



Contribution to the Theory of Uncertain Systems Alliances

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Abstract. Basic concepts and ideas of uncertain systems alliances are briefly overviewed. Then the strength of the alliance joints is analyzed via medium mutual information measure. This approach can help to estimate the value/importance of respective bonds within the alliance, which is important in alliances with significant uncertainty.

Keywords: Systems alliance · Uncertainty · Interface · Fuzzy approach · Medium mutual information

1 Introduction

The concept of systems alliances proposed already in 2001 Vlček [1]. He introduced it on the idea that an alliance of two or more parts/modules originates as:

- a product of random encounter
- an outcome of processes of contamination and immunity
- a construct.

Generally, the systems alliance does not hold common characteristics of species (genetic code), or common goals, or common identity, i.e. it can hardly be identified as a system. Components of the alliance have to be well distinguishable; they must contain well-defined interfaces. They also do not require identification as systems or automata. The membership of components within the alliance is typically dynamic one. Their members generally mediate holistic goals in alliances, if any.

The principles of the alliance forming have been explained via the concepts of information power (IP) and of multilingual translation efficiency respectively. An illustration of basic phenomena resulting in the emergence of alliance could be based on the concepts of interface sharing (IS), and of irregularities conjugation (IC) as well [2–6].

Many alliances can be represented in suitable language model. The language model of the alliance makes possible quantitative evaluation of its processes.

Basic ideas of this construct are as follows:

- Any module/system, which takes part in the alliance, is expressed in respective multi-grammar language. The relations within these modules are expressed via mutual translations of particular languages of modules elements.

- The relations of any module participating in alliance against the super – system (SS) are performed in mutual translation of boundary elements languages into the language of the respective SS.
- The mutual relations of the modules taking part in alliance are reflected in mutual translation of the languages of the respective boundary systems elements.
- The alliance manifests itself in more complete and/or more efficient translation of the alliance language into the language of SS in comparison with the independent translations of the languages of modules taking part in alliance.

Problems arise due to incompleteness of grammars of respective languages.

Synergic phenomena of IS and IC are so significant that they could be used as the definition characteristics of the alliance [5, 6]. An emergence of these phenomena results in the improvement of the regularity of the interfaces either mutually among the modules/systems which take part in the alliance or between the alliances as a whole and its neighborhood (environment, forming together with considered alliance the super – system SS). The secondary effect is an improvement of the efficiency of the resources utilization.

Interfaces (IF) are in fact the only parts of alliances which one has firmly keep under the control and which are almost identified in detail. Among interfaces, the significant position is hold by so-called critical interfaces (IFs). Critical interfaces within the alliance can be modeled via finite deterministic automata, or hard subsystems. These models make possible to take into consideration uncertainties which could be linked with irregularities of interfaces. The same models are capable to present the irregularities conjugation as well. Similar models can also represent the effects of alliance control.

More sophisticated and more powerful model of alliance interfaces stem from the concept of quantum-like subsystems [4, 7]. Quantum superposition of states or even the concept of entanglement are suitable tools how to record non - orthogonal interface parameters and resulting phase sensitivity of the respective IF (Fig. 1).

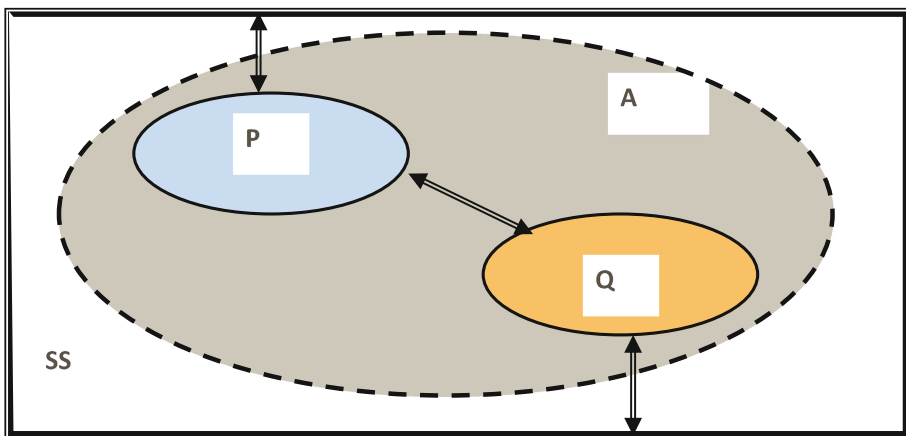


Fig. 1. Simplified presentation of systems alliance, consisting of two parts - “Amoeba model”; (P and Q are constituent elements - modules of alliance A; SS represents super – system, i.e. the environment the alliance exists within, the arrows indicate various types of relations between parts)

- The forming and existence of alliance is causally joined with the receiving and processing of information. In the dissipative environment it is the condition sine qua non for the plain existence of alliance.
- The impact of the information received can be measured utilizing the concept of IP, which is defined as the integral response of the alliance “systems time” to the information received [5].
- At this level, we cannot distinguish if the resulting effect is the randomizing or ordering one. In alliances, it means whether or not the sharing of interface increases or decreases its regularity. (It can be distinguished at the higher level of abstraction and within the frame of constructive approach only) [5, 11].
- The results of experiments on various real processes (laser cooling; traffic control; social preferences...) support the idea that the ordering can easily be flipped into chaos, et vice versa. The results are very sensitive on “phase” i.e. on the actual time delay with which information received is transformed into the sequence of events (run of system time).
- This knowledge resulted in the trials to construct “phase sensitive” systems modeling methodology, which could be able to respect this effect. There is probably not a matter of chance that this methodology has some significant similarities with the models of quantum physics.

2 Uncertainty in Alliances

Let turn the attention to more general situations when some systems alliances consist from parts being in principle either nonlinear or uncertain (or both). Such cases appear almost ever, when the complexity of alliance exceeds certain limit. This is typical namely for alliances consisting of a mixture of various kinds of components and involving as the elements living bodies e.g. human. In this case, one has to face strong uncertainty factor resulting from the extremely high complexity of human brain and inability to define firmly its actual state and properties. Actually, nobody is able to define actual state and structure of any human brain and there are (and never had been) two identical brains (in structure and in behavior). Moreover, one is not able to recognize perfectly not only the state of certain human brain, but also the stage of interaction in certain human group. Therefore, one must consider the interaction of human intelligence with technical components of some alliance as highly uncertain. Anyway, the uncertain within the alliance have to be considered also other factors, like the impacts of environmental influences, some functional dependences on impacts of various independent variables (namely the time).

As an example of one important factor appeared considerably newly found circa-dial dependence of the system controlling daily biological rhythms regulating the activity of more than 40% of human genes, which orchestrates the activity of eating, blood temperature and blood pressure. Now there is known, that almost any cell in the human body contains its own circa-dial clock machinery, the master circa-dial clock system then synchronizes almost all of them. A tiny brain region called the suprachiasmatic nucleus controls the level of hormones responsible for the control of sleep – wake

cycles. These Chrono - types differ so widely, that no two people have identical intervals of activity distribution. They differ about 8 h or even more. The sleep can cause numerous ill effects e.g. disturbances in distribution of the sleep hormone, melatonin.

3 Systems Alliance Cohesion

A grouping of components/systems creating an alliance has certain necessary attributes, particularly these:

- It forms a meta-structure
- Its parts/subsystems communicate with each other on a level of symbolic information
- Its parts are often complex systems showing “intelligent behavior”

Within the framework of intelligent behavior of the systems that create an alliance, one should hold the following typical features in mind:

- The subsystems have their own identity
- They create a model of the world
- They create a system of values for themselves
- They have goal-oriented behavior.

To use the term alliance most of these attributes have to be valid. One cannot say that a car is an alliance of mechanical, hydraulic and electrical systems. On the other hand, a pack of wolves hunting alone stag is an alliance. The particular members of the pack can live individually, they have common behavior and the particular members of the pack have to communicate with each other while hunting (of course nonverbally in this case).

The fundamental criteria for judging, whether the agreement of independent systems' behavior with the same goals is coincidental, or whether it is an alliance's coordinated behavior lies in the strength of bonds (joints) among a particular subsystems, i.e. in an exchange of information among them. The co-ordination means exchanging information. The strength of bonds among particular subsystems is in fact the rate of co-ordination - from zero co-ordination of mutually independent systems to the strongest co-ordination, when the particular subsystems are tied via deterministic function relations. In this case, we do not speak about the alliance but about the system built up from original alliance's members.

The question arises whether one cannot explicit the vague definition of the term being discussed with the help of formal methods measuring the strength of the bonds among particular subsystems creating the “possible alliance”, in other words whether one can identify the alliance with certain method?

4 Co-ordination Measure Quantification

Suppose that one has two systems and that one will evaluate the activities of both systems by observing or measuring the two quantities. The quantity X describes the behavior of the system A, while the quantity Y describes the behavior of the system B.

One can judge their coordinated synergy according to the behavior of both quantities. Of course, the number of quantities can be higher.

If the behaviors of the quantities are somehow similar, there will be some kind of co-ordination. Mostly the basic statistic methods are used and one measures the rate of similarity by the correlation coefficient. If it is small, one can say that the quantities are non-correlated, if it is close to one, one says that the quantities are strongly correlated. However, the term non-correlated does not mean independent. The often-mentioned correlation coefficient can be zero and the quantities are in a deterministic function relation. The situation introduced above can take place if the described relations are non-linear.

The principle of independence is defined precisely by the means of distribution of the quantities mentioned above.

If the quantities are independent, then it holds true:

$$P(X, Y) = P(X)P(Y),$$

where $P(X)$, $P(Y)$ are marginal probabilities of both quantities and $P(X, Y)$ is their joint combined probability.

If the relation:

$$P(X, Y) < P(X)P(Y),$$

is valid, then there is some kind of dependence between the quantities.

One can determine the strength of this dependence (the strength of the mutual bond) accurately if one determines what information of Y the X quantity bears, and vice versa. The fact, what information of Y the X quantity bears, is determined by the medium mutual information set by the relation:

$$T(X : Y) = H(X) + H(Y) - H(X, Y)$$

where $H(X)$ and $H(Y)$ are marginal entropies

$$H(X) = - \sum_{i=1}^n P(x_i) \log P(x_i)$$

and

$$H(X, Y) = - \sum_{i=1}^n \sum_{k=1}^m P(x_i, y_j) \log P(x_i, y_j)$$

is the combined joint entropy.

The medium mutual information can be extended on a multi-dimensional case of more variables. The medium mutual information is a non - negative symmetric but unfortunately it does not meet the triangle inequity and thus it does not have property of metrics and one cannot directly use it as a distance between the particular possibilities

of the alliances' ordering. As one will see below, it is possible to get around this disadvantage by means of a proper procedure.

How can one use the rate of mutual dependence for discovering mutual bonds among such subsystems that one would consider being an alliance? One can use a method that was developed in the general theory of systems many years ago for the detection or identification of the structure of a system [7–9].

Imagine a very simple system consisting of three variables X, Y, Z. One could call the system of these three quantities an alliance if all of them are bound with strong bonds, if the three of them all are mutually dependent. If the three of them all are independent or two of them are dependent and the third one independent, then one will not be able to speak about the alliance of three systems described by these quantities.

Let all of the possible orderings of dependencies among the mentioned quantities – the possible structures of the system be created.

- $K_0 = (X, Y, Z),$
- $K_1 = (X, Y; Y, Z),$
- $K_2 = (X, Z; Y, Z),$
- $K_3 = (X, Z; X, Y),$
- $K_4 = \{X, Y; Z\},$
- $K_5 = \{X, Z; Y\},$
- $K_6 = \{Y, Z; X\},$
- $K_7 = \{X; Y; Z\}.$

K_0 is the most complex system in this case; the system of three independent variables K_7 is the simplest system. One can order the particular possible structure in such a way that the elements of a more simple structure are subsets of the elements of a more complex structure. This means that element K_0 will have the highest rate of ordering while element K_7 will have the lowest one.

The ordered set with the biggest and the smallest element creates a union. If one demonstrates the ordering by means of a chart, one will get the so-called Hasse diagram of the union mentioned above (Fig. 2).

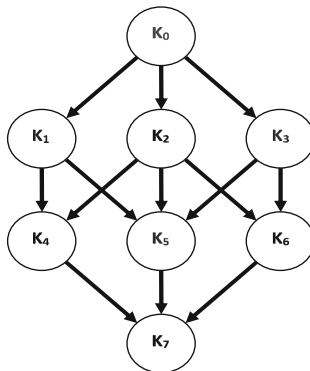


Fig. 2. Hasse diagram of the union of the candidates of structure

In this case, K_1 is a subset of K_0 , K_4 is a subset of K_1 and K_7 . If one moves along any edge of the Hasse diagram in the direction of the arrows, the following element will be always the subset of the preceding element. This feature of the union enables us to use medium mutual information for discovered structures. When moving along the edge of the Hasse diagram the medium mutual information will also meet - apart from the features mentioned above - the triangle inequity and has, therefore, the features of a metric. The difference between the medium mutual information, providing all three quantities are mutually independent, and a genuinely measured medium mutual information will show which candidate of the structure is closest to the candidate K_0 and thus one can consider this structure to be the searched structure of the system.

In practice it is processed in this way: one finds out the values of the combined joint probabilities $P(x_i, y_j, z_k)$, where x_i, y_j, z_k are particular classes of the values of these variables. If one summarizes over all i one will get the joint probability $P(y_j, z_k)$, so:

$$P(y_j, z_k) = \sum_{i=1}^n P(x_i, y_j, z_k)$$

Analogously one attains the marginal probabilities $P(y_j), P(z_k)$

$$P(y_j) = \sum_{k=1}^r P(y_j, z_k)$$

and

$$P(z_k) = \sum_{j=1}^m P(y_j, z_k)$$

In such a way, one can find all of the necessary marginal and joint probabilities so that one could set the medium mutual information for the particular candidates of the structure. From the measured joint probability, one set the medium mutual information for the candidate of the structure K_0 . The difference between the medium mutual information $T(K_0)$ and medium mutual information of any further candidate of the structure will be the distance of this candidate from the system with the structure being searched. Let remark that this way of looking for a structure is one of the oldest but also most illustrative. It is suitable for the identification of systems with a relatively small number of variables. In the course of time a large number of other methods has been developed that use somewhat different approaches and that are more effective in the case of a larger number of variables.

The essential question deals with the variables themselves. By now, one had silently assumed that the variables of the system are in metrological terminology metric variables. One deals with those variables mostly in physical disciplines. However, one can unambiguously assign numeric values to values of variables based on some measuring

procedure and a specific unit is defined. (One is not going to deal with a precise definition of a specific unit - the specific unit is for example weight, electric charge, power etc.). The measured values of these quantities can be easily selected according to their size into classes and one can estimate the probabilities of their incidence.

5 A Task of Empirical Data Evaluation

Here however another problem occurs. With the exception of theorizations, if one works with empirical data, one never knows their actual probabilities. One only estimate the probabilities based on empirically established relative frequencies. How the estimation of probability will differ from the real probability and how the estimation will be deflected depends on many circumstances: how many samples one will have for disposal, into how many classes one will divide the range of possible values that the relevant variable acquires, how many dimensional probabilities one will estimate etc. The deflection of the estimation for a small number of samples can cause quite large errors, which can lead to very incorrect images of the system's structure.

It is interesting that these methods were at first used in practice in the scientific disciplines like psychology, where the experimenters struggled with shortage of sufficient experimental data all the time. The problem of an accurate comparison of medium mutual pieces of information set based on inaccurate estimations of probabilities was transferred onto testing statistic hypotheses. E.g. "the quantities are independent" type towards the alternative "the quantities are not independent" ones by means of statistic tests (χ^2 - test, F-test) which one has a lot of experience with within the experimental research. This enables to decide how many samples one will need at a certain anticipated complexity of the system and a number of possible quantum levels of particular quantities.

These information measures based on probabilities are of a big advantage in that they are not limited only to metric quantities, which can be unambiguously quantified. In many economic systems, biological, psychological and similar systems, one encounters quantities whose values are only able to be ordered – ordinal quantities where for example, one of the most important quantities in economics – utility belongs.

One can say that this activity will bring a bigger utility than the other one but one is not able to say "by how much". There is no unit for utility. In humanistic and socio-economic systems are frequently encountered quantities, whose values one is not even able to order. Mostly one assigns mere verbal designation to the values and one speaks them as about the nominal quantities.

For example, in various sociological systems alliances the mood of the subject plays a role. If one has three degrees of mood "I am angry", "I am nervous", "I am weary", it is hard for us to find any ordering. (If one assigns numbers to these values, they mostly have meaning of mere designation. It is of no use making any mathematical operations with them. For instance, inventory numbers, numbers of health insurance etc.) Also in these cases, the entropic rates for searching for relations among the alliance's members work satisfactorily. They would work very well if one is able to

place the values of the mentioned nominal quantities into respective classes. These quantities, e.g. type of mood play an important part in the creation of electoral alliances of citizens, in market research etc. Nevertheless, classification of a certain mood into a proper class is on the one hand very subjective and on the other hand very vague.

Here one encounters a different type of uncertainty than statistic uncertainty, the certainty of vagueness type. One terms the values verbally but the uncertainty lies in the semantics of a verbal label. The uncertainty lies in what all one terms with the given word. In other words, the classes into which one classifies the particular values of nominal quantities do not have sharp limits. This type of uncertainty can be detected by means of fuzzy sets and one will show that the relations among variables can be searched for in the same way as was done by now. The basic idea of detection of vagueness by means of fuzzy sets lies in the fact that the uncertainty of belonging into a class is defined via the function of affiliation. One can formalize the semantics of a respective value of a nominal variable by means of a suitable function of affiliation. Then one works with the so-called linguistic variable.

6 Fuzzy Approaches

For example, the relation among the elements of alliances of stock market speculators, who by means of artificial demand or sale influence prices of shares, are among such variables. The relations are in the form of regulations (logic implications) of the type: “if the A company shares are rising and the B company shares are sluggish then let us buy the C company shares”. One can illustrate how it is possible to identify the existence of such relations and consequently the existence of alliances.

One will proceed from a classic definition of probability. The probability of the E phenomenon is defined as:

$$P(E) = \int_{\Omega} \chi_E(\omega) dP$$

where χ_E is a characteristic function of the set of E phenomena. Ω is a selective space and P is a probability rate.

At sharp phenomena, one assumes that the phenomenon with a certainty either belongs or does not belong into respective set (class of values). The characteristic function therefore gains values “0” or “1”.

The probability of the fuzzy phenomenon is:

$$P(F) = \int_{\Omega} \mu_F(\omega) dP$$

where the characteristic function for the fuzzy set is created by the function, which gains values from the interval (0, 1). The probability of the fuzzy value gives us the

probability of a random gaining uncertain value. For the discrete selective space $\Omega = \{\omega_1, \omega_2, \dots, \omega_n\}$ one will get for the probability of the fuzzy phenomenon

$$P(A) = \sum_{i=1}^n \mu_A(\omega_i)P(\omega = \omega_i)$$

Similarly as for sharp quantities, one can only estimate the probability from data. At crisp data, one has to estimate to be based on frequencies; at fuzzy data one estimates to be based on the so called pseudo-frequencies.

The pseudo-frequency fuzzy value incidence is defined as:

$$N(\omega_1) = \sum_{i=1}^n \mu_A^i(\omega_1)$$

The pseudo-frequency of values belonging into the fuzzy set A is a sum of degrees of pertinence with which the particular values belong into the fuzzy set. It will not be a problem to find the estimation for a one-dimensional pseudo-frequency. For searching for relations identifying a possible alliance one need to estimate the joint probabilities.

7 Discussion

The results of alliance behavior are very sensitive on “phase” i.e. on the actual time delay with which information received is transformed into the sequence of events (run of system time). This knowledge resulted in the trials to construct “phase sensitive” systems modeling methodology, which could be able to respect this effect. There is probably not a matter of chance that this methodology has some significant similarities with the models of quantum physics [10].

Now let turn the attention to more general situations, when some alliances consist of parts being either nonlinear or uncertain (or both). As it was already mentioned, such situations appear in the case the complexity of alliance exceeds (e.g. in number of its elements) certain limit. This is typical situation namely for alliances consisting of specific elements – human beings with their highly complex information systems based on neural systems/brains. Here one has to face strong uncertainty factor resulting from the extremely high complexity of each human brain and un-ability to define firmly its actual properties, inner states respectively.

However, as uncertain for systems alliance operation have to be considered also other factors, like the impacts of environmental influences, some functional dependences or impacts of various independent variables (namely the time) and also the change of requirements on alliance properties and behavior. The last aspect can be considered as typical especially for alliances involving the living components as their properties and behavior can change in the course of gaining their experience and level of their knowledge, which of course can be for each such component different [11].

New tools/models utilizing synergy of Computer Science (Big data, Internet of things), Artificial Intelligence (Holonc or neural networks) and Systems Theory (Complex Systems/Alliances behavior) can help solving these complex tasks of tackling omnipresent uncertainty.

The authors are convinced that the ideas, concepts and methodologies of uncertain systems alliances could be fruitful also for the studies and applications of agent/holonc nets.

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