Stability and Large Deformations of Slender Structures Supported by Soil Materials

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Abstract. The stability and geometric nonlinearities of slender structures are a major topic in structural design. While this topic is most relevant in the field of Structural Engineering, e.g. for slender steel or concrete structures, only few applications take the role of soil-structure-interaction explicitly into account. The focus of this paper is placed on the impact of soil support and its modelling for the buckling analysis based on examples both for pile foundations and for railway track stability. The general interaction between steel design and the geotechnical input is addressed.

1 Introduction: Structural Stability and Soil-Structure-Interaction

Based on the early work of *Leonhard Euler* for critical loads of axially compressed columns the investigation of the stability of slender constructions belongs to the standard design criteria to be proved. For some types of structures, these investigations of structural stability are combined with considerations on the subsoils or soil-like materials contribution. To name a few, technical questions include the global buckling of pile-like foundations, but also local buckling phenomena of shell-like foundations such as in suction buckets or monopiles. Very similar to global buckling considerations of piles embedded in soft soil (see e.g. Vogt et al. 2005) is the lateral stability of ballast track originating from significant axial forces occurring along the rail. (Ballast will be addressed as a soil material in the following chapters.) To complete the list of topics, the buckling of silo structures for granular materials is specifically interesting and has been intensively investigated in the past e.g. in the DFG Collaborative Research Centre SFB 219 in Karlsruhe University. However, piles and track will be the main focus in this paper. With respect to the piles the paper considers especially the view of the steel designer.

For any structural member contacting adjacent soil material the latter can be both: a significant loading and a supporting element. The interaction to the subsoil is often modelled using springs. The characteristics of theses springs can be linear or nonlinear. While it is undisputed that soil seldomly behaves linear, the role of the calculation procedure for the structural stability has a decisive impact on the need for a linearization of the stiffness. For the most common numerical linear buckling analysis a full linearization of stress-deformation behavior will be necessary.

However, if for the steel design material and geometrical nonlinearities as well as imperfections are considered, soil behavior in any complexity can be modelled as well, see e.g. Gottschalk (2017) who investigated the buckling of suction buckets considering also nonlinear springs as well as Hübner (2007) who investigated buckling of embedded tubular piles using also a Hypoplastic model for the derivation of springs or lately Bakroon et al. (2018), considering pile tip buckling and deformations using a full ALE concept. In this context the paper discusses a few examples on stability and large deformations of slender structures supported by soil materials.

In the load-deformation diagram buckling will show for the occurrence of a horizontal tangent or a bifurcation point in the solution path. In the example from Fig. 1, it can be assumed that in the case of large deflections the cross-section is plasticized, and a plastic hinge is formed in the middle. A limit load occurs below the critical load at which no load increase is possible. The load-deformation diagram results from the consideration of geometrical and material nonlinear behavior. The limiting load is dependent on the amplitude of the initial disturbance $w_{0,i}$. The soft transition results from a successive plasticizing process of the cross-section.
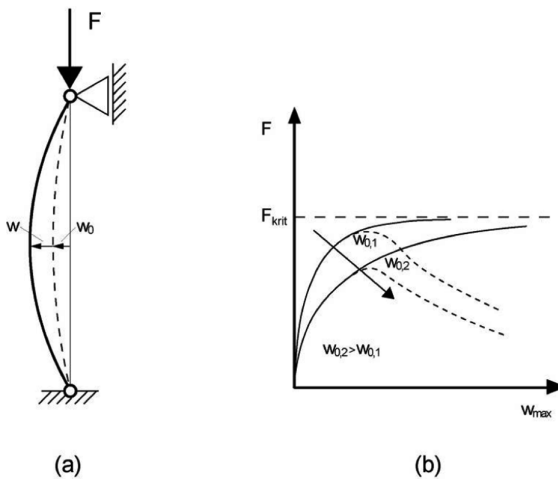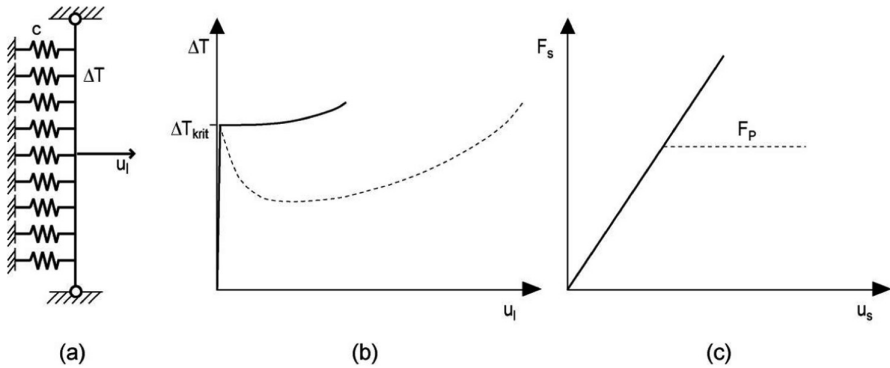

Fig. 1. Limit loads for an imperfect rod including material plasticity

A similar behavior can be observed when a nonlinear spring reaches the plasticity limit (Fig. 2) and is described in the next chapter for the lateral stability of a track model.


Fig. 2. (a) model of axially constrained rod, (b) critical temperature for linear (solid line) and elastic-plastic (dotted line) support, (c) characterizing function of the considered stiffness, see Van (1997).

2 Stability of Railway Track

The continuously welded rail track is prone to lateral shift and track buckling in case of large track irregularities and external loading. The geometric non-linearity here results from the pressure-loaded track with track irregularities, while the nonlinear material behavior is primarily due to the sublinear increase of the lateral bedding resistance with the lateral load force until a limit resistance is reached. Figure 3 shows an example of a one-sided distortion of the track and the scheme for its modelling in a FE environment while Fig. 4 adds the definitions for the misalignment. Some fundamental comprehensive work for the understanding and modelling of this design situation has been done by Van (1997) and Kish et al. (1998).

The lateral resistance (QVW from German Querverschiebewiderstand) depends on the specific track construction, especially the type of sleeper but also largely on the maintenance conditions (see Fig. 5 and Van 1997). For common B70 sleepers lateral resistance should be in the range of 8 kN (Chatkeo 1985), as assumed here.
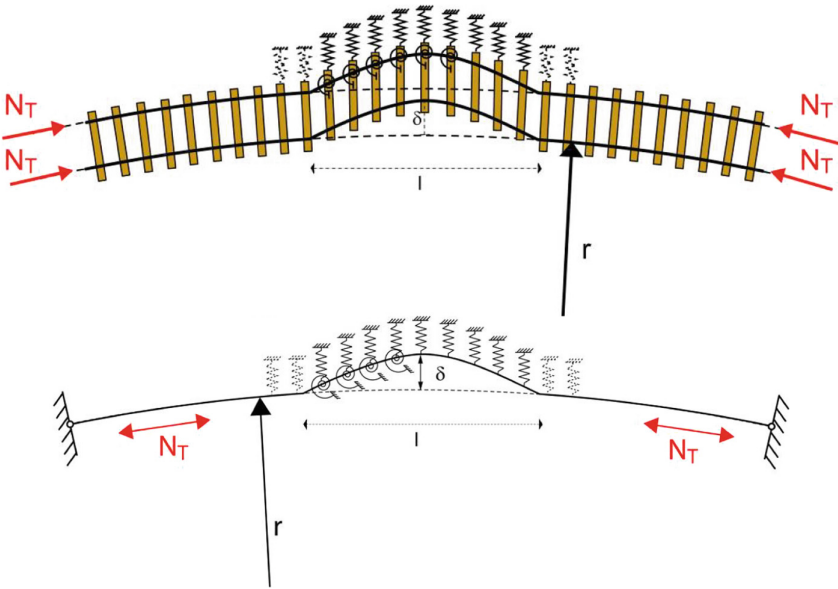
Fig. 3. (Top) an example of a one-sided distortion of the track and (bottom) the scheme for its modelling in a FE environment
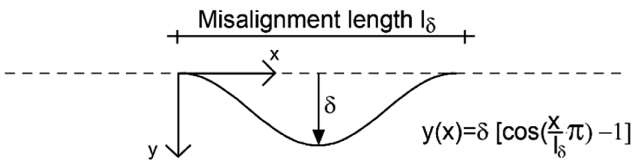
Fig. 4. Definitions for the initial lateral misalignment: Misalignment shape, misalignment length l and misalignment amplitude δ
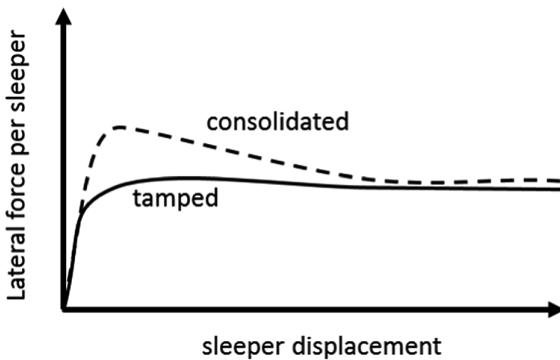
Fig. 5. Lateral resistance of individual sleepers: consolidated state (dashed line) and after tamping (solid line)

The transverse displacement resistance profile QVW1 is calculated from the values for a B70 sleeper, which is suitable for a good (prior to track maintenance) to averaged (after track maintenance) track. The resistance approaches asymptotically the peak resistance of 8.1 kN for all approaches. QVW2 is a bilinear curve, QVW3 and QVW4 result from the fact that QVW1 in direction can be stretched or compressed by a factor of 2. The assumptions for lateral resistance are summarized in Fig. 6.
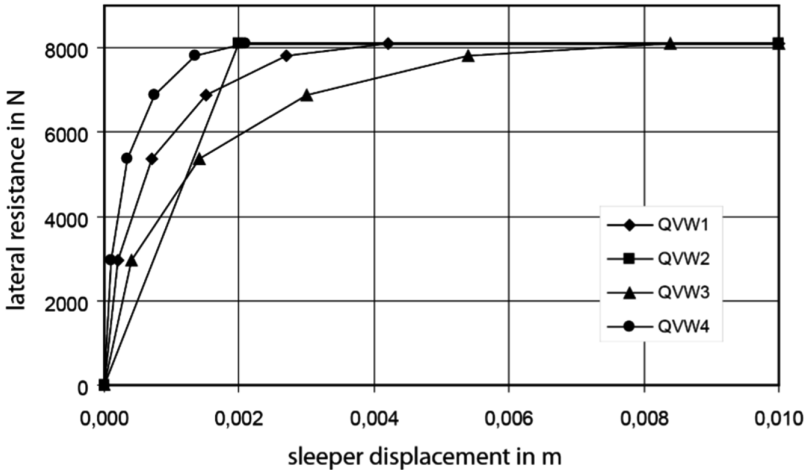

**Fig. 6.** Sleeper resistance with different approaches for load displacement curvature

The overall model consists of the elements shown in Figs. 3 and 4 and is further explained below.

- Lateral bedding: For the description of lateral bedding, springs are used located at each rail support point. The characteristic curves are simplified as non-linear-elastic
- Rails: In reality, the rails are usually continuously welded. For the Model it is sufficient to model a section. The rail will be 48 meters long and constrained at its ends. The rails are modelled as Bernoulli beam. Nonlinear geometric effects are taken into account. The two rails are merged. Investigations by Van (1997) show that this results in a negligible deviation.
- Track geometry: The track geometry is determined by the specification of the curve radius and the track direction error (shape, length and amplitude), see Fig. 4.

For a set of parameters, a study was conducted to better understand the critical lateral track stability limit with respect to geometric parameters and imperfections, the curvature of the lateral track resistance based on a steady increase of axial compression in the rail due to temperature increase (Baeßler 2008). Table 1 summarizes the standard parameters for the model.

Table 1. Standard parameters of the mechanical model

Parameter	Value	Dimension
Radius r	5 000	M
Track length l_G	48	M
Sleeper distance	0.6	M
Torsional stiffness c_T	170 000	Nm/rad per m
Amplitude of misalignment δ	0.005	M
Length of misalignment $l_δ$	10.0	M
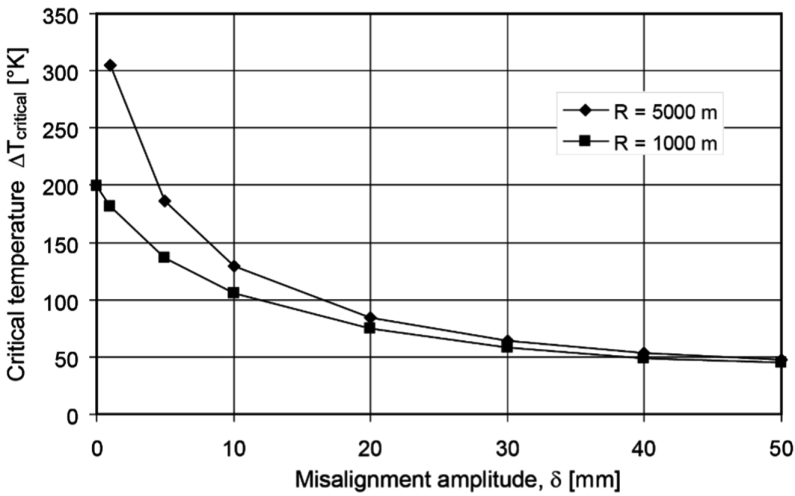
Fig. 7. Critical temperature for track buckling depending on misalignment amplitude δ and track radius.

For the track stability analysis, the critical temperature is important when buckling is observed. In principal rail temperature and axial loading of the rail are connected by thermal expansion and the longitudinal constraining of the rail. Thereby, a critical track buckling load is associated with a critical temperature ΔT. Track radius and misalignment amplitude have been varied in the investigation. As can be expected any deviation from a straight track e.g. narrow radius or extend of misalignment is highly important and reduces the critical temperature, see Figs. 7. The misalignment should be limited as much as possible.
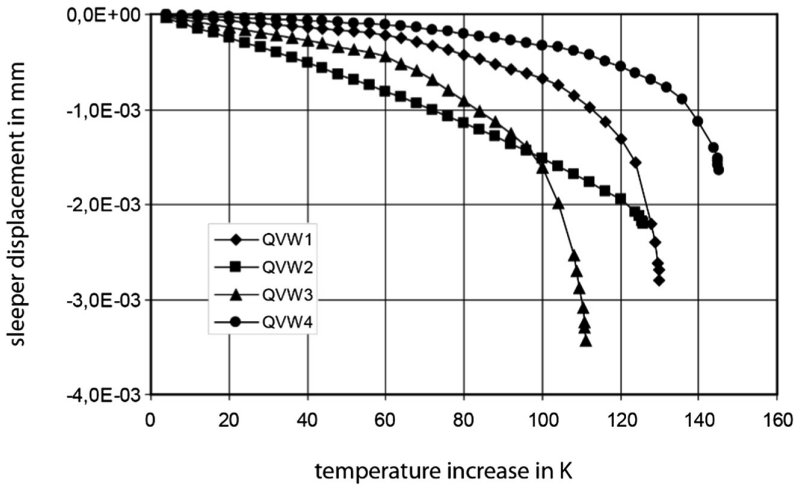
Fig. 8. Sleeper displacement vs. temperature increase for different lateral resistance (QVW)

Furthermore, the general importance of the soil support for the lateral stability of the track is obvious, see Fig. 8. However, as can be found from the parameter study the peak resistance is not sufficient for verification but also the resilience is decisive. A softer spring with the same plasticity limit decreases the critical temperature by more than 30% in the study.

3   Stability of Slender Piles in Soft Soil: Global Buckling

Slender piles embedded in soft soil are mechanically very similar to track: a beam like structure, axially loaded and supported by soil material.

From a general point of view, the problem of pile buckling concerns the stability of the slender pile, either before or during installation in the ground and thereafter during its application as a structural foundation element. In this context, the term global buckling can usually be used to describe the transverse deformation of the pile profile (e.g. Euler's buckling) without significant deformation of the cross section. For piles, fundamental investigations on the buckling of slender piles in soft soils are described in Vogt et al. (2005). For an elastic bedding the analytical solution of *Engesser* for single bars is given there. It is stated that an undrained shear strength c_u of 10 to 15 kPa has been assumed to be sufficient to prevent buckling. However, the aforementioned work describes that also the low stiffness for soft soils can be decisive.

Global buckling goes beyond the single column as e.g. specifically in Offshore Structures like Jackets with piles, where the buckling verification of the global structure includes the piles.

4   Stability of Soil Supported Offshore Piles: Local Buckling

The bearing capacity of shell structures strongly depends on the buckling behavior. Local buckling usually refers to the particular form of buckling for shell elements and thin-walled structural components subject to in-plane compressive stresses, where the mechanical instability results in sudden out-of-plane deformations. The slenderness, the type of loading, boundary conditions and imperfections are important determining factors. Typical examples of cylindrical shell structures are support structures of Wind Energy Converter (WEC) like tubular steel towers and Monopiles. Typically, buckling analyses are performed by applying the stress-based approach, which refers to a few documented base cases of loading, boundary conditions and imperfections (e.g. DIN EN 1993-1-6). Alternatively, a numerically based approach can be applied. The main intention of using the numerical approach is to represent boundary conditions and stress distributions at a more detailed and realistic level and finally to come to more economic designs.

In particular the stress-based approach does not provide documented reference cases for lateral supported tubular slender piles. A very common solution is to neglect the effect of the soil and to accept a lower bearing capacity due to buckling.

One option to consider more complex load situations and boundary conditions is to evaluate the overall slenderness λ_{ov}, where F_{Rpl} is the plastic buckling load calculated by a material non-linear analysis (MNA) and F_{Rcr} is the elastic buckling load, calculated by a linear buckling analysis (LBA).

$$\lambda_{ov} = \sqrt{\frac{F_{Rpl}}{F_{Rcr}}} = \sqrt{\frac{r_{Rpl}}{r_{Rcr}}}$$

Alternatively, the load factors r can be used to determine the overall slenderness λ_{ov}. For an axially loaded pile the overall slenderness is comparable to the slenderness λ_x. High values for λ indicate a low buckling resistance. In a first step, the result of these two buckling analyses (LBA, MNA) is the ideal buckling load. The effect of imperfection is considered in a second step by tabulated values which refer to certain tolerance class and determine the real buckling load respectively the real buckling resistance of the structure.

Imperfections can be also considered in the numerical analysis. A geometrically and materially non-linear analysis including imperfections (GMNIA) must be performed therefor. A challenging task is to cover all possible imperfections and to find the lowest real buckling resistance. Typically, eigenform related imperfections cause the lowest buckling resistance. Therefore, numerous buckling eigenforms must be analyzed to find the design driving case.

The potential of the numerical approach shall be described for a simplified case. An embedded Monopile of 8 m diameter is 80 m long and over 35 m laterally supported by soil, see Fig. 9.
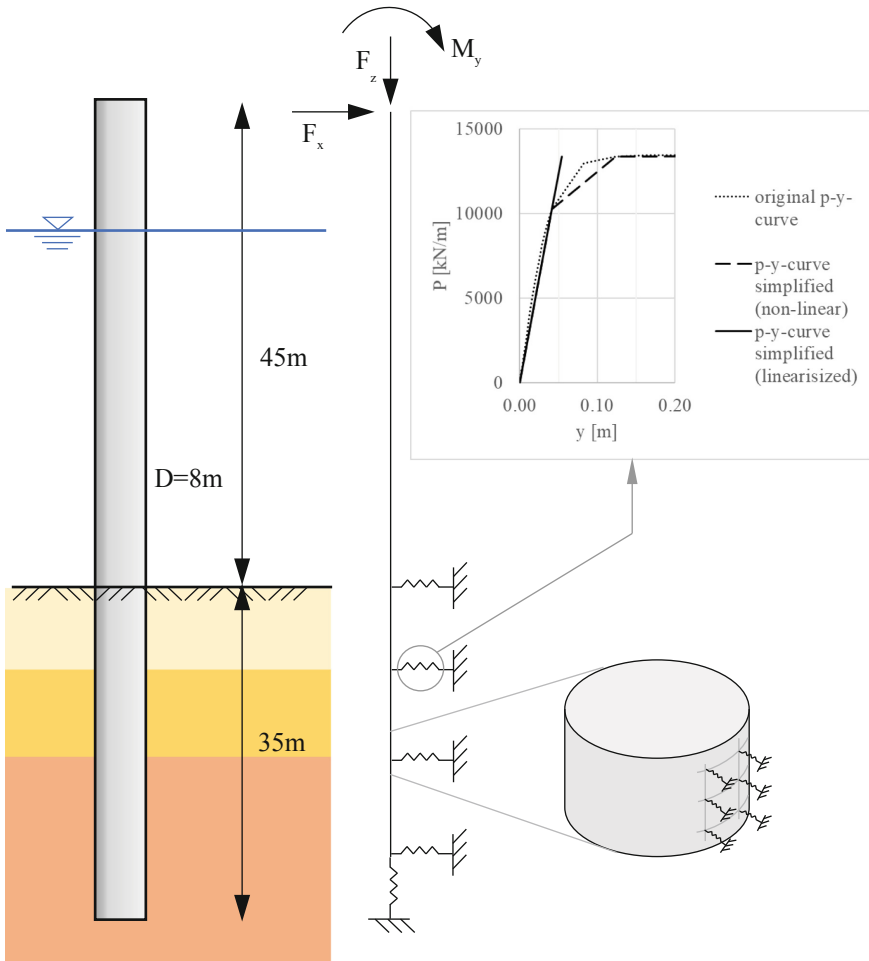
**Fig. 9.** Monopile – boundary conditions and model simplifications

The soil profile consists of a loose sand layer (0.5 m) on top and several stiff and very stiff clay layer which alternate with sand layer down to the pile tip level (35 m). The soil is represented by linear Winkler springs, which have been derived from non-linear p-y-curves. Linearization may not be applicable, e.g. for weak soil layer. In these cases deformations of the steel structure exceed the limit, where linear soil resistance can be assumed. Then non-linear Winkler springs must be applied, which increases the numerical complexity. T-z curves which usually represent the skin friction of the pile have not been applied here. They would decrease the axial load in the pile towards the pile tip. The t-z effect on buckling near the seabed is considered negligible. The structure is loaded by a vertical and a horizontal load and a bending moment on the top. The magnitude of imperfections has been chosen to be in line with the quality class B of DIN EN 1993-1-6.

At first, a linear buckling analysis (LBA) is performed. The lowest buckling eigen-modes indicate the potential failure modes with respect to the applied load case (Fig. 10). Other load cases may lead to other failure modes which are not considered here.
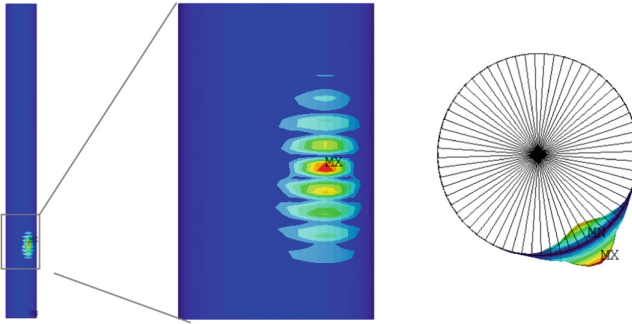
Fig. 10. Local buckling eigenmode (deflections) referring to the lowest load factor

Further analyses considering non-linear material properties (MNA) and geometrical non-linear deformations (GMNA) are performed. Based on the failure mode (LBA) an imperfect geometry is created which considers a maximum allowable deflection of the pre-buckled shape as well. Finally, a GMNIA provides the bearing capacity of the imperfect system under the given load case.

LBA   MNA   GMNA   GMNIA

$r_{Rcr} = 1.98$ $r_{Rpl} = 0.58$ $r_{GMNA} = 0.40$ $r_{GMNIA} = 0.39$
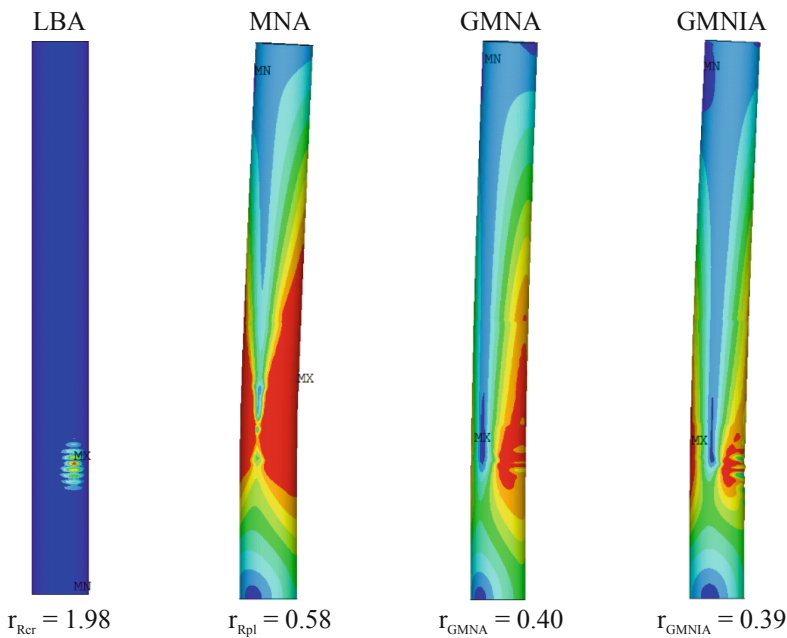
Fig. 11. Load factors r for different types of analyses

The load factors r shown in Fig. 11 describe the level of the buckling load with respect to the applied set of loads in the analyses (LBA, MNA, GMNA and GMNIA). The load factors respectively the buckling load decrease with the introduction of materially and geometrically non-linear characteristics and finally the imperfections. The overall slenderness λ_{ov} as the result of an LBA and MNA can be compared to the slenderness λ_x of the stress-based approach. One reason for the difference is the lateral resistance of the soil, which has been considered in the numerical model only.

Numerical approach	Stress-based approach
$\lambda_{ov} = \sqrt{\dfrac{r_{Rpl}}{r_{Rcr}}} = \sqrt{\dfrac{0.58}{1.98}} = 0.54$	$\lambda_x = \sqrt{\dfrac{f_{y,k}}{\sigma_{x,Rcr}}} = \sqrt{\dfrac{355}{1750}} = 0.45$

The numerical approach shown for one single situation (load case, imperfection) must be applied on numerous combinations including collapse affine and construction related imperfections as well, which is not shown here. The load factor $r_{GMNIA} = 0.39$ applied on the loads of the investigated load set describes the lowest elastic-plastic buckling loading of the structure. In comparison to this the load factor of the stress-based method is about 0.24 and does not consider any lateral resistance of the soil layer. The GMNIA approach allows for a consideration of the soil support and shows potential reserves in design.

Finally, the result of the GMNIA should be verified on known problems to confirm the reliability of this approach. Very often this verification is not possible due to missing comparative data. It is also recommended to evaluate different magnitudes of imperfections to confirm that the lowest buckling load factor has been calculated.

The numerical approach allows to include complex boundary conditions like soil embedment. The effect of soil embedment has been shown only for one load set and one shape of imperfection. For a detailed analysis numerous load sets and imperfections must be investigated. The potential of this approach with respect to the stress-based method has been indicated. It is expected that a more economic design is achievable by using the numerical and more complex approach instead of relying on the stress-based concept.

5 Pile Tip Buckling of Tubular Steel Piles

Damages to the pile, particularly localized at the pile tip, often arise during any of the many phases of pile installation. The topic is addressed for an offshore application.

For instance, the operations involving a high risk for pile tip damage include the general lifting and handling on deck, the lowering into the pile-sleeve cone, the transitory pile-in-sleeve state and the pile driving itself whenever the pile tip impacts on any obstacles within the subsoil (e.g. large boulders, stiff soil strata, etc.) (Aldridge et al. 2005). Since it is often not easy to discern from literature if shell buckling was involved in investigations and reported failures (or if it was only a propagation of initial

damage or imperfection) the term "pile tip buckling" is partly used as an umbrella term for pile tip deformations and their propagation during installation. Buckling of the lower end of the pile is mainly driven by the pile tip resistance. However, for the case of large pile diameters as currently used for monopile foundations, the assumption of a tip resistance that is equally distributed all over the tip surface becomes increasingly questionable. In this respect, the scientific literature shows that the buckling of cylindrical shells depends largely on the boundary conditions of the support. Typically, these supports (foundations) are designed to provide a defined boundary condition which is then considered in the buckling analysis and design of the superstructure shell. For piles, this boundary condition depends on the (geo) mechanical response of the foundation and would have to be provided by a specific geotechnical analysis considering the local soil conditions. In practice however, this requirement is often circumvented by adopting simple assumptions which allow a safe but also economic design of the pile.

One option to include Pile Tip Buckling scenarios is to vary the pile tip resistance, see Fig. 12.
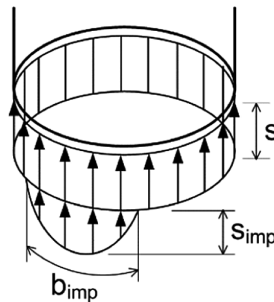
Fig. 12. Equally-distributed tip resistance s and increased tip resistance b_{imp} to consider an unequal distribution of tip resistance

However, one of the key open aspects to be further investigated is the buckle propagation during pile driving and the consequent collapse of the cross section, as outlined in (BUMP 2018) and widely reported for various offshore platforms (see e.g. Alm et al. 2004; Erbrich et al. 2011; Senders et al. 2013). Such phenomena are characterized by the progressive accumulation of large pile deformations (see Fig. 13, as opposed to the sudden failure which is typical for the shell stability buckling problems. While the latter are usually analysed by means of GMNIA approaches the former phenomena must be treated explicitly as a transient process instead.
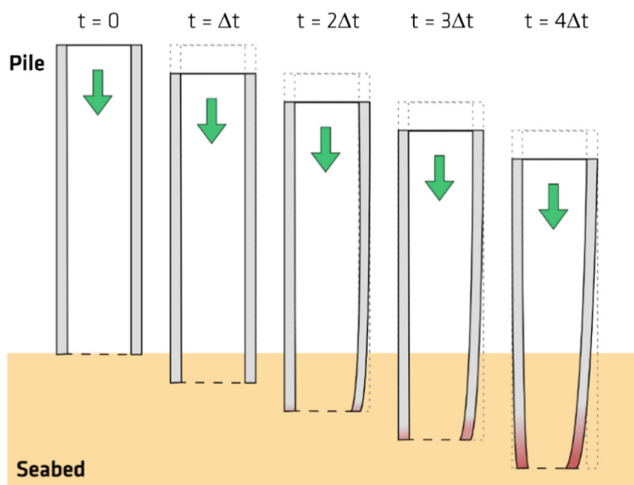
Fig. 13. Pile buckling propagation during dynamic driving

Currently no design rules for Pile Tip Buckling are available except for rough prescriptions for the wall thickness as the API-criterion which might eventually be very costly for larger Monopiles.

6   Conclusions/Summary

Stability and large deformations of slender structures are a major topic in structural design and can be decisively influenced by structure-soil-interaction. The paper addresses several subtopics where structural stability depends on the soil support. Firstly, the investigation of lateral stability of an axially compressed ballasted railway track is summarized. The rail itself is an extremely slender structure. So, its lateral stability under relevant compression loading is completely dependent on a stable lateral support of the ballast. Ballast support is modelled as local springs. Neither the mechanical ultimate strength of the sleeper ballast interaction nor the initial stiffness is sufficient to fully describe the lateral shift until buckling of the track. The use of nonlinear springs improves the results and is still numerically feasible.

Due to its similarity considering the mechanical system slender piles in soft soil have been addressed afterwards addressing the work of *Vogt et al. (*2005*)*. The problem of global buckling for pile structures is also very relevant for complete foundation structures as for offshore jackets, usually implying simplification as linear springs for the pile soil interaction.

In contrast to global buckling local buckling deals with local buckling phenomena of tubular shells including large radial deformations. Local buckling has to be considered for all tubular members of the steel structure. Considering a Monopile several scenarios have to be addressed that are relevantly influenced by the structure-soil-interaction:

(a) The embedded pile is significantly loaded close to the mudline during operation. In this paper the buckling load has been exemplarily analysed for an 80 m Monopile (D = 8 m, t = 55–65 mm) which is embedded in soil over 35 m. Based on one load set an LBA, MNA, GMNA and GMNIA as well as a stress-based buckling analysis were performed.
(b) Buckling is triggered during installation when local imperfections in the structure or in the resistance of the soil occur
(c) Pile Tip Buckling as a synonym for large radial deformations occurs as a transient process of buckle propagation during installation.

It must be concluded that further research on soil-structure-interaction would help to optimize large monopiles with respect to buckling considerations. In this respect it is an open question to what extend soil can be linearized or needs to be incorporated using more sophisticated models.

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