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Behavioral Operational Research in Portfolio Selection

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3.1 Introduction

Portfolio selection is part of the finance that discusses asset choice and diversification to improve investor's wealth. Despite conventional finance, behavioral finance does not consider the investor to be completely rational. Instead, it discusses the effects of psychological factors on decision-making. Hence, Behavioral Operational Research (BOR) is the way to bridge between Operational Research and behavioral finance.

The term *portfolio selection* (optimization) first gained attention, when Markowitz (1952) proposed his famous model. In the past decades, many portfolio selection models have been developed using basic operational research ideas. However, behavioral finance scientists have criticized those models, because they do not contain human attitudes and behavioral biases. For example, some scientists such as Thaler (1999) explain the importance of mental accounting, which is a behavioral bias of investors, who have multiple parallel accounts (goals) in

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their minds. Following this idea, the term *behavioral portfolio* was first introduced by Shefrin and Statman (2000). Considering the *mental accounting* concept, Das et al. (2010) introduced the Mental Accounting model (MA), which is extended in many studies: considering derivatives and non-normal returns by Das and Statman (Das et al. 2010; Baptista 2012; Momen et al. 2016, 2019a; Alexander et al. 2017).

Most of the above studies have considered behavioral effects on portfolio selection from two different points of view: *elemental* and *structural effects*. By definition, elemental effects only influence components of portfolio models, while structural effects change the way the portfolio is designed as a whole. In the following sections, we first discuss the effects of behavior on portfolio selection, and then we present some models that have taken those effects into consideration.

3.2 Effect of Behavior on Portfolio Selection

In this section we explain elemental and structural effects of behavior on portfolio selection, respectively. We first address individual elements and how they affect portfolio selection, and then we mention different issues that influence structure of portfolio selection.

3.2.1 Elemental Effects

Over the course of past decades many elements have been introduced for portfolio selection. However, the most prominent elements include expected return of investments, risk of investments and more recently behavioral biases of investor (as indicated in BOM literature; see Kunc, Chapter 1). We address all of them in the following paragraphs.

3.2.1.1 Behavioral Biases

Sjöberg and Engelberg (2009) discuss the effect of psychological factors on investment decision-making. Moreover, researches such as Brandt and Wang (2003) and Holt and Laury (2002) open up the possibility

that psychological factors may have an effect on decision-making of investors. Furthermore, studies such as (Fellner 2009; Pan and Statman 2012; Pompian 2012) discuss more specifically the impact of behavioral biases on investment decisions.

Researchers have distinguished numerous biases (e.g. Bailey et al. 2011). Some of the most influential behavioral biases are *overconfidence* (one's irrational belief in his strength of intuitive reasoning, judgment and cognitive abilities), *ambiguity aversion* (investor hesitation when probability distributions of events seem uncertain), *self-control* (investor inclination to spend now at the expense of saving for future) and *framing* (investor responds to similar situation differently due to difference in context). These behavioral biases have many unpleasant consequences for investors, such as: lower expected utility (Odean 1998), excessive trading (Barber and Odean 2001), lower returns (Bailey et al. 2011) and leaving market (Odean 1999). More importantly, they generate deviations from normative models assuming perfect economic rationality (as indicated in BOM literature; see Kunc, Chapter 1).

3.2.1.2 Risk

There are two questions in modeling risk for portfolio selection, first what risk measure should be used in modeling the risk of assets, second how the investor attitude toward risk should be addressed. In order to answer the first question, some studies use *pure risk* (Siebenmorgen and Weber 2003), and *risk premium* (Cillo and Delquíe 2014), others mostly consider a form of *Value-at-Risk* (VaR) (Shefrin and Statman 2000; Berkelaar et al. 2004; Das et al. 2010; Alexander and Baptista 2011; Brunel 2011; Baptista 2012; Alexander et al. 2017).

These risk measures have the benefit of conceptual and calculation simplicity (Leavens 1945; Garbade 1986; Jorion 2007). However, none of them are proper coherent risk measures (Artzner et al. 1999). Specially, VaR as the most popular behavioral risk measure is not coherent, because it is not subadditive. This is an important drawback for a behavioral risk measure (Kalyvas et al. 1996), because behavioral portfolios are built based on the assumption that people have different mental

accounts, and if a risk measure is not subadditive, it may produce mental account sub-portfolios for which the sum of risks is less than the risk of investor total holdings. This is against the idea that diversification does not increase risk.

The next drawback of these risk measures is related with the second above mentioned question, because these risk measures include investor attitude toward risk by deriving a single fixed risk aversion coefficient. There are three main issues regarding the use of risk aversion coefficients. First, they are fixed during the time, second, they are fixed in different mental accounts (goals), and third, they are fixed in encountering various levels of gain and loss. However, Pratt and Arrow in an early introduction of risk aversion came up with the hypothesis that risk aversion changes (Halek and Eisenhauer 2001), and others such as (Brunnermeier and Nagel 2008; Guiso and Paiella 2008; Sahm 2012) also confirmed this hypothesis in different ways. Moreover, there are studies that indicate people have varying attitude toward different levels of gain and loss (Kahneman and Tversky 1979; Fennema and Wakker 1997), and also risk aversion cannot be fixed throughout mental accounts (Das et al. 2010).

3.2.1.3 Expected Return

Some portfolio models use standard estimators for expected return (Shefrin and Statman 2000; Berkelaar et al. 2004; Nevins 2004; Das et al. 2010; Alexander and Baptista 2011; Brunel 2011; Baptista 2012; Alexander et al. 2017; Momen et al. 2017a), which suffer from the two following issues. First, the standard estimators are sensitive to return outliers. As Fabozzi et al. (2010) argue, sometimes even one outlier such as an extreme return affects the expected return, which is unfavorable in modeling. Second, behaviorally biased investors do not usually completely rely on statistical estimators; instead they tend to follow their own attitudes toward expected returns. Therefore, it is very appealing for them to have a model that includes their own estimates (Fabozzi et al. 2010).

3.2.2 Structural Effects

There can be many structural effects of behavior on portfolio selection; here, we present two effects that have been documented recently: prescription effect, and mental accounting effect.

3.2.2.1 Prescription Effect

Portfolio models rely on analysis of humans for inputs. It means that inputs such as expected return of assets and their covariance matrix should be estimated and inserted to the models by humans. According to Raiffa (1968) there are three types of analyses: *normative*, *descriptive* and *prescriptive*. (1) Normative analysis is concerned with the rational solution to the problem at hand. It defines an ideal that actual decisions should strive to approximate. (2) Descriptive analysis is concerned with the manner in which real people actually make decisions. (3) Prescriptive analysis is concerned with practical advice and tools that might help people achieve results more closely approximating those of normative analysis.

Based on the above definitions, the unsatisfying performance of investors due to behavioral traits can be explained as follows (Russo and Schoemaker 1992; Odean 1999): investors and their advisors incorporate different types of analyses. Advisors as rational agents perform normative analysis to achieve acceptable performance, while the instinctive behaviors of investors are in the descriptive analysis domain. Therefore, none of their views can provide a satisfying portfolio. A satisfying portfolio is based on prescriptive analysis, this is called prescription effect.

3.2.2.2 Mental Accounting Effect

Investors with mental accounting bias do not consider their portfolios as a whole. Instead, they consider their portfolios as collections of mental sub-portfolios (mental accounts) where each sub-portfolio is associated with a goal and each goal is evaluated by deviation from a

threshold also known as a risk measure. According to Shefrin (2010), mental sub-portfolios are segmented in a narrow framing process, which overlooks interdependencies between mental accounting structures. Hence, the segmentation of sub-portfolios is rarely optimal. However, if an investor wants to consider mental accounting, he has to find a new structure that is able to contain several mental accounts at the same time.

3.3 Behavioral Portfolio Models

As stated in the previous section, from a BOR point of view, portfolio models can be designed by considering elemental effects and structural effects. In this section, we present models for containing these two effects.

3.3.1 Models for Elemental Effects

In Sect. 3.2.1, three elemental effects were presented. In this section, models for each of those three effects are depicted in the same order.

3.3.1.1 Models for Behavioral Biases

Pompian (2012), Pan and Statman (2012), and Nordén (2010) find behavioral biases to be among psychological factors that impact the risk attitude. On the other hand, behavioral biases are affected by past returns as stated by Chen and Kim (2007) and Statman et al. (2006). Following these results and Grable et al. (2006), varying risk attitude depends on behavioral biases and investors latest realized return from the market.

In order to represent a relationship between risk attitude (α), behavioral biases and latest realized return of investor, an obvious option is to consider multivariate linear regression (Momen et al. 2017b):

$$\alpha = \Theta\mathbf{B} + \varepsilon \quad (3.1)$$

where $\Theta = [\theta_0 \theta_1 \cdots \theta_M]$, $\theta_i (i = 0, 1, \dots, M)$ is the i -th regression coefficient, $\mathbf{B} = [1 b_1 \cdots b_M]'$, $b_j (j = 1, \dots, M)$ is the j -th behavioral bias, M is the number of behavioral biases under consideration, and ε is the error term.

Equation 3.1 can be used to derive constraints in the model that calculates risk aversion based on behavioral biases. However, the problem is that regression does not infer or present causality between variables (Scheines et al. 1998). To infer causality based on data, one should follow causation methods such as the PC algorithm (Scheines et al. 1998), which results in a network of relations between desired variables. The algorithm decides on independence of pairs of variables (e.g. behavioral biases) by using conditional independence tests (Spirtes et al. 2000). Therefore, we define $S = [b_1 b_2 \cdots b_M \alpha r^T]'$ and $s_i (i = 1, \dots, M + 2)$ as the i -th element of S , where $b_k (k = 1, \dots, M)$ is the k -th behavioral bias, r^T is the latest realized return, α is an indicator of risk attitude (confidence level in the risk measure), and M is the number of behavioral biases under consideration. Equation (2) summarizes the output of the PC algorithm (Momen et al. 2019b):

$$\mathbf{A} \times \mathbf{S} = \mathbf{S}_0 \tag{3.2}$$

where $\mathbf{S}_0 = [b_{01} b_{02} \cdots b_{0M} \alpha_0 r_0^T]'$ is the vector of intercepts, \mathbf{A} is an $(M + 2) \times (M + 2)$ matrix, the ij element (a_{ij}) of which is defined as follows.

$$a_{ij} = \begin{cases} 0 & , \text{ No edge from } s_i \text{ to } s_j \\ 1 & , \quad i = j \\ -\text{edge coefficient from } s_i \text{ to } s_j & , \quad \text{Otherwise} \end{cases} \tag{3.3}$$

The above formulations are intended to draw a relationship between risk attitude and behavioral biases of the investor.

3.3.1.2 Models for Risk

A proper risk measure for behavioral portfolio selection should be coherent, and should contain investor attitudes. There is a category of

risk measures with these qualities, which are called spectral risk measures (SRM) and defined as (Acerbi 2002):

$$M_{\phi_a} = - \int_0^1 \phi_a(p) q_p dp \quad (3.4)$$

where q_p is the p payoff quantile, $\phi_a(p)$ is an investor specific weighting risk aversion function, and $p \in [0, 1]$.

SRMs relate the risk measure to the subjective risk aversion of the investor. More precisely, the SRM is a weighted mean of the quantiles of payoff distribution, where the weights are related to the investor risk aversion function (Dowd and Cotter 2007).

The most well-known spectral risk measure is the Conditional Value at Risk (CVaR) (Rockafellar and Uryasev 2000). However, CVaR leaves little space for investor views on the risk measure (Grootveld and Hallerbach 2004). Hence, Dowd et al. (2008) assess several risk measures, and argue in favor of exponential risk measures.

3.3.1.3 Models for Expected Return

Black and Litterman (1991) have proposed a method for estimating robust inputs that are able to contain investor views (Silva 2009). Asset proportions derived from such a model are less sensitive to the model input variations (Fabozzi et al. 2007). The Black-Litterman estimator of return (r_{BL}) is defined as follows:

$$r_{BL} = \left[(\tau \Sigma)^{-1} + P' \Omega^{-1} P \right]^{-1} \left[(\tau \Sigma)^{-1} \Pi + P' \Omega^{-1} q \right] \quad (3.5)$$

where Π is expected excess return vector, Σ is covariance matrix of returns, τ is a small scalar ($\tau \ll 1$) in $\Pi = \mu + \epsilon_{\Pi}$, $\epsilon_{\Pi} \sim \text{Normal}(0, \tau \Sigma)$, where μ is the unknown true expected return of assets, which is often estimated using equilibrium expected return. q is a $K \times 1$ vector of investor views, P is a $K \times N$ matrix in $q = P\mu + \epsilon_q$, $\epsilon_q \sim \text{Normal}(0, \Omega)$ and Ω is a $K \times K$ matrix expressing the confidence in the views, N is the number of available

assets, K is the number of assets that the investor has views on their expected returns. One can interpret $\tau \Sigma$ as investor confidence in the precision of his estimates for the equilibrium expected returns, and Ω as his confidence in the accuracy of his views on individual asset returns.

3.3.2 Models for Structural Effects

After presenting models for elemental effects in the previous section, in this section, we introduce models for the two structural effects that have been discussed before.

3.3.2.1 Models for Prescription Effect

When dealing with the modeling of a portfolio for a client (investor), advisors or investment advisory companies either rely on their own market perception to build a model for their client or trust the client's perception of market. In the first case, the literature shows that the relationship between advisor and client will be terminated prematurely, since clients cannot rely on advisors understanding the market for a long time. This is usually due to the client's irrational understanding of his abilities to outperform the market. However, in the second case, as clients usually have less experience and information about the market than advisors, they often end up losing their money, which again ruins their relationship with the advisor. A third option for advisors is to measure the behavioral biases of their clients and use them as proxies to balance between first two options. In this way, the model will decide whether the advisor should rely on his own perceptions or not, and if so to what extent.

As shown in Fig. 3.1, the conceptual model of Prescriptive Portfolio Selection (PPS) reveals that its parameters are trade-offs between advisor and investor, or in other words between normative and descriptive analyses. In order to estimate parameters using this model, we cannot solely rely on any of these analyses, and the best thing we can do is to compromise, which is provided by prescriptive analysis. Therefore, a proxy can be used to compromise between normative and descriptive

analyses. This proxy can be a behavioral bias (or several) that distinguishes between normative and descriptive analyses. For example, Momen et al. (2017a) derive a model that balances risk and return with overconfidence bias and find out that investors have no significant preference between results of their own perceptions and the proposed model (Prescriptive Portfolio Selection).

3.3.2.2 Models for Mental Accounting Effect

In most available portfolio selection models, decision variables are defined as the proportion of assets in the portfolio. However, in order to include mental accounting effect, we define decision variables (w_{ij}) as proportion of asset $_j$ in sub-portfolio i (SP_i). This definition helps us to model behavioral portfolio selection that usually includes more than one sub-portfolio in a collective manner; hence, the name *Collective Mental Accounting* (CMA) has been proposed (Momen et al. 2019a). CMA is defined as follows:

w_i : Vector of asset weights in sub-portfolio i ($w_i = [w_{i1}, w_{i2}, \dots, w_{in}]$)

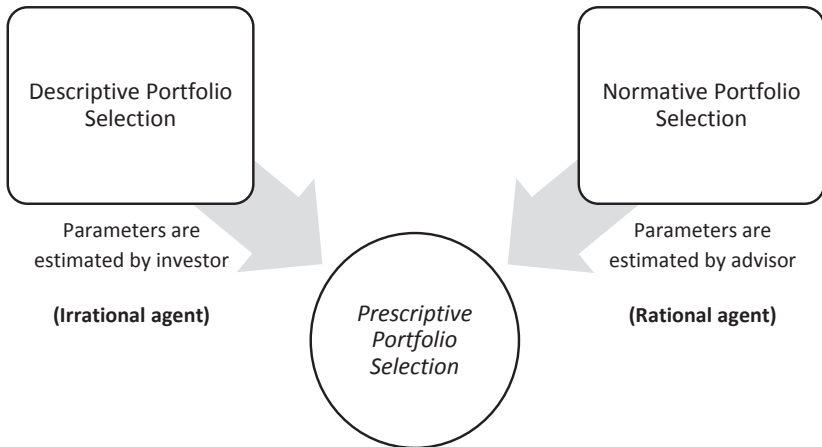


Fig. 3.1 Conceptual model for prescription effect

$\sum_i w_{ij}$: Proportion (weight) of asset j in the aggregated portfolio (total asset holdings)

Ω_i : Proportion (weight) of wealth allocated to sub-portfolio i , if exogenously determined

n : Number of assets

m : Number of sub-portfolios

The expected return for the portfolio is defined as $\sum_{j=1}^n r_j \sum_{i=1}^m w_{ij}$, where r_j is the expected return for asset j . In CMA, there is a risk measure (as constraint) for each sub-portfolio, which is defined as $\Pr [r(\text{SP}_i) \leq H_i] \leq \alpha_i$ for sub-portfolio i , where $r(x)$ is the random variable of expected return for portfolio x , and α_i is the maximum probability of not reaching the threshold for sub-portfolio i (H_i). Therefore, the basic CMA model is as follows:

$$\text{Max} \sum_{j=1}^n r_j \sum_{i=1}^m w_{ij}$$

$$\Pr [r(\text{SP}_i) \leq H_i] \leq \alpha_i; i = 1, 2, \dots, m$$

$$\sum_{j=1}^n \sum_{i=1}^m w_{ij} = 1 \quad (3.6)$$

The above model has the capability to calculate the proportion of each sub-portfolio endogenously. In other words, there is no need to pre-specify the proportion of an investor's wealth for each mental sub-portfolio. Anyway, some researchers such as Baptista (2012) believe sub-portfolio proportions should be defined exogenously, because some experienced or confident investors may not like to rely on mathematical model outputs solely, and prefer to define their sub-portfolio weights themselves. Therefore, in CMA, one can pre-specify sub-portfolio proportions by including them in a simple mathematical constraint such as $\sum_{j=1}^n w_{ij} = \Omega_i; i = 1, 2, \dots, m$, where Ω_i is the exogenously defined proportion for sub-portfolio i . One can rewrite $\Pr [r(\text{SP}_i) \leq H_i] \leq \alpha_i$ as:

$$rw_i^T - k(\alpha_i)\sqrt{w_i\Gamma w_i^T} \geq H_i \quad (3.7)$$

where r represents the vector of expected returns, w_i is vector of asset weights in sub-portfolio i (i.e. $w_i = [w_{i1}, w_{i2}, \dots, w_{in}]$), Γ denotes covariance matrix, and X^T is the transpose of X . Therefore, the CMA model can be rewritten as:

$$\text{Max} \sum_{j=1}^n r_j \sum_{i=1}^m w_{ij}$$

$$rw_i^T - k(\alpha_i)\sqrt{w_i\Gamma w_i^T} \geq H_i; i = 1, 2, \dots, m \quad (3.8)$$

Now we introduce the semi-definite programming (SDP) representation of the CMA model, thus it can be solved by methods such as the interior-points or spectral-bundle, efficiently. The derivation and proof of the followings are available in Momen et al. (2019a). We define all VaR constraints in a semi-definite matrix S as follows:

$$S := \begin{bmatrix} S_1 & 0 & \dots & 0 \\ 0 & S_2 & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \dots & 0 & S_m \end{bmatrix} \succcurlyeq 0 \quad (3.9)$$

where $S_i; i = 1, 2, \dots, m$ is a semi-definite representation for the i -th VaR constraint, and is proven to be as follows:

$$S_i := \begin{bmatrix} \left(\frac{rw_i^T - H_i}{k(\alpha_i)}\right)I_n & Qw_i^T \\ w_iQ^T & \left(\frac{rw_i^T - H_i}{k(\alpha_i)}\right) \end{bmatrix} \succcurlyeq 0; i = 1, 2, \dots, m \quad (3.10)$$

where I_n is the identity matrix of size n , $Q^TQ = \Gamma$. Therefore, the SDP representation for the CMA model is as the following:

$$\text{Max} \sum_{j=1}^n r_j \sum_{i=1}^m w_{ij}$$

$$S \succcurlyeq 0 \tag{3.11}$$

where e_n is a vector of n elements all equal to 1, and X, Y is a natural inner product between X and Y matrices.

There are many applications for the CMA model. Some studies such as Statman (2004) argue that in behavioral portfolios people are risk averse in one layer of portfolio pyramid and they are risk seeker in another layer. This means that models should be able to accommodate various types of risk measures in one model for behavioral portfolio selection. Since CMA includes all sub-portfolios in one model, it is possible to consider different measures of risk for each sub-portfolio.

There are many cases that an investor wants to impose upper or lower bounds on his portfolio such as $f(w_1) + f(w_2) + f(w_3) \leq \beta$, these are only possible in a standalone model such as CMA. It allows investors to be conservative in some sub-portfolios, and speculative in other ones, without changing the model entirely. For example, it is possible to use a very conservative risk measure such as the worst-case VaR for one sub-portfolio and conventional VaR for another sub-portfolio.

With the above logic, it is also possible for investors to impose various other arbitrary constraints on different sub-portfolios. For instance, an investor can ban short selling in one sub-portfolio ($w_{1j} \geq 0, j = 1, 2, \dots, n$) while permitting it in other ones (w_{ij} free in sign, $j = 1, 2, \dots, n; i = 2, \dots, m$).

3.3.3 Concluding Remarks

We discussed the role of BOR in portfolio selection in two steps: first we introduced effects of behavior, then we presented at least one BOR remedy for each of the presented effects. We classified effects in two categories: elemental and structural.

In elemental effects, we revealed the effects of behavioral biases, risk, and expected return on portfolio selection. More specifically, we mentioned the effects of behavioral biases on investment performance, and presented multiple regression and causation methods to include them in the BOR modeling. We also revealed the issues that exist regarding the modeling of risk in portfolio selection, and introduced spectral risk measures as a broad way of including investor attitudes toward risk while keeping the risk measure coherent. Moreover, we brought up the issue of robustness in expected returns along with the need for investor ideas to be included in estimating the expected returns. In order to resolve these issues, we proposed the use of Black-Litterman robust estimators instead of standard estimators.

As structural effects, we presented the prescription and mental accounting effects. Prescription effect is addressed by showing the usefulness of a prescriptive portfolio selection that makes a balance between views of investors and their advisors. Mental accounting effect is derived from behavioral finance and psychology which conclude that investors have several simultaneous mental accounts (investment goals), instead of just one. We presented Collective Mental Accounting model with a semi-definite programming representation to contain all mental accounts.

In summary, here we tried to emphasize the importance of considering behavioral aspects in modeling portfolio selection. We showed that in modeling portfolio selection, the modeler should be aware of two distinct types of behavioral effects, which could be dealt with by BOR. The goal was to provide a helicopter view on the modeling process along with some issues and remedies to complete the picture. Based on this chapter, one can see the future behavioral improvements of portfolio models either in structure of models or in the details of elements. With this concept in mind, we can expect future contributions to be more converged toward the goal of better models that capture behavioral issues and account for limitations of normative models integrating BOR practice (as indicated in Kunc, Chapter 1).

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