# 3

# Statistical and Econometric Analysis of the Cycle

A vast (and daunting) array of statistical and econometric techniques are available to the researcher for deriving business cycle metrics. Again doing justice to this literature is incredibly difficult but it is useful to cover the broad approaches adopted and how they relate to the theoretical models discussed in the previous chapter.

## 3.1 Aggregated or Disaggregated Data

In this section we draw heavily on the discussion in Chadha et al. (2019) to which the reader is directed for more detail. As Chadha et al. (2019) note there are two key issues with regard to business cycle determination.

The first issue is whether one should look at many disaggregated series to analyse the business cycle or whether a single aggregate measure of output such as GDP should be analysed. Many classic analyses of business cycle dating in the United Kingdom typically used a wide variety of series. Burns

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and Mitchell's (1946) chronology of the UK business cycle up until 1938, from 1792 on annual basis and from 1848 on a monthly basis, was based on 141 time series covering not only production indices and activity series, but also commodity prices, asset prices, interest rates and money and credit series. A number of chronologies followed in this tradition such as Ashton (1959) for the C18th, Gayer et al. (1953) for the period 1792–1848, and Rostow (1972) who covered the years from 1788 to 1914.

The obvious problem is that the range of indicators may provide conflicting answers to key turning points in the business cycle. Burns and Mitchell (1946) note that "*there were cases in which the turning points were widely scattered, and others in which they were concentrated around two separate dates.*" Their approach was to derive "specific cycles" in each time series and then combined and "weighted" to determine the "reference cycle" for the overall economy. As noted by Romer (1994) there was a large degree of subjective judgement involved in weighting together different series and the method used was left relatively vague.

At that time, aggregate measures of economic activity, such as real GDP, were still in their infancy, particularly given the focus on not only the C20th, but also the C19th. So in this sense Burns and Mitchell like other researchers had no other choice than to use a range of indicators. As Harding and Pagan (2002) point out this is indeed what Burns and Mitchell would have used had it been available:

> Aggregate [economic] activity can be given a definite meaning and made conceptually measurable by identifying it with gross national product.
>
> Unfortunately, no satisfactory series of any of these types is available by months or quarters for periods approximating those we seek to cover. (Burns and Mitchell 1946)

Given the increasing availability of national accounts data, later business cycle chronologies tended to use aggregate measures of economic activity such as GDP to identify turning points. In the post-WW2 period, the Central Statistical Office (CSO 1993), now the Office for National Statistics (ONS), maintained a quarterly "reference chronology", covering from 1958 to 1992, based on turning points in real GDP. The Organisation for Economic Co-operation and Development (OECD 2019) continue to produce a set of turning points for the

United Kingdom using (de-trended) real GDP and a version of the Bry and Boschan (1971) algorithm we discuss later. This chronology extends back to 1955 on a monthly basis. And leading research institutes, such as the Centre for Economic Policy Research (CEPR 2019) and National Bureau of Economic Research (NBER 2019), focus on real GDP (and some its components) and employment.

Focusing on an aggregate measure of economic activity, such as real GDP, clearly has advantages. As we discuss in Chapter 4 real GDP can be measured in three different ways. On the expenditure side, it combines together consumption, investment and government spending by domestic residents plus net spending by overseas residents on exports (exports minus imports) using their share in total expenditure to weight them together. On the output side, it combines the production of the agricultural, industrial and services sectors weighted by their respective shares in the value added they contribute to the economy. These components are, in turn, aggregates of many more sub-components. And on the income side, GDP can be measured as the sum of employment earning and profits delated by the GDP deflator to give a measure of real output. So GDP weights together many different indicators of activity according to their overall importance in the economy.

However there remain issues with using GDP as a sole representative indicator. First, as we go back in time GDP data may only exist at an annual frequency when precise business cycle dating requires quarterly or monthly data especially if that dating is ultimately used to inform monthly and quarterly policy decisions. This can be ameliorated by using monthly and quarterly indicators to interpolate or "temporally disaggregate" aggregate measures of GDP using methods such as those suggested by Chow and Lin (1971). So in some sense this is just a method that combines the aggregate and disaggregated approaches but in a way that is constrained to match the aggregate annual data.

Second, there may be reasons why GDP or output is not the best summary measure for business cycle analysis. Output might be the best measure for assessing inflationary pressures but it may not provide the best measure of social welfare in the economy. Unemployment, GDP per person or productivity may be better indicators of overall welfare. Here of course one can adopt Burns and Mitchell's approach which establishes a reference cycle based on GDP and then specific cycles for other variables of interest can then be benchmarked against that.

Thirdly one might not consider cycles to be important for welfare unless it is broadly spread across different sectors. Zarnowitz (1985), building on Lucas' definition of the cycle, argues that business cycles represent expansions and contractions that consist of recurrent serially correlated and *cross-correlated movement in many economic variables.* So the dispersion of cyclical or serially correlated movements across many activities is more important than one narrowly concentrated in a few industries or sectors.

Finally each of the three approaches to measuring GDP themselves often provide conflicting answers. For example, in the UK the output, income and expenditure measures suggest a different profile for the slowdown in productivity at the end of the C19th and the start of the C20th. Therefore, it is not clear that focusing on an aggregate measure of economic activity gets around the issue of measurement error. Often the only thing that can be done is to average the estimates from the three approaches. However, using *balanced* estimates of real GDP would help to ameliorate this problem, where estimates from the three approaches are weighted together based on a subjective assessment of the reliability of its underlying components (Sefton and Weale 1995). The subjective element of course remains important.

We discuss the availability of GDP data and the importance of some of these issues in the UK in Chapter 4.

## 3.2 Classical Versus Growth Cycles—To De-trend or Not to De-trend?

The second issue is whether business cycle metrics should be derived from the level of activity—the classic cycle—or whether it should be applied to de-trended data and generate "growth cycles". Burns and Mitchell (1946) were clear they were interested in expansions and contractions in the level of activity and this methodology is currently still used by the NBER Business Cycle Dating Committee which meets to determine the chronology of US business cycles. However as Romer (1994) discusses the NBER chronology appears to have shifted over time. Prior to 1927 the dating appears to be based on de-trended data and this can have significant impact on business cycle chronology and metrics. For example, Romer (1994) shows how this shift in procedure is largely behind the result that US recessions appear to get shorter over time and generates an alternative chronology prior to 1927 based on an algorithm that closely matches the NBER dating of post-war US cycles.

So which method is to be preferred? Harding and Pagan (2002) are quite clear "there is no need to perform a de-trending operation to analyse the business cycle". They note the wide variety of de-trending methods available to researchers each of which might produce a different chronology and metrics. A number of recent chronologies published by researchers such as Romer (1994), Davis (2006), Berge and Jordà (2013), and Jordà et al. (2013) follow their approach and prefer to base them on data in levels. This is also true of the modern chronologies published by the CEPR and NBER.

However some researchers are interested in business cycles for analysing the degree of inflationary pressure in the economy and de-trended measures of output or "output gaps" are a key ingredient in Phillips curve analysis that links inflation to activity. More generally policymakers may be interested in ironing out inefficiencies in the economy. Growth cycles tend to be correlated with fluctuations in unemployment and monetary and fiscal policy might be set to offset those movements. So in this respect growth cycles are a more useful concept to analyse for policymakers. However one has to be careful to de-trend output in the right way. As discussed earlier, the correct concept of trend for the New Keynesian model is the level of output that would prevail under flexible prices and it is not clear any of the statistical methods of de-trending output do this effectively as we will see.

The metrics one derives from the classic and growth cycle are different in a number of respects. First, classic cycles tend to have much longer expansion phases than compared with contraction phases. Typically in many countries recessions have been short and sharp and so the metrics for the two phases appear highly asymmetric. Growth cycles on the other hand tend to be more symmetric as the contraction phase applies to any period where growth is below the estimated trend and not just to absolute falls in activity.

Also as Romer (1994) notes, growth cycles based on de-trended output tend to have peaks that appear earlier than classic cycles and troughs that appear later, if the profile of output is relatively smooth. This is because output, although increasing, may slow relative to trend before it falls in absolute terms. Similarly contractions in growth cycles will persist beyond the period of falling output until output growth returns to trend.

As a result of these differences it is important to decide on which metrics are important for the purposes to which business cycle measurement is being put. Classical cycles are more judgement-free and so in some sense the business cycles facts that will emerge will be firmer. That means they may be more useful for comparing turning points across time such as in Romer's analysis. But classical cycles may be less useful in themselves for policy purposes when judgements about trend inevitably have to be made.

We discuss methods of deriving business cycle metrics under both methods in the next two sections.

## 3.3 Methods of Determining Turning Points in Classic Cycles

The classic approach to chronicling the business cycle is to identify turning points such as peaks and troughs. This then defines two phases. The *expansion phase* is the period following the trough of the cycle to the next peak. The *contraction* phase is then the period following the peak to the trough. The full cycle is then the combined expansion and contraction phase. There are two general approaches to detecting turning points in classic cycles. The first takes what is effectively a graphical approach supported by algorithms to censor and refine the turning points obtained. The second involves applying a statistical model to the data.

### 3.3.1 The Graphic or Algorithmic Approach

Under the simplest algorithmic approach a set of candidate peaks, $P_t$, and troughs, $T_t$, can be identified by looking at changes or the first difference in the level of output. A peak period P is defined if output is lower both before and after that period. A trough is defined if output is higher either side of that period.

$$P_t = 1 \text{ if } y_t \geq y_{t-1} \text{ and } y_t > y_{t+1}$$
$$T_t = 1 \text{ if } y_t \leq y_{t-1} \text{ and } y_t < y_{t+1}$$

Figure 3.1 shows this graphically for the case of a smooth cycle.

It may seem odd that the analysis of a classical cycle's turning points is based on the change or growth of output, given the distinction made earlier with growth cycles. But as stressed by Harding and Pagan (2002), the rules above are not about locating a cycle in the growth rate they are just an input into the dating process of the classical cycle in levels.

For annual data the application of this dating rule is relatively straightforward and this process may then be sufficient with no further iterations unless the changes are very small and there are "flat points" at

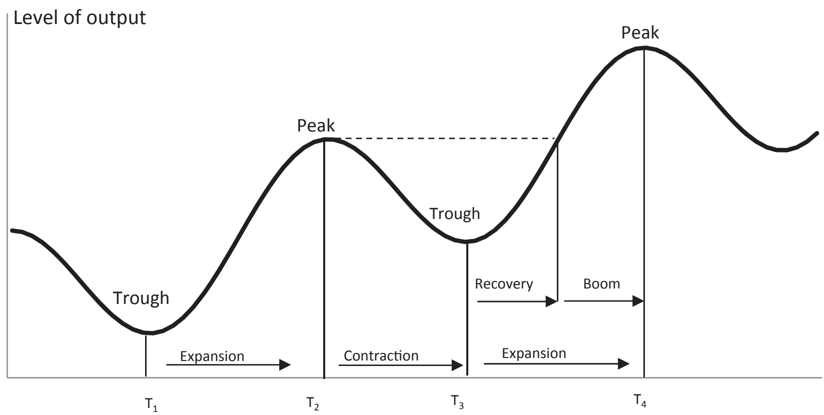


**Fig. 3.1** Classical business cycle dating

peaks or troughs where one has to decide whether output has peaked or troughed at the start or end of the flat point.

For monthly and quarterly data one might want also to place some restrictions on the length of the cycles given volatility and measurement error in the data. And, when those rules are in place certain other features then hold such as making sure peaks and troughs alternate. This process requires an explicit algorithmic procedure to implement. The most popular algorithm is that developed by Bry and Boschan (1971). They specify a set of rules that apply to monthly seasonally adjusted series:

- First a contraction or expansion phase must have a duration of no less than 5 months. This is the source of the popular idiom known as a "technical" recession which involves two consecutive quarters of negative growth.
- Second a full cycle, on both a Peak-to-Peak and Trough-to-Trough basis must have a duration of at least 15 months.

However further censoring rules may then apply to the data. For example a two-quarter contraction maybe succeeded by a quarter of growth, which in turn is followed by a quarter or quarters of decline. Such "double-dip" recessions are common. This raises the question of whether the contraction should be dated up to the earlier or later trough. A metric that can be used is the absolute output gain in the intervening quarter of growth. If it is larger (smaller) than the absolute output loss in the subsequent quarter or quarters of decline, we select the earlier (later) trough as marking the end of the contraction.

A further issue is the notion of the *recovery* phase of the cycle which forms part of the expansion phase. Figure 3.1 demonstrates this in the expansion between time periods $T_3$ and $T_4$. The recovery point is where, following a period of contraction, output recovers to reach its previous peak, beyond that is the "boom" period. However it may be the case that output may not have returned to its previous peak by the time the next contraction occurs. The recovery is "interrupted". This then leads to issues of how to treat "mini" peaks and troughs that occur in between the previous trough and the recovery point.

### 3.3.2 The Statistical Approach—Markov-Switching Methods

Another way to detect business cycle turning points is through an explicit statistical model of the data and then using the estimated parameters of the model to try and obtain a business cycle chronology based on the probability of being at a particular turning point. A popular method of doing this is the Markov-switching approach developed by Hamilton (1989). This approach is well able to capture the asymmetry observed in the data that troughs are shorter and sharper than expansions and assumes no prior knowledge of turning points (unlike the earlier method of Neftci 1982).

Under this approach mean or underlying growth in the economy is in one of two states ($S$): a high-growth or expansion state ($S=1$), or a low-growth/contraction state where $S=0$. And the economy randomly switches between these two unobserved states. By specifying the statistical model underlying this switching one can try and determine from the data which state the economy is in. Using a simple autoregressive or AR(2) process for output we can show an example of this approach:

$$y_t = y_{t-1} + \mu_{st} + \phi(y_{t-1} - y_{t-2} - \mu_{st-1}) + v_t$$

where $\mu_{st}$ is the mean growth rate that switches between states:

$$\mu_{st} = \alpha_0 + \alpha_1 S_t$$

The probability $P$ of being in one state or the other follows a first-order Markov process. This defines the probability of being in a particular state conditional on the existing state:

$$P\left[S_t = 1|S_{t-1} = 1\right] = p$$

$$P\left[S_t = 0|S_{t-1} = 1\right] = 1 - p$$

$$P\left[S_t = 0|S_{t-1} = 0\right] = q$$

$$P\left[S_t = 0|S_{t-1} = 1\right] = 1 - q$$

Hamilton then develops an approach for estimating the parameters of this statistical model. Which state an economy is in at any particular time remains unobservable in the Hamilton framework. The probability of being in a particular state at a particular time can be calculated based on the estimated parameters of the model and the evolving path of the process. The key metric for determining a turning point is through evaluating the conditional probability based on the observed movements of output at that point $P[S_t = 1|y_t, y_{t-1}, y_{t-2}]$ which Hamilton's filtering process determines. He augments this with a smoothing procedure using future values of y to ensure the path of probabilities remains smooth. One then needs to set a benchmark probability to determine whether the economy is one state or another. Hamilton (1989) suggests using the rule that a quarter is part of a low-growth, or recessionary, period if the smoothed conditional probability $P[S_t = 1|y_{t+2}, y_{t+1}, y_t, y_{t-1}, y_{t-2}] < 0.5$.

## 3.4 Methods of De-trending—Growth Cycles

The basic method to determine metrics for growth cycles is to remove both the trend and irregular (white noise) components from a series to leave the cyclical component(s). There are three popular and related procedures for de-trending a series on a univariate basis, which are then often extended to a multivariate basis where common trends and cycles between variables can be identified. These three methods are discussed in turn but can be shown to derive from a more general model and as a result the implicit restrictions in the three models can be identified and discussed.

Essentially a series can be decomposed into a trend component ($\tau$) a cyclical component ($c$) and a white noise component $\omega$ where $\omega_t = N(0, \sigma_\omega)$ is normally distributed and may contain measurement error in the data. For quarterly and monthly data an additional seasonal component ($s$) can be added

$$y_t = \tau_t + c_t + \omega_t + s_t$$

Many of the differences between de-trending methods depend on how the trend and cyclical components are specified and the extent to which the trend and cycle are assumed to be correlated.

### 3.4.1 The Unobserved Components Model

The Unobserved Components (UC) model was popularised by Harvey (1989) and Harvey and Jaeger (1993). In this model the trend and the cyclical components are assumed to be uncorrelated phenomena that are driven by different "shocks" or stochastic processes.

The trend component itself is typically written as a random walk with drift.

$$\tau_t = \tau_{t-1} + \mu_t + \epsilon_t$$

which can be written as:

$$\tau_t = \mu t + \sum_{j=0}^{t} \varepsilon_{t-j}$$

where the trend has both a stochastic and deterministic trend component. The term $\mu t$ represents the deterministic component of the trend which grows at a constant "drift" rate $\mu$ whereas $\sum_{j=0}^{t} \epsilon_{t-j}$ represents the "stochastic trend" based on the cumulative effect of "permanent" shocks that affect the trend level of output and other variables in economy. Again those permanent shocks have a distribution: $\varepsilon_t = N(0, \sigma_\varepsilon)$.

However this model can be generalised further where the drift term itself can be stochastic. This means there can be "shocks" to both the trend growth rate and trend level of output. This is known as the "local linear trends" model.

$$\mu_t = \mu_{t-1} + \eta_t$$

where again the error term is a mean-zero normally distributed variable, $\eta_t = N\left(0, \sigma_\eta\right)$

Another way of modelling the trends is as a sequence of one-off deterministic regime shifts in either the level or deterministic trend component:

$$\tau_t = \mu + \mu * D_t + \tau_{t-1}$$

$$\tau_t = \mu t + \mu * \sum_{i=0}^{t} D_{t-i}$$

where $D$ is a vector of impact dummy variables and $\sum_{i=0}^{t} D_{t-i}$ a vector of one-zero step-dummies. $D$ can also be specified so there are one-off shifts in the slope of the deterministic trend. This is known as a "linear segmented trend" model (see Perron and Wada [2009] and Crafts and Mills [2017]). Essentially this involves determining separate growth regimes with a different trend growth rate. The timing of regimes can either be determined statistically or by assumption based on known features of the economy or using natural breakpoints such as wars.

In the UC model the cyclical component is typically written as an autoregressive moving average model or "ARMA" model driven by a different set of shocks $\upsilon_t$. In this case

$$\Psi(L)c_t = \Phi(L)\,\upsilon_t$$

where $\upsilon_t$ is a set of shocks $\upsilon_t = N(0, \sigma_\upsilon)$ which generate dynamic or cyclical (serially correlated) effects through the distributed lag matrices $\Psi(L)$ and $\Phi(L)$. So for example $\Phi(L)\upsilon_t$ is a distributed lag of current and past cyclical shocks $= \Phi_0\upsilon_t + \Phi_1\upsilon_{t-1} + \ldots$

The simplest model of the cycle that can generate serially correlated movements and periodic cycles is the stationary AR(2) model discussed earlier when discussing the multiplier-accelerator model.

$$c_t = \psi_1 c_{t-1} + \psi_2 c_{t-2} + \upsilon_t$$

The nature of the cycle generated depends on the roots ($z$) of this second order difference equation.

$$z_1, z_2 = \frac{-\psi_1 \pm \sqrt{\psi_1^2 + 4\psi_2}}{2}$$

If the roots are real

$$\psi_1^2 + 4\psi_2 > 0$$

then the impact of a shock $\upsilon_t$ will gradually and asymptotically tend to zero. In this case a "cycle" may only be observed because the shock process is white noise and negative shocks are likely to follow positive shocks. But this would be in line with the Lucas definition of the cycle which refers to serially-correlated movements around a trend.

If on the other hand the roots are complex

$$\psi_1^2 + 4\psi_2 < 0$$

then a shock $\upsilon_t$ will generate a periodic cycle of its own with periodicity or length of cycle given by

$$k = \frac{2\pi}{\cos^{-1}\left[\psi_1/(2\sqrt{-\psi_2})\right]}$$

In this case what we observe as the cycle will be a mixture of overlapping waves responding to positive and negative shocks as discussed in Chapter 2.

An alternative approach is where the cyclical component can be set up explicitly as a trigonometric function where the cycle can be expressed as a mixture of sine and cosine waves dependent on two parameters $\alpha$ and $\beta$ with a given frequency $\lambda$

$$c_t = \alpha \cos \lambda t + \beta \sin \lambda t$$

$$c_t^* = -\alpha \sin \lambda t + \beta \cos \lambda \tau$$

which are combined recursively to produce a time-varying stochastic cycle given by

$$\begin{bmatrix} c_t \\ c_t^* \end{bmatrix} = \rho \begin{bmatrix} \cos \lambda & \sin \lambda \\ -\sin \lambda & \cos \lambda \end{bmatrix} \begin{bmatrix} c_{t-1} \\ c_{t-1}^* \end{bmatrix} + \begin{bmatrix} \upsilon_t \\ \upsilon_t^* \end{bmatrix}$$

where $\rho$ is a damping factor < 1 to ensure the process is stationary. This is effectively an ARMA(2) model which reduces to a simple AR(1) process if $\lambda = 0$ or $\pi$. But in general, this framework assumes the cycles are periodic by definition.

A satisfactory description of cyclical movements may require more than one cyclical component operating at different frequencies, for example a business cycle frequency of 2–8 years and a credit cycle component of 8–20 years. Simultaneous estimation of these different cyclical components can be achieved within the UC framework. So in principle $c_t$ can be set up as a set of $N$ trigonometric cycles.

$$c_t = \sum_{j=1}^{N} c_{j,t}$$

The parameters of the model are typically estimated using the Kalman Filter and a key restriction in the "standard" UC model is that the covariance between the unobserved shocks driving the cycle and the shocks driving the trend is zero.

$$\mathrm{Cov}(\varepsilon_t \upsilon_t) = 0$$

Many applications of the UC model with this restriction trend to produce a relatively smooth trend component and relatively large cyclical components.

### 3.4.2 The Beveridge-Nelson Decomposition

The Beveridge-Nelson (1981) decomposition (BN) starts from a similar underlying specification to the UC model. The trend component is typically set up in the same way but in this case the cyclical component is a moving average process resulting from the same stochastic source as the trend. In this case:

$$y_t = \tau_t + c_t$$

$$\tau_t = \tau_{t-1} + \mu_t + \varphi(1)\epsilon_t$$

$$c_t = \tilde{\varphi}(L)\epsilon_t$$

So the trend and cycle are perfectly and (negatively) correlated in this model. Decompositions using this approach typically produce more variable trends and much smaller cyclical components. We will see this is very evident in the UK data. Morley et al. (2003) show that the Beveridge and Nelson (1981) decomposition is in fact equivalent to that of the trend-cycle decomposition from a general UC model that allows for correlation between the shocks driving the trend and cycle. In other words the standard UC model employed by many researchers effectively imposes what in many cases is a testable zero correlation restriction on the shocks driving trend and cycle. Morley et al. (2003) show this zero correlation restriction can be rejected for US quarterly data. Once the restriction of a zero correlation between the shocks driving trend and cycle is relaxed the UC and BN models are essentially the same. As Grant and Chan (2017) discuss both methods in this case will in general deliver cycles that exhibit what many business cycle researchers find unpalatable—cycles that are noisy and small in amplitude.

### 3.4.3 HP and Band-Pass Filters—Non-parametric Methods

An alternative approach to deriving trends and cycles is to use a non-parametric filtering approach. In this case the specification of the trend and cycle components are not determined a priori and then estimated but rather are derived by meeting some other a priori criteria. One well known non-parametric method is the Hodrick-Prescott or HP filter. The Hodrick-Prescott decomposition is based on the smoothing problem initiated by Bohlmann (1899) and Whittaker (1923). The trend is the solution to the following problem

$$\min_{\tau}\left[\sum_{t=1}^{T}(y_t-\tau_t)^2+\lambda\sum_{t=1}^{T}\left(\Delta^2\tau_t\right)^2\right]$$

where λ is a fixed constant that penalises variability in the trend component. The larger the value of λ, the smoother is the associated HP trend. Hodrick and Prescott (1980, 1997) highlight that λ may be viewed as the noise-to-signal ratio under certain restrictive conditions. They then suggest setting $\lambda = 1600$ for US quarterly data and 100 for annual data. Grant and Chan (2017) show that a general unobserved component with a more flexible specification for the trend and which allows for correlation between trend and cycle components nests the HP filter as a special case. Harvey and Jaeger (1993) show that the HP filter is the optimal linear estimator of the trend in the basic UC model

$$y_t = \tau_t + \nu_t$$

$$\tau_t = \tau_{t-1} + \mu_t + \epsilon_t$$

$$\mu_t = \mu_{t-1} + \eta_t$$

$$\lambda = \frac{\sigma_\nu}{\sigma_\eta}$$

Notice, however, that this rationalisation has the following assumptions: (i) the series is integrated of order 2; (ii) the cyclical component is a white noise process; and (iii) that the chosen value of the parameter λ corresponds to the ratio of the variance of the irregular component to the variance of the innovation in the trend component. This is one reason why HP filters may produce spurious cycles if the smoothing parameter chosen or other implied restrictions are different to that implied by the unrestricted estimates of the UC model. As an example of the HP filter producing spurious cycles Harvey and Jaeger (1993) and Harding and Pagan (2005) show that if output follows a random walk, which would be interpreted in the UC framework as a pure stochastic trend, and one applies the standard HP filter to data generated from the random walk process one will generate what looks like a cycle with significant serial correlation.

Band-pass filters are another popular way of non-parametric de-trending but take a slightly different approach to the HP filter. The band-pass filter essentially takes a two-sided weighted moving average of the data where the weights are chosen so that cycles in a particular band, given by a specified lower and upper bound, are allowed through and the remaining cycles are filtered out. Essentially for business cycle analysis a business cycle band of 2–8 years is typically chosen. Baxter and King (1995) and Christiano and Fitzgerald (1999) both provide approximations to the ideal band-pass filter. These filters have also been argued to generate spurious cycles (e.g. Benati 2001). Because the band-pass filter is both a low pass and a high pass filter, it leads to cycles that are typically smoother than the HP filter, which acts only as a high pass filter and does not cut off higher frequency movements below say two years.

A feature of both types of filters is whether the filter is "one-sided" or "two-sided". A one-sided filter only uses information up to time t when assessing the state of the cycle at time t. A two-sided filter however uses both past and future observations to assess the state at any given moment in time. For the purposes of developing forecasting models, e.g. Phillips curve models that use the output gap to forecast inflation, a one-sided filter is a more appropriate method to de-trend output. This is because when carrying out forecasts in real time future information about the time series is unavailable, so one needs to construct a model based on what forecast information set that will be available at the time. For the purposes of retrospectively analysing business cycles however a two-sided filter is more appropriate as it takes into account all the information we have about the time series although there will still be an issue about the end point of the series where the filter will be missing the information about future values.

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