Geomechanical Influences of Interface Dilatancy

A. P. S. Selvadurai(✉)

Department of Civil Engineering and Applied Mechanics, McGill University, Montreal, QC H3A 0C3, Canada
patrick.selvadurai@mcgill.ca

Abstract. This paper examines the influence of dilatant processes that can occur at a discontinuity of finite dimensions located at a compressed elastic geological interface, due to relative shear movement in the plane of the discontinuity. The dilatant phenomena will result in displacements in a direction normal to the shear movement and these displacements will be influenced by the elasticity of the geological materials and the compression at the interface. The dilatant displacements will also create a zone where there is loss of contact at the unilaterally constrained interface. The resulting problem is examined by appeal to results of the mathematical theory of elasticity. The influence of dilatant processes on the development of shear at the elastically constrained interface is examined by considering the procedures proposed by D.W. Taylor to analyze dilatant phenomena. The mathematical developments illustrate the combined influence of elastic constraints and interface compression on the amplification of the shear stress generated at the finite region. In the absence of dilatancy, the developments reduce to the classical result involving only Coulomb friction.

1 Introduction

The mechanics of geologic interfaces, particularly faults and fractures, is important to the field of engineering geosciences dealing with stability of geologic strata, the development of tectonic motion, movements of pre-existing fractures and the interaction of constructed underground facilities. In the study of earthquake generating mechanisms during movement at a transform fault (Fig. 1), the limiting stresses and the movements necessary to rupture a locked-in region are important input parameters for developing earthquake models.

The literature dealing with contact mechanics is extensive (covering nearly six thousand references dealing with diverse areas of geomaterial interfaces, geologic fault zones, tribology, wear, biomechanics, contact mechanics, etc.) and a complete review is beyond the scope of this article. Historical studies related to the mechanics of contact between surfaces can be found in the volumes by Bowden and Tabor [1] and Hisano [2]. Other developments that emphasize engineering applications, mathematical modelling, computational modelling and experimental aspects of contact mechanics are given in several texts and review articles on contact mechanics and these developments are documented in articles by Selvadurai and Boulon [3], Selvadurai [4], Selvadurai and Yu [5], Selvadurai and Atluri [6] and Selvadurai et al. [7].

T. Triantafyllidis (Ed.): *Recent Developments of Soil Mechanics and Geotechnics in Theory and Practice*, LNACM 91, pp. 44–53, 2020.
https://doi.org/10.1007/978-3-030-28516-6_3

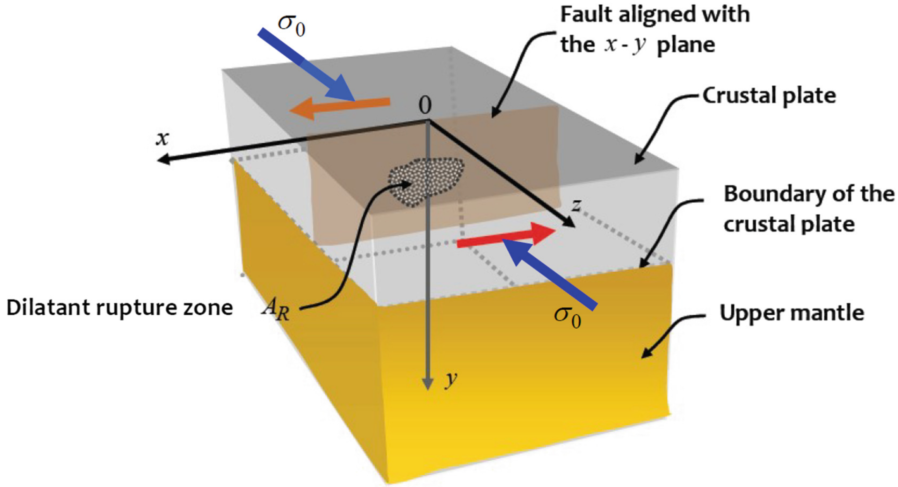

Fig. 1. Relative movement at a transform fault

In this article, we examine a rather idealized problem related to elastic isotropic geomaterial halfspace regions in *smooth contact*, pre-compressed by a normal stress σ_0 and containing a circular patch that possesses both *frictional* and *dilatant* mechanical properties. The Coulomb friction can be a result of the contact at the local scale of the idealized Euclidean surfaces and dilatant effects can occur due to surface irregularities. The dominant mechanism is asperity ride-up but the modelling can account for the deterioration of the dilatancy angle that can be identified with asperity breakage and damage, and indentation fracture (Selvadurai [8]). From a geo-environmental point of view, the dilatant movement at fault zones can lead to an increase in the aperture at contacts, which can enhance fluid flow through the fracture [5, 9–12]. The objective of this study is to develop a convenient analytical result that can be used to estimate the development of shear stresses at the *frictional-dilatant* circular patch during the relative shearing movement at the otherwise frictionless interface.

The problem posed here can be quite complicated if the influence of the frictional contact at the entire surfaces of the fracture and the dilatant effects of the circular patch are considered simultaneously. Frictional “asperity” patches surrounded by frictionless or stress free regions have been considered in the literature but these studies do not examine the possible development of dilatancy in the frictional patches. We have developed a theoretical approach for the study. However, it is unlikely that such a complete frictional interface contact problem will be amenable to analytical treatment, which is the basis for this study. The approach adopted here is to examine the mechanics of the pre-compressed *dilatant circular patch during shear*, while maintaining frictionless behaviour over the interface region exterior to the circular patch. The rationale for assuming frictionless behaviour exterior to the circular patch is to emphasize the constraint imposed by the dilatant patch and to evaluate the role that dilatancy at the patch has on limiting the shear capacity of the dilatant region. Observations of the property of dilatancy in geomaterials date back to the classical studies of Osborne Reynolds (1842–1912) that relate to volume expansion in the mass of a granular material during shear. The specific problem of interface dilatancy relates to the volume expansion that can take place at contacting surfaces due to the local geometric structure. The problem of interface dilatancy has several geomechanical applications, particularly those related to the load carrying capacity of embedded structural elements such as anchors and rock fractures. With interface dilatancy, the influences are restricted to the contact region and the adjacent domains can exhibit continuum properties consistent with the geologic material behaviour. The analysis of interface dilatancy lends itself to mathematical approaches that deal with the extended domain and the frictional-dilatancy response at the contacting regions.

We assume that the dilatant processes come into effect when the interface experiences a differential shear displacement in its plane. The dilatant movement is accommodated through the uniform displacement of the circular patch normal to its plane. The expansion can, of course, exhibit a variation over the circular region, but to preserve the simplicity of the model, we assume that the dilatant patch exerts a *uniform displacement* normal to its plane. This normal displacement can cause separation at the initially mated frictionless surfaces exterior to the circular patch, through its indentation into the deformable geologic media. The extent of separation will depend on (i) the elasticity characteristics of the geologic material, (ii) the magnitude of the dilatant displacement and (iii) the far-field normal stresses acting on the interacting surfaces. The paper presents an analysis of the elasticity problem associated with the dilatant patch and evaluates the normal stresses generated in the patch region during its dilatant expansion. This result is used in conjunction with a virtual work formulation to obtain a relationship for the shear stress at which the circular patch will experience failure or rupture.

2 Mathematical Modelling of the Contact Problem

We examine the problem of the smooth contact between two halfspace regions compressed by a normal stress σ_0 and containing a frictional-dilatant circular patch (Fig. 2). The dilatant behaviour at the circular patch can be caused by the relative shearing action between the halfspace regions and, in the process, the region can induce an indentation orthogonal to the shearing displacements.

We consider the elastostatic interaction between the frictionless interface containing the dilatant circular patch of radius a in a situation where the frictionless interface experiences a *total* relative movement $2\Delta u$ in its plane. (i) In general, the dilatancy-induced displacement normal to the circular patch can be variable within the dilatant region; for the purposes of developing a convenient analytical result, we assume that the induced normal displacement at the frictional interface has a constant value Δv imposed on both halfspace surfaces (Fig. 2). (ii) The dilatant behaviour of the circular patch will cause separation between the halfspace regions that are in smooth contact under the action of the compressive stress σ_0. (iii) The configuration of the boundary of the zone of separation can be elliptical in shape depending on the extent of shear. In this model, however, we assume that the boundary of separation can be approximated by a circular profile of radius b, which is an unknown (Fig. 2) and needs to be

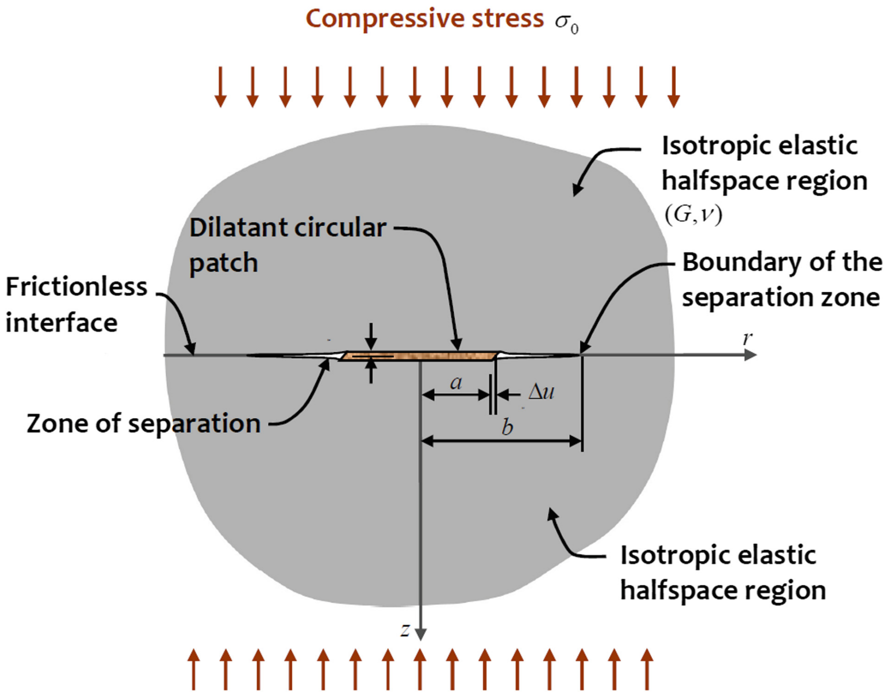

Fig. 2. Indentation of the pre-compressed halfspace regions by the dilatancy in the circular patch.

determined by solving a unilateral contact problem for the two halfspace regions [4]. To develop results for (a) the separation region during dilatancy-induced indentation at the circular region and (b) the force developed by the indentation displacement, we consider the following auxiliary problems:

(i) The internal pressurization of an annular crack of internal radius a and external radius b located in an elastic infinite space, by a pressure σ_0 and governed by the following three-part axisymmetric mixed boundary value problem referred to a halfspace region:

$$\begin{aligned} &\sigma_{rz}(r,0) = 0; \quad r \geq 0 \\ &u_z(r,0) = 0; \quad 0 \leq r \leq a \\ &\sigma_{zz}(r,0) = -\sigma_0; \quad a < r < b \\ &u_z(r,0) = 0; \quad b \leq r < \infty \end{aligned} \tag{1}$$

An approximate solution to this problem was developed by Selvadurai and Singh [13] and the important results relate to (a) the evaluation of the Mode I stress intensity factor at the outer boundary of the annular crack $(K_I^{\sigma_0})$, and (b) the resultant force developed in the ligament region $0 \leq r \leq a$.

(ii) The second auxiliary problem relates to the indentation of a penny-shaped crack of radius b by a smooth rigid circular indenter of radius a and thickness $2\Delta v$. This problem can also be posed as an axisymmetric three-part mixed boundary value problem referred to a halfspace region:

$$\begin{aligned} &\sigma_{rz}(r,0)=0; \quad r\geq 0 \\ &u_z(r,0)=\Delta v; \quad 0\leq r\leq a \\ &\sigma_{zz}(r,0)=0; \quad a<r<b \\ &u_z(r,0)=0; \quad b\leq r<\infty \end{aligned} \tag{2}$$

Approximate solutions to this mixed boundary value problem were developed by Selvadurai and Singh [14], and Selvadurai [15, 16]. Here also, the results of interest to the analysis of the dilatant patch problem are the Mode I stress intensity factor at the tip of the penny-shaped crack $(K_I^{\Delta v})$ and the force that is induced on the inclusion due to the indentation.

The location of the zone of separation due to the dilatancy-induced expansion at the unilaterally constrained interface can be obtained by the constraint of vanishing of the combined stress intensity factor obtained from the auxiliary problems described above. This constraint gives the result

$$\left(\frac{G\Delta v}{2\sigma_0 a(1-v)}\right)cF_{\Delta v}(c)-F_{\sigma_0}(c)=0 \tag{3}$$

where

$$F_{\Delta v}(c)=\left[\begin{array}{l}\frac{4c}{\pi}+\frac{16c^2}{\pi^3}+c^3\left(\frac{64}{\pi^5}+\frac{4}{3\pi}\right)\\ +\ c^4\left(\frac{80}{9\pi^3}+\frac{256}{\pi^7}\right)+c^5\left(\frac{448}{9\pi^5}+\frac{1024}{\pi^9}+\frac{4}{5\pi}\right)\end{array}\right] \tag{4}$$

$$F_{\sigma_0}(c)=\left[\begin{array}{l}1-\frac{4c}{\pi^2}-\frac{16c^2}{\pi^4}-c^3\left(\frac{1}{8}+\frac{64}{\pi^6}\right)\\ -\ c^4\left\{\frac{16}{3\pi^4}+\frac{4}{\pi^2}\left(\frac{1}{24}-\frac{8}{9\pi^2}+\frac{64}{\pi^6}+\frac{4}{9\pi^3}\right)\right\}\\ -\ c^5\left\{\frac{16}{\pi^4}\left(\frac{1}{24}-\frac{8}{9\pi^3}+\frac{64}{\pi^6}+\frac{8}{9\pi^2}\right)+\frac{256}{9\pi^6}-\frac{4}{15\pi^2}\right\}+\mathrm{O}(c^6)\end{array}\right] \tag{5}$$

and $c(=a/b)<1$. The lowest root of (3) gives the extent of the zone of separation. Omitting the details it can be shown that the force generated at the contact zone of the indenting region $0\leq r\leq a$, with a separation region $a\leq r\leq b$ and a re-established contact zone $b\leq r\leq\infty$, can be evaluated in the form

$$P_N=\sigma_0\pi a^2+\frac{4aG\Delta v}{(1-v)}P_N^{\Delta v}-\sigma_0\pi a^2P_N^{\sigma_0} \tag{6}$$

where

$$P_N^{\Delta v}=\left[\begin{array}{l}\left(1+\frac{4c}{\pi}\right)+\frac{16c^2}{\pi^4}+c^3\left(\frac{64}{\pi^6}+\frac{16}{9\pi^4}-\frac{8}{9\pi^2}\right)\\ +\ c^4\left(\frac{256}{\pi^8}+\frac{64}{9\pi^4}\right)+c^5\left(\frac{10240}{\pi^{10}}+\frac{9600}{225\pi^6}+\frac{92}{225\pi^2}\right)\end{array}\right] \tag{7}$$

$$P_N^{\sigma_0} = \begin{pmatrix} -\frac{8}{\pi^2 c} + \left\{1 - \frac{32}{\pi^4}\right\} + 8c\left\{\frac{1}{\pi^2} - \frac{48}{\pi^6}\right\} \\ -\frac{c^2}{9\pi^8}\left\{4608 + \pi^3(32 - 64\pi + 3\pi^3)\right\} \\ -\frac{4c^3}{45\pi^{10}}\left\{23040 + \pi^3\left(-320 + 480\pi + 15\pi^3 + 6\pi^5\right)\right\} \\ -\frac{c^4}{675\pi^{12}}\left\{5529600 + \pi^3\left(-76800 + \pi\begin{bmatrix} 192000 \\ +\pi^2\left(3600 + \pi\left\{-320 + 3168\pi + 45\pi^3\right\}\right)\end{bmatrix}\right)\right\} \end{pmatrix} \tag{8}$$

3 The Shear of the Dilatant Circular Patch

In general, at the dilatant zone the response will be elasto-plastic. In a strict sense, the analysis of the shear rupture problem during the generation of frictional-dilatant phenomena should be examined by appeal to a theory of plasticity applicable for elasto-plastic phenomena with specified failure criteria and non-associated flow rules (Davis and Selvadurai [17]). Here, we focus on the evaluation of the peak rupture stress that can be generated at the circular patch when dilatancy is present. To estimate the limiting response, we utilize the procedure presented by D.W. Taylor [18] for the analysis of dilatancy processes in granular materials. In essence, when examining failure processes associated with dilatancy effects, Taylor proposed a criterion that neglects the elastic energy storage processes at the direct contact zone (see also Christian and Baecher [19]). (To a certain extent, this argument is consistent with the limit analysis concepts proposed by Drucker and Prager [20]; see also Davis and Selvadurai [17] and Ichikawa and Selvadurai [21].) The basic hypothesis involves the relationship between (i) the *work done by the shearing forces and normal forces* and (ii) the *energy dissipated at the frictional-dilatant region*, expressed in terms of force resultants rather than exact distributions over the contact zone. The work component W consists of the work of the shear force (P_T) acting at the onset of rupture and the work of the normal force (P_N) induced by dilatancy on the upper and lower surface: i.e.

$$W = 2P_T(\Delta u) + 2P_N(-\Delta v) \tag{9}$$

The energy dissipated at the dilatant circular patch is given by

$$D = 2P_N\,(\Delta u)\tan\varphi \tag{10}$$

where φ is the contact friction angle and Δv is the dilatant displacement. We note that the work of forces and the dissipation on both faces of the circular patch have to be included in the formulation. In Taylor's hypothesis, the conventional association between Δv and Δu is through the linear relationship $\Delta v = \Delta u \tan\alpha_0$, where α_0 is the constant dilatancy angle. The work of Selvadurai et al. [7], extends Taylor's definition of dilatant displacements to include dilatancy effects of the form

$$\Delta v = a\left(\frac{\Delta u}{a}\right)^2 \tan\alpha \tag{11}$$

and with variation in the dilatancy angle α is described by

$$\tan\alpha = \exp\left(-\lambda\left|\frac{\Delta u}{a}\right|\right)\tan\alpha_0 \tag{12}$$

where λ is a non-dimensional parameter and the modulus sign accounts for the invariance of the dilatancy angle on the sense of the shear displacement. Selvadurai et al. [7] show that the representation (12) correlates well with experimental observations of the mechanics of dilatant geologic interfaces. Omitting details, it can be shown that the shear stress developed on the dilatant circular patch can be expressed in the form

$$\begin{aligned}\tau_D = \sigma_0&\left(1+\frac{4G}{(1-\nu)\sigma_0\pi}\left(\frac{\Delta u}{a}\right)^2\exp\left(-\lambda\left|\frac{\Delta u}{a}\right|\right)\tan\alpha_0 P_N^{\Delta v} - P_N^{\sigma_0}\right)\\ &\times\left(\tan\varphi + \frac{\Delta u}{a}\exp\left(-\lambda\left|\frac{\Delta u}{a}\right|\right)\tan\alpha_0\right)\end{aligned} \tag{13}$$

and the radius of the separation zone is now obtained from the smallest positive root of the characteristic equation

$$\left(\frac{G}{\sigma_0(1-\nu)}\right)\left(\frac{\Delta u}{a}\right)^2\exp\left(-\lambda\left|\frac{\Delta u}{a}\right|\right)\tan\alpha_0\, cF_{\Delta v}(c) - 2F_{\sigma_0}(c) = 0 \tag{14}$$

The result (13) for the shear stress generated at failure at the dilatant circular patch can be compared with the analogous result for the non-dilatant case, which corresponds to the interface with purely Coulomb friction. The shear stress amplification factor (*SSAF*) is given by

$$\begin{aligned}SSAF = \frac{\tau_D}{\tau_C} = &\left(1+\frac{4G}{(1-\nu)\sigma_0\pi}\left(\frac{\Delta u}{a}\right)^2\exp\left(-\lambda\left|\frac{\Delta u}{a}\right|\right)\tan\alpha_0 P_N^{\Delta v} - P_N^{\sigma_0}\right)\\ &\times\left(1+\frac{\Delta u}{a}\exp\left(-\lambda\left|\frac{\Delta u}{a}\right|\right)\frac{\tan\alpha_0}{\tan\varphi}\right)\end{aligned} \tag{15}$$

In the limiting case when $\alpha_0 \to 0$, the result (15) reduces to unity assuming that when $c \to 1$, $P_N^{\sigma_0} \to 0$. This assumption is invoked in view of the nature of the series approximations used in the solutions of the three-part mixed boundary value problems defined by (1) and (2). Figures 3 and 4 illustrate the typical variations in the shear stress amplification with (i) the shear displacement, (ii) the Coulomb friction angle, (iii) the dilatancy angle, (iv) the dilatancy degradation parameter and (v) the magnitude of the in situ compressive stress relative to the shear modulus. The basic approach can also be used to estimate the in-plane load carrying capacity of flat anchors [22] that are created by pressure grouting techniques (Fig. 5).

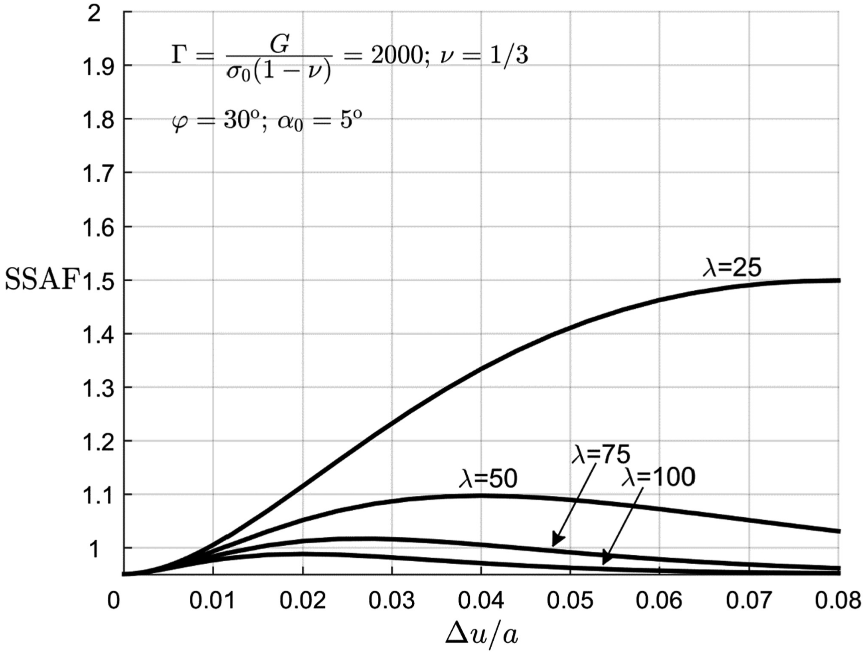

Fig. 3. The variation in the shear stress amplification factor with the relative shear movement

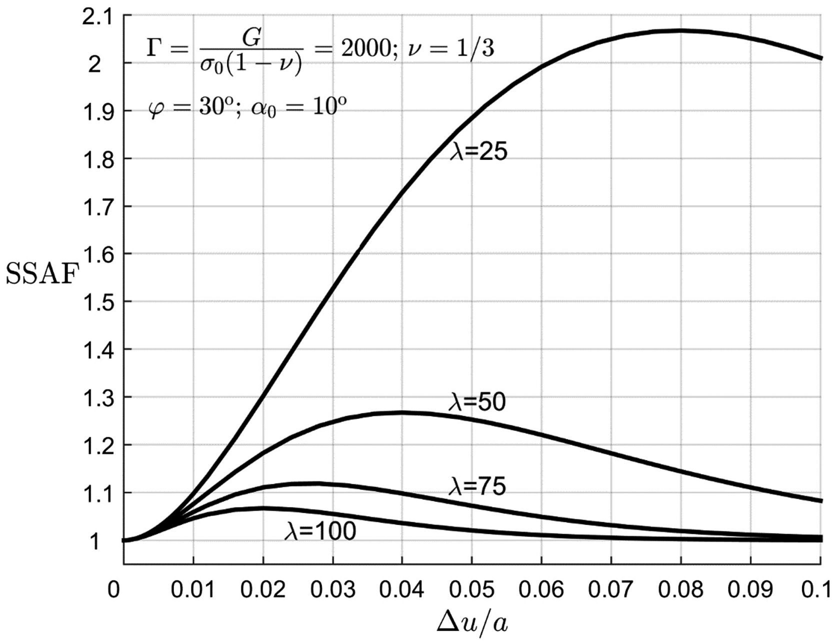

Fig. 4. The variation in the shear stress amplification factor with the relative shear movement

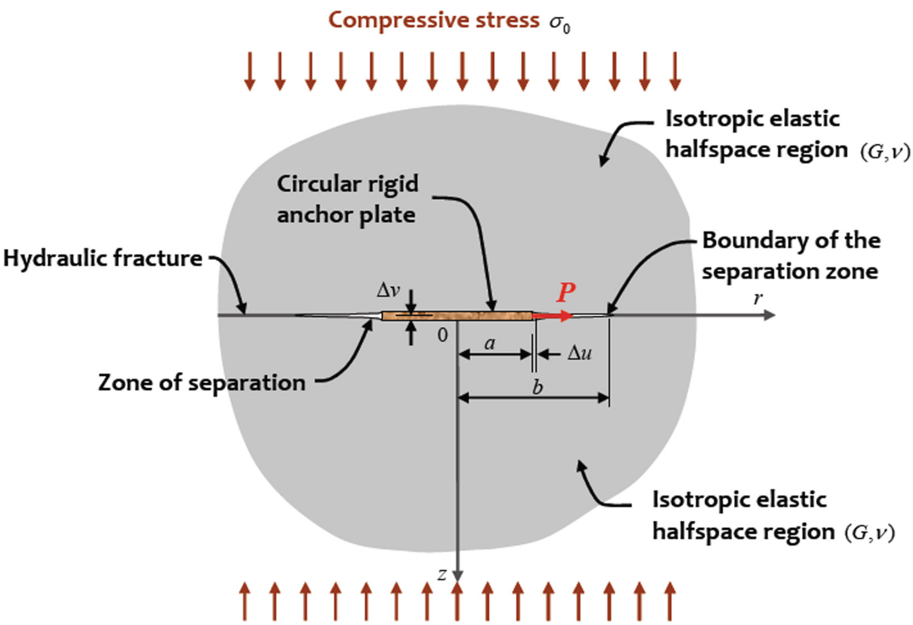

Fig. 5. In-plane loading for a flat anchor embedded in a fracture

4 Concluding Remarks

Dilatant processes at geological interfaces can have a strong influence on the shear stresses that are needed to rupture the region exhibiting dilatancy. The analysis of the shear rupture of an elastically and unilaterally constrained interface is non-routine since it requires the evaluation of the zone of separation induced by dilatancy effects with appeal to the solution of three-part mixed boundary value problems in elasticity theory applicable to a halfspace region. The paper demonstrates the efficient use of solutions available in the literature to evaluate the separation zone and the forces that are generated at the contact zone during dilatational displacement over a circular contact zone. The enhancement of the shear stresses needed to create shear rupture is demonstrated. This observation has implications for the interpretation of the seismic moment calculations, which currently accounts for only the relative shear displacements. It is shown that the analysis can also be extended to include interface dilatancy degradation that can result from asperity breakage during shear, resulting in a progressive decrease in the dilatancy angle with shear.

Acknowledgements. The author is grateful to the research support received by the Natural Sciences and Engineering Research Council of Canada and the James McGill Research Chairs program. The author would also like to acknowledge the contributions made by Dr. P.A. Selvadurai, Post-Doctoral Fellow at ETH Zurich and Dr. A.P. Suvorov, Assistant Professor, Moscow State University for Civil Engineering.

References

1. Bowden, F.P., Tabor, D.: Friction and Lubrication of Solids. Clarendon Press, Oxford (1986)
2. Hisano, M.: Friction at the Atomic Level - Atomistic Approaches in Tribology. Wiley-VCH, Weinheim (2018)
3. Selvadurai, A.P.S., Boulon, M.J. (eds.): Mechanics of Geomaterial Interfaces, Studies in Applied Mechanics, vol. 42. Elsevier, Amsterdam (1995)
4. Selvadurai, A.P.S.: On an invariance principle for unilateral contact at a bi-material elastic interface. Int. J. Eng. Sci. **41**, 721–739 (2003)
5. Selvadurai, A.P.S., Yu, Q.: Mechanics of a discontinuity in a geomaterial. Comput. Geotech. **32**, 92–106 (2005)
6. Selvadurai, A.P.S., Atluri, S.N. (eds.): Contact Mechanics in the Engineering Sciences - Material Characterization, Micromechanical Processes and Modelling, Geosciences. Tech Science Press, Forsyth (2010)
7. Selvadurai, A.P.S., Selvadurai, P.A., Suvorov, A.P.: Contact mechanics of a dilatant region located at a pre-compressed elastic interface. Int. J. Eng. Sci. **133**, 144–168 (2018)
8. Selvadurai, A.P.S.: Fracture evolution during indentation of a brittle elastic solid. Mech. Cohes.-Frict. Mater. **5**, 325–339 (2000)
9. Boulon, M.J., Selvadurai, A.P.S., Benjelloun, H., Fuega, B.: Influence of rock joint degradation on hydraulic conductivity. Int. J. Rock Mech. Min. Sci. **30**, 1311–1317 (1993)
10. Nguyen, T.S., Selvadurai, A.P.S.: A model for coupled mechanical and hydraulic behaviour of a rock joint. Int. J. Numer. Anal. Methods Geomech. **22**, 29–48 (1998)
11. Selvadurai, A.P.S., Nguyen, T.S.: Mechanics and fluid transport in a degradable discontinuity. Eng. Geol. **53**, 243–249 (1999)
12. Selvadurai, A.P.S.: Normal stress-induced permeability hysteresis of a fracture in a granite cylinder. Geofluids **15**, 37–47 (2015)
13. Selvadurai, A.P.S., Singh, B.M.: The annular crack problem for an isotropic elastic solid. Q. J. Mech. Appl. Math. **38**, 233–243 (1984)
14. Selvadurai, A.P.S., Singh, B.M.: On the expansion of a penny-shaped crack by a rigid disc inclusion. Int. J. Fract. **25**, 69–77 (1984)
15. Selvadurai, A.P.S.: On an integral equation governing an internally indented penny-shaped crack. Mech. Res. Commun. **12**, 347–351 (1985)
16. Selvadurai, A.P.S.: Separation at a pre-fractured bi-material geological interface. Mech. Res. Commun. **21**, 83–88 (1994)
17. Davis, R.O., Selvadurai, A.P.S.: Plasticity and Geomechanics. Cambridge University Press, Cambridge (2003)
18. Taylor, D.W.: Fundamentals of Soil Mechanics. Wiley, New York (1948)
19. Christian, J.T., Baecher, G.P.: D.W. Taylor and the foundations of modern soil mechanics. J. Geotech. Geoenvironm. Eng. **141**, 02514001 (2015)
20. Drucker, D.C., Prager, W.: Soil mechanics and plastic analysis or limit design. Q. Appl. Math. **10**, 157–165 (1952)
21. Ichikawa, Y., Selvadurai, A.P.S.: Transport Phenomena in Porous Media. Springer, Berlin (2012)
22. Selvadurai, A.P.S.: In-plane loading of a rigid disc inclusion embedded in a crack. Int. J. Solids Struct. **36**, 1701–1714 (1999)