



# Nonlinear Finite Element Simulation and Analysis of Double Circular Arc Spiral Bevel Gear Nutation Drive

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**Abstract.** This paper describes the nonlinear finite element analysis model of double circular helical bevel gear nutation drive, the model considering the changes of elastic contact deformation, friction force and the number of meshing tooth of gear, reflecting the actual driving situation of the gear in nutation transmission and dynamic loading. The dynamic meshing of double circular arc spiral bevel gears at different rotate speeds is simulated by nonlinear finite element analysis software, and vibration displacement, dynamic meshing stress and contact force of nutation gear and output gear are analyzed. In order to improve the stability of nutation drive and reduce the vibration and noise, the nonlinear dynamic meshing analysis of nutation drive cannot be ignored.

**Keywords:** Nutation drive · Double circular arc · Dynamic meshing · Nonlinear finite element

## 1 Introduction

Nutation drive is a new type of motion in machinery. Compared with traditional gear transmission, Nutation gear drive has the characteristics of high transfer efficiency, large reduction ratio and steady transmission, it is usually used in robot joint reducer [1–4]. In nutation system, rotors and wobbling bodies [5] rotate about a point and are characterized by a kinematical constraint that prevents them from rotating around the axis. In the pericyclic mechanical transmission system [6], the nutation gear provides high reduction rate and large load bearing capacity. The multi-body model and dynamic balance conditions of nutation system was discussed in paper [7]. A new type of motor [8–10] which with high torque was developed based on the principle of nutation. Nutation drive with rolling tooth [11, 12] was described, it also has a large deceleration ratio. Finally, the development of nutation coordinate system, meshing equation and 3D tooth surface model of double circular spiral bevel gears [13–18] optimizes the performance of nutation drive.

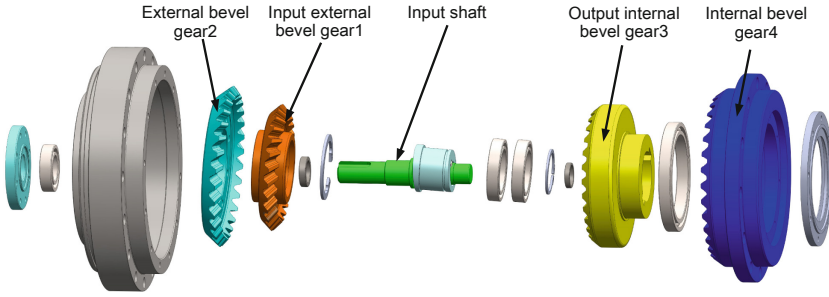
Concerning gear simulation research, multi-body dynamic [19] or static method [20] are often used. These methods can obtain the meshing stiffness or load stress, etc.

However, the elastic gear and continuous transmission process are not considered, so these methods cannot accurately calculate the dynamic response of the whole meshing process. Finite element method is used for analysis and calculation the contact stress of spiral bevel gears [21, 22], and dynamic finite element simulation method is usually used for dynamic meshing analysis of gears [23, 24], and carrying out the dynamic transmission error, tooth surface meshing force, vibration acceleration and other dynamic responses. These studies attach great importance on gear transmission, but there is still not specific research on dynamic nonlinear finite element of nutation gear yet.

In this paper, a dynamic model of double circular spiral bevel gear nutation drive is depicted, gear are set as elastic bodies, and the continuous meshing contact deformation is nonlinear. The model considers the friction, elastic contact deformation and the change of the number of teeth. Dynamic meshing simulation of two-stage spiral bevel gear nutation reducer is carried out. The relative vibration displacement of input external bevel gear and output internal bevel gear, the equivalent stress of dynamic meshing between tooth root and addendum, the position distribution of contact stress on tooth surface during dynamic loading, and the changing rule of dynamic meshing force with time were obtained. It has great significance for improving transmission precision, tooth load capacity and transmission stability of gear nutation.

## 2 Finite Element Dynamic Analysis Model

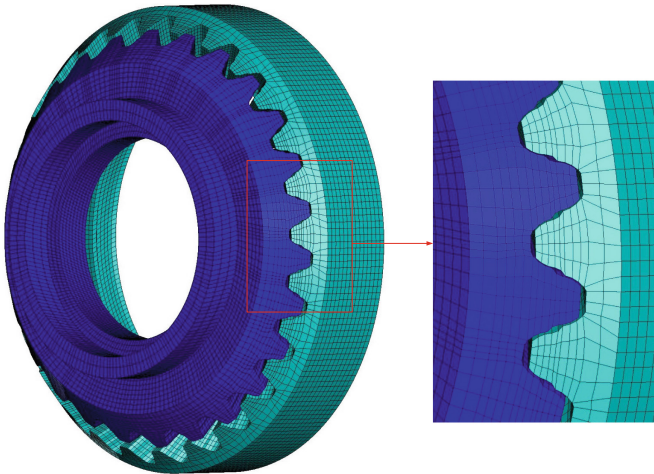
In this paper, nonlinear finite element analysis of the two-stage double arc spiral bevel gear nutation reducer is shown in Fig. 1. Main parameters of input external bevel gear 1 and output internal bevel gear 3 in the secondary nutation drive are presented in Table 1. We built a dynamic analysis model in finite element analysis software Ansys/LS-Dyna. In order to obtain high quality mesh and save computing time, element solid164 and shell163 are adopted for mesh division in finite element analysis model. The solid164 is a hexahedron element with six tetrahedrons and the element shell163 has 12 degrees of freedom. Using solid164 elements to divide elastic gear, and shell163 elements are used to divide the internal ring surface of gear, and set it as rigid body, and elastic gears are driven by rigid body. We use the explicit dynamics algorithm, and tooth surface of the input nutation gear and output gear are defined as surface to surface automatic contact. The torque was added to the internal bevel gear, and the rotate speed was set on the input external bevel gear. Elastic modulus was set as 2.06 Gpa, Poisson's ratio 0.269, density 7850 kg/m<sup>3</sup>, and friction coefficient was set as 0.02. Adjusting element size and mesh density, the appropriate mesh was obtained, and final dynamic model is illustrated in Fig. 2. A total of solid analysis model has 116704 elements with 140126 nodes.



**Fig. 1.** Assembly model of two-stage double arc nutation reducer

**Table 1.** Main design parameters of gear

Parameter	Nutation gear	Output gear
Spiral angle $\beta$	25°	25°
Nutation angle $\phi$	5°	5°
Number of teeth	26	28
Pitch cone angle $\delta$	47.19°	127.8°
Normal module $m_n$	2	2
Cone distance $R$	50 mm	50 mm



**Fig. 2.** Finite element dynamic analysis model

### 3 Simulation Experiment Analysis

In order to verify the accuracy of the model, we set simulation time as 3 ms to solve the dynamic model, the change of the relative circumferential vibration displacement of the nutation input gear 1 and output external bevel gear 3 with time are shown as Fig. 3. In 0–1 ms, with the improving of rotation speed, vibration amplitude and the offset of relative displacement average value increases dramatically, and nutation gear is in a bilateral impact state. At various rotating speeds, the vibration displacement amplitude of nutation drive is small, in the order of  $10^{-3}$ , this indicates that the double circular spiral bevel gear nutation drive has higher transmission accuracy and the precision of the model is proved.

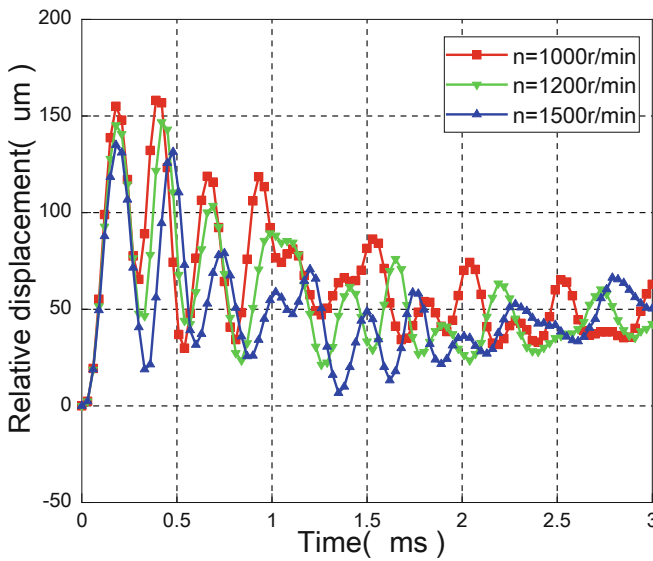
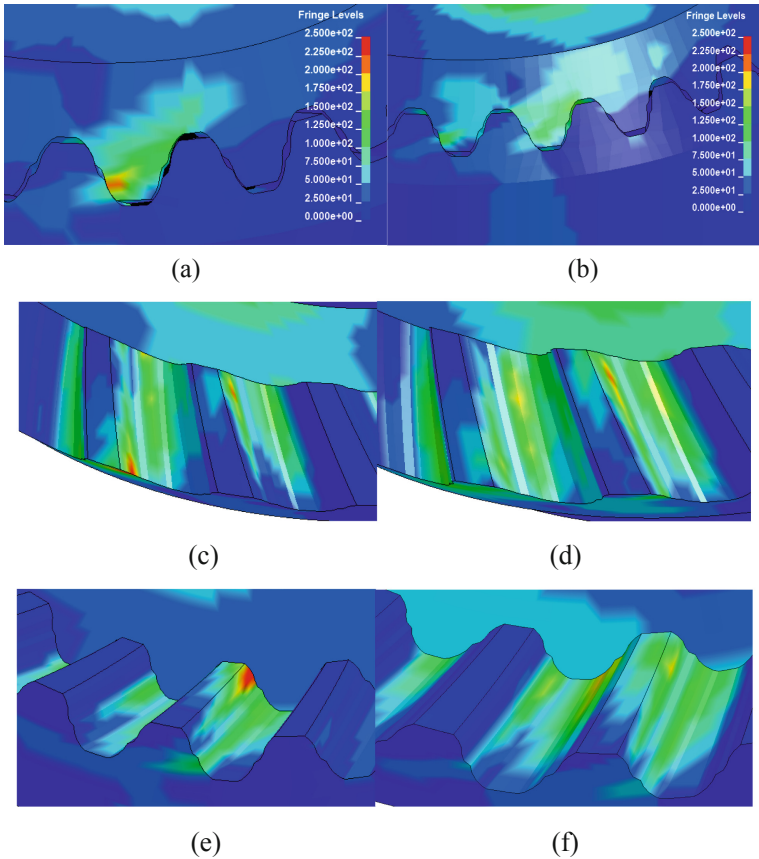


Fig. 3. Relative displacement time domain with time

Nutation output bevel gear with oscillates is up and down, it is subjected to impact stress during meshing. Contact stress distribution during the steady-meshing process of nutation gear and output gear are depicted as Fig. 4 with range level. The input speed was set as 1500 r/min and simulation time as 0.05 s. At different rotating time, the performance of contact stress is diverse. Contact stress position of nutation input external bevel gear and output internal bevel gear from single meshing to triple

meshing are shown as Fig. 4(a)–(b). In Fig. 4(c)–(d), stress distribution of single tooth and triple tooth of nutation input gear were shown. The contact stress distribution position of output internal bevel gear meshing is shown as Fig. 4(e)–(f).



**Fig. 4.** Tooth surface meshing stress distribution

For contact stress patterns of dedendum and addendum, elements at crown and root of nutation gear and output gear are selected respectively. The location and number of the element are shown in Fig. 5. The variation of meshing equivalent stress of the elements with time is indicated in Fig. 6. Since the gear is a continuous elastomer, the gear tooth may still be subjected to equivalent stress within non-meshing time.

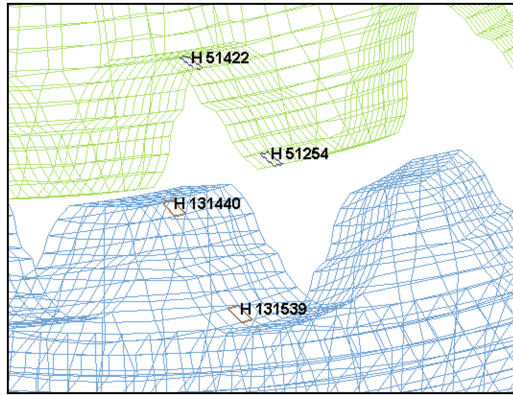


Fig. 5. Element location and number

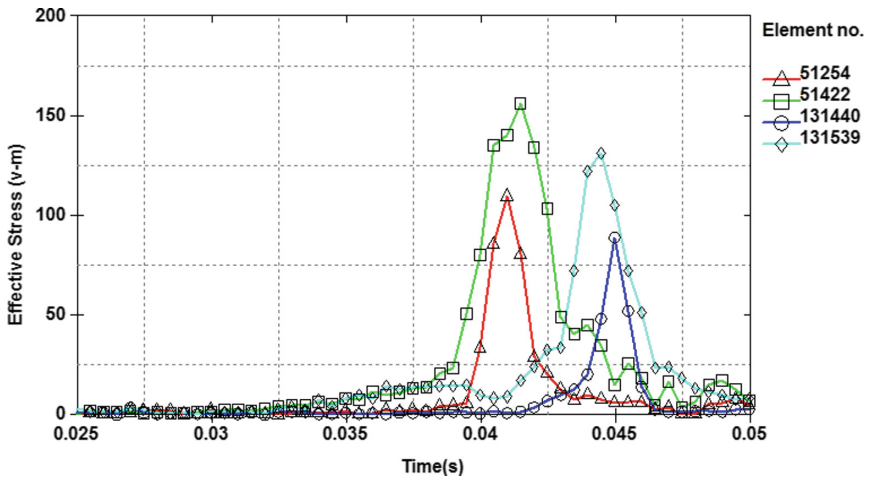


Fig. 6. Element equivalent stress

In nutation drive, meshing points changes with time and the meshing force changes periodically, it can be seen from Fig. 7 that the respective influence on the dynamic meshing force of the gear caused by the different input speed, the input speed was set to 1000 r/min, 1200 r/min, 1500 r/min and 2000 r/min. According to the results, with the increase of input speed, the vibration frequency of contact force gose up.

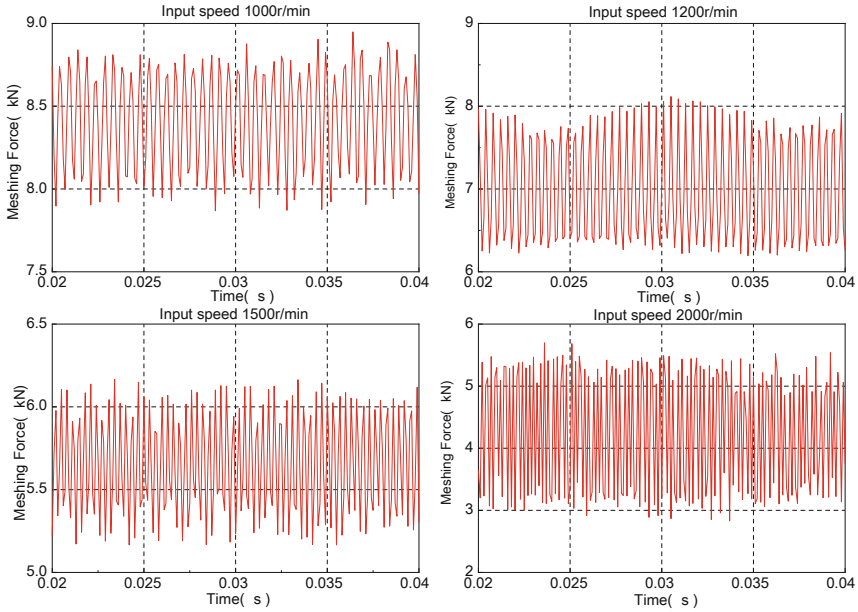


Fig. 7. Dynamic meshing force at different input speeds

## 4 Discussion and Summary

A 3D dynamic finite element analysis model of double circular spiral bevel gear nutation drive is established, and the dynamic analysis model was solved by finite element analysis software. The basic dynamic meshing rules at different input speeds are obtained. Through the analysis of contact stress and contact force of nutation input external bevel gear and output internal bevel gear, the result proves that the nutation gear meshing transmission is stable and has high load bearing capacity.

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