Chapter 6
Conclusion

Game theory is a mathematical framework for strategy analysis and design as well as for optimal decision-making under conflict and behavioral uncertainty. On the one hand, game theory plays a key role for modern economics; on the other, it suggests possible approaches and solutions for complex strategic problems in various fields of human activity.

The logical methods of optimal strategy design in mathematical terms date back to the beginning of the seventeenth century. The problems of production and pricing in oligopolies, i.e., the classical problems of game theory, were studied in the nineteenth century by Cournot [225, 226] and Bertrand [204, 205]. The idea of a game as a mathematical model for a conflict of interests appeared at the beginning of the past century in the works of Lasker, E. Zermelo, and E. Borel [209]. Pioneering results on game theory were published since the 1920s, but a systematic treatment was first presented in 1944 by J. von Neumann and O. Morgenstern in their monograph *Theory of Games and Economic Behavior* [262]. The title and content of this book indicated that game theory was claiming for a revolution in economic sciences with its novel approach. Thus, the year 1944—the first edition of the book—is generally considered as the birth of game theory.

Further development of game theory was associated with the name of American mathematician Nash [257, 258], who formulated the principles of decision dynamics. The cited monograph by von Neumann and Morgenstern became well-known mostly owing to an exploration of zero-sum games, in which a win of one party means a simultaneous loss of the other. However, equal attention in the book was paid to the games with non-opposing interests. Nash analyzed different management strategies in economics and business as well as different behavioral strategies and arrived at an important conclusion. With such strategies, one party is always gaining while the other losing, i.e., they yield victors and vanquished. Nash was wondering: is it possible to find an equilibrium in which nobody wins and also nobody loses? Such strategies would revolutionize negotiations, resolution of conflicts and design of other compromise decisions.

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Nash created analysis methods for the games in which *all* parties simultaneously win or lose. An example of such a game is wage negotiations between a labor union and an employer. This situation may result in a lengthy strike (affecting both parties) or a mutually beneficial agreement. Nash modeled a situation (the so-called Nash equilibrium or noncooperative equilibrium, as we know it today) in which both parties use optimal strategies, thereby achieving a stable equilibrium. The players are interested in keeping this equilibrium, since any unilateral deviation would worsen their condition.

Nevertheless, the concept of Nash equilibrium is selfish: it guides each player towards maximization of his/her/its *own* payoff only. The constructive criticism of this selfish approach motivated V. Zhukovskiy and his postgraduate student K. Vaisman to adequately consider the interests of all other parties of a conflict, even at the cost of neglecting the individual interests of each player. In 1994 they formulated the altruistic concept of Berge equilibrium, which is the subject of this book. Here are three English proverbs related to Berge equilibrium.

(a) It is better to give than to take.[1]
(b) Share and share alike.[2]
(c) Live and let live.[3]

Game theory has been evolving through different stages, with different levels of interest from the scientific community. In the 1950s, game-theoretic methods seemed to be very promising, but all excitement gradually faded in the 1960s–1970s, despite the considerable mathematical results established then. However, the 1980s saw an increased utilization of game-theoretic methods in different applications, and today one would hardly find any field of economics and business science (micro- and macro-economics, finance, marketing, management, etc.) which can be studied without a basic background in game theory [109–116, 128, 147, 152, 161–167, 175–188, 190, 193, 196].

In the course of development, game theory has become a general logical-mathematical theory of conflicts. Game-theoretic methods allow us to analyze different conflicts (phenomena and processes), to outline and predict the behavioral scenarios for all conflicting parties, as well as to suggest recommendations on conflict resolution and elimination of dangerous consequences.

During the two or three recent decades, the value of game theory and the interest in game-theoretic research have significantly increased in many fields of economic and social sciences. It is no exaggeration to state that game theory is vital for modern

[1]Originates from *The Bible*, Acts 20:35: "In all things I have shown you that by working hard in this way we must help the weak and remember the words of the Lord Jesus, how he himself said, 'It is more blessed to give than to receive.'"

[2]Give equal shares to all. Daniel Defoe appears to be the first to have used this phrase in *The Life and Strange Adventures of Robinson Crusoe* (1719): "He declar'd he had reserv'd nothing from the Men, and went Share and Share alike with them in every Bit they eat."

[3]People should accept the way other people live and behave, especially if they do things in a different way.

economics. In the present time the scientific community is devoting much research effort to extend the scope of game theory. On the one hand, this theory forms a rather abstract branch of mathematics; on the other, a rather efficient analysis tool for economic, political, legal, military, technical and other problems. Applications of game-theoretic methods are found in agriculture, medicine, ecology, sports, anthropology, psychology, pedagogy, sociology, and others.

In modern economics and business science, game theory has a wide variety of applications. Game-theoretic tools and approaches may be fruitful in situations connected with strategic decision-making, competition, cooperation, risks and uncertainty. At the macro-level, game theory is used for decision-making processes in international trade, competition, taxation, protectionist practices and cartelization (e.g., OPEC), including an assessment of contributions for each party and further allocation of profits. At the micro-level, game theory assists, e.g., in advertising cost optimization in a competitive market, efficient production organization or auction design. Using game-theoretic methods, one may choose business partners for joint projects, construct behavioral scenarios for competitors, as well as find mechanisms of interregional interactions and income allocation schemes. Game-theoretic models are widespread in planning and prediction, strategic development design, pricing, negotiations, in particular, coordination of mutual interests and relations of partners, asset owners, employees and employers, and other economic agents. Moreover, game-theoretic methods are used to analyze the behavior of criminal gangs and political struggle.

Game theory provides

– a formal and clear language to analyze different economic phenomena, processes and systems;
– possible tools to check intuitive or rational decisions and solutions in terms of their consistency and applicability to a given problem;
– principles, criteria and methods to find optimal solutions.

A classical and most remarkable example of successful application of game-theoretic methods was the Federal Communications Committee (FCC) spectrum auction held in 1994 [148]. The organizers intended to collect at least \$ 3.5 million but, with the help of game theory experts, the real revenues reached approximately \$ 8 billion [238, 252, 253, 259–265, 270–277, 279, 283–289, 306].

Nowadays, the number of publications (papers, monographs, textbooks) on game theory is into tens of thousands [149, 150]. Despite its long history, game theory has become appreciated by the scientific community only relatively recently. The pioneering research of future Nobel laureates J. Nash, R. Selten, L. Hurwitz, R. Myerson and others took place in the 1950s. Yet the first Nobel Prize in Economic Sciences for the advances in game theory was awarded in 1994, which was the first indication of wide scientific recognition. Since then, during a period of less than 15 years, the Nobel Prize in Economic Sciences was awarded seven times for game-theoretic research; in particular, in 2005 jointly to R. Aumann and T. Schelling "for having enhanced our understanding of conflict and cooperation through game-theory analysis."

As a matter of fact, an explicit polarization can be observed in the monographs and textbooks on game theory. In a considerable part of these publications, the authors give a detailed description of the mathematical framework of game theory, including solution concepts, principles and models, restricting themselves to a few abstract examples. As a result, it may seem that game theory has nothing to do with real economic problems. Such books are characterized by a high level of formal abstraction and a considerable simplification of real situations, which makes the corresponding game-theoretic models unsuitable in practice [193]. This gap between theory and practice often appears in light assumptions and conclusions, which are not accompanied by good interpretations in the context of a given problem or not reduced to specific managerial decisions or behavioral strategies in a given situation.

This state of affairs explains the existing scepticism of practitioners (economists and managers) towards game theory. Another reason of the scepticism that restricts the use of game theory is the relatively high complexity of this theory. The main complexity consists in its logic rather than its mathematical framework.

Another considerable part of the literature is focused on outlining economic situations that can be described by games, without a proper consideration of methods and tools to find solutions. Such an approach conceals the rich capabilities of game theory. As a result, practitioners have a clear idea that this theory is applicable, but do not fully comprehend how. In other words, the practical results included in the monographs and textbooks on the subject are either trivial, or very complicated [147, p. 246].

The authors of this book are far from overestimating the capabilities of game theory, which is often done by some researchers. Game-theoretic models represent a tool that should be properly handled and applied whenever possible. Like any other models, games provide a more or less adequate approximation of real situations and events. This does not mean, however, that the models cannot be efficient in practical problems. Game theory itself is neither a universal description of real life in mathematical terms, nor a universal solution procedure for all problems. For a successful application of game theory, one needs to be facilitated with its logical-mathematical framework and also with the subject under study.

Indeed, game theory and its postulates may seem rather abstract or even unsuitable. But we believe that the major application of game theory is the development of a special "strategic vision" of a current situation, often nonformalizable yet facilitating a qualitative, complete and rigorous analysis.

As a counterweight to the generally accepted selfish Nash equilibrium, this book is devoted to a new solution concept for noncooperative games—the altruistic Berge equilibrium. For over twenty years since its appearance, Berge equilibrium has been facing different troubles. *First*, the sudden death of K. Vaisman at the age of 35, who was the initiator and enthusiast of this concept; *second*, the negative review of Shubik [269] of Berge's book [202] in which the main idea of this equilibrium was described; *third*, the unclear usefulness of Berge equilibrium in real problems (where and how can it be applied?); *fourth*, the easiness of deriving theoretical results on Berge equilibrium in two-player games (for such games, Berge equilibrium design is reduced to Nash equilibrium calculation if the players exchange their payoff functions); and *fifth*, the absence of an authoritative researcher to lead this direction of investigations. These reasons (and probably others not mentioned here) suspended the elaboration of a constructive theory of Berge equilibria at the stage of accumulation of facts, revelation of properties, comparison with Nash equilibrium, and analysis mostly in the context of matrix games.

In our opinion, the forthcoming stage of development will be associated with an heuristic approach to the mathematical theory of Berge equilibrium. No doubt, at this stage it is necessary to answer the following questions of paramount importance:

1^0. How should a Berge equilibrium be constructed?
2^0. Does a Berge equilibrium exist?

Actually, these questions are directly addressed in the present book for the *static setup* of noncooperative N-player games (such games have no dynamics and are time-invariant).

One central result of the book is that the Germeier convolution is involved in answering question 1^0. More specifically, the problem is reduced to a saddle point calculation for a special Germeier convolution (Sect. 2.8.3) of the players' payoff functions, which is efficiently constructed using the original noncooperative game: the minimax strategy at this saddle point is the Berge equilibrium in the original game.

This technique has allowed us to answer question 2^0 about the existence of a Berge equilibrium. Moreover, our existence theorem takes into account the internal instability of the set of Berge equilibria (there may exist two Berge equilibria such that the players' payoffs in one equilibrium are strictly greater than in the other, see Example 2.4.1). To deal with this, the concept of Berge equilibrium has been augmented by Pareto maximality with respect to other Berge equilibria, yielding the so-called Berge–Pareto equilibrium (Definition 2.9.1). Theorem 2.9.1 establishes the existence of a Berge–Pareto equilibrium in mixed strategies for continuous payoff functions and compact strategy sets of all players.

Another central result consists in laying the theoretical foundations of Berge equilibrium design under interval strategic uncertainty, a novel direction of Berge equilibrium-related research. We suggest two decision approaches under such conditions. *First*, the formal definition of a strongly-guaranteed Berge equilibrium, which is reduced to instantaneous minimization of each payoff function and further transition to the game of guarantees. *Second*, the formal definition of Slater-minimal guarantees [82, 83] for each situation, with the same transition to the game of guarantees [33, 109–114]. This approach involves a two-level hierarchical game in which the lower level is formed by strategic uncertainties (under the information discrimination of all players). Both definitions lead to an appropriate modification of the maximin. The latter and former approaches yield existence theorems in mixed strategies, see Theorem 3.5.1 and also the end of Sect. 3.5.3.

Finally, the applications to the Cournot and Bertrand oligopoly models are described in Chap. 4, including the case of import as an uncertain factor.

Note that the material presented in Chap. 4 settles the issue regarding the use of Berge equilibrium in real problems. In addition, as explained in the Preface, the concept of Berge equilibrium completely matches the Golden Rule of ethics: "Behave to others as you would like them to behave to you."

Finally, we should emphasize that the approach adopted in Chap. 3 is not the only possible one. Even for the antagonistic case of noncooperative games, there exist other principles (minimax regret, pessimism–optimism) as well as other criteria (Laplace–Bayes, Hodges–Lehmann, BK-criterion, P-criterion [137]), each having certain advantages and shortcomings. We have not considered Berge equilibrium for differential positional games, although the existence theorem for the separate dynamical system was established earlier in [72] under an appropriately modified formalization of the players' strategies and motions generated by them. (Also see numerous publications of Zhukovskiy's scholars on dynamic programming-based Berge equilibrium design for specific multistage games arising in competitive economics). Other applications-relevant models not covered by this book include differential positional games with time delay, multistage positional setups of the games, and many more. The above-mentioned problems are waiting for thorough study, and the reader will certainly discover many interesting facts getting deeper into them. "On deep paths of mystery unknown creatures leave their spoor."[4]

[4] A fragment from *Ruslan and Lyudmila*, a poem by Aleksandr S. Pushkin, (1799–1837), a Russian poet, novelist, dramatist, and short-story writer. Considered as the greatest poet and founder of modern Russian literature.