Teaching Mathematics in the Digital Era: Standards and Beyond

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1 Introduction

In all technologically advanced societies today, the question of whether information and communication technology (ICT) should be used in education, and especially in mathematics education, is no longer an issue. Nevertheless, the question of how to use technology effectively to improve mathematics learning and/or teaching remains pertinent. In a literature review on the "barriers to the uptake of ICT by teachers," Jones (2004) points out that "effective training is crucial if teachers are to implement ICT effectively in their teaching" (p. 8). This clearly raises the issue of (pre-service) teacher education (TE) and (in-service) teacher professional development (TPD) regarding the use of digital technology.

We began researching the issue of mathematics TE/TPD with respect to technology use in 2016 by surveying research on the uses of technology in upper secondary level mathematics education (Hegedus et al., 2017). Our review of a number of TE/TPD initiatives in the literature revealed that most cases report disappointment with the outcomes of these initiatives. One of the main explanations for this disappointment is the discrepancy between teachers' needs and TE/TPD contents (Emprin, 2010). While teacher educators often showcase successful examples of ICT use that they themselves designed and implemented, they rarely address the ways in which teachers can implement these activities. Likewise, Lagrange and Dedeoglu (2009) point to a disparity between teachers' expectations in terms of ICT use and the ICT potentialities revealed by research and presented by teacher educators. These findings

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point to the need for ICT competency standards to delineate the specific knowledge and skills (mathematics) teachers need to integrate ICT effectively in the classroom.

We therefore searched for existing documents governing teachers' knowledge about teaching mathematics with technology (Tabach & Trgalová, 2017; Trgalová & Tabach, 2018). In analysing these documents, we referred to the *TPACK model* (Mishra & Koehler, 2006). This model, which is widely used in research about teachers' professional knowledge related to ICT, depicts how teachers' technological knowledge interacts with their pedagogical and content knowledge in successfully integrating ICT into teaching. We articulated the TPACK model with the theoretical construct of *double instrumental genesis* (Haspekian, 2011) outlined below (Sect. 2). Our preliminary findings, which were based on our analysis of international ICT frameworks for teachers (e.g., ISTE, 2008; UNESCO, 2011) as well as of several national standards (e.g., USA, Australia, Israel or France), show that only a few such standards exist at either the national or the international level for mathematics teachers, or even for teachers in general. Moreover, most of the existing standards are not specific either to subject matter or to school level. The theoretical lens of the TPACK combined with the double instrumental genesis concept reveal that some standards overemphasize technological knowledge (TK) while others are not sufficiently precise to inform teacher education programs despite taking all categories of TPACK knowledge into consideration. On the other hand, some standards emphasize the need to develop teachers' awareness of the added value of technology in terms of its impact on students' understanding of mathematics. Our theoretical frame overlooked this dimension.

The aim of our research reported in this chapter is twofold: (1) to pursue our investigation of existing ICT standards for (mathematics) teachers by expanding it to national policies and institutional frameworks in several OECD countries and Australia; and (2) to define a conceptual framework for capturing various dimensions of teachers' professional knowledge and skills oriented toward the use of digital technology.

The chapter is organized as follows. We begin by describing the theoretical framework (Sect. 2) we adopted in our research, followed by a description of the methodology we used for analysing existing ICT-related policies and frameworks (Sect. 3). In Sect. 4 we report the findings of our research and in Sect. 5 we further discuss the findings and propose a conceptual framework for defining teachers' ICT competencies.

2 Theoretical Framework

We sought to capture not only the cognitive dimension but also other dimensions, such as the affective dimension, which have been deemed important in relation to ICT integration. Hence, we chose to replace the TPACK model with the *pedagogical technology knowledge* (PTK) framework (Thomas & Hong, 2005; Thomas & Palmer, 2014). The PTK framework (Fig. 1) includes a number of teacher factors

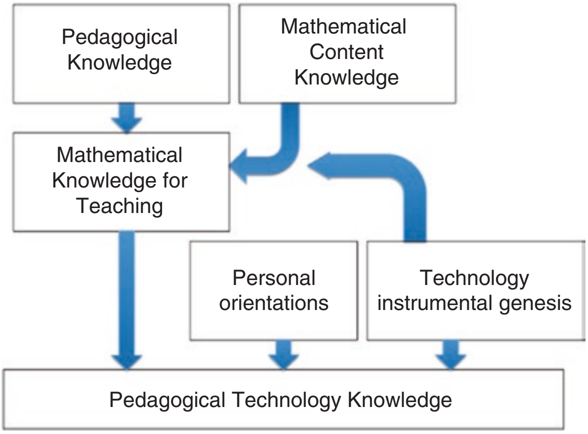

Fig. 1 Pedagogical technology knowledge framework (Thomas & Palmer, 2014)

intrinsic to the production of knowledge for teaching with technology, namely: teachers' instrumental genesis with respect to technology, mathematical knowledge for teaching (Ball, Thames, & Phelps, 2008), and teachers' personal orientations and goals (Schoenfeld, 2011), especially beliefs about the value of technology and the nature of learning mathematical knowledge as well as teachers' confidence in using technology.

We consider PTK to be an appropriate framework for examining mathematics teachers' technology-related knowledge for several reasons. First, it was developed within mathematics education specifically with mathematics teachers in mind, as indicated in reference to MKT (Ball et al., 2008), which is specific to mathematics education. Moreover, it further specifies important components of teachers' knowledge, such as knowledge of curriculum or students. Second, it includes an affective component by referring explicitly to teachers' orientation, which we consider to be an important dimension of teachers' professional competence, as noted by scholars such as Lynch, Russell, Evans, and Sutterer (2009) and Blömeke and Delaney (2012). Finally, the PTK framework explicitly refers to the technology-related instrumental genesis (Artigue, 2002). In other words, it acknowledges the process of using technological tools to achieve a set of goals, thus creating instruments in an ongoing process.

Nevertheless, we suggest two modifications to the PTK framework. First, instead of "technology instrumental genesis" we introduce the *double instrumental genesis* approach (Haspekian, 2011). In accordance with Rabardel's instrumental approach (2002), a user develops an *instrument* from an *artefact* used to accomplish a given task by elaborating usage schemes. This process is called *instrumental genesis*. The concept of *double instrumental genesis* acknowledges that teachers must develop two instruments from a given ICT tool (artefact): a mathematical instrument in a *personal instrumental genesis* (i.e., understanding how the tool transforms mathematics, being able to solve mathematical tasks with the tool, and so on) and a didactic instrument in a *professional instrumental genesis* (i.e., ability to use the tool to teach mathematics).

Second, we adapt Ball et al.'s (2008) categories of "mathematics knowledge for teaching" (MKT) to technology. Out of six knowledge areas in the MKT model, we adapt the following four to technology:

- specialized content knowledge that, in a technological environment, presents specificities related to the mathematics embedded in technology and thus needs to be redefined as *specialized digital content knowledge* (SDCK);
- knowledge of content and students, which in a technological environment includes additional aspects that may be formulated as *knowledge of digital content and students* (KDCS);
- knowledge of content and teaching that in a technological environment may be referred to as *knowledge of digital content and teaching* (KDCT);
- knowledge of content and curriculum in a digital environment, e.g., knowledge of prescribed use of ICT that should be redefined as *knowledge of digital content and curriculum* (KDCC).

We refer to the resulting model as "mathematical digital knowledge for teaching" (MDKT – Fig. 2).

These two modifications of the PTK lead to what we refer to as the "mathematical knowledge for teaching with technology" (MKTT) framework (Fig. 3).

To summarize, our proposed theoretical model of mathematical knowledge for teaching with technology (Fig. 3) comprises three domains: teachers' orientations (affective domain), teachers' knowledge (cognitive domain) and teachers' double instrumental genesis related to technology.

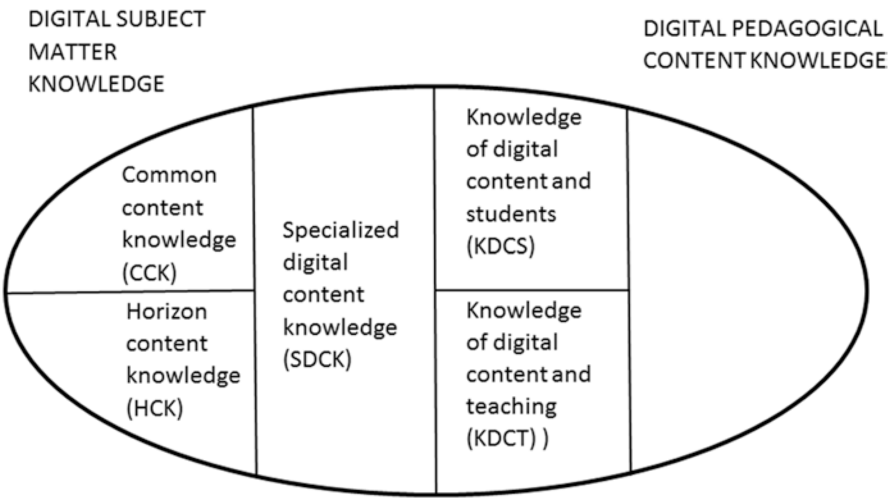

Fig. 2 Mathematical digital knowledge for teaching. (Adapted from Ball et al.'s mathematical knowledge for teaching to technological environment)

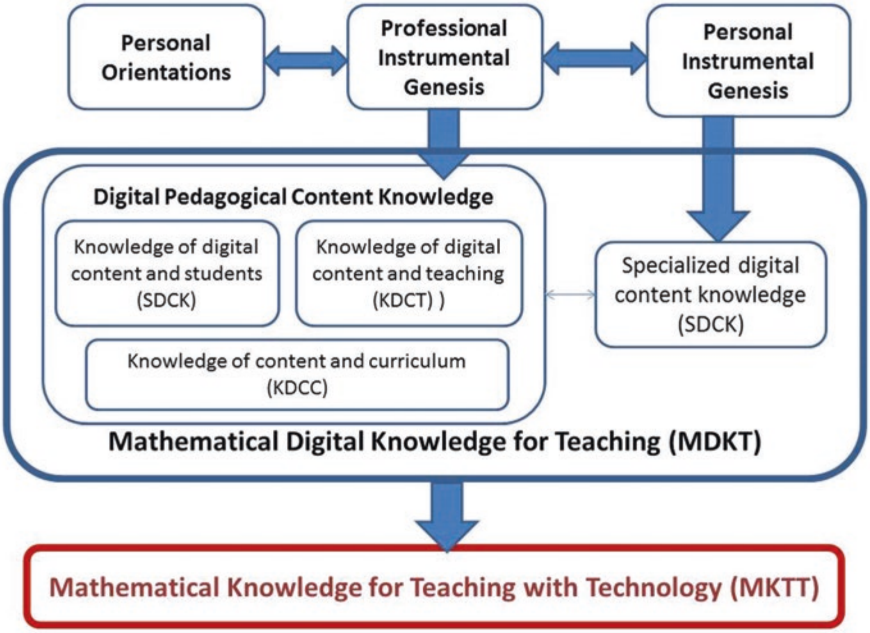

Fig. 3 Mathematical knowledge for teaching with technology framework

3 Methods

In this section, we describe the methodology we used to analyse several national and international policies. The analysis aims at highlighting specific ICT-related components made explicit in these policies that fall under the three main aspects outlined in our theoretical model (Fig. 3): cognitive domain, affective domain and double instrumental genesis perspective.

3.1 Data Sources

In pursuing our research aim, we looked for documentation written in English about standards or frameworks describing teachers' ICT-related knowledge, competencies and/or skills. We used the terms "knowledge," "skills," and "competencies," as these are the terms used by policymakers in official documents in various countries. We searched for current institutional documents, that is, documents at the national or international level that explicitly focus on teachers and teaching in digital environments. For example, in the US national documentation we found the *Standards for Preparing Teachers of Mathematics* (Association of Mathematics Teacher Educators, 2017), a comprehensive document aimed specifically at teachers specializing in mathematics and organized by grade levels and stages in the teachers' careers. However, the ICT component in this document was minor so we did not include it in our data set.

In the following sub-sections, we analyse two documents that describe frameworks for teaching with technology at the continental level: Europe (*European Framework for Digital Competence of Educators*) and Australia (*Australia National Framework for Professional Standards for Teaching*). We also analyse two national policies from two countries with a strong focus on ICT in education: Ireland (*Ireland Digital Strategy for Schools – Enhancing Teaching, Learning and Assessment*) and Norway (*Professional Digital Competence Framework for Teachers in Norway*).

3.2 Data Analysis

Although each document describes a web of connections between various elements, for the purpose of our analysis we attempted to separate these connections as follows. While reading the documents, we attempted to connect the statements made in each document to the components of our theoretical framework. Some statements referred to the *knowledge* base needed by teachers. This knowledge base might refer to mathematical knowledge, pedagogical knowledge or one of the six knowledge areas in the MDKT. Other statements focused on teachers' values, emotions or attitudes relevant to ICT integration. We grouped these under *personal orientations*. A third category in the documents referred to teachers' competencies—usually described as an ongoing process that refers to what teachers can do with technology for their own needs. We refer to this category as *personal instrumental genesis*. On the other hand, this category also noted that for their students to benefit from ICT as an integral part of their learning, teachers must search for digital resources, select the appropriate resources based on pedagogical and mathematical considerations, and create documents to be used by the students in class. We see this category as teachers' *professional instrumental genesis*.

4 Findings

4.1 European Framework for Digital Competence of Educators (DigCompEdu)

In 2017, the European Joint Research Centre[1] released a document introducing a framework for the digital competence of educators (Redecker, 2017). The framework was developed in response to

[1] The Joint Research Centre (JRC) is the European Commission's science and knowledge service. It aims to provide evidence-based scientific support to the European policymaking process.

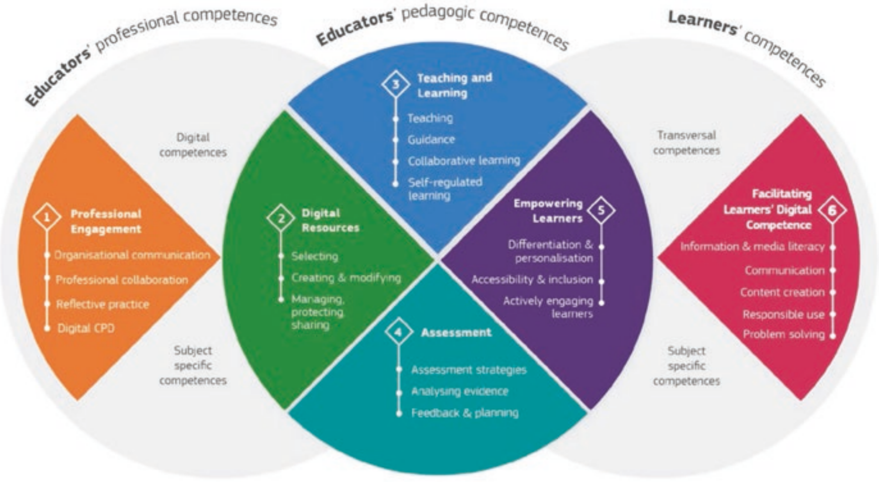

Fig. 4 DigCompEdu framework (Redecker, 2017, p. 19)

> the growing awareness among many European Member States that educators need a set of digital competences specific to their profession in order to be able to seize the potential of digital technologies for enhancing and innovating education (Redecker, 2017, p. 6).

The framework builds on analysis of existing national and international frameworks and self-assessment tools to obtain "educator-specific digital competences" (p. 9).

In this document, digital competence is defined as

> the confident, critical and creative use of ICT to achieve goals related to work, employability, learning, leisure, inclusion and/or participation in society (p. 90).

The framework identifies 22 elementary competencies organized into six areas that cover all facets of the education profession (Fig. 4). Area 1 concerns educators' use of digital technology in professional communication and collaboration with colleagues, learners, parents and other actors in education, as well as for their own individual development. Area 2 covers competencies needed for effective and responsible use, creation and sharing of digital resources. Area 3 focuses on the management and orchestration of digital technology in teaching and learning. Area 4 depicts competencies needed for using technology to enhance assessment. Area 5 details competencies needed for using learner-centred strategies with technologies. Finally, Area 6 focuses on the specific pedagogic competencies required to facilitate students' digital competence. According to the authors, Areas 2–5 constitute the core of the framework, which details

> educators' digital pedagogic competence, i.e. the digital competences educators need to foster efficient, inclusive and innovative teaching and learning strategies (p. 16).

We therefore focus on these areas, which we analyse through the lens of our theoretical framework.

Personal orientation. The DigCompEdu framework seems to overlook the need to foster the personal orientation dimension. In particular, it takes for granted that educators, even those who are newcomers and whose proficiency level[2] with technology is lowest, are aware of technology's potential for enhancing teaching and learning. Although confidence with technology use constitutes the very essence of the definition of digital competence, the framework does not consider this an issue and assumes that development of professional digital competence will result in increased confidence:

> The proficiency statements are designed to celebrate achievements and to encourage educators to develop their competences, by indicating small steps that will eventually, step by step, increase their confidence and competence (p. 28).

Personal instrumental genesis seems to be considered only in Area 1 in relation to technologies for communication and collaboration. Pedagogical digital competencies in Areas 2–5 appear to build on the educators' "existing digital competence" that they will apply "in the pedagogical realm" (p. 30).

The competencies described in Areas 3 and 4 are related to *professional instrumental genesis.* More specifically, competence 3.1, which is titled "teaching" and is deemed to be fundamental to the entire framework, can be considered to cover professional instrumental genesis of technology for teaching and learning. It covers competencies to be used in the following activities:

> To plan for and implement digital devices and resources in the teaching process, so as to enhance the effectiveness of teaching interventions. To appropriately manage and orchestrate digital teaching strategies. To experiment with and develop new formats and pedagogical methods for instruction (p. 21).

Similarly, competence 4.1 titled "assessment strategies" can be considered a professional genesis of technology for assessment purposes. It covers competencies to be mobilized in the following activities:

> To use digital technologies for formative and summative assessment. To enhance the diversity and suitability of assessment formats and approaches (p. 21).

(Mathematics) digital knowledge for teaching. Although the framework is not subject-matter-specific, Areas 3 and 4 refer to the content to be taught. Competencies that imply knowledge of content and teaching with technology (KDCT) can be seen in the following activities:

> To structure the lesson so that different (teacher-led and learner-led) digital activities jointly re-inforce the learning objective. To set up learning sessions, activities and interactions in a digital environment. To structure and manage content, collaboration and interaction in a digital environment. To consider how educator-led digital interventions—whether face-to-face or in a digital environment—can best support the learning objective. To reflect on the effectiveness and appropriateness of the digital pedagogical strategies chosen and flexibly adjust methods and strategies. (3.1 – Teaching, p. 52)

[2] The DigCompEdu framework considers six stages of digital competence development: newcomer (A1), explorer (A2), integrator (B1), expert (B2), leader (C1) and pioneer (C2). For more information, see Redecker (2017, pp. 28–29).

Competencies implying knowledge of digital content and students (KDCS) manifest themselves in the activities as follows:

> To set up learning activities in digital environments, having foreseen learners' needs for guidance and catering for them (3.2 – Guidance, p. 54),

or

> To assist learners in identifying areas for improvement and jointly develop learning plans to address these areas (4.3 – Feedback and planning, p. 66).

Finally, competencies implying pedagogical knowledge and technology are addressed in Area 3: "to use classroom technologies to support instruction, e.g. electronic whiteboards, mobile devices"; "to experiment with and develop new formats and pedagogical methods for instruction (e.g. flipped classroom)" (3.1 – Teaching, p. 52); "to experiment with and develop new forms and formats for offering guidance and support, using digital technologies" (3.2 – Guidance, p. 54).

4.2 *Australia National Framework for Professional Standards for Teaching*

In 2003, the Ministerial Council on Education, Employment, Training and Youth Affairs in Australia published a document defining the *National Framework for Professional Standards for Teaching* (MCEETYA, 2003). The aim of the framework was to provide

> an architecture within which generic, specialist and subject-area specific professional standards can be developed at National, and State and Territory levels (p. 2).

The framework seeks to define

> what constitutes quality teaching and facilitates the articulation of the knowledge, understandings, skills and values for effective teaching through development of standards at the local level (p. 5).

The framework is based on four professional elements—Professional Knowledge; Professional Practice; Professional Values; and Professional Relationships—and comprises four career dimensions: Graduation; Competence; Accomplishment; and Leadership, as illustrated in Fig. 5.

This framework, however, does not refer to digital context. Five years later, a *Joint Ministerial Statement on Information and Communications Technologies in Australian Education and Training: 2008–2011* was published. This short document[3] states that

> Australia will have technology enriched learning environments that enable students to achieve high quality learning outcomes and productively contribute to our society and economy.

[3] See https://files.eric.ed.gov/fulltext/ED534395.pdf

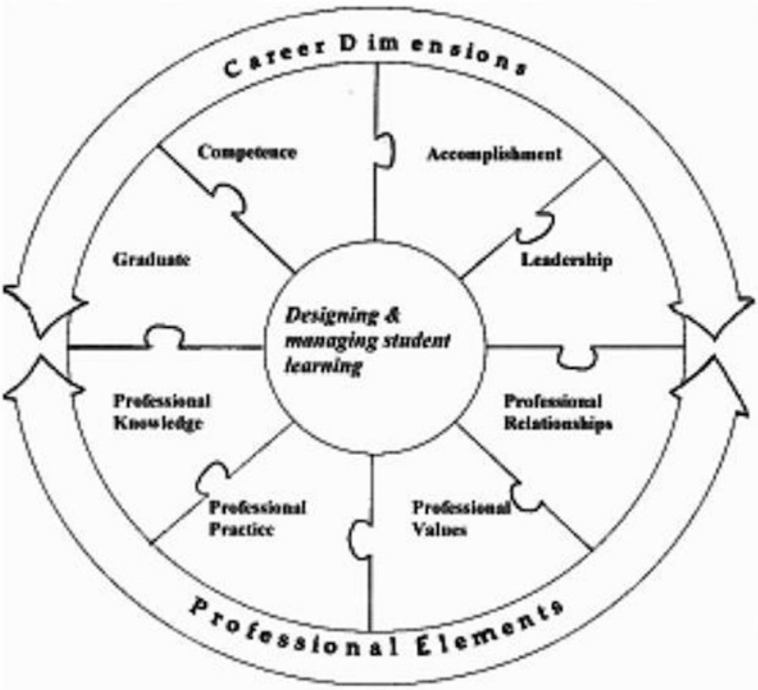

Fig. 5 Australia national framework for professional standards for teaching

This document also led to new teaching considerations. Education Services Australia (ESA) published *Pedagogies and Digital Content* (Baker, 2009), a document that includes a historical overview of Australian efforts to define criteria for teachers at the national level and at the local level of countries in which such development took place. This comprehensive overview is guided by six research questions, one of which is of interest for our study:

> What skills and capabilities will teachers need in order to access and use repositories of suitable, exciting, culturally appropriate, discoverable and affordable digital content? (p. 1).

The Australian strategic plan adopted the UNESCO ICT Competency Framework (2011), which it saw as a good basis upon which to build a more specific plan and as a basis for assessing progress. The strategic plan also used the UNESCO ICT Competency Framework as a text suggesting the next steps to be taken. The authors of the Australian strategic plan claim that the UNESCO's ICT Competency Standards for Teachers

> define the ICT-related skills required of teachers in primary and secondary schools. They take as their starting point the assumption that new technologies require new teacher roles, pedagogies, and approaches to teacher training (p. 26).

This interpretation has implications regarding the impact of ICT use on

> the skills required of teachers in managing classrooms in which such ICT-related interaction and collaboration are used for teaching and learning (p. 26).

Although the terms "skills" and "competencies" are central, they are not defined in the document.

The *National Framework for Professional Standards for Teaching* (MCEETYA, 2003) was developed to provide a

> basis for agreement on and consistency around what constitutes quality teaching and facilitates the articulation of the knowledge, understandings, skills and values for effective teaching through development of standards at the local level (p. 24).

We now analyse the document via our *Mathematical Knowledge for Teaching with Technology Framework* (Fig. 3).

Personal orientation. The term "values" appears in the declared aim. While this notion is not further developed, we believe it resonates with the teachers' personal orientation towards technology integration.

We could not find any specific reference to *personal instrumental genesis* or to *professional instrumental genesis* in the document.

In terms of *subject-specific knowledge*, we did not find any explicit reference to specialized digital content knowledge (SDCK), yet we did find references to the other three types of knowledge.

Knowledge of content and students (KDCS). The plan states that teachers must be familiar with theories about students' learning that take into account the use of ICT. Such familiarity will allow teachers to develop a detailed understanding of how young people learn, and in particular provide them with a clear understanding of the role of the teacher in leading this endeavour.

Knowledge of content and teaching (KDCT) and *knowledge of content and curriculum* (KDCC). The document relates to both of these knowledge areas, as it sees teachers as the designers of learning activities: "using a range of techniques, tools, practices and resources" (p. 25). To do so, teachers must be familiar with the curriculum in digital environments and at the same time must understand content and teaching in these environments. As noted by Baker (2009), however, the document does not anticipate that

> teachers might need to encourage cross-disciplinary thinking, social interaction and the use of digital media, or be able to provide students with state-of-the-art tools in technology-rich learning environments (p. 25).

4.3 Ireland Digital Strategy for Schools: Enhancing Teaching, Learning and Assessment

In 2015, the Department of Education and Skills in Ireland published a comprehensive strategic plan for ICT integration in K-12: *Digital Strategy for Schools: 2015–2020. Enhancing Teaching, Learning and Assessment* (Strategy Development Group, 2015). The plan includes four themes: Teaching, Learning and Assessment Using ICT; Teacher Professional Learning; Leadership, Research and Policy; and ICT Infrastructure. In other words, this strategic plan views teachers as central actors in the successful implementation of ICT in schools. The second theme is the focus of our interest in this chapter.

In line with Butler, Leahy, Shiel, and Cosgrove (2013), the strategic plan makes the following basic assumption:

> The concept of teaching and learning through the use of ICT is highly complex. The introduction of ICT into a learning environment does not in and of itself bring about change in pedagogical practice (p. 19).

Hence, a major effort is needed to provide teachers with various PD opportunities for professional development.

The Ireland Digital Strategic Plan adopted the UNESCO ICT Competency Framework (2011). On the one hand, this framework provides teachers, school principals and PD providers with a landscape for examining and evaluating the current situation and outlines future steps in various directions. On the other hand, the framework must be adapted to the Irish context.

The strategic plan uses the terms "practice," "knowledge and skills" and "confidence" with respect to the need to support teachers' PD. Yet only knowledge is made explicit to some extent. The meaning of teacher "practice," "skills" and "confidence" is taken for granted and no definitions are provided for these terms. With respect to knowledge, the plan refers to the TPACK framework (Mishra & Koehler, 2006) as a means of approaching the knowledge teachers need. This is a rather naïve view of this framework. As noted by Voogt, Fisser, Pareja Roblin, Tondeur, and van Braak (2012), this framework has three different interpretations: T(PCK) as extended PCK; TPCK as a unique and distinct body of knowledge; and TP(A)CK as the interplay between three domains of knowledge and their intersections. From the way the strategic plan describes TPACK, it seems the authors implicitly adopt the last view, acknowledging that besides

> the three types of knowledge required by a teacher for effective pedagogical practice in a technology enhanced learning environment namely, technological knowledge, knowledge of curriculum content and pedagogical knowledge, [e]qually important to the model are the interactions between these bodies of knowledge. When teachers effectively integrate these areas of knowledge, they can embed ICT effectively into their practice (p. 29).

We now analyse the second theme in the strategic plan—teachers' professional learning—via our *Mathematical Knowledge for Teaching with Technology Framework* (Fig. 3).

Personal orientation. The Ireland Digital Strategic Plan acknowledges that teachers need to develop "confidence to embed ICT more into their practice" (p. 31). Moreover, the plan states that supporting and building this confidence should be an ongoing activity in PD "throughout a teacher's career" (p.32).

Personal instrumental genesis. Some reference to this issue is apparent in the following quote: "ALL teachers should have the requisite knowledge and skills to integrate ICT effectively into their practice" (p. 30). While this does not state directly that this knowledge refers to teachers' ability to use ICT for their own needs, it is the closest thing to such a statement.

Professional instrumental genesis. The strategic plan acknowledges the fact that technology itself has developed and changed at a rapid pace. Hence, learning how to integrate ICT to promote students' learning as a single event is not sufficient. Rather, learning must continue throughout a teacher's professional lifetime. Moreover, the plan encourages PD developers to provide "teacher professional learning in a range of formats," emphasizing that "teacher professional learning needs to be rooted in classroom practice" (p. 31).

Pedagogical knowledge is referred to in this statement about ICT implementation:

> all forms of teacher professional learning should highlight a range of pedagogical practices that support the active use of ICT by learners in a range of settings (p. 32).

In contrast, no references are made to mathematical knowledge or any other subject-matter knowledge. The strategic plan is general and addresses teachers of all school subjects. Nevertheless, the document does hint at the need to develop subject-matter knowledge by recommending the use of PD in "subject-department approaches" (p. 31). The strategic plan also lacks references to MKT or any other subject-specific knowledge: specialized digital content knowledge, knowledge of content and students, knowledge of content and teaching, and knowledge of content and curriculum.

Note that this strategic plan is aimed at the 5 years period from 2015 to 2020. The plan lacks any specificity in terms of grades or levels, yet it does define the following indicators for its success (Fig. 6).

4.4 Professional Digital Competence Framework for Teachers in Norway

The Norwegian Centre for ICT in Education was established in 2010 as an agency under the direct authority of the Norwegian Ministry of Education and Research. Its mission is "to help ensure that ICT is used to improve the quality of education,

GOAL: TEACHER PROFESSIONAL LEARNING – INDICATORS OF SUCCESS

- **Use of ICT for teaching, learning and assessment is embedded at each stage of the continuum of teacher education i.e. Initial Teacher Education, Induction and Continuous Professional Learning.**
- **Department and Teaching Council policies on teacher education recognise the role and potential of ICT to enhance teaching and learning.**
- **Department-funded support services and related bodies have embedded the use of ICT in CPD design, development and delivery.**
- **Guidance and examples of good practice on the effective, critical and ethical use of ICT for teaching, learning and assessment are provided to and shared by teachers.**

Fig. 6 Indicators for success (e-Digital Strategy Development Group, 2015, p. 30) learning outcomes and learning strategies for young children, pupils and students." In 2017, the Centre released a document outlining a framework for digital competence among teachers (Kelentrić, Helland, & Arstorp, 2017). According to the authors, the framework builds on competence areas of the teaching profession "viewed from a digital perspective" (p. 3). The framework comprises seven competence areas that contain descriptions of knowledge, skills and competence. We begin by examining the meaning the framework assigns to the terms "skills" and "competence" (note that the term "knowledge" is not defined in the document).

The glossary annexed to the document includes the following definition of competence:

> **Competence** means acquiring and using knowledge and skills to master challenges and solve tasks in familiar and unfamiliar contexts and situations. Competence entails understanding, and the capacity for reflection and critical thinking (p. 11).

Digital competence is further defined as

> the confident, critical, and creative use of ICT to achieve goals related to work, employability, learning, leisure, inclusion and/or participation in society. Digital competence is a transversal key competence which enables the acquisition of other key competencies. It is related to many of the so-called '21st Century skills', which should be acquired by all citizens, to ensure their active participation in society and the economy (p. 11).

This definition clearly draws on the definition provided by the EU in its DigComp framework. Finally, **digital skills**

> involve being able to use digital tools, media and resources efficiently and responsibly, to solve practical tasks, find and process information, design digital products and communicate content. Digital skills also include developing digital judgement by acquiring knowledge and good strategies for the use of the Internet (p. 14).

The Norwegian framework for the digital competence of teachers (Fig. 7) is organized into seven competence areas covering the various facets of the teaching profession. The authors indeed claim that:

> All of the areas of competence are equally important, but it is the sum of the competence areas that makes up a professional, digitally competent teacher (p. 14).

In the following sections we analyse the Norwegian framework through the lens of our theoretical framework.

Personal orientation. This dimension of teachers' competence is missing from the framework. The assumption may be that a positive attitude toward the use of technology in education appears to be self-evident and is not an issue.

Personal instrumental genesis. The framework acknowledges the need for teachers to develop their own digital skills, which they are expected to do during their primary and secondary education. Indeed, entering students in teacher education programs are expected to have already developed basic digital skills

> so they can search for and process information, produce and communicate online, as well as exercise digital judgement (p. 1).

Professional instrumental genesis is emphasized in the framework as a lifelong process that begins during the teachers' initial teacher education:

Fig. 7 Professional digital competence framework for teachers (Kelentrić et al., 2017, p. 3)

> In order to be capable of developing pupils' basic skills and specialised knowledge, teachers must develop their own professional digital competence during their initial teacher education, and later, through continuing professional education and development, during their teaching career (p. 1).

(Mathematics) digital knowledge for teaching (MDKT). Although the framework is not subject-matter specific, two competence areas explicitly mention the content to be taught: (a) subjects and basic skills and (b) pedagogy and subject didactics.

The subjects and basic skills area states that:

> A professional, digitally competent teacher understands how digital developments are changing and expanding the content of subjects. The teacher understands how the integration of digital resources into learning processes can help to achieve competence aims in a subject, and to address the five basic skills. […] At the same time, the teacher needs to understand what pupils' digital skills entail, and how they can be fostered in the subjects (p. 4).

Understanding "how digital developments are changing and expanding the content of subjects" requires specialized digital content knowledge (SDCK), while understanding "how the integration of digital resources into learning processes can help to achieve competence aims in a subject" implies knowledge of content and teaching with technology (KDCT). Moreover, understanding "what pupils' digital skills entail and how they can be fostered in the subjects" requires knowledge of digital content and students (KDCS).

According to the pedagogy and subject didactics area:

> A professional, digitally competent teacher possesses pedagogical knowledge, as well as knowledge of subject didactics relevant to the practice of their profession in a digital environment. Based on this, the teacher integrates digital resources into their planning, organisation, implementation and evaluation of the teaching in order to foster pupils' learning and development (p. 7).

This area thus addresses pedagogical as well as content knowledge in relation to digital technology (MDKT).

5 Discussion

We begin this section by discussing the meanings of the central terms—e.g., knowledge, skills, and competency—we found in each document. We follow this by suggesting refinements in our theoretical framework (Fig. 3) based on new findings reported in the previous section. Finally, we discuss further directions for study.

5.1 Central Terms

One notable observation stemming from our analysis is related to the inconsistencies in vocabulary across the four documents. Table 1 outlines the terms mentioned in the documents of the various institutions and clearly shows that each document uses its own terms. We were surprised to find that in some documents these terms are not defined at all (denoted by shaded cells in Table 1). In particular, the definition of "knowledge" in the Ireland document is indirect—the authors refer to the TPACK as a framework to define it. The term "competence" is also problematic. For example, the Norway framework is organized around seven competence areas, yet each of these competence areas is decomposed into three components: knowledge, skills and competence. This illogical and confusing definition of the term "competence" makes it difficult to grasp the meaning assigned to it.

The UNESCO framework defines competence as "the skills, knowledge and understanding needed to do something successfully" (UNESCO, 2011, p. 92). This same concept was also defined by the EU DigComp framework. Moreover, the Norwegian framework's definition draws on that of the EU in the following way:

> Competence means acquiring and using knowledge and skills to master challenges and solve tasks in familiar and unfamiliar contexts and situations. Competence entails understanding, and the capacity for reflection and critical thinking.

This definition broadens the one given by UNESCO.

Only the Norwegian framework defines digital skills as involving

Table 1 Various terms used in the documents to designate what teachers need in order to integrate ICT

	Australia	Ireland	Norway	EU	UNESCO
Knowledge	+	+	+		
Understanding	+				
Skills	+	+	+		
Competence/ competency			+	+	+
Activity as an expression of a competence				+	
Capabilities	+				
Practice		+			
Confidence		+			
Values	+				

+ Means that the term appears in the document; shaded cell means that the term is not defined in the document; empty cell means that the term does not appear in the document

> being able to use digital tools, media and resources efficiently and responsibly, to solve practical tasks, find and process information, design digital products and communicate content. Digital skills also include developing digital judgement by acquiring knowledge and good strategies for the use of the Internet (p. 14).

We found a comparison between competency and skills in an OECD document:

> A competency is more than just knowledge or skills. It involves the ability to meet complex demands, by drawing on and mobilising psychosocial resources (including skills and attitudes) in a particular context. For example, the ability to communicate effectively is a competence that may draw on an individual's knowledge of language, practical IT skills and attitudes towards those with whom he or she is communicating (OECD, 2003, p. 4).

It seems that both skills and competency are related to taking action—in our case actions taken by the teacher in a technological environment. Yet skills seem to be "less" than competency. Knowledge is a basis for both skills and competency, but the connections between these concepts are not specified. Clearer definitions of the basic terms in each frame are necessary.

5.2 Refining Our Framework

In our model of teachers' professional knowledge and skills for teaching mathematics with technology, cognitive and affective domains as well as personal and professional instrumental genesis emerge as relevant in terms of capturing what teachers must develop to be able to use technology efficiently.

Nevertheless, we found that two of the documents we analysed—EU (Redecker, 2017) and Australia (Baker, 2009)—placed major emphasis on teachers' ability to search in digital repositories for suitable resources, to select those resources best suited to their students' needs, to create new digital resources themselves or with their team members, to share their resources with their peers, as well as to evaluate the resources' efficiency and appropriateness with respect to the learning objective. This is a rather new aspect not put forward in documents we studied previously. In the documentational approach to didactics (Gueudet & Trouche, 2009; Trouche, Gueudet, & Pepin, 2018) that draws on the instrumental approach, this facet of teachers' work with (digital) resources is called *documentation work*. We believe that this important part of teachers' professional work is captured by our framework in the professional instrumental genesis. Moreover, it is linked to teachers' knowledge both of content and of students, which in a technological environment includes additional aspects that may be formulated as knowledge of digital content and students (KDCS) and knowledge of content and curriculum in a digital environment (KDCC). With respect to our framework, we thus deem documentation work as a way to operationalize what we refer to as professional instrumental genesis.

In our framework, we contend that personal orientation impacts the way a teacher will use (or not use) digital technology. We assume that a positive opinion of the potential of technology for teaching and learning is a prerequisite (but of course not sufficient) for successful technology integration. The EU DigCompEdu framework seems to suggest that the development of digital competence will increase educators' confidence in using technology in teaching and learning. Moreover, the Ireland document considers confidence as something that needs to be nurtured throughout a teacher's professional lifetime.

People who enter the teaching profession today are considered to be digital natives (Prensky, 2001). Hence one might assume that those in this population have a positive attitude toward technology and a high level of personal digital mastery, so that personal orientation and personal instrumental genesis are self-evident. This seems to be the case in the Norwegian framework that expects student teachers to have acquired basic digital skills during their primary and secondary education. Yet from our experience with teacher educators, we know that personal instrumental genesis of mathematics-specific technology is far from self-evident among most young teachers. Consequently, in our view, it is an important component of teachers' ICT competence.

The findings reported in Sect. 4 have led us to refine our MKTT framework. We contend that the cognitive and affective dimensions along with the double instrumental genesis are important components of competent technology use for mathematics teachers. Moreover, consistent with the authors of the OECD (2003) document, we acknowledge that competence is more than merely knowledge and skills, as competence draws on knowledge, skills and attitudes. Therefore, although here we have chosen to refrain from entering the debate about the definition of competence, we believe that our model depicts what we propose calling *digital competencies for teaching mathematics with technology* (Fig. 8).

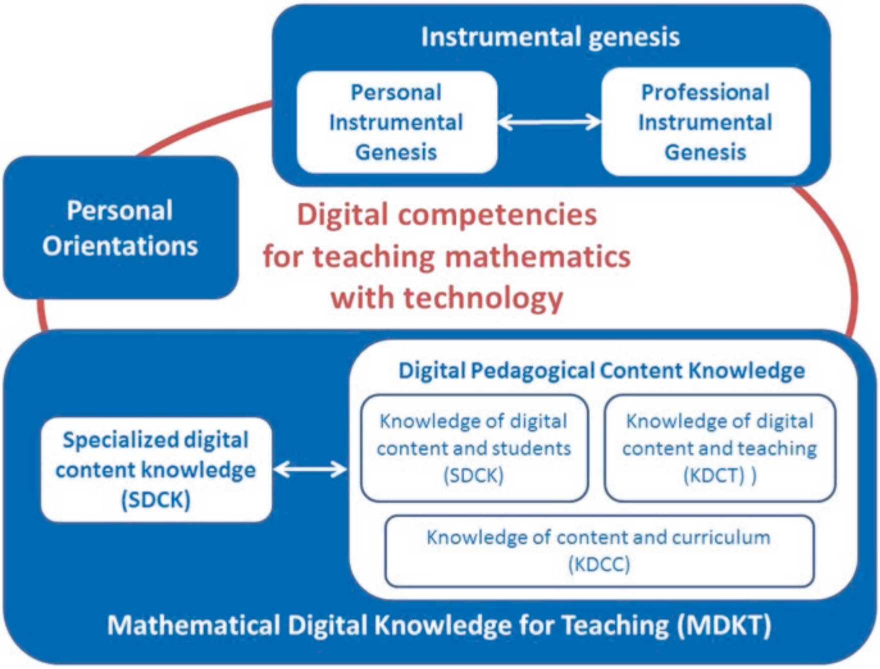

Fig. 8 Digital competencies for teaching mathematics with technology

5.3 Further Directions for Study

Our previous research has led us to appeal to the mathematics education community to advocate the issue of elaborating ICT standards for mathematics teachers:

> We call on the mathematics education research community to consider elaborating sets of standards for teaching with ICT for different age groups and school subjects so as to allow for the promotion of the professional level of instrumental genesis (Trgalová & Tabach, 2018, p. 351).

Our analysis of the documents in this chapter indicates that these issues are still under-developed. Standards for teacher competencies that are both subject-specific and age-specific are still missing. The P21[4] took a first step in this direction in that, together with the NCTM and MAA, it developed examples of student competencies to be achieved by the end of the 4th, 8th and 12th grades in mathematics. This approach needs to be expanded to define the work required of teachers to achieve these competencies. We believe that leaders of mathematics education in every country should undertake similar steps.

[4] P21 Partnership for 21st Century Learning, www.P21.org

Our proposed digital competencies for teaching mathematics with technology framework (Fig. 8) was developed and used in a dialectical process of implementation and refinement. Until now, however, the framework has only been implemented to study policy documents. As a next step in our research endeavour, we seek to implement the framework on data stemming from empirical studies involving the work of mathematics teachers both before and after teaching an ICT-based mathematics lesson in order to determine whether and to what extent the framework is suitable for evaluating teachers' ICT competencies.

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