Conflict Avoidance Within Max-Plus Fault-Tolerant Control: Application to a Seat Assembly System



Marcin Witczak D, Paweł Majdzik D, Bogdan Lipiec D and Ralf Stetter D

Abstract Flexibility and agility are central requirements for future manufacturing systems (especially assembly systems), because in most industries the product variety and the fluctuations in demand are still increasing. An increase of the degree of flexibility allows more efficient activities aiming at following the dynamically evolving markets. Such systems should be able to react to changes of product, demands, increased varieties of products requirements concerning reduced delivery times and increased product quality. Therefore, a strong focus on the flexibility of manufacturing and assembly systems leads to economic advantages for industrial companies in terms of the system investment cost. In particular, the cost related to the reconfiguration of the system.

1 Introduction

In order to fulfill all above requirements, Flexible Manufacturing Systems (FMSs) have to contain typical layers, such as devices layer (industrial robots, conveyor belts, vision systems, sensors etc.), control layer (robot controllers, programmable automation controllers, inverters), and visualization layer (human interface machine).

M. Witczak (🖂) · P. Majdzik · B. Lipiec

Institute of Control and Computation Engineering, University of Zielona Góra, 50 Podgórna Str., 65-246 Zielona Góra, Poland e-mail: m.witczak@issi.uz.zgora.pl

P. Majdzik e-mail: p.majdzik@issi.uz.zgora.pl

B. Lipiec e-mail: b.lipiec@issi.uz.zgora.pl

R. Stetter Faculty Mechanical Engineering, University of Applied Sciences Ravensburg-Weingarten, Weingarten, Germany e-mail: ralf.stetter@hs-weingarten.de

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Manufacturing and assembly systems which employ Automated Guided Vehicles (AGVs) are one important means for enabling flexible operation and agile reconfiguration. However, such systems only allow a smooth and economical operation, if elaborate control and diagnosis systems are present. Today, the Model Predictive Control (MPC) was identified as prominent control concept addressing this challenge, because both the model and the control commands can be continuously updated by using the moving horizon approach. The application of MPC produces two main advantageous aspects. The first advantageous aspect is that the MPC delivers a design procedure for the controller and that it easily can be tuned. The second advantageous aspect is that MPC is able to deal with constraints concerning the inputs and outputs of complex systems. The specific advantages of MPC allow meeting the various precedent requirements regarding manufacturing and assembly processes in industrial companies systems [1-3]. On a certain control level, it is sensible to describe such manufacturing and assembly systems as Discrete Event Systems (DESs) [4]. DESs are event-driven dynamical systems whose state transitions of a DES indicate the physical phenomenon that causes the change in state [5, 6]. One of the primary approaches to evaluate the performance of FMSs is the simulation. The most important advantages of this approach are that it can be used for arbitrary classes of DESs, however this approach requires tedious simulation runs and cannot provide an understanding of the dependence of parameters. The other approach allows calculating and analyzing the system performance using an algebraic model, e.g. max-plus algebra model.

This chapter illustrated a novel control scheme that is based on the general idea to apply the max-plus algebra for MPC (compare [7]). Up to now this idea only allowed to describe DESs without resource conflicts. The novel control scheme presented in this chapter also allows to control DES with such conflicts. It is another advantageous aspect that the novel scheme includes a representation of uncertain discrete event systems which are influenced by internal and unobservable events (compare [8]). Additionally, the scheme includes an active fault-tolerant control framework which allows to identify faults and to accommodate these faults accordingly [9]. It is important to note that the application of this kind of elaborate control and diagnosis scheme can be enabled and eased it the control and diagnosis system is consciously designed. Guidelines for this kind of design can be summarized under the notion "Design for Control"; the next section is focusing on this topic.

The most important features related to the design for control are presented in Sect. 2. In Sect. 3 an overview of the assembly system is given. The modelling of the assembly system is described in Sect. 4. Section 5 presents modelling of AGVs, and in Sect. 5.2 describe predictive control of two AGVs. Section 6 is devoted to fault tolerant control.

2 Design for Control

In recent years, research projects were initiated which aim at the development of design guidelines which aim at support the development of technical systems that enable and ease control. These guidelines were summarized under the notion "Design for Control (DfC)". The term "Control" includes a large number of different activities with the aim to manage, command, direct or regulate the behaviour of technical systems. One example for a DfC guideline is the recommended use of over-actuation, i.e. the application of more or stronger actuators than directly necessary [10]. It is rather obvious that the possibility for control actions is enhanced by means of this kind of over-actuation. The focus of this chapter is a complex system with many AGVs. Today, complex systems are usually realized with modules which reduce the complexity and allow reuse. One insight concerning the DfC is that structures should be congruent, i.e. for modular technical system also the control system should be modular [10]. Another important insight is that modules should contain local intelligence for local control loops [10]. For the operation of AGVs it is sensible to distribute control and diagnosis tasks to the individual AGVs and even their components in order to optimize control and diagnosis speed and to avoid excessive requirements for the communication between AGVs. Additionally, in order to reduce complexity, it is sensible to realize planning, control and diagnosis systems with certain hierarchies (compare [11]). Figure 1 shows a hierarchical and distributed control and diagnosis concept with an FTC-MPC layer.

Meanwhile it is an established fact that control and diagnosis systems will only be applied in industrial companies, if they are an integral part of the production system



Fig. 1 Hierarchical and distributed control and diagnosis concept

information infrastructure (compare [11]), namely the Enterprise Resource Planning (ERP) system and Manufacturing Execution System (MES). Enterprise resource planning (ERP) is an integrated computer-based system used to manage internal and external resources including tangible assets, financial resources, materials, and human resources. On the next level below are Manufacturing Execution Systems (MES). This level takes this planning output of the ERP and executes this plan in the production. A fault-tolerant control system, as proposed in this chapter, needs to communicate with the MES.

The rapid development of information technology of the last decades enables to intensify the collection and processing of all kinds of data and information in production systems. The culmination of this data collection and processing is the so-called "Digital Twin" of the production system. A digital twin can be defined in the following manner [12]: a digital twin is an integrated multi-physics, multi-scale, probabilistic simulation of a complex technical system which employs the best available physical models, sensor readings, sensor information updates, etc., to mirror the life of its corresponding twin—the real technical system. A digital twin consists of three parts [13]:

- the original technical system in real space,
- a digital product in a virtual space and
- the connection of data and information which links the two spaces.

Digital twins are virtual images of physical objects or systems. Digital Twins of manufacturing and assembly systems significantly contribute to the required transparency and to near real-time production control [14]. The compulsory precursor of the digital twin is the Internet of Things (IoT). Digital Twins dispose of four essential entities:

- sensors which allow a detailed, far reaching monitoring of current status
- · connectivity, which realizes a networks between the modules of the systems
- · defined data structures enabling analytic functionalities
- a user interface that visualizes the relevant data and information

Examples for realized digital twins are digital twin driven product manufacturing in shop floors and product services [15]. The Digital Twins concept and its additional digital functions enable the monitoring and control of real counterparts—real technical systems. In addition, digital twins communicate with each other and with higher architectural levels. For the AGV system under consideration, this general concept is shown in Fig. 2.

In Fig. 2 it is visible that the hierarchical and distributed control structure of the original technical system is represented in the digital twin. A continuous update is necessary at all levels. Between the levels information flows are present such as sensor readings and assignments (here the term "assignments" is a general term for information or commands such as required schedules or arrival times at certain points in spaces). These information flows also have to be represented in the digital twin and require continuous update.



Fig. 2 Digital twin

This chapter concentrates on a fault-tolerant control framework that is located on the highest level of Fig. 2. In the next sections the exemplary system is illustrated and the modelling possibilities for this level are explained.

3 Overview of the Assembly System

The considered manufacturing system consists of two main parts (see Fig. 3). The first part constitutes an assembly system that produces the car's seats. The second one is a transportation system that transmit the seats from the assembly system to the high storage warehouse. One of the most flexible transport means for in-plant transportation are AGVs. AGVs dispose of further advantageous characteristics such as comparatively low investment costs and relatively small expenditures for elements of the plant infrastructure. The objective of this section is to describe the individual production tasks in the assembly system (see Fig. 4). The assembly system can be considered as the system belonging to the class of DES. The entire description of DES should contain the following parameters:

- *k* event counter;
- R_i *i*th processing unit;
- d_i ith processing time;
- $u_i(k)$ time instant at which the product is transferred to the system's *i*th input;
- $x_i(k)$ time instant at which *i*th processing unit starts to carry out a demanded task;
- $y_i(k)$ time of delivering the *i*th product;
- $t_{i,j}$ time for transportation from the *i*th to the *j*th processing unit;
- $t_{in,i}$ time for transportation from the *i*th input storage to the *i*th processing unit.



Fig. 3 Overview of the manufacturing system



- R_1 assembly station lower seat frame, R_2 assembly station back rest seat,
- R_3 assembly station conection frame and back rest,
- R_4 station assessment of quality,



Experts in automotive industry expect a profound change of the use of cars in the next decade. The next levels of autonomous driving will enable drivers and passengers to use the space in their cars in a completely different manner. This influences nearly all components of the care interior and especially the seats. Future seat will need to dispose of integrated safety systems such as seat integrated belts and airbags. Additionally, even more comfort features such as climate control, massage functions and personal audio will be integrated in the seats. This will lead to heavier seats which require more space. This will also lead to changes in-factory transportation systems. One possibility to address the difficult transportation tasks of future seats are AGVs, which are flexible enough for a large product variety and fluctuations of demand. A prospective seating assembly systems is shown in Fig. 4. The process starts with two parallel assembly stations-one for the assembly of the lower part of the seat (resource R_1 with processing time d_1) and one for the assembly of the back rest (resource R_2 with processing time d_2). Both parts are then delivered to a common assembly station which connects the two parts (resource R_3 with processing time d_3). Subsequently, seats are transported to the station 4 (R_4 with processing time d_4), where the quality of final products are checked. The finished seats are then

transported to the storage zone. From this station, several AGVs transport the seats to the loading zone (i.e. the automatic warehouse).

Having the precedent formal description of the system and the mathematical background that is introduced in next section, it is possible to determine the mathematical model of the assembly system and the redundant AGVs.

4 Modelling of the Assembly System

This section explains the main mathematical concepts describing the max-plus algebra formalism and to present the max-plus algebra linear space equation of the assembly system (described in the previous section, Fig. 4).

4.1 Max-Plus Algebra Formalism

It is possible to define the basic structure of the so-called max-plus algebra (\mathbb{R}_{max} , \oplus , \otimes) as formulated subsequently:

$$\mathbb{R}_{max} \triangleq \mathbb{R} \cup \{-\infty\},\$$

$$\forall a, b \in \mathbb{R}_{max}, a \oplus b = \max(a, b),$$

$$\forall a, b \in \mathbb{R}_{max}, a \otimes b = a + b,$$

(1)

where \mathbb{R}_{max} is the field of real numbers.

The first operator \oplus describes the max-plus algebraic addition while the second operator \otimes stands for the max-plus algebraic multiplication.

The fundamental characteristics of these max-plus algebra operators may be formulated in the subsequent form:

$$\forall a \in \mathbb{R}_{max} : a \oplus \varepsilon = a \text{ and } a \otimes \varepsilon = \varepsilon,$$

$$\forall a \in \mathbb{R}_{max} : a \otimes e = a,$$
 (2)

In these equations $\varepsilon = -\infty$ and e = 0 act as neutral elements for both the max-plus algebraic addition and for the max-plus algebraic multiplication operators.

It is important to note that the max-plus algebra operations are associative, commutative and distributive in the same manner as in conventional algebra. Thus, the subsequent properties can be formulated: associativity of addition $\forall a, b, c \in \mathbb{R}_{max} : (a \oplus b) \oplus c = a \oplus (b \oplus c),$ commutativity of addition $\forall a, b \in \mathbb{R}_{max} : (a \oplus b) = b \oplus a,$ (3) associativity of multiplication $\forall a, b, c \in \mathbb{R}_{max} : (a \otimes b) \otimes c = a \otimes (b \otimes c)$

u solution for multiplication for $u, v, v \in \mathbb{C}$ as $max : (u \otimes v) \otimes v = u \otimes (v \otimes v)$

Two important aspects of max-plus algebra are that it does not have additive inverses and it is idempotent. This is why max-plus algebra is considered a semiring and not a ring. For matrices $X, Y \in \mathbb{R}_{max}^{m \times n}$ and $Z \in \mathbb{R}_{max}^{n \times p}$

n

$$(X \oplus Y)_{ij} = x_{ij} \oplus y_{ij} = \max(x_{ij}, y_{ij}),$$
(4)

$$(\mathbf{X} \otimes \mathbf{Z})_{ij} = \bigoplus_{k=1} x_{ik} \otimes z_{kj} = \max_{k=1,\dots,n} (x_{ik} + z_{kj}).$$
(5)

The publications [16, 17] contain further details and definitions concerning the formalism of max-plus algebra.

4.2 Max-Plus Linear System

One of the challenges of the work with DESs has its origin in the fact that DESs necessitate a non-linear description, if they are modelled in conventional algebra. Nevertheless, it was possible in recent years to find a specific class of DES that are named max-plus linear systems. Linear max-plus models only enable the synchronization of tasks but do not allow an occurrence of concurrency. Consequently, DESs can be modelled in the subsequent form employing the max-plus algebra formalism:

$$\boldsymbol{x}(k+1) = \boldsymbol{A} \otimes \boldsymbol{x}(k) \oplus \boldsymbol{B} \otimes \boldsymbol{u}(k+1), \tag{6}$$

$$\mathbf{y}(k) = \mathbf{C} \otimes \mathbf{x}(k),\tag{7}$$

the index *k* serves as event counter and:

$\mathbf{x}(k) \in \mathbb{R}_{max}^n$	designates the state, which contains the time instants corresponding
	to the internal events occurring at k ,
$r(L) = \mathbb{D}^{r}$	designates the input vector which contains the time instants come

- $u(k) \in \mathbb{R}^{r}_{max}$ designates the input vector, which contains the time instants corresponding to input events occurring at k,
- $y(k) \in \mathbb{R}_{max}^{m}$ designates the output vector, which contains the time instants corresponding to the output events occurring at *k*,
- $A \in \mathbb{R}_{max}^{n \times n}$ designates the state transition matrix, $B \in \mathbb{R}_{max}^{n \times r}$ designates the control matrix and $C \in \mathbb{R}_{max}^{m \times n}$ designates the output matrix.

The basic challenge in the development of described assembly system is to design and implement an appropriate synchronization rules for all tasks, both processing and transportation tasks. Generally, two essential synchronization modes, i.e. a mutual exclusion mode and a rendez-vous mode, can be distinguished. The mutual exclusion mode requires that at the same time only one task can perform its operation on the shared resource. The randez-vous mode involves the case where two or more tasks have to finish its operations so that the next operation can start its performance.

In the system described above, two of the modes of synchronization rules, which were mentioned earlier in this chapter, are present. The first mode of synchronization rules describes the phenomenon that any processing unit may start performing its intended operation on a next product (in the k + 1th iteration) as soon as the earlier processing operations on the previous product have been successfully carried out (in the *k*th iteration). This mode of synchronization concerning the R_1 unit (see Fig. 4) can be expressed by the subsequent equation:

$$x_1(k+1) = \max(x_1(k) + d_1, \ u_1(k+1) + t_{in,1})$$
(8)

It is obvious that this kind of synchronization has to applied for each assembly station in the system (9).

The second mode of synchronization represents the randes-vous mode and concerns the unit R_3 (see Fig. 4) that is used by tasks from two assembly cycles. Taking into account the structure of described system, the operations on R_1 and R_2 have to be finished in order to the assembly operation on unit R_3 can be started. This mode of synchronization is represented by:

$$x_3(k+1) = \max(x_1(k+1) + d_1 + t_{1,3}, x_2(k+1) + d_2 + t_{2,3}, x_3(k) + d_3)$$

In one takes the preceding assumptions as well as the modes of synchronization into consideration, it is possible to describe the system from Fig. 4 using the following model:

$$\begin{aligned} x_1(k+1) &= \max(x_1(k) + d_1, \ u_1(k+1) + t_{in,1}) \\ x_2(k+1) &= \max(x_2(k) + d_2, \ u_2(k+1) + t_{in,2}) \\ x_3(k+1) &= \max(x_1(k+1) + d_1 + t_{1,3}, \ x_2(k+1) + d_2 + t_{2,3}, \ x_3(k) + d_3) = \\ \max(x_1(k) + 2d_1 + t_{1,3}, \ x_2(k) + 2d_2 + t_{2,3}, \ x_3(k) + d_3, \\ u_1(k+1) + d_1 + t_{in,1} + t_{1,3}, \ u_2(k+1) + d_2 + t_{in,2} + t_{2,3}) \\ x_4(k+1) &= \max(x_3(k+1) + d_3 + t_{3,4}, \ x_4(k+1) + d_4) = \\ \max(x_1(k) + 2d_1 + d_3 + t_{1,3} + t_{3,4}, \ x_2(k) + 2d_2 + d_3 + t_{2,3} + t_{3,4}, \\ x_3(k) + 2d_3 + t_{3,4}, \ x_4(k+1) + d_4), \ u_1(k+1) + d_1 + d_3 + t_{in,1} + \\ t_{1,3} + t_{3,4}, \ 2(k+1) + d_2 + d_3 + t_{in,2} + t_{2,3} + t_{3,4}) \end{aligned}$$
(9)
$$\bar{y}(k) = x_4(k) + d_4 + t_{out,1}$$

One may also describe the equations listed above using a compact form (6)–(7) while a detailed description of the system matrices is given in (10).

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$$A = \begin{bmatrix} d_{1} & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & d_{2} & \varepsilon & \varepsilon \\ 2d_{1} + t_{1,3} & 2d_{2} + t_{2,3} & d_{3} & \varepsilon \\ 2d_{1} + d_{3} + t_{1,3} + t_{3,4} & 2d_{2} + d_{3} + t_{2,3} + t_{3,4} & 2d_{3} + t_{3,4} & d_{4} \end{bmatrix},$$

$$B = \begin{bmatrix} t_{in,1} & \varepsilon \\ \varepsilon & t_{in,2} \\ d_{1} + t_{in,1} + t_{1,3} & d_{2} + t_{in,2} + t_{2,3} \\ d_{1} + d_{3} + t_{in,1} + t_{1,3} + t_{3,4} & d_{2} + d_{3} + t_{in,2} + t_{2,3} + t_{3,4} \end{bmatrix},$$

$$C = \begin{bmatrix} \varepsilon & \varepsilon & d_{4} + t_{out,1} \end{bmatrix},$$

$$(10)$$

In consideration of the fact that an analytical description of the system is present, the processing and transportation times can be incorporated within an analytical description (Eq. (11)), which are: $d_1 = 1$, $d_2 = 2$, $d_3 = 2$, $d_4 = 1$, $t_{1,3} = 4$, $t_{2,3} = 1$, $t_{3,4} = 2$, $t_{in,1} = 2$, $t_{in,2} = 1$, $t_{out,1} = 2$.

$$A = \begin{bmatrix} 1 & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & 2 & \varepsilon & \varepsilon \\ 6 & 5 & \varepsilon & \varepsilon \\ 10 & 9 & 6 & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 2 & \varepsilon \\ \varepsilon & 1 \\ 7 & \varepsilon \\ 11 & 8 \end{bmatrix}, \qquad C = \begin{bmatrix} \varepsilon & \varepsilon & \varepsilon & 4 \end{bmatrix}$$
(11)

4.3 Handling Constraints

For the sake of describing the full functionality of the system, it is inevitable to generate a set of constraints which limit system behavior. The constraints of the system can be described in the subsequent form:

• The first constraint describes the fact that the system has to follow a predefined trajectory. It is possible to define this trajectory by employing scheduling constraints of the subsequent form:

$$t_{ref,j}(k) \ge x_j(k), \quad j = 1, \dots, n.$$
 (12)

In this expression, $t_{ref, j}(k)$ stands for the upper bound of $x_j(k)$ at the time instant k.

• The second constraint is directly linked to the second mode of synchronization. It facilitates the avoiding of tasks which are waiting (see Sect. 4.2):

$$\forall i \in \{1, 2, \dots, n\} \ \left((x_i(k+1) - (x_i(k) + d_i)) \le 0 \right), \tag{13}$$

In this expression, n denotes the size of system; this size is equal to the number of present processing units.

• The third constraint is directly linked to the performance of the AGV,

$$\bar{u}_i \ge u_i(k+1) - u_i(k), \quad i = 1, \dots, r.$$
 (14)

It is important to note that the upper bound \bar{u}_i stands for the maximum velocity the AGV may achieve. A crossing of this limit may lead to a dramatically increase of the energy consumption of the drives of the AGV.

• The fourth and last constraint is concerning the change rate:

$$u_j(k+1) - u_j(k) \ge z_j, \quad j = 1, \dots, r.$$
 (15)

In this expression, $z_i > 0$ designates the upper bound of the change rate.

One additional obvious constraint is the fact that the time to reach any individual assembly station for k + 1 needs to be larger than or at least equal to the one for k.

4.4 Constrained Model Predictive Control

Current industrial production systems require constraints and certain control quality measures. One central advantage of MPC is its natural ability of dealing with constraints, therefore it is an ideal candidate to address the challenges of current industrial production systems. The framework, which is proposed in this chapter, could be developed on the basis of the general MPC strategy for max-plus linear systems described in [7]. In the proposed scheme, MPC and max-plus algebra are applied in order to reduce the number of conflict tasks. The core of the problem is to find the input sequence $u(k), \ldots, u(k + N_p - 1)$ minimizing the cost function J(u)

$$J(u) = -\sum_{j=0}^{N_p - 1} \sum_{i=1}^r q_i u_i (k+j),$$
(16)

where $q_i > 0$, i = 1, ..., m denotes a positive weighting constant, while N_p designates the prediction horizon. It is a core advantage of (16) compared to the quadratic criteria employed in the case of continuous systems that no time-consuming quadratic programming is required. On the contrary, an efficient linear programming framework may be applied, because of the linear constraints (12)–(15).

The first inevitable step that leads to a possible computational framework is to make sure that no direct influence of $x(k + 1), ..., x(k + N_p - 1)$ to the scheduling constraints (12) exists. In order to achieve this, let:

$$\tilde{\boldsymbol{x}}(k+N_p-1) = \boldsymbol{M} \otimes \boldsymbol{x}(k) \oplus \boldsymbol{H} \otimes \tilde{\boldsymbol{u}}(k), \tag{17}$$

where

$$\tilde{\boldsymbol{u}}(k) = \begin{bmatrix} \boldsymbol{u}(k+1) \\ \boldsymbol{u}(k+2) \\ \vdots \\ \boldsymbol{u}(k+N_p-1) \end{bmatrix}, \quad \tilde{\boldsymbol{x}}(k+N_p-1) = \begin{bmatrix} \boldsymbol{x}(k+1) \\ \vdots \\ \boldsymbol{x}(k+N_p-1) \end{bmatrix}. \quad (18)$$

On the basis of the description of the DES formulated in (6)–(7), it may be shown that:

$$H = \begin{bmatrix} B & \varepsilon & \cdots & \varepsilon \\ A \otimes B & B & \cdots & \varepsilon \\ \vdots & \vdots & \ddots & \vdots \\ A^{\otimes N_p - 2} \otimes B & A^{\otimes N_p - 3} \otimes B & \cdots & B \end{bmatrix}, \qquad M = \begin{bmatrix} A \\ A^{\otimes 2} \\ \vdots \\ A^{\otimes N_p - 1} \end{bmatrix}.$$

It is possible to formulate the intended optimization strategy in a straight-forward manner. An initial condition x(k) needs to be determined. Starting from this condition, the optimal input sequence $\tilde{u}(k)^*$ may be found by means of solving:

$$\tilde{u}(k)^* = \arg\min_{\tilde{u}(k)} J(u), \tag{19}$$

considering the constraints (12)–(15).

All associated constraints need to be provided, before the scheme can be applied to the assembly system (Fig. 2). The first logical step are the scheduling constraints:

$$t_{ref}(0) = [1, 2, 7, 11]^{T},$$

$$t_{ref}(1) = [3, 3, 8, 12]^{T},$$

$$\vdots$$
(20)

The prediction horizon was set to $N_p = 4$ along with $q_1 = q_2 = 1$ shaping the cost function (16) The goal of this example is to show the performance of the scheme in case of a chosen schedule (t_{ref}) under a resource conflict. As can be observed in Fig.6, a conflict on R_3 arises for k = 4. The proposed scheme along with suitable constraints allows an appropriate control of the process tasks by optimized acceleration/deceleration of the pre-product providing time to R_1 from u_1 and/or to R_2 from u_2 . While analyzing Fig.5, it can be observed that an appropriate control of u_1 at k = 3 allows avoiding the conflict described above.



Fig. 5 Evolution of control variable with proposed strategy (dashed line) and without it (solid line)



Fig. 6 The states $x_3(k)$ and $x_4(k)$ with MPC (dashed line) and without it (solid line)

5 Modelling of the AGVs

As it was described in Sect. 3, the overall system consists of two parts, where the second one constitutes the transportation system that is based on AGVs. AGVs are responsible for delivering given final products (seats) from the assembly outlet towards appropriate point of the warehouse. The warehouse has high-rise shelves on which pallets with products are stored. Between the shelfs are aisles for automated

forklifts. The advantages of such high storage warehouse are: good access to articles, economical use of space and pressure-avoiding storing of the goods ([18]).

The unique design of the AGVs allows unlimited manoeuvring possibilities (see [19] for a comprehensive explanation). AGVs system ensures the high flexibility and relatively large fault-tolerance. They can theoretically drive in the zone in front of the warehouse and can supply and receive products on palettes to and from dedicated transfer stations. The feeding system consists of three control levels with a hierarchical control structure. The lowest level controls the continuous base-line including physical and virtual sensors. An middle control level is applied for detailed path planning. The highest control level called "supervisory control level" is responsible for dispatching AGVs and for controlling transportation times. This supervisory control level is in the core subject of the research described in this section.

Because of safety requirements, AGVs have to move along a designed lanes which are intended to forward and backward movement.

$$\mathbb{M}(k) = [c(k), b(k), d(k), p(k)],$$
(21)

where:

- c(k) denotes the item packing and transportation time from the outlet of the production system to p(k) transfer station;
- b(k) denotes the item unpacking and transportation time from p(k) transfer station to the production outlet;
- d(k) is the minimum acceptable time difference between delivering k 1th item to p(k 1) transfer station and kth item to p(k) transfer station, respectively;
- p(k) is a unique number identifying the transfer station, i.e., $p(k) \in \{1, ..., n_s\}$ where n_s is the number of transfer stations.

Moreover, the sequence of the items which have to be transported from the production outlet to the transfer stations are supplied by MES:

$$\mathbb{M}(0), \mathbb{M}(1), \dots, \mathbb{M}(N_p - 1), \tag{22}$$

where N_p stands for production horizon. It should be noted that each *k*th item have to be delivered to the p(k) transfer station according to an assumed time schedule:

$$x_{ref}(0), x_{ref}(1), \dots, x_{ref}(N_p - 1),$$
 (23)

In order to achieve this aim, the schedule of n_v AGVs has to be dispatched with along a sequence of item outlet delivery times:

$$y(0), y(1), \dots, y(N_p - 1).$$
 (24)

These delivery times stand for the time of providing the kth item at the outlet of the production system. In this chapter, the performance of the AGV-based transportation system is measured in the following form:

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$$J(y) = -\sum_{k=0}^{N_p - 1} y(k).$$
 (25)

This preceding function needs to be minimized taking into consideration the scheduling constraint (23) while also considering the overall performance of the AGVs. Resulting from this, the largest possible sum of (24) needs to be obtained, which guarantees the satisfaction of (23). It is important to note that (25) may also be defined in a different fashion, e.g. by means of allowing the maximization of the consecutive differences y(k + 1) - y(k). This arrangement may provide the maximum spread between consecutive item outlet delivery times. For the sake of simplicity and clarity, this chapter concentrates on an transportation system consisting of two AGVs.

An mathematical description of two AGVs has to be defined employing an extended max-plus algebra which can be based on the max-plus algebra presented in Sect. 4. Additionally, for obtaining a sequence (24) which maximizes (25) taking into consideration the scheduling constraint (23), the model predictive control employing the max-plus algebra description is used. The preceding approach assumes that the actual transportation times of the *i*th AGV, which carries the *k*th item, are equal to their nominal values, even though the second AGV transportation times are set to zero, i.e.:

if
$$c_1(k) = c(k)$$
, $b_1(k) = b(k)$ then $c_2(k) = 0$, $b_2(k) = 0$ (26)

if
$$c_2(k) = c(k)$$
, $b_2(k) = b(k)$ then $c_1(k) = 0$, $b_1(k) = 0$ (27)

Transportation delays for which the actual measured transportation times $c_i(k)^m$ and $b_i(k)^m$ are not equal c(k) and b(k) respectively, are considered to be *faults*. This process may be formally described as:

if $b_k(k)^m = b(k)$ then $f_{i,k}(k) = 0$

if
$$c_i(k)^m = c(k)$$
 then $f_{i,c}(k) = 0$
else $f_{i,c}(k) = c_i(k)^m - c(k)$ (28)

else
$$f_{i,b}(k) = b_i(k)^m - b(k)$$
 $i \in \{1, 2\}$ (29)

5.1 Mathematical Description of AGVs

This section aims to deliver a mathematical description which will enable the faulttolerant control of the AGV system. The core of this section is a mathematical description of twin AGVs which allows a real-time determination of their time schedule on a given horizon N_p . The initial step can be a definition of the main variables:

 $x_i(k)$ denotes the time instant at which the *i*th AGV is ready to transport the *k*th item, $i \in \{1, 2\}$;

- $x_3(k)$ denotes *k*th item delivery time at the p(k) transfer station;
- $v_i(k)$ denotes decision variable that associates *i*th AGV with the transportation of *k*th item; $v_i(k) \in \{e, \varepsilon\}, i \in \{1, 2\}.$

Note that $v_i(k) = e$ means that the *i*th AVG transports the *k*th item while $v_i(k) = \varepsilon$ means an opposite situation. On the basis of the variables, which were defined precedently, the time-evolution of $x_i(k)$ for each AGV can be described in the subsequent form:

$$x_1(k) = \max(x_1(k-1) + b_1(k-1) + c_1(k-1), y(k) + v_1(k)),$$

$$x_2(k) = \max(x_2(k-1) + b_2(k-1) + c_2(k-1), y(k) + v_2(k)).$$
(30)

with the associated constraints

$$b_{1}(k) = \max(e, b(k) + v_{1}(k)),$$

$$b_{2}(k) = \max(e, b(k) + v_{2}(k)),$$

$$c_{1}(k) = \max(e, c(k) + v_{1}(k)),$$

$$c_{2}(k) = \max(e, c(k) + v_{2}(k)).$$

(31)

and

$$v_1(k) = e \Leftrightarrow v_2(k) = \varepsilon$$

$$v_2(k) = e \Leftrightarrow v_1(k) = \varepsilon$$
(32)

Note that from (42) follows that only one AGV, i.e. *i*th AGV can transport *k*th item from the production outlet towards p(k)th transfer station. Subsequently, the *k*th item delivery time at p(k)th transfer station obeys:

$$x_3(k) = \max(x_1(k) + c_1(k) + v_1(k), x_2(k) + c_2(k) + v_2(k), x_3(k) + d_3(k))$$
(33)

On the basis of (31) it is possible to show that

$$c_{1}(k) + v_{1}(k) = \max(e, c(k) + v_{1}(k)) + v_{1}(k) = c(k) + v_{1}(k)$$

$$c_{2}(k) + v_{2}(k) = \max(e, c(k) + v_{2}(k)) + v_{2}(k) = c(k) + v_{2}(k)$$
(34)

and for this reason (35) can be condensed to:

$$x_3(k) = \max(x_1(k) + c(k) + v_1(k), x_2(k) + c(k) + v_2(k), x_3(k-1) + d_3(k))$$
(35)

Through a detailed analysis of (30)–(35), it becomes obvious that the only employed mathematical operators are + and max. On that account, among the different available DES modelling techniques [20–22], the max plus algebra [7, 23] is apparently

the most suitable one. Employing the preceding notation, it may be proposed to reformulate the model (30)–(35) into the subsequent form:

$$x(k) = A(v(k-1), v(k), k) \otimes x(k-1) \oplus B(v(k), k) \otimes y(k),$$
(36)

where: $\mathbf{x}(k) = [x_1(k), x_2(k), x_3(k)]^T$, $v(k) = [v_1(k), v_2(k)]$, $\mathbf{A}(v(k-1), v(k), k) \in \mathbb{R}_{max}^{n \times n}$ and $\mathbf{B}(v(k), k) \in \mathbb{R}_{max}^{n \times r}$ stand for the state transition matrix and the control matrix, respectively.

For a convenient application (and with a slight abuse of the notation conventions) the above matrices will be denoted by $A_v(k)$ and $B_v(k)$. On that account, substituting (30) into (35) leads to:

$$x_{3}(k) = \max(x_{1}(k-1) + b_{1}(k-1) + c_{1}(k-1) + c_{1}(k) + v_{1}(k),$$

$$x_{2}(k-1) + b_{2}(k-1) + c_{2}(k-1) + c_{2}(k) + v_{2}(k),$$

$$y(k) + c(k) + v_{1}(k), y(k) + c(k) + v_{2}(k),$$

$$y(k) + c(k) + v_{3}(k), \dots, y(k) + c(k),$$

$$x_{3}(k-1) + d(k))$$
(37)

Combining (30) and (37) makes it possible to derive the matrices $A_v(k)$ and $B_v(k)$ that are given by (38)

$$A_{\nu}(k) = \begin{bmatrix} b_{1}(k-1) + c_{1}(k-1) & \varepsilon & \varepsilon \\ \varepsilon & b_{2}(k-1) + c_{2}(k-1) & \varepsilon \\ b_{1}(k-1) + c_{1}(k-1) + c_{1}(k) + v_{1}(k) & b_{2}(k-1) + c_{2}(k-1) + c_{2}(k) + v_{2}(k) & d_{3}(k) \end{bmatrix},$$

$$B_{\nu}(k) = [v_{1}(k), v_{2}(k), c(k)]^{T}$$
(38)

5.2 Model Predictive Control of Two AGVs

The main focus of this section is the generation of a sequence (24), which maximizes the cost function (25) taking into account the scheduling constraint (23). This section concerns the determination of the item delivery time sequence (24) for a predefined production horizon N_p . This item delivery time sequence minimizes (25) and should be determined considering both the scheduling constraints (23) and the performance of a set of n_v AGVs.

The developed framework employs a general MPC paradigm for max-plus linear systems as presented by de Schutter and van den Boom [7]. This paradigm was extended with the decision variables $v_i(k)$, i = 1, 2. The main challenge is to identify the input sequence $y(k), \ldots, y(k + N_p - 1)$ on a moving horizon $k, \ldots, k + N_p - 1$. This identification necessitates a slight modification of the cost function (25) that was previously introduced:

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$$J(y) = -\sum_{j=0}^{N_p - 1} y(k+j).$$
(39)

Consequently, the central task is obtaining $y(k), \ldots, y(k + N_p - 1)$ for each k. A initial step towards the computational framework is the derivation of predictions of $\mathbf{x}(k+1), \ldots, \mathbf{x}(k+N_p-1)$. This step may be realized by means of defining

$$\tilde{y}(k) = \begin{bmatrix} y(k) \\ y(k+1) \\ \vdots \\ y(k+N_p - 1) \end{bmatrix}, \quad \tilde{x}(k) = \begin{bmatrix} x(k) \\ x(k+1) \\ \vdots \\ x(k+N_p - 1) \end{bmatrix}, \quad (40)$$

$$\tilde{v}(k) = \begin{bmatrix} v(k) \\ v(k+1) \\ \vdots \\ v(k+N_p - 1) \end{bmatrix}, \quad v(k) = [v_1(k), v_2(k)]^T.$$

as well as a recursive application of (36). The next step, which precedes the development of the entire algorithm, is the introduction of a complete set of constraints, which is required during repetitive optimization cycles on $k \dots, k + N_p - 1$:

Transportation: the transportation is defined by (31)–(34) and concerns the transportation times of a set of AGVs:

$$b_{1}(k) = \max(e, b(k) + v_{1}(k)),$$

$$b_{2}(k) = \max(e, b(k) + v_{2}(k)),$$

$$c_{1}(k) = \max(e, c(k) + v_{1}(k)),$$

$$c_{2}(k) = \max(e, c(k) + v_{2}(k)).$$

(41)

Concurrency: concurrency is defined by (42) and pertains selecting the AGV transporting the *k*th item:

$$v_1(k) = e \Leftrightarrow v_2(k) = \varepsilon$$

$$v_2(k) = e \Leftrightarrow v_1(k) = \varepsilon$$
(42)

Scheduling: scheduling is defined by (23) and concerns a required items delivery time:

$$x_3(k) \le x_{ref}(k). \tag{43}$$

Production performance: production performance is closely connected to the maximum rate of change of the production outlet delivery time: Conflict Avoidance Within Max-Plus Fault-Tolerant Control ...

$$y(k+1) - y(k) \ge y_z(k),$$
 (44)

where $y_z(k) \ge 0$ is the production performance upper bound. Based on the preceding constraints, the complete optimization problem can be condensed to:

$$\left(\tilde{y}(k)^*, \tilde{v}(k)^*\right) = \arg\min_{\tilde{y}(k), \tilde{v}(k)} J(y), \tag{45}$$

under (41)-(44).

To conclude, the developed control strategy for two AGVs disposes of the structure given by Algorithm 6 with *fault-tolerance* capabilities.

Algorithm 6: May	(-plus MPC	for two	AGVs
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Step 0: $\begin{bmatrix} Set \ k = 1, \ N_p, \ v(0); \\ Step 1: \\ Get \ M(k), \dots, \ M(k + N_p - 1), \ y_z(k) \dots, \ y_z(k + N_p - 1) \text{ and} \\ x_{ref}(k), \dots, \ x_{ref}(k + N_p - 1) \text{ from MES;} \\ Step 2: \\ Measure the state \ x(k - 1) \text{ and obtain } \tilde{y}(k)^* \text{ and } \tilde{v}(k)^* \text{ by solving the constrained} \\ optimization problem (45); \\ Step 3: \\ Use the first vector elements of \ \tilde{y}(k)^* \text{ and } \tilde{v}(k)^* (i.e., \ y(k)^* \text{ and } v(k)^*) \text{ and feed them into} \\ \text{the system (30);} \\ Step 4: \\ Stek = k + 1 \text{ and go to Step 1;} \\ \end{bmatrix}$

In addition to the rather elegant recursive description of (17) and the linearity of the cost function (39), it is possible to observe that any optimization constraint having the form $a = \max(b, c)$ may be transformed into a set of equivalent linear constraints, i.e., $a \ge b$, $a \ge c$. This fact obviously indicates that the optimization problem can be reduced to mixed-integer linear programming.

6 Fault-Tolerant Control of AGVs

The objective of this section is to provide an answer to the subsequent research question: *How to manage large inconsistencies, which may lead to the significant transportation delays and possible violation of the scheduling constraints?* This question concerns the accommodation of the possible faults, which are defined by (28). The consequence of these possible faults (28) may be a severe violation of the scheduling constraints (43). This violation can result in an infeasibility of the overall

optimization problem (17). In order to address this problem field, a time varying relaxation variable $\alpha(k) \ge 0$ may be incorporated into (43) which results in:

$$\mathbf{x}_3(k) \le x_{ref}(k) + \alpha(k). \tag{46}$$

In this context, it is intended that $\alpha(k)$ is as little as possible in order to achieve a small divergence from the time schedule that was initially desired. For the purpose of obtaining the optimal values of $\alpha_j(k)$, a new cost function can be proposed:

$$J(\alpha) = \sum_{j=0}^{N_p - 1} \alpha(k+j).$$
 (47)

Consequently, it is possible to introduce a new FTC-oriented cost function:

$$J(y,\alpha) = (1-\beta)J(y) + \beta J(\alpha).$$
(48)

In this equation, $1 \le \beta \le 0$ is a constant that can be set by the control engineer and which can be adjusted to reflect the higher importance of either J(y) or $J(\alpha)$, respectively. By defining $\tilde{\alpha}(k) = [\alpha(k), \dots, \alpha(k + N_p - 1)]^T$, it is possible to rewrite the optimization problem as:

$$\left(\tilde{y}(k)^*, \tilde{v}(k)^*\right) = \arg\min_{\tilde{y}(k), \tilde{\psi}(k), \tilde{\alpha}(k)} J(y, \alpha), \tag{49}$$

under (41)–(44) and (46). The consideration of the precedent optimization problem allows to propose an entire FTC algorithm, which updates the matrices $A_v(\cdot, \cdot, \cdot)$ and $B_v(\cdot, \cdot)$ together with associated constraints depending on fault estimates. FTC algorithm ensures that the optimization problem is always feasible. If the current performance of an AGV set is insufficient to attain $x_{ref}(k)$, it is optimally relaxed and the closest schedule to the original infeasible one is obtained.

6.1 Performance Evaluation

The central aim of this section is the evaluation of the reliability of *Algorithm 2* in chapter "Cyclic Two Machine Flow Shop with Disjoint Sequence-Dependent Setups". For the sake of simplicity and clarity, it was applied to an transportation system consisting of two AGVs. The desired schedule is given by:

$$x_{ref} = [1, 2, 3, 7, 10, 2, 15, 16, 20]^T.$$
 (50)

In this case, the nominal transportation times were set equal to one minute, i.e. b(k) = c(k) = 1. It is important to point out that the assumed schedule (50) is not



Fig. 7 Comparison of x_3 and x_{ref} (left) and the associated item outlet delivery time (right)



evenly distributed, consequently it is not possible to realize the scheduling in a simple ad hoc manner. Additionally, a dedicated fault scenario was assumed, which consists of a one minute transportation delay $f_{2,c} = 1$ of one of the AGVs during its first operation. Figure 7a shows (50) (red line) along with the actual item delivery time x_3 (blue line). Figure 7b contains the respective item outlet delivery times. It is obvious that the item delivery time is larger than the desired schedule for k = 2 only. Most notably, for all remaining event counters a desired schedule is achieved. Additionally, from the Gantt diagram (Fig. 8) it is evident that the second robot operates for k = 2, and therefore, according to the fault scenario a one minute delay occurs. However, the predictive FTC algorithm is able to identify this fault and can calculate a desired AGV work schedule that is able to eliminate this delay for subsequent event counters.

7 Remarks and Conclusions

The central research objective of this chapter was to clarify if and and how a faulttolerant interval max-plus algebra model predictive control framework can be applied for controlling flexible assembly systems which include resource conflicts. The main research contribution was the proposition of a unified FTC MPC procedure which

guarantees an optimal allocation of transportation tasks among a set of two AGVs. In particular, one of the objectives was to describe this AGV system by means of an interval max-plus algebra framework along with appropriate constraints. The proposed analytic description of two AGVs system had to be able to consider two basic properties of concurrent tasks: synchronization and concurrency. The underlying optimization criteria takes into account all transportation tasks according to a given MES-based time schedule. An decisive advantage is the fact that the cost function is linear but not quadratic. This property allows the application of the proposed approach in an on-line mode even for medium or large scale AGVs systems. The framework allows either to avoid resource conflicts or at least to minimize the possible negative influences of this kind of conflicts. The performance of this framework could be illustrated on the example of a seat assembly system, which represents all important functionalities and levels of industrial production systems. It was discussed in detail, how the design of the control and diagnosis system can enable the respective control and diagnosis task. This discussion was based on established guidelines for Design for Control. The research results allow to avoid resource conflicts in the seat assembly system and the consequences of remaining conflicts could be minimized.

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