\left(L(H)\right) = H - \theta L - P_L(\theta+\pi)(H-L). \tag{3}$$

According to the "Threshold rule" obtained in [4,5,23] risk-neutral taxpayers refuse their evasion from H to L levels of income if probability P_L satisfies the condition

$$P_L \geq P^* = \frac{\theta}{\theta+\pi}. \tag{4}$$

Due to the limited budget of the tax authority, the optimal values of auditing probabilities P^* are extremely rare reached in real life. Therefore the tax authority needs to find additional methods to stimulate taxpayers to be honest. As one of these ways the dissemination of information about increased probability of tax auditing in the population of taxpayers can be considered. In particular, this information can take the form of a message "$P_L \geq P^*$". It means that disseminated information contains the statement that the share of the taxpayers randomly selected for auditing will be at least equal to the value P^* which is critical for the agents' decision to evade or not.

Construction of the model of information spreading needs some additional statements:

- The first statement is that the considered population is the set of size n_H, which consists only of the taxpayers with high level H of income. Therefore we suppose that in the case when there is no information the total population evades. Thus we can denote the number of evaders as n_{ev} and determine it in this case as $n_{ev} = n_H$.
- When information has been injected at the initial time moment in the population then the number of taxpayers informed about increased probability of tax auditing becomes $n^0_{inf} = n^0_{nev}$. These agents are informed directly and therefore they decided not to evade. In each time moment $n_H = n_{ev}(t) + n_{nev}(t)$ (or $\nu_{nev}(t) + \nu_{ev}(t) = 1$, where ν_{nev}, ν_{ev} are the shares of evaders and non-evaders correspondingly), $t \in [0, T]$.

Moreover, at the initial time moment, the expected income of tax authority can be defined as the total tax revenue TTR^N_0 in the absence of information, which includes only payments of agents with true income level L:

$$TTR^N_0 = n_L\theta L + n_H\,(\theta\,L + P_L\,(\theta + \pi)(H - L)) - n\,P_L\,c, \tag{5}$$

where c is the unit cost of auditing.

At the final time moment T the total tax revenue TTR^N_T is computed in the assumption that the information was spread and the system has come to a steady state:

$$\begin{aligned} TTR^N_T = n_L\theta L + n_H\,&\left(\nu^T_{nev}\theta H + \nu^T_{ev}\,(\theta L + P_L(\theta + \pi)(H - L))\right) \\ &- n(P_L\,c + \nu^0_{inf}c_{inf}), \end{aligned} \tag{6}$$

where ν^T_{nev} is the share of honest taxpayers, who do not evade at the moment $t = T$, ν^T_{ev} is the share of agents, who continue to evade taxation at the moment $t = T$, ν^0_{inf} is the value (fraction) of the informational injection at the initial time moment ($\nu^0_{inf} = \nu_{inf}(t_0)$) and c_{inf} is the unit cost of such injection, it is assumed that $c_{inf} << c$.

2.2 Model with Agents with Different Risk-Propensity

However, the real life brings more interesting effects to the interactions between tax authority and taxpayers. Thus, in considered population there can exist individuals with different risk-propensity: risk-averse, risk-neutral and risk-loving. It means that total population of taxpayers should be divided into three subgroups, as it was supposed in [15]. These subgroups differ from each other by the various agents' behaviour profiles in the similar external conditions. Therefore their response on the same information should be different. At the initial time moment in the absence of information only taxpayers with low level of income and risk avoiding agents (whose share is ν_a) pay. It means that in this case the total tax revenue is

$$TTR^R_0 = n_L\theta L + n_H\,(\nu_a\theta H + (1 - \nu_a)P_L(\theta + \pi)(H - L)) - nP_L\,c. \tag{7}$$

At the final moment T when the information was spread and the system has come to a steady state the total tax revenue TTR_T^R is

$$TTR_T^R = n_L\theta L + n_H\left(\nu_a\theta H + (1-\nu_a)(\nu_{nev}^T\theta H + \nu_{ev}^T(\theta L + P_L(\theta+\pi)(H-L)))\right) \\ -n(P_L\, c + \nu_{inf}^0 c_{inf}). \tag{8}$$

3 Network Model

In this paragraph we formulate some basic assumptions of the network evolutionary model. Generally, in evolutionary game, a population of agents, who possess different types of behaviour, is divided into several subpopulations, in compliance with a number of types of behaviors. In the entire population pairwise interactions are defined by the bimatrix games, which describe all possible communications between randomly matching players. In contrast to original formulation of the evolutionary game, where evolution of the population is described through the set of random meetings in the well mixed population, we suppose that the social connection of each agents or taxpayer can be represented by the networks with different topology. Therefore, the interagent interactions are feasible only between connected taxpayers. Hence the evolution process differs from the ordinary evolutionary game. Following [8,20] we consider the process of disseminating information about inspections in the taxpayers' network as an evolutionary process in the population of economic agents. We design the algorithms of propagation based on the imitation rules [21,24]. In the current paper we consider three algorithms for selecting neighbors as an exampled agent in the imitation dynamics. The first one is random choice: an opponent agent is randomly selected from a set of agents who connected with the active agent. The second is based on the most influential neighbor: the opponent with the most direct connections to other agents is selected from the set of neighboring agents. If there are several agents with an equally large number of links, the choice between them is random. The third one is based on the neighbor with the highest income: an exampled neighbor is selected if his/her income, according to the results of the first iteration step, was the greatest.

3.1 Instant Games

In this subsection we consider a structured population of economic agents (taxpayers). As in [21,24] the instant communications between taxpayers is defined by two-players symmetric bimatrix game $\Gamma(A, B)$. Every taxpayer can choose one of two behaviors $X = \{ev, nev\}$, where ev is the behaviour "not evade", nev is the behaviour "to evade". Payoff matrix of the instant game between connected agents has structure of one of the following classical games: the Prisoner's Dilemma, the Stag Hunt game, the Hawk-Dove game. Since the structure of these bimatrix games is well-known then they are appropriated to estimate an impact of network structure and imitation rules. Further, an instant interaction can be modelled by using special structure of the game.

However we use the modification of Prisoner's Dilemma game [19], which can be described by the bimatrix game, presented below, where a payoff matrix of the first player is A and payoff matrix of the second player is symmetric $B = A^T$ [24]:

$$\begin{array}{c|cc} & C & D \\ \hline C & (\overline{u}+SW),(\overline{u}+SW) & (\overline{u}-SW,u(L(H))) \\ D & (u(L(H)),\overline{u}-SW) & (\overline{u},\overline{u}) \end{array}$$

where C is the strategy "to cooperate", which can be interpreted "to pay taxes" in our case, D is the strategy "to defeat" which is equal to "to evade" in the studied model, $\overline{u} = 1/2u(L(L))+1/2u(H(H))$ is the average profit of the "mean" agent, SW is social welfare, obtained for the participation in social consolidation. The payoff matrix describes possible meetings between two honest taxpayers with strategies C, two evaders, who use strategies D, as well as two asymmetric situation profiles define interactions between honest taxpayer and evader. Hereafter in the next two games we follow the same technique.

In the Stag Hunt game [22] a payoff matrices A and $B = A^T$ are formed in the following way:

$$\begin{array}{c|cc} & S & I \\ \hline S & (\overline{u}+SW,\overline{u}+SW) & (0,\overline{u}-SW) \\ I & (\overline{u}-SW,0) & (\overline{u}-SW,\overline{u}-SW) \end{array}$$

where strategy S corresponds to social strategy "to hunt a stag" in classical form of the game, but in our case this strategy means to "to pay taxes", analogously strategy I corresponds to individual strategy "to hunt a hare" in original game, in our interpretation this a strategy recommends taxpayer "to evade".

The case of Hawk-Dove game is based on the payoff matrix A, $B = A^T$:

$$\begin{array}{c|cc} & F & D \\ \hline F & (\frac{u(L(H))-(\theta+\pi)(H-L)}{2},\frac{u(L(H))-(\theta+\pi)(H-L)}{2}) & (\overline{u}+SW,0) \\ D & (0,\overline{u}+SW) & (\frac{\overline{u}+SW}{2},\frac{\overline{u}+SW}{2}) \end{array}$$

where strategy F originally corresponds "to be a Hawk", which means that agent demonstrates an aggressive behaviour, in our case this strategy leads taxpayer "to evade", strategy D is "to be a Dove" and forces agent to follow a passive behaviour, in our case this behaviour is "to pay taxes". Additionally, we assume that the condition $u(L(H)) << (\theta+\pi)(H-L)$ should be satisfied and it works for the large values of the parameters θ and π or if the value of the difference $(H-L)$ is large.

3.2 Imitation Rules on the Networks

As it was mentioned above in the Sect. 3 an evolutionary process occurs on the indirect network $G = (N, K)$, where $N = \{1, \ldots, n_H\}$ is a set of economic agents and $K \subset N \times N$ is an edge set (each edge in K represents two-players symmetric game between connected taxpayers) [8,20]. It is assumed that the taxpayers choose strategies from a binary set $X = \{ev, nev\}$ and receive payoffs according to the matrix of payoffs. Each instant time moment agents use a single strategy against all opponents and thus the games occurs simultaneously. The strategy state: $x(T) = (x_1(t), \ldots, x_{n_H}(t))^T$, where $x_i(t) \in X$ is a strategy of taxpayer i, $i = \overline{1, n_H}$, at time moment t. Aggregated payoff of agent i will be defined as in [20]:

$$u_i = \omega_i \sum_{j \in M_i} a_{x_i(t), x_j(t)}, \tag{9}$$

where $a_{x_i(t), x_j(t)}$ is a component of payoff matrix, $M_i := \{j \in L : \{i, j\} \in K\}$ is a set of neighbors for taxpayer i, weighted coefficient $\omega_i = 1$ for cumulative payoffs and $\omega_i = \frac{1}{|M_i|}$ for average payoffs. Vector of payoffs of the total population is $u(t) = (u_1(t), \ldots, u_{n_H}(t))^T$.

The state of population will be changed according to the rule, which is a function of the strategies and payoffs of neighboring agents:

$$x_i(t + i) = f(\{x_j(t), u_j(t) : j \in N_i \cup \{i\}\}). \tag{10}$$

This rule dictates a method of imitation or adaptation of agents to the changes in his/her environment, which means that taxpayer can change her behaviour if at least one neighbor has the better payoff. As an example of such dynamics we can use the proportional imitation rule [21,24], in which each agent chooses a neighbor randomly and if this neighbor received a higher payoff by using a different strategy, then the agent will switch with a probability proportional to the payoff difference. The proportional imitation rule can be presented as:

$$p\left(x_i(t + 1) = x_j(t)\right) := \left[\frac{\lambda}{|M_i|}(u_j(t) - u_i(t))\right]_0^1 \tag{11}$$

for each agent $i \in K$ where $j \in M_i$ is a uniformly randomly chosen neighbor, $\lambda > 0$ is an arbitrary rate constant, and the notation $[z]_0^1$ indicates $\max(0, \min(1, z))$.

In our work, we define three modifications of imitation rules, in compliance with the method of choice an exampled agent, taxpayers' payoffs and distribution of risk-statuses of taxpayers over the entire population. The main assumption of the work is the application of these imitation rules only to the subgroup of risk-neutral taxpayers, as far as, in our hypothesis, this group is the most influenceable on the tax collections in the entire population. Fractions of risk-loving and risk-averse taxpayers are fixed at the initial time moment.

– **Rule 1.** *Random neighbor.* When a taxpayer i receives an opportunity to revise her strategy then she chooses an exampled agent at random with equal probability to all connected neighbors.

- **Rule 2.** *Neighbor with the highest payoff.* When agent i receives an opportunity to revise her strategy then she considers current payoffs of all taxpayers and chooses an agent (or a set of agents) with maximum payoff. If there are several agents with maximum payoff then an exampled agent is chosen at random between this subset.
- **Rule 3.** *The most influenceable neighbor.* Firstly, taxpayer i estimates a number of connections of all nearest neighbors and selects an exampled agent from the set of agents with the maximum number of links. If there are several agents with maximum links, then an opponent is selected at each iteration of the dynamic process at random between the subset of influenceable agents.

4 Numerical Simulations

In current section we present numerical simulations which illustrate the theoretical approach described above. As the initial distribution of the population's propensity to risk, Gaussian distribution quantiles or curves obtained in the works [6, 11] can be considered. In the current study, to conduct numerical simulations we use the following results of psychological research on risk-addiction [18]: the number of risk-loving agents from the general population of taxpayers is 18%, risk-neutral agents are 65%, risk-avoidants are 17%. In all experiments we work with the distribution of the income among the population of Russian Federation in 2018 [25]. According to the model we consider only two levels of income are accessible for each taxpayer: low and high (L and H). After the unification of groups with different levels of income according to the economic reasons, we calculate the average levels of income L and H (the mathematical expectations of the uniform and Pareto distributions [13]) and receive the corresponding shares of the population (see Table 1).

Table 1. Two modeled groups and average income

Group	Income interval (rub./month)	Average income (rub.)	Share of population (%)
L	Less 25000	$L = 12500$	51
H	More 25000	$H = 50000$	49

For all experiments we fix the values of parameters: $\nu_a = 17\%$ is a share of risk-averse taxpayers in population, tax rate is $\theta = 13\%$, penalty rate is $\pi = 13\%$, optimal value of the probability of audit is $P^* = 0.5$, actual value of the probability of audit for those who declared L is $P_L = 0.1$, unit cost of auditing is $c = 7455$ (rub.), as a unit cost of information injection we consider $c_{inf} = 10\%c = 745.5$ (rub.).

The matrixes of the following games are used as scoring matrices: the Prisoner's Dilemma, the Stag Hunt game, the Hawk-Dove game. Each agent evaluates his profit using these matrices and information about the strategies of the neighboring agents, with whom he has connections. Two algorithms for calculating this rating are considered:

- Cumulative: the sum of the profits from each interaction is computed;
- Average: the sum of the profits from interactions divided by their number.

If we take for $x_i(t)$ the i-th agent' profit at iteration t, then the $(t+1)$-th iteration can be considered as final if: $\sqrt{\sum_{i=1}^{n}(x_i(t) - x_i(t+1))} \leq 3 \cdot 10^{-2}$.

The experiment for the same initial distribution was repeated a 10^2 times, after which a report was generated.

Three examples below present different combination of instant games, protocols and method of computing of total tax revenue. In all examples initial distribution of evaders and honest taxpayers is the same: $\nu_{ev}^0 = 13$, $\nu_{nev}^0 = 12$, and the size of total population is $N = 25$. This size of population have been chosen to simplify perception of the figures. The created software product allows carrying out similar experiments for a population of larger sizes. However, the conclusions obtained from these experiments are hampered by a sharply increasing number of cases that need to be analyzed.

Figures 1–2, 4–5 and 7–8 demonstrate the evolution of the shares of taxable population during the iteration process. We use the following notation:

- agents with strategy "to pay taxes" are drown by squares, agents who use strategy "to evade" are drown by circles respectively;
- the risk status of agents is displayed by using the colors in the figures: risk-averse taxpayers are green nodes, risk-neutrals are red and risk-loving are blue.

To present the dynamic process in the population we use following modification of the network:

- strongly connected network, where the probability of link formation is 1/10;
- weakly connected network, where the probability of link formation is 1/3;
- random graph, where the probability of link formation is $1/k, k \in N$.

Example 1. In this example the following combination of parameters has used: graph is grid, instant game is Prisoner's Dilemma and imitation rule is the most influenceable neighbor. The pictures 1–2 present initial and final states of the system subject to cumulative method of computing of agent's profit.

Series of numerical experiments have produced a set of data for estimation of the influence of various parameters on the level of TTR^R. Some series of experiments present the dependence of the total tax revenue on the number of agents who chose the strategy "to pay". Experiments for the cumulative profit of the dynamics of TTR^R, which depends on the share of honest taxpayers, demonstrate more stable results and hence we use these examples as more illustrative. Below in Fig. 3 the mentioned dynamics are presented for the series of experiments with the cumulative type of payoff.

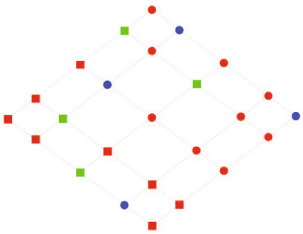

Fig. 1. Experiment I. Initial state of the population is $\nu_{ev}^0 = 13$, $\nu_{nev}^0 = 12$, initial total tax revenue is $TTR_0^R = 52082.86$;

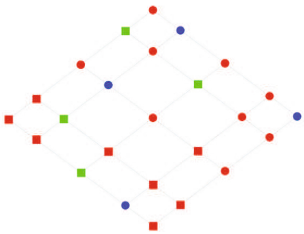

Fig. 2. Experiment I. Final state of the population is $\nu_{ev}^T = 15$, $\nu_{nev}^T = 10$, method of computing of agent's profit: cumulative, final total tax revenue $TTR_T^R = 65210.50$.

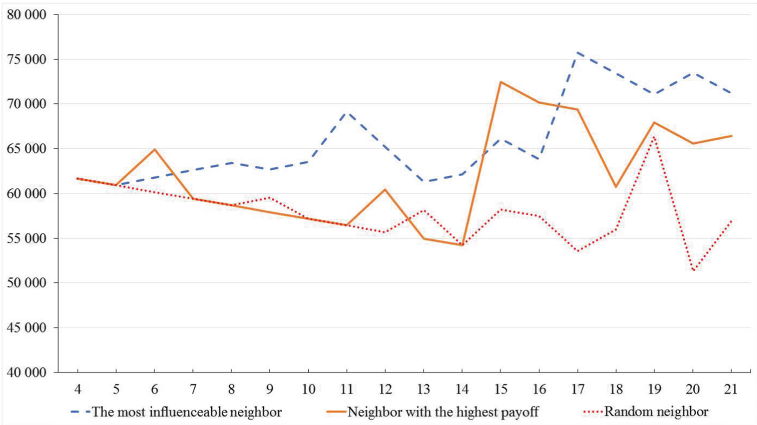

Fig. 3. Experiment Series I. The dynamics of TTR^R depending on the number of agents who pay. The ordinate axis represents the values of TTR^R, and the abscissa axis shows the number of honest taxpayers.

In Example 1 we have series of experiments for the cumulative payoffs, where the maximum values of TTR_T^R obtained by using different imitation rules for the corresponding initial and final data are following:

- The most influenceable neighbor: $TTR_T^R = 75758.7$, initial state of the population is $\nu_{ev}^0 = 8, \nu_{nev}^0 = 17$; final state of population is $\nu_{ev}^T = 6, \nu_{nev}^T = 19$;
- Neighbor with the highest payoff: $TTR_T^R = 72490.78$, initial state of the population is $\nu_{ev}^0 = 10, \nu_{nev}^0 = 15$; final state of population is $\nu_{ev}^T = 9, \nu_{nev}^T = 16$;
- Random neighbor: $TTR_T^R = 66336.52$, initial state of the population is $\nu_{ev}^0 = 6, \nu_{nev}^0 = 19$; final state of population is $\nu_{ev}^T = 11, \nu_{nev}^T = 14$.

Dynamics of the total revenue demonstrate different behavior, for example, imitation rules "the most influenceable neighbor" and "neighbor with the highest payoff" increase the total revenue with the increasing of the share of honest taxpayers, whereas imitation rule "random neighbor" decrease the value of TTR^R.

Example 2. The next example presents results of simulations for strong connected network with "Stag hunt" game as an instant game and imitation rule is "the neighbor with highest payoff".

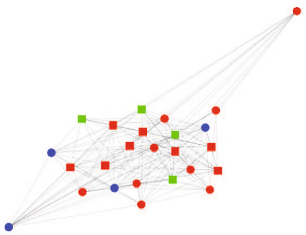

Fig. 4. Experiment II. Initial state of the population is $\nu_{ev}^0 = 13$, $\nu_{nev}^0 = 12$, initial total tax revenue is $TTR_0^R = 52082.86$;

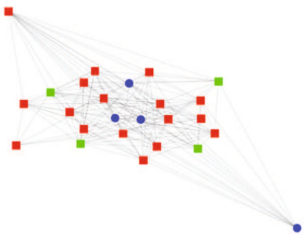

Fig. 5. Experiment II. Final state of the population is $\nu_{ev}^T = 4$, $\nu_{nev}^T = 21$, method of computing of agent's profit: cumulative, final total tax revenue $TTR_T^R = 82657.93$.

Similar to the previous series of experiment, we represent the dynamics of TTR^R in Fig. 6.

In the current series of experiments for the cumulative payoffs, the maximum values of TTR_T^R obtained by using different imitation rules for the corresponding initial and final data are following:

- The most influenceable neighbor: $TTR_T^R = 83403.43$, initial state of the population is $\nu_{ev}^0 = 14, \nu_{nev}^0 = 11$; final state of population is $\nu_{ev}^T = 4, \nu_{nev}^T = 21$;
- Neighbor with the highest payoff: $TTR_T^R = 84894.43$, initial state of the population is $\nu_{ev}^0 = 16, \nu_{nev}^0 = 9$; final state of population is $\nu_{ev}^T = 4, \nu_{nev}^T = 21$;
- Random neighbor: $TTR_T^R = 85639.93$, initial state of the population is $\nu_{ev}^0 = 17, \nu_{nev}^0 = 8$; final state of population is $\nu_{ev}^T = 4, \nu_{nev}^T = 21$.

The second example presents a very interesting dynamics of TTR^R, from the initial share of non-evaders to $\nu_{nev} = 7, \ldots, 9$ the value of total revenue archives its peaks and then decreases. However the general trend is the increasing of total tax revenue for all imitation rules.

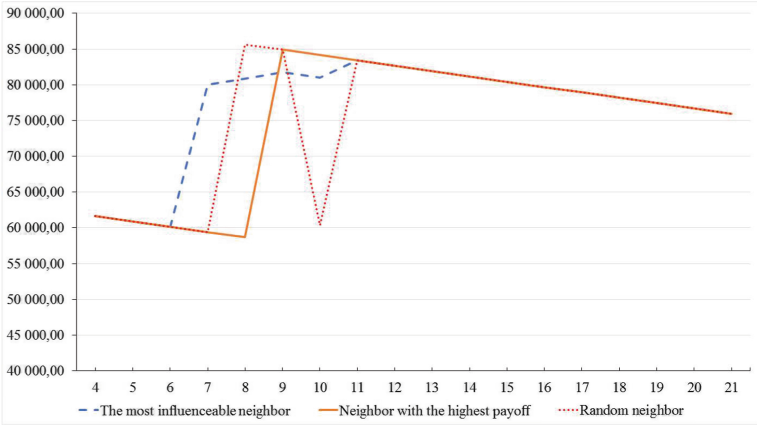

Fig. 6. Experiment Series II. The dynamics of TTR depending on the number of agents who pay. The ordinate axis represents the values of TTR^R, and the abscissa axis shows the number of honest taxpayers.

Example 3. The next example presents results of simulations for weakly connected network with "Hawk Dove" game as an instant game and imitation rule "random neighbor".

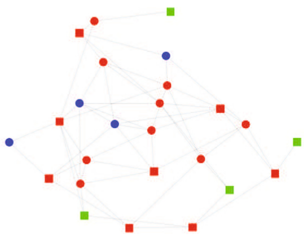

Fig. 7. Experiment III. Initial state of the population is $\nu^0_{ev} = 13$, $\nu^0_{nev} = 12$, initial total tax revenue is $TTR^R_0 = 52082.86$;

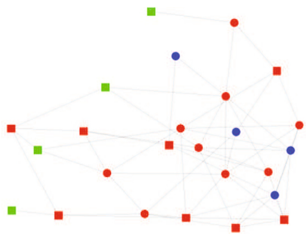

Fig. 8. Experiment III. Final state of the population is $\nu^T_{ev} = 7$, $\nu^T_{nev} = 18$, method of computing of agent's profit: Cumulative, final total tax revenue $TTR^R_T = 77899.54$.

The dynamics of TTR^R on the number of non-evaders is represented in Fig. 9.

In the current series of experiments for the cumulative payoffs, the maximum values of TTR^R_T obtained by using different imitation rules for the corresponding initial and final data are following:

- The most influenceable neighbor: $TTR^R_T = 75948.43$, initial state of the population is $\nu^0_{ev} = 4, \nu^0_{nev} = 21$; final state of population is $\nu^T_{ev} = 4, \nu^T_{nev} = 21$;

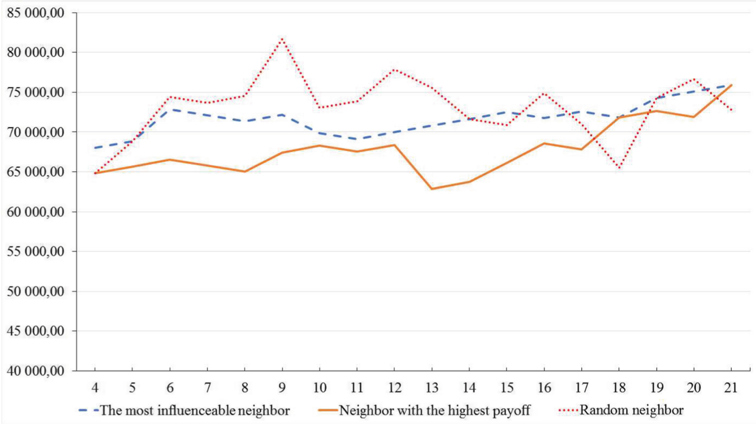

Fig. 9. Experiment Series III. The dynamics of TTR^R depending on the number of agents who pay. The ordinate axis represents the values of TTR^R, and the abscissa axis shows the number of honest taxpayers.

- Neighbor with the highest payoff: $TTR_T^R = 75948.43$, initial state of the population is $\nu_{ev}^0 = 4, \nu_{nev}^0 = 21$; final state of population is $\nu_{ev}^T = 4, \nu_{nev}^T = 21$;
- Random neighbor: $TTR_T^R = 81722.17$, initial state of the population is $\nu_{ev}^0 = 16, \nu_{nev}^0 = 9$; final state of population is $\nu_{ev}^T = 6, \nu_{nev}^T = 19$.

Here all dynamics fluctuate around mean value with weak tendency of increasing total revenue.

4.1 Experimental Results

By using the model of information spreading, we run 18 different cases of the initial distribution of risk statuses in the taxpayer population represented by networks with different configurations were considered in the series of numerical experiments. All experiments include grid, strongly connected and weakly connected random graphs. Evolutionary process of spreading information in the population of taxpayers is based on the bimatrix games such as "Prisoner's Dilemma", "Stag Hunt", "Hawks and Doves". For each initial distribution corresponding simulation was repeated 10^2 times to obtain statistically significant data to analyze the structure of possible scenarios. In addition, each experiments was performed for two types of payoffs: cumulative and average.

The series of experiments statistically demonstrate that regardless of network topology and the imitation rule, which is used for the Prisoner's Dilemma game, the strategy of evasion is stable. It is chosen by the majority of taxpayers in the most number of experiments.

At the same time, the game "Hawks and Doves" presents a completely opposite agents' behaviour: for any combination of imitations protocols, graphs and payoffs, agents prefer strategy "to pay". The results of the game "Stag Hunt" are generally similar to the results of the "Hawks and Doves" game. However, in the case of a weakly connected graph, we receive a tendency of the equiprobable (in a statistical sense) choice of both strategies by using "The Most Influential Neighbor" and "Neighbor with the highest payoff" imitation rules. The largest difference in the choice of strategies is demonstrated by the use of the Random Neighbor protocol.

It has also been revealed that the structure of the bimatrix game has the most influence on the final distribution of taxpayers. Regardless of the choice of imitation rules, the obtained results are completely comparable.

The most important result of numerical simulation is the ability to present the conclusions which type of network topology, imitation rules and type of bimatrix game improves the final attitude to the risk of the economic agents and, therefore, increases the level of declared income of taxpayers', in case of injected information circulates in the taxable population.

In addition, numerical simulations demonstrate two trends in changes of the total tax revenue of the system. Firstly, if the number of taxpayers, which pay taxes prevails over the number of evaders in the final distribution then a value of TTR_T^R increases in comparison with the initial total tax revenue TTR_R^0. Secondly, in some rare cases, total tax revenue grows even if a number of evaders is larger then a number of honest taxpayers in final distribution. This occurs because the collection of taxes and penalties increases, however, the probability of such event is small.

In some numerical experiments the dynamics of total tax revenue TTR_T^R have demonstrated unspecified results depending on the initial injection of information $\nu_{inf}^0 = \nu_{nev}^0$. For example, comparison of the results for different types of games or different topology of networks presents a tendency to decrease in total revenue TTR_T^R if the number of those who chose the strategy "to pay" increases. Whereas, analysis of the dynamics of TTR_T^R under various imitation rules, on the contrary, shows a tendency to increase the value of total tax revenue. Some simulations bring dynamics of TTR_T^R with large deviations from mean values of TTR_T^R. Hence, it is necessary to run additional series of experiments to receive more data sets to examine the trends more exact and that is the frame work for further research.

Comparison of the practical factors allows us to assess the efficiency of the proposed method of simulation the tax collection. This scenario analysis helps users to consider the process of information dissemination as a tool to improve the incentive and fiscal functions of the tax system. Additionally, the reduction of risk-loving agents helps to provide one of the fundamental principles of a tax system – the principle of fair taxation.

5 Conclusions

In the present work, the question of stimulating the taxable population to fair tax payments through the dissemination of information about future audits was studied.

The current study presents an impact of information dissemination about future audits over a taxable population to fair tax payments. This problem was formulated as the model, which combines ideas of applying the "Threshold rule" and evolutionary dynamics to imitating the actual information dissemination process. Moreover, evolutionary dynamics are considered not on the population of the randomly interacted agents but on the structured population which described by the network with different topology.

References

1. Antocia, A., Russua, P., Zarrib, L.: Tax evasion in a behaviorally heterogeneous society: an evolutionary analysis. Econ. Model. **10**(42), 106–115 (2014)
2. Antunes, L., Balsa, J., Urbano, P., Moniz, L., Roseta-Palma, C.: Tax compliance in a simulated heterogeneous multi-agent society. In: Sichman, J.S., Antunes, L. (eds.) MABS 2005. LNCS (LNAI), vol. 3891, pp. 147–161. Springer, Heidelberg (2006). https://doi.org/10.1007/11734680_11
3. Boure, V., Kumacheva, S.: A model of audit with using of statistical information about taxpayers' income. Vestnik SPbGU **10**(1–2), 140–145 (2005). (in Russian)
4. Boure, V., Kumacheva, S.: A game theory model of tax auditing using statistical information about taxpayers. Vestnik SPbGU **10**(4), 16–24 (2010). (in Russian)
5. Chander, P., Wilde, L.L.: A general characterization of optimal income tax enforcement. Rev. Econ. Studies **65**, 165–183 (1998)
6. Friedman, M., Savage, L.J.: The utility analysis of choices involving risk. J. Polit. Econ. **56**(4), 279–304 (1948)
7. Goffman, W., Newill, V.A.: Generalization of epidemic theory: an application to the transmission of ideas. Nature **204**(4955), 225–228 (1964)
8. Gubar, E., Kumacheva, S., Zhitkova, E., Kurnosykh, Z.: Evolutionary behavior of taxpayers in the model of information dissemination. In: Constructive Nonsmooth Analysis and Related Topics (Dedicated to the Memory of V.F. Demyanov, CNSA 2017) Proceedings, pp. 1–4. IEEE Conference Publications, St. Petersburg (2017)
9. Gubar, E., Kumacheva, S., Zhitkova, E., Kurnosykh, Z., Skovorodina, T.: Modelling of information spreading in the population of taxpayers: evolutionary approach. Contrib. Game Theory Manage. **10**, 100–128 (2017)
10. Gubar, E.A., Kumacheva, S.Sh., Zhitkova, E.M., Porokhnyavaya, O.Yu.: Propagation of information over the network of taxpayers in the model of tax auditing. In: 2015 International Conference on Stability and Control Processes in Memory of V.I. Zubov (SCP 2015) Proceedings, pp. 244–247. IEEE Conference Publications, St. Petersburg (2015)
11. Kahneman, D., Tversky, A.: Advances in prospect theory: cumulative representation of uncertainty. J. Risk Uncert. **5**, 297–323 (1992)
12. Kandhway, K., Kuri, J.: Optimal control of information epidemics modeled as Maki Thompson rumors. In: Preprint submitted to Communications in Nonlinear Science and Numerical Simulation (2014)
13. Kendall, M.G.: A. Stuart, Distribution Theory, Nauka, Moscow (1966). (in Russian)
14. Kumacheva, S.S.: Tax auditing using statistical information about taxpayers. Contrib. Game Theory Manage. **5**, 156–167 (2012)
15. Kumacheva, S.S., Gubar, E.A.: Evolutionary model of tax auditing. Contrib. Game Theory Manage. **8**, 164–175 (2015)

16. Kumacheva, S., Gubar, E., Zhitkova, E., Tomilina, G.: Evolution of risk-statuses in one model of tax control. In: Petrosyan, L., Mazalov, V., Zenkevich, N. (eds.) Frontiers of Dynamic Games. Static and Dynamic Game Theory: Foundations and Applications, pp. 121–138. Birkhauser, Cham (2018)
17. Nekovee, A.M., Moreno, Y., Bianconi, G., Marsili, M.: Theory of rumor spreading in complex social networks. Phys. A **374**, 457–470 (2007)
18. Niazashvili, A.: Individual differences in risk propensity in different social situations of personal development. Moscow University for the Humanities, Moscow (2007)
19. Owen, G.: Game Theory. Saunders Company, Philadelphia (1968)
20. Riehl, J.R., Cao M.: Control of stochastic evolutionary games on networks. In: 5th IFAC Workshop on Distributed Estimation and Control in Networked Systems, pp. 458–462. Philadelphia, PA, USA (2015)
21. Sandholm, W.H.: Population Games and Evolutionary Dynamics. The M.I.T. Press, Cambridge (2010)
22. Skyrms, B.: The Stag Hunt and the Evolution of Social Structure. Cambridge University Press, Cambridge (2003)
23. Vasin, A., Morozov, V.: The Game Theory and Models of Mathematical Economics. MAKS Press, Moscow (2005). (in Russian)
24. Weibull, J.: Evolutionary Game Theory. The M.I.T. Press, Cambridge (1995)
25. The web-site of the Russian Federation State Statistics Service. http://www.gks.ru/