Fuzzy Systems: A Human Reasoning Approach Using Linguistic Variables

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Abstract. The term "Fuzzy" means vague or imprecise or uncertain or inexact. Fuzzy Sets enable us to accept the vagueness and lack of precision. Fuzzy Sets are used when classical/crisp representation cannot make the decision for a problem. Fuzzy Set Theory is a vast field and relies heavily on mathematical equations. This paper is an attempt to capture its essence without getting overwhelmed by the complexity of details. In this paper, we have confined our discussions about Fuzzy Sets and its operations in contrast to Classical Sets. We have started with Classical Sets followed by a discussion on how a Classical Set fails in some of the problems and how Fuzzy Sets overcome those issues. Then we have briefly discussed the Linguistic Variables and how it is more practical and realistic than a binary reasoning. For the sake of simplicity and understanding, we have tried to avoid mathematical equations wherever possible.

Keywords: Fuzzy Sets · Membership function · Classical Sets · Linguistic Variables

1 Introduction

Fuzzy [1–3] Sets deal with the information that is vague, imprecise, uncertain, ambiguous, inexact, or probabilistic [4] in nature. *Fuzzy Sets* are associated with the membership function whose range is between 0 and 1. *Fuzzy Sets* are used in the development of Fuzzy Control Systems to make the appropriate decision. *Fuzzy Sets* differs from classical/crisp sets with the help of membership function and its operations. A *Linguistic Variable* [6] is a variable whose value is given by a word or a sentence rather than a numeric value.

In this paper, we have compared *Fuzzy Sets* with *Classical Sets* and established its superiority in terms of practical usage. We have documented the theoretical foundations of the subject matter after going through some notable works in the field. This work is aimed at presenting a lucid overview of the concepts to a naïve reader.

After this brief Introduction in Sect. 1, Sect. 2 contains a discussion on *Classical Sets* followed by the formal discussion on *Fuzzy Sets* in Sect. 3. Section 4 contains a discussion on *Classical Sets* Operations in comparison with a *Fuzzy Set*s Operations in Sect. 5. In Sect. 6 there is a brief discussion on Linguistic Variable and last Sect. 7 is our conclusion.

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2 Classical Sets

Before starting with *Fuzzy Sets* let us first take a brief look at classical/crisp sets. A set is defined as a collection of well-defined objects.

For example:

– Collection of even integers.
– Collection of prime numbers.
– Collection of vowels.

The *Classical Set* is defined in such a way that the universe of discourse (X, as a collection of objects all having the same characteristics) is divided into two groups: member and non-member.

Suppose an object x in a *Classical Set* A. This object x is either a member or a non-member of the given set A.

Let the universe of discourse be A. A collection of elements within a universe are called sets and collection of elements within a set is called subsets.

For a *Classical Set* A in universe X:

– An object x is a member of set A (x ϵ A), i.e., x belongs to A.
– An object x is a non-member of set A ($x \notin A$), i.e., x doesn't belong to A.

The *Classical Set* allows membership of elements in binary form i.e. 0 or 1.

Mathematically, the *Classical Set* can be defined as:

Let U be the universal set.

A *Classical Set* 'A' of the set U is characterized by

$$A = \{(x, x \in U)\}$$

Where A = *Classical Set*, x = value, and U = universal set.

Let us take an example of two glasses where one is empty and another is full of water. By looking at Fig. 1 below, we can easily identify which glass is full and which one is empty. There is no ambiguity in the decision.

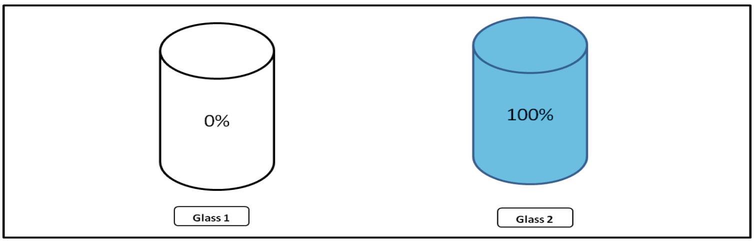

Fig. 1. Classical Sets example

3 Fuzzy Sets

Fuzzy Set [1] Theory was first proposed by Lotfi A. Zadeh in 1965. The word “fuzzy” means “ambiguous”. The *Fuzzy Set* Theory is a generalization of a Classical or Crisp Set Theory. A *Fuzzy Set* is a class of object with the continuity of value of membership. This set is characterized by a membership function which is assigned to each object with a different membership value ranging from 0 to 1.

Let us take an example where we have four glasses of equal size. Each of these glasses, are respectively 0%, 20%, 80%, and 100% filled with water. Looking at the Fig. 2 below, we cannot decide whether a glass is full or empty. There is an ambiguity. The glass 1 which has 0% water is empty and the glass 4 which is 100% filled with water is full. But the glass 2 which is 20% filled and the glass 3 which is 80% filled is neither full nor empty. So, we can’t say that both are full or empty; there is an ambiguity.

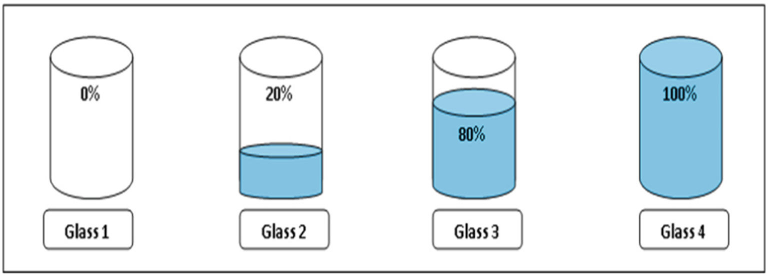

Fig. 2. Fuzzy Sets example

So, *Fuzzy Sets* viz. ‘Empty’ and ‘Full’ will allow the ambiguity with the help of degrees or grades of membership as follows:

Full {Glass 1(0), Glass 2(0.2), Glass 3(0.8), Glass 4(1)}

Empty {Glass 1(1), Glass 2(0.8), Glass 3(0.2), Glass 4(0)}

Here, Glass 1(0), Glass 2(0.2) … Glass 4(1) form a membership function which contains membership value like 0, 0.2, 0.8 etc. lying between 0 & 1(including 0 &1).

It can be graphically represented as (Fig. 3):

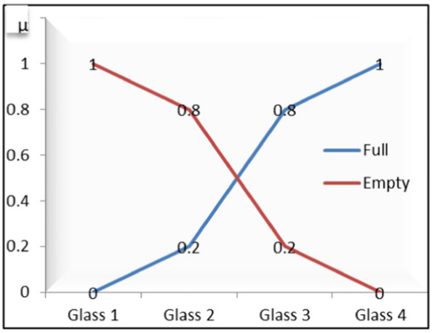

Fig. 3. Graphical representation

Mathematically, *Fuzzy Set* can be defined as:
Let U be the universal set.
A *Fuzzy Set* 'A' of the set U is characterized by its membership function μ defined by

$$A = \{(x, \mu(x)) : \; x \in U\}$$

Where A = *Fuzzy Set*, x = value, μ(x) = membership function and U = universal set.

It means that for every x ϵ U, x does also belong to A by an amount μ(x) where $0 \leq \mu(x) \leq 1$.

4 Classical Set Operations

Before understanding a *Fuzzy Set* operation lets briefly overview the basic *Classical Set* operations (Fig. 4).

The basic operations of the *Classical Set* are as follows:

Union
It contains all the elements of set A or set B. It is also called as Logical 'OR'.

$$A \cup B = \{x | A \text{ or } x \in B\}$$

Example:

Set A = {a, b, c}
Set B = {c, d, e}
Set A U B = {a, b, c, d, e}

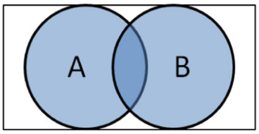

Fig. 4. Classical union operation

Intersection
It contains the common elements that are present in set A and set B. It is also called as Logical 'AND' (Fig. 5).

$$A \cap B = \{x | x \in A \text{ and } x \in B\}$$

Example:

Set A = {a, b, c}
Set B = {c, d, e}
Set A ∩ B = {c}

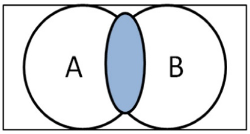

Fig. 5. Classical intersection operation

Complement
It contains the entire element but accepts the elements which are present in set A. It is also called as Logical 'NOT' (Fig. 6).

$$\bar{A} = \{x | x \notin A,\ x \in X\}$$

Example:

Set X = {a, e, i, o, u}
Set A = {a}
Set $\bar{A}$ = {e, i, o, u}

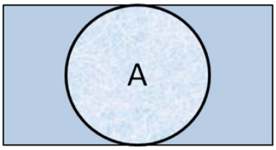

Fig. 6. Classical complement operation

5 Fuzzy Set Operations

Now the basic operation of *Fuzzy Set* is similar to the *Classical Set* but there is a difference in calculating the value of operations.

Basic operations of *Fuzzy Set* are:-

Union
The membership function of Union of two *Fuzzy Set* A and B ($\mu A \cup B$) is a maximum of membership function of μA and μB.

The Union operation of *Fuzzy Set* is equivalent to the Binary operation 'OR'.

$\mu A \cup B\ (x) = \max[\mu A(x), \mu B(x)]$ for all $x \in U = \mu A(x) \vee \mu B(x)$;

Here $\vee$ is the symbol for maximum (Fig. 7).

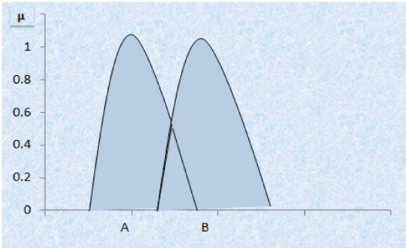

Fig. 7. Fuzzy union operation

Intersection

The membership function of Intersection of two *Fuzzy Set* A and B ($\mu A \cap B$) is a minimum of membership function of μA and μB.

The Intersection operation of *Fuzzy Set* is equivalent to the Binary operation 'AND'.

$\mu A \cap B\ (x) = \min[\mu A(x), \mu B(x)]$ for all $x \in U = \mu A\ (x) \wedge \mu B(x)$;

Here $\wedge$ is the symbol for minimum (Fig. 8).

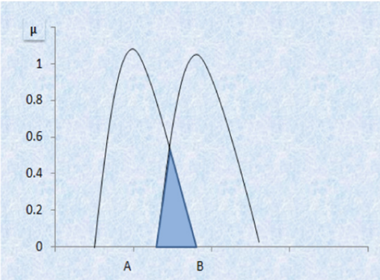

Fig. 8. Fuzzy intersection operation

Complement

The membership function of Complement of a *Fuzzy Set* A ($\mu \bar{A}$) is a negation of membership function of μA.

The Complement operation of *Fuzzy Set* is equivalent to the Binary operation 'NOT' (Fig. 9).

$$\mu \overline{A}(x) = 1 - \mu A(x)$$

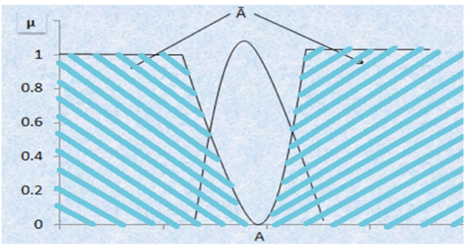

Fig. 9. Fuzzy complement operation

6 Linguistic Variables

It is one of the important concepts in *Fuzzy Logic* which played a vital role in the Fuzzy System. The *Linguistic Variable* [6] means a variable whose values are in words or a sentence in a natural language instead of numerical value. For example, a word like

Temperature is the *Linguistic Variable* if its value is linguistic rather than a numerical value i.e. very cold, cold, warm, hot, very hot, etc., rather than 5, 15, 25, 35, 45… Here *Linguistic Variable* temperature represents the temperature of a room.

Linguistic Variable can also be defined by: (x, T(x), U, M) where x is the name of the variable, T(x) is term-set i.e. the set of linguistic values assigned to x, U is the universe of discourse, and M is the semantic rule associated with each variable (membership) i.e. it defines the membership function of each fuzzy variable; for example M (cold) = the fuzzy set for temperature below 25 with membership of μ_{cold}.

Consider an example where x: temperature is defined as a *Linguistic Variable*, T (temperature): {very cold, cold, warm, hot, very hot}, U: {0, 60}, M: define the membership function (μ) of each variable, for instance, M (hot) = *Fuzzy Set* for temperature above 35° with membership of μ_{hot} (Fig. 10).

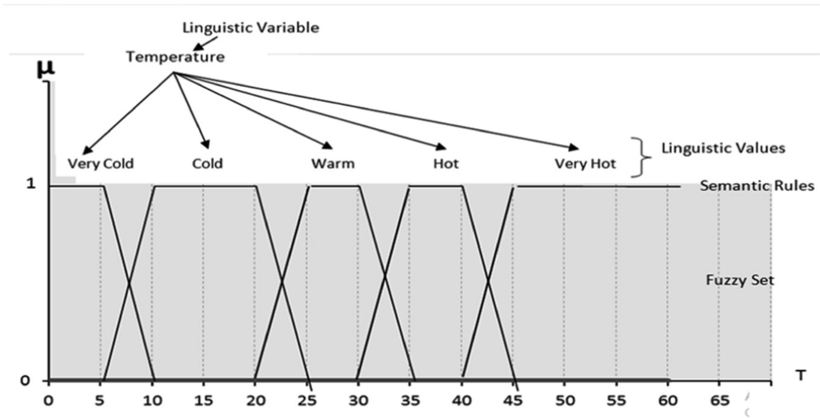

Fig. 10. Linguistic Variables

In the above example Linguistic Variable, Temperature is having a Linguistic values such as very cold lies between the range [0, 10], cold lies between the range [5, 25], warm lies between the range [20, 35], hot lies between the range [30, 45], and very hot lies between the range [40, 60].

The concepts of Linguistic Variable provide a mean of approximate characterization of concepts which are complex or not well-defined to be responsive to describe in quantitative terms. When the information which is not exact nor very in exact than *Linguistic Variable* offer a more realistic framework for human reasoning than the two valued logic i.e. true or false.

The main applications of the *Linguistic Variable* is especially in the fields of artificial intelligence, human decision processes, pattern recognition, psychology, law, medical diagnosis, information retrieval, economics and related areas.

7 Conclusion

In the above discussion, we have presented the theory of *Fuzzy Sets* in contrast to the classical/crisp set in simple terms. Also, we have discussed the *Linguistic Variables* that make things more realistic. The concepts of *Fuzzy Sets* pave the way for various advanced theories [7, 8] and implementations that has been documented extensively. Though the works related to these areas are widespread now, they are still grabbing the attention of the researchers and are frequently being cited. The industry is coming up with various kinds of innovative implementations based on *Fuzzy Logic*.

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