Chapter 5
Ascending Order Constraints Sensitivity Optimal Design Method for Steel Structure

Junchen Guo and Xin Zhao

Abstract Traditional structural design method usually adopts the process of trial and error methodology to try to search a "feasible solution" that satisfy the structural design requirements. This process is complex and inefficient, and it can't get the optimal solution. Therefore, integrating computers and advanced optimization ideas into engineering structures to make structural design quickly, accurately, and intelligently is an important in the field of engineering structure design. This paper develops constant incremental sensitivity analysis method that regard sensitivity coefficient as the guide of member cross-sectional optimal design and redistributes structural material under design constraints to improve material efficiency. Based on this analysis method, ascending order constraints sensitivity optimal design method is developed for steel structures. Taking the sensitivity analysis results as a reference standard, the optimal design method can optimize the structural members and minimize the overall structural cost. To verify the accuracy and the optimization ability of the proposed design method, a utility program was developed. At last, a single plane frame structure is exemplified in this paper to discuss the applicability of the optimization design method. The results of program design and manual design are compared and analyzed which confirm the accuracy and effectiveness of the proposed optimal design method.

Keywords Ascending order constraints sensitivity · Optimal design · Steel structure

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5.1 Introduction

Traditional structure design usually adopts the process of modeling-analysis-calculation-artificial modify to try to search a "feasible solution" that satisfied the structural design requirements. This process is complex and inefficient, and it can't get the most reasonable solution. Therefore, integrating computers and advanced optimization ideas into engineering structures to make structures design quickly, accurately, and intelligently is an advance research in the field of engineering structure design.

Intelligent design needs to be based on a certain design method. The design method can be divided into optimal design and compliant design according to the different paths from the initial design model to the optimized design model. Optimal design is based on the experience of selecting the original component size with large redundancy and conservative design. The design method can reduce the size of the component to reduce the design redundancy gradually, and check the design compliance the requirements of the standard. Compliant design is to select the original component size that is negative in design redundancy and initial component size doesn't satisfy the design constraints. The design method increases the component size to improve the design redundancy gradually, so that the structure just satisfied the requirements of the standard [1, 2].

Based on the constant incremental sensitivity analysis (CISA) method [3, 4], this paper proposes ascending order constraints sensitivity optimal design (AO-CSOD) method and ascending order constraints sensitivity compliant design (AO-CSCD) method. A single plane frame structure is exemplified in this paper to discuss the applicability of the methods AO-CSOD method and AO-CSCD method and the results of the two methods are compared. Finally, the factors that cause the difference between the AO-CSOD method and AO-CSCD method are analyzed and summarized.

5.2 Theoretical Basis

5.2.1 Ascending Order Constraints

Constraints can be divided into driven constraints and validation constraints according to the order in which they are used. Drive constraint is a kind of design constraint which needs to consider the influence of design variables in the optimal design, which is considered as the constraint condition in the process of solving design variables. Validation constraint is a design constraint introduced as a verification condition in optimal design [5].

Constraints can be divided into global constraints, assembly constraints, component constraints, sectional constraints and detailing constraints. We can sort constraints according to a certain hierarchy of relationships to our optimization priorities.
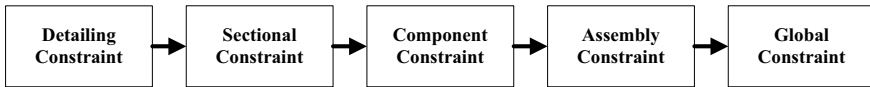
Fig. 5.1 Ascending order constraints

First, component constraints, sectional constraints and detailing constraints are optimized as driven constraints, when those constraints of beams or columns were selected as the driven constraints which satisfied the requirements of the standard, Then, we can check other components whether they can satisfy the requirements of the standard, if not, the unsatisfied constraints are further optimized as driven constraints.

When the component constraints, sectional constraints and detailing constraints are satisfied (retaining a certain degree of redundancy in the optimal design), the constraints are ascended, and then the assembly constraints and the global constraints are used as validation constraints. If the constraints don't satisfy the requirements of the standard, the constraints are further optimized as driven constraints until all of the constraints satisfy the requirements of the standard. After a series of optimization with ascending order constraints, the redundancy of the structure will be greatly reduced (Fig. 5.1).

5.2.2 Redundancy

Redundancy is a measure of the margin of a performance metric to the design standard limitations and can be defined as formulas (5.1) and (5.2).

$$R_{ei} = (g_i^u - g_i)/g_i^u \tag{5.1}$$

$$R_{ei} = (g_i - g_i^l)/g_i^l \tag{5.2}$$

where R_{ei} is the redundancy of the design constraint 'I', g_i is design constraint 'I', g_i^u is the upper limitation of the design constraint 'I', g_i^l is the lower limitation of the design constraint 'I', When the constraint limitation is the upper limitation, take the formula (5.1), and when the constraint limitation is the lower limitation, take the formula (5.2).

"Over-redundancy" means that the structure has a large optimization space. "Under-redundancy" means that the design indexes of the structure don't satisfy the requirements of the standard, and the "proper-redundancy" indicates that the design of the structure is satisfactory. In this paper, the proper redundancy is 0–10%, more than 10% is over-redundancy. When all the constraints of the structure satisfy the requirements of the standard, the smaller the redundancy of the structure is, the better the structure is.

Under Constrained	→AO-CSCD→	Over Constrained	←AO-CSOD←	Over Constrained

Fig. 5.2 The AO-CSCD method and the AO-CSOD method

5.2.3 Optimal Design Method and Compliant Design Method

The design method from the primitive design with over-constrained (over-redundancy) to the properly-constrained (proper-redundancy) is called optimal design method. At present, the optimal design method is widely used in the optimal design. But the optimal design method also has some shortcomings. The interaction between design constraints in optimal design methods often leads to conservative results.

If the designers want to reduce the redundancy under the condition of satisfy the requirements of the standard, the whole process can be inverted, so this paper also proposes a compliant design method. The compliant design optimizes the primitive structure from under-constrained to properly-constrained (Fig. 5.2).

5.2.4 The CISA Method

The CISA method is to increase the material volume or material cost of each component group in turn and calculate the change value of design constraints as the sensitivity coefficient of each component group. The formula (5.3) is used to calculate the sensitivity coefficient [6–8].

$$s_{i,k} = \Delta g_i / \Delta v_k \qquad (5.3)$$

where $S_{i,k}$ is the constant incremental sensitivity coefficient of the design constraint 'I', Δg_i is the change value of design constraint 'I', Δv_k is the change value of design variable k.

The CISA method developed in this paper is used in the optimal design and compliant design. We determine the optimal position (beam or column or other component) by the ratio of the increment of structural material (change the section in the standard steel form) to the value of change in the global constraints (story drift). This can better consider the coupling effect between different components such as beams and columns. The redundancy of the structure is reduced effectively and the working efficiency is improved.

The CISA method is shown in Fig. 5.3, where the black up arrow indicates an increase in the section size of the component, and the black down arrow indicates a decrease in the section size of the component.
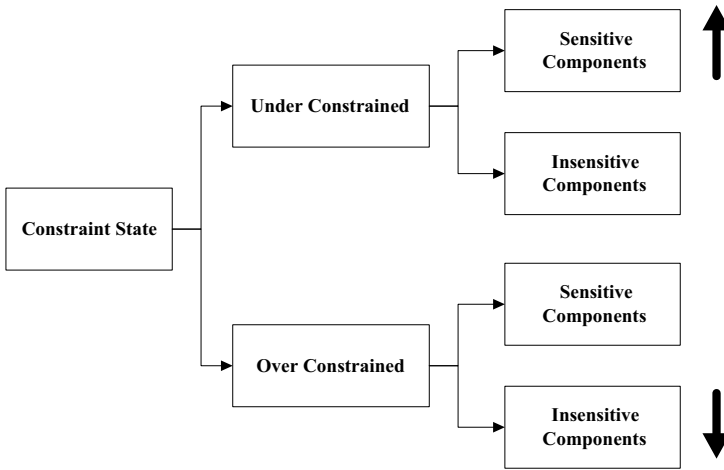
Fig. 5.3 The CISA method

5.2.5   The AO-CSCD Method

The AO-CSCD method combines the ascending order constraints method with the compliant design method, and further proposes the ascending order constraints sensitivity compliant design method.

The flow of AO-CSCD method is shown in Fig. 5.4.

5.2.6   The AO-CSOD Method

The AO-CSOD method combines the ascending order constraints method with the optimal design method and applies compliant design method described. Then further proposes the ascending order constraints sensitivity optimal design method.

The flow of AO-CSOD method is shown in Fig. 5.5.

5.3   Case Study

5.3.1   A Single-Story Planar Frame Structure

A single-story steel frame model with a height of 4 m and a bay of 6 m was selected. Beams and columns adopted the standard H-section of steel table, and the column bottom was hinged to the ground. The material is Q234B steel, the density is 7850 kg/m^3,

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┌──────────────┐          ┌─────────────────────────────────────┐
│              │          │  The Structural system is determined.│
│  Primitive   │──────────│  The redundancy of the pre-selected  │
│  Design      │          │  component size is under-constrained, │
│              │          │  and don't satisfy the requirements   │
└──────────────┘          │  of the standard.                     │
        │                 └─────────────────────────────────────┘
        ▼                                   │
┌──────────────────────┐                    ▼
│ The compliant design │          ┌─────────────────────────────────────┐
│ of the components    │          │      Component constraints           │
│ (the redundancy      │──────────│      Sectional constraints           │
│ slightly inadequate) │          │      Detailing constraints           │
│                      │          │      Normalize constraints           │
└──────────────────────┘          │  Beams and columns can increase the  │
   │          Ascending           │  size of the sections at the same time│
 Satisfy      order               └─────────────────────────────────────┘
              constraints                       │
   ▼                                            ▼
┌──────────────────────┐   CISA   ┌─────────────────────────────────────┐
│ The compliant design │          │ Increase the section size of the     │
│ of the assembly      │──────────│ components with higher sensitivity   │
│ (the redundancy      │          │ coefficient                          │
│ slightly inadequate) │          │ Makes the redundancy slightly        │
└──────────────────────┘          │ inadequate.                          │
   │          Ascending           └─────────────────────────────────────┘
 Satisfy      order                             │
              constraints                        ▼
   ▼                                 ┌─────────────────────────────────────┐
┌──────────────────────┐   CISA      │     Check all the constraints        │
│ The compliant design │─────────────│ Optimize the section size of the     │
│ of the global        │             │ Components or assemblies with higher │
│ constraint           │             │ sensitivity coefficient to the       │
└──────────────────────┘             │ global constraints                   │
   │                                 └─────────────────────────────────────┘
   ▼
┌──────────────────────┐
│ Calculate the        │
│ redundancy           │
└──────────────────────┘
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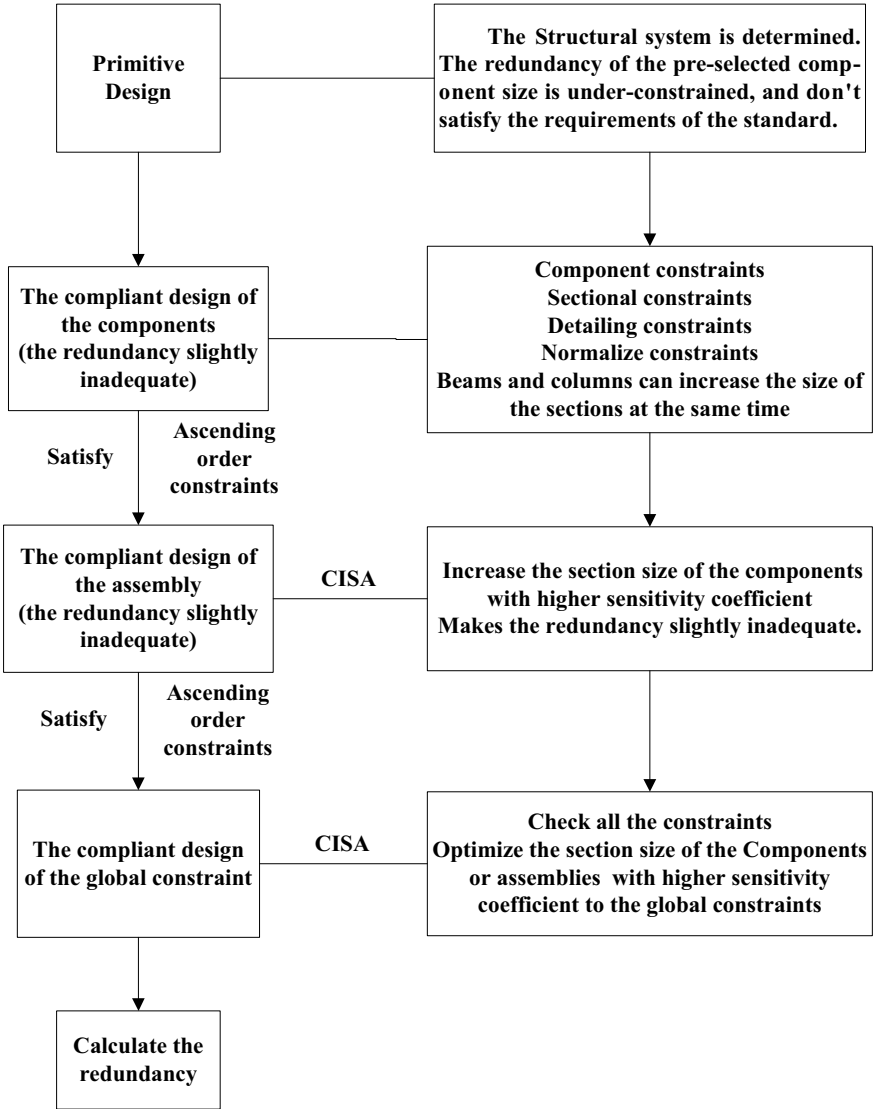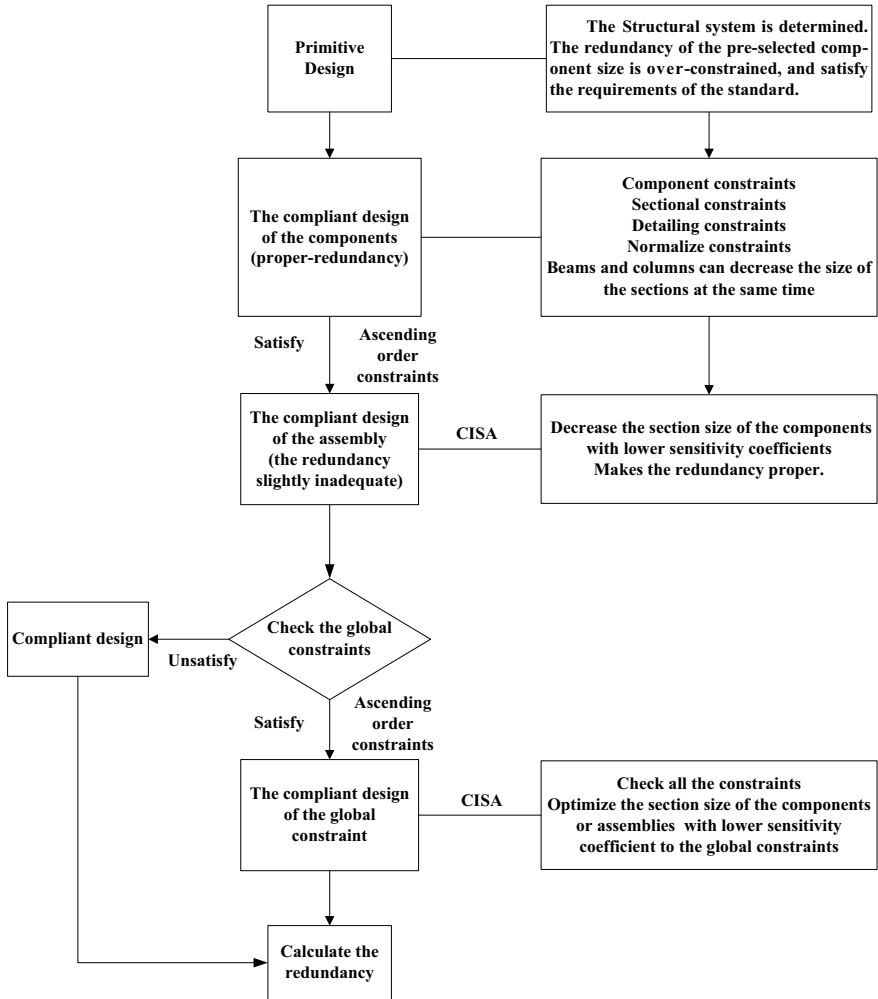
Fig. 5.4 The AO-CSCD method

Fig. 5.5 The AO-CSOD method

the elastic modulus is 210 GPa, the Poisson's ratio is 0.3, the yield strength is 235 MPa and the design values of the strength of material is 215 MPa (Fig. 5.6).

As shown in Tables 5.1 and 5.2, select the calculation parameters.

The limitation of constraints is shown in Table 5.3.

The AO-CSOD method. Primitive design: The redundancy of the component size is over-constrained. Column: $400 \times 400 \times 13 \times 21$. Beam: $550 \times 200 \times 10 \times 16$.

The primitive design parameters of AO-CSOD are shown in Table 5.4.

The AO-CSCD method. Primitive design: The redundancy of the component size is under-constrained. Column: $100 \times 100 \times 6 \times 8$. Beam: $100 \times 50 \times 5 \times 7$.

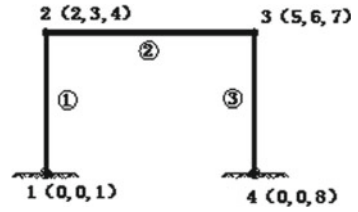**Fig. 5.6** A single-story
planar frame structure

Table 5.1 Calculation parameters

Superimposed dead load (N/m)	Live load (N/m)	Basic wind pressure (kN/m²)	Shape coefficient	Wind pressure height coefficient	Bay (m)
27,000	12,000	0.75	1.3	1	6

Table 5.2 Calculation parameters

Maximum of seismic influence coefficient	Damping ratio	Characteristic site period	Slenderness ratio
0.24	0.02	0.4	100

Table 5.3 Limitation of constraints

Slenderness ratio	Beam deflection (live load)	Beam deflection (combination of loads)	Stress ratio (beam)	Stress ratio (column)	Story drift
100	0.0020	0.0025	0.9000	0.9000	0.0040

The primitive design parameters of AO-CSCD are shown in Table 5.5.

The design section adopts the international hot-rolled H steel table GB/T 11263-2010. the section steel table is arranged according to the section area from large to small. In the design process, beams and columns increase and decrease sections in turn according to this list.

Table 5.4 The primitive design parameters of AO-CSOD

Project		Slenderness ratio	Beam deflection (live load)	Beam deflection (combination of loads)	Stress ratio (beam)	Stress ratio (column)	Story drift	Material dosage (kg)
Primitive design	Value	27.72	1.5×10^{-4}	4.9×10^{-4}	0.5639	0.3801	0.0020	1926
	Redundancy	72.27%	92.69%	80.56%	37.34%	57.77%	50.55%	

Table 5.5 The primitive design parameters of AO-CSCD

Project		Slenderness ratio	Beam deflection (live load)	Beam deflection (combination of loads)	Stress ratio (beam)	Stress ratio (column)	Story drift	Material dosage (kg)
Primitive design	Value	112.9	0.0559	0.1821	21.34	10.80	0.1978	191
	Redundancy	−12.90%	−2694%	−7183%	−2277%	−1100%	−4845%	

5.3.2   The Optimization of Component Constrained

It is not necessary to consider incremental sensitivity coefficient under component constraints, but to optimize the structure to proper-redundancy according to the requirements of detailing constraints, sectional constraints and normalize constraints.

The constraints used in the case are slenderness ratio, deflection of beam (live load), deflection of beam (characteristic combination of loads), stress ratio of beam, stress ratio of column.

Because of the opposite arrangement of sections between the AO-CSOD method and the AO-CSCD method, the section area may change little when the sections increase or decrease, but the parameters such as moment of inertia are quite different from each other. So, they do not converge to the same solution in the calculation.

Optimal results under component constraints in AO-CSOD method are shown in the Table 5.6 that the column is $400 \times 200 \times 8 \times 13$, the beam is $250 \times 255 \times 14 \times 14$.

Optimal results under component constraints in AO-CSCD method are shown in the Table 5.7 that the column is $446 \times 150 \times 7 \times 12$, and the beam is $250 \times 255 \times 14 \times 14$.

Table 5.6 Optimal results under component constraints in AO-CSOD method

Project		Slenderness ratio	Beam deflection (live load)	Beam deflection (combination of loads)	Stress ratio (beam)	Stress ratio (column)	Story drift
AO-CSOD	Value	45.53	2.1×10^{-4}	7.1×10^{-4}	0.6668	0.8392	0.0047
	Redundancy	54.47%	89.23%	71.52%	25.91%	6.76%	−16.62%

Table 5.7 Optimal results under component constraints in AO-CSCD method

Project		Slenderness ratio	Beam deflection (live load)	Beam deflection (combination of loads)	Stress ratio (beam)	Stress ratio (column)	Story drift
AO-CSCD	Value	45.53	2.6×10^{-4}	8.5×10^{-4}	0.8263	0.8392	0.0047
	Redundancy	54.47%	87.07%	65.93%	8.187%	6.120%	−18.10%

5.3.3 The Optimization of Global Constraints and Sensitivity Analysis

Take the AO-CSOD method as an example.

It can be seen from the above data that after the component constraint optimization is completed, the story drift didn't satisfy the design requirements, therefore, execute the global constraints design part in the AO-CSCD method.

After executing the global constraint optimization, we carry on the CISA method. In this case, the story drift under wind load and the story drift under seismic action are used as global constrained. The optimal design process contains 2 cycles. The sensitivity coefficients of beams and columns are shown in the Fig. 5.7.

Taking the first cycle of compliant design as an example, as we can see in Fig. 5.7, the sensitivity coefficients of the columns under the action of seismic and wind load are higher than the sensitivity coefficients of the beams, so we increase the section size of the columns. If the sensitivity coefficients of the columns under the action of seismic and wind load are lower than the sensitivity coefficients of the beams, then we increase the section size of the beams. If appear the sensitivity coefficient points to the optimized object is inconsistent under two kinds of action, then optimizes section size of the columns and beams at the same time.
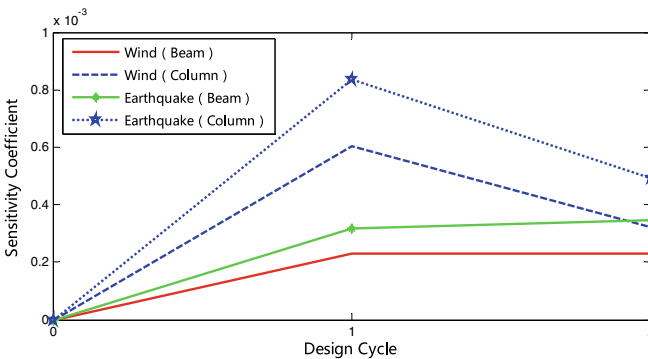

Fig. 5.7 The sensitivity coefficients of beams and columns

Optimal results of AO-CSOD method.
Column: 300 × 300 × 10 × 15. Beam: 400 × 200 × 8 × 13.
Optimal results of AO-CSCD method.
Column: 300 × 300 × 10 × 15. Beam: 470 × 150 × 7 × 13.

5.3.4 Comparison and Analysis of Design Results

The optimization design results of the AO-CSOD method and the AO-CSCD method are compared with the primitive design results. The results are compared with the manual design results at the same time. Manual design results are shown in the Table 5.8.

Column: 338 × 351 × 13 × 13. Beam: 496 × 199 × 9 × 14.

The results of the AO-CSOD method are shown in the Table 5.9.

The results of the AO-CSCD method are shown in the Table 5.10.

Comparing the results of the AO-CSOD method and the AO-CSCD method, we can find that the material dosage of the AO-CSOD method is smaller than that of the AO-CSCD method, and comparing the two design results and the manual design results, it is obvious that both design results are better than the manual design results, and the design efficiency has been greatly improved, from half an hour to just a few dozen seconds.

Table 5.8 Manual design results

Project		Slenderness ratio	Beam deflection (live load)	Beam deflection (combination of loads)	Stress ratio (beam)	Stress ratio (column)	Story drift	Material dosage (kg)
Manual design	Value	33.21	1.5×10^{-4}	5.1×10^{-4}	0.6018	0.6447	0.0032	1297
	Redundancy	66.78%	92.36%	79.75%	33.13%	28.37%	59.15%	

Table 5.9 The results of the AO-CSOD method

Project		Slenderness ratio	Beam deflection (live load)	Beam deflection (combination of loads)	Stress ratio (beam)	Stress ratio (column)	Story drift	Material dosage (kg)
AO-CSOD	Value	36.84	3.1×10^{-4}	0.0010	0.7800	0.7892	0.0039	1127
	Redundancy	63.16%	84.50%	59.04%	18.98%	18.98%	1.671%	

Table 5.10 The results of the AO-CSCD method

Project		Slenderness ratio	Beam deflection (live load)	Beam deflection (combination of loads)	Stress ratio (beam)	Stress ratio (column)	Story drift	Material dosage (kg)
AO-CSCD	Value	36.84	2.6×10^{-4}	8.6×10^{-4}	0.8182	0.7238	0.0038	1072
	Redundancy	63.16%	87.02%	65.78%	9.092%	19.58%	3.995%	

5.4 Conclusion

This paper proposes the AO-CSOD method and the AO-CSCD method based on the CISA method,

The AO-CSOD method and AO-CSCD method are applied to the optimization of a single-story steel frame structure. The results of the two methods are compared and analyzed. The main conclusions are as follows:

(1) In the AO-CSOD method and the AO-CSCD method, we can effectively identify the components and their positions that need to be optimized base on the CISA method. The efficiency of optimization has been effectively improved, and the workload of optimal design has been greatly reduced.
(2) Due to the characteristics of the algorithm, the optimization effects of the AO-CSOD method are not better than that of the AO-CSCD method. However, if a structure system is complicated, it is more reasonable to start the design from the over-redundancy, so we combine the compliance design method in the AO-CSOD method, and the AO-CSOD method can also converges to the approximate optimal solution.
(3) The AO-CSOD method and the AO-CSCD method can achieve automatic configuration at design progress and have higher design efficiency compared with current manual optimal design. The approximate optimal solutions can be obtained in a short time, and they can save the amount of structural materials and obtain better economic and social benefits.

References

1. Isenberg, J., Pereyra, V., Lawver, D.: Optimal design of steel frame structures. Appl. Numer. Math. **40**(1), 59–71 (2002)
2. Tocher, J.L., Karnes, R.N.: The impact of automated structural optimization on actual design. AIAA J. (1971)
3. Adelman, H.M., Haftka, R.T.: Sensitivity analysis of discrete structural systems. AIAA J. **24**(5), 823–832 (1986)

4. Yu, T.Y., Zhao, X.: Virtual work sensitivity method for the optimization design of tall buildings. In: 13th East Asia-Pacific Conference on Structural Engineering and Construction (EASEC-13), Sapporo, Japan (2013)
5. Austin, F.: A rapid optimization procedure for structures subjected to multiple constraints. In: AIAA/ASME/SAE 18th Structures, Structural Dynamics and Materials Conference, San Diego, Calif., pp. 71–79 (1977)
6. Brayton, R.K., Spence, R.: Sensitivity and Optimization. Elsevier, New York (1980)
7. Barrar, C.D.: Structural Optimization Using the Principle of Virtual Work and an Analytical Study on Metal Buildings. Virginia Tech, Virginia (2009)
8. Zhao, X., Dong, Y.M., Yu, T.Y.: Sensitivity analysis of material distribution to structural period for super tall buildings. In: IASS-SLTE 2014 Symposium, pp. 99–104 (2014)