Chapter 1
Tribute to Ali H. Nayfeh (1933–2017)

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Abstract Ali H. Nayfeh has been the most influential scholar and scientist of the contemporary era of nonlinear dynamics in mechanics and engineering. Upon summarizing his publications and achievements, due to space restriction attention is only paid to his successful activity as a books' author, discussing specific/novel aspects and highlighting some common underlying methodological features.

Keywords Ali Nayfeh · Perturbation methods · Nonlinear oscillations · Structural mechanics

1.1 Introduction

In the last 40 years, Ali H. Nayfeh has been the most influential, worldwide recognized, scholar and scientist in the area of nonlinear dynamics applied to mechanics and engineering. He embodied a rare mixture of scientific training and expertise/interests. Indeed, upon starting as a fluid dynamicist and an applied mathematician (60s and early 70s), he turned into a combined applied mathematician, dynamicist and physicist (70s and 80s) and then a comprehensive dynamicist, with resumed and enhanced attention to structural mechanics (90s and the new millennium). He worked in many scientific areas, dealing with perturbation techniques, nonlinear oscillations, aerodynamics, flight mechanics, acoustics, ship motion, hydrodynamic stability, nonlinear waves, structural dynamics, experimental dynamics, linear and nonlinear control, micromechanics, and providing outstanding contributions. During four decades he was also a brilliant educator for thousands of student, and wrote a meaningful number of important books.

Due to lack of space, it is impossible to dwell here on all aspects of Nayfeh's scientific activity, thus choosing to solely focus on his contributions as a scholar and

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a successful books' author, while leaving to a more extended paper [16] a general discussion on his many original contributions as a scientist.

Upon summarizing his life, publications and achievements [2, 4, 18] in Sect. 1.2, Nayfeh's books are comprehensively discussed in Sect. 1.3, by analyzing specific features and novel aspects of a number of them, and by highlighting some underlying common methodological features. Section 1.4 provides a summary portrait of Ali Nayfeh as an outstanding scholar and scientist in nonlinear dynamics, at the crossroad between applied mathematics and engineering.

1.2 Life, Publications, Achievements: A Short Summary

Ali H. Nayfeh was born on December 21, 1933 in Shuwaikah, Palestine, to an illiterate and poor family which, nonetheless, highly encouraged him to acquire education and maximum possible knowledge. Owed to the harsh conditions in his country and lack of higher-education institutes, Ali worked as a teacher of mathematics in small villages and towns for ten years. At the age of 26 he won a scholarship to study in the USA, and in only 5 years moved from a 1 year junior college up to getting Bachelor's Degree in engineering science (1962), Master of Science (1963), and Ph.D. in Aeronautics and Astronautics (1964) at Stanford University.

For six years (1964–1970) he worked in the aerospace industry, upon which he was appointed at Virginia Tech as a Professor (1971) and then as a University Distinguished Professor (1976). Nayfeh:

- Wrote: (i) 10 books in perturbation methods and nonlinear dynamics, several of them with thousands of citations, considered as the most valuable and fundamental references in their fields, translated into Russian, Chinese, and German, and used as textbooks in top schools; (ii) nearly 480 papers in refereed journals.
- Was the Founder and Editor of two fundamental journals, *Nonlinear Dynamics* and *Journal of Vibration and Control*.
- Advised 69 Ph.D. students: many of them became prominent scholars, department chairs and deans in top ranked institutes all over the world.
- Established the highly successful series of 13 Conferences on *Nonlinear Vibrations, Stability, and Dynamics of Structures* at Virginia Tech (1986–2010).
- Received honorary doctorates from Politechnika Szczecinska, Poland, Technical University of Munich, Germany, and Marine Technical University of St. Petersburg, Russia, along with an incredible number of Awards, including:

1995 AIAA Pendray Aerospace Literature, "for seminal contributions to perturbation methods, nonlinear dynamics, acoustics, and boundary-layer transition."
1996 ASME Den Hartog, "for lifetime contributions to the teaching and practice of vibration engineering."
2005 ASME Lyapunov (first recipient), "for lifelong contributions to the field of nonlinear dynamics."

2008 ASME Thomas Caughey (first recipient), “for significant contributions to the field of nonlinear dynamics through practice, research, teaching, and outstanding leadership.”

2014 Benjamin Franklin Medal in Mechanical Engineering, “for developing novel methods to model complex engineering systems in structural dynamics, acoustics, fluid mechanics, and electromechanical systems.”

Ali always aimed at contributing to the development of science in the Arab World. He established the college of engineering at King Abdel Aziz University in Jeddah, Saudi Arabia (1976), and an engineering college in Jordan where he served as Dean and Vice-president for engineering affairs for four years (1980). He also helped establishing a new graduate and internationally reputable program in mechanics in Tunisia (2002).

Upon retiring from Virginia Tech, he volunteered at the University of Jordan, under the condition of working without pay, helping scientists and researchers, and providing advice and consultations; he even established and funded a modern school in his birth village of Palestine, to offer the best education and produce a new generation of brilliant scientists.

Quoting from [18]: “Anyone who has met Dr. Nayfeh knows well his inexhaustible energy and deep desire for knowledge; and more importantly, his passion to share and spread his knowledge with others. … He was a brilliant scientist, a distinguished teacher, an inspiring motivator, a great community leader, and an amazing and lovely human. He will be truly and deeply missed.”

1.3 Nayfeh’s Books

Nayfeh wrote several books, generally clearly distinct from each other and of high impact. They are summarized in Fig. 1.1, by grouping them in three main areas, with well-identified and coherent features.

The first group is concerned with analytical methods, and includes all books on asymptotic techniques, with special emphasis on the method of multiple time scales (MMS) which was Nayfeh’s most originally addressed theme within the realm of applied mathematics. Within this overall context, the transition can be recognized from a theoretically-oriented approach [6] to a more introductory treatment of mathematical aspects [7], with also clear educational purposes [8], up to a modern popularization of the MMS [12] taking advantage of symbolic algebra, as well as the independent treatment of a method [9] playing an important role in nonlinear dynamics.

Books in the second group are more physically-oriented, and also more varied. Indeed, they range from the effective use of asymptotic techniques for the analysis of weakly nonlinear archetypal oscillators [13], to the presentation of computational and geometrical concepts, techniques and tools of the modern theory of dynamical systems also allowing to deal with strongly nonlinear and complex phenomena [11], up to the analysis of the involved interaction phenomena, regular or non-regular, which

	ANALYTICAL METHODS	NONLINEAR PHYSICAL ASPECTS	STRUCTURAL MECHANICS
1970-79	*Perturbation Methods* (1973) companion →	*Nonlinear Oscillations* (1979) with D.T. Mook	
1980-89	*Introduction to Perturbation Techniques* (1980) *Problems in Perturbations* (1985)		
1990-99	*Method of Normal Forms* (1993) *Perturbation Methods with Mathematica, Maple* (1999) with C.M. Chin	*Applied Nonlinear Dynamics* *Analytical, Computational and Experimental Methods* (1995) with B. Balachandran	
2000-09		*Nonlinear Interactions* *Analytical, Computational and Experimental Methods* (2000)	*Linear and Nonlinear Structural Mechanics* (2004) with P.F. Pai

Fig. 1.1 Ali Nayfeh's books

characterize the nonlinear dynamics of multi-mode models reliably representative of actual engineering systems [10].

The third group consists of a book [14] which, pushing forward the interest towards distributed parameter systems already apparent in [10], deals more generally with structural mechanics issues in terms of modelling, dynamic analyses, and exemplary phenomenological aspects.

In the following, the main features and novel aspects of some of these books with respect to their publication time are discussed, by citing a few author's sentences from the corresponding prefaces (as per the writer's choice) and by summarizing a few characterizing aspects.

1.3.1 Specific and Novel Features

Although being certainly linked with each other, Nayfeh's two fundamental books on perturbation methods were driven by a different perspective.

- *Perturbation Methods* (1973). "The different *techniques are described using examples* which start with model simple ordinary equations that can be solved exactly and *progress toward complex* partial differential equations", with "examples drawn from *different branches of physics and engineering*", and "the different techniques described as formal procedures *without any attempt at justifying them rigorously*."

- *Introduction to Perturbation Techniques* (1980). In contrast with the former book, where "*coincise* and *advanced* material" is dealt with, this second one presents the same material "in *elementary* way." Indeed, "a*s a result of teaching perturbation methods for eight years* to first-year and advanced graduate students at Virginia Polytechnic Institute and State University, I have *selected a limited number of techniques and amplified their description* considerably. Also I have attempted to answer the questions most frequently raised by my students. … A new chapter is devoted to the determination of the adjoints of homogeneous linear equations and the *solvability conditions* of linear inhomogeneous problems."

However, Nayfeh's research activity on mathematical aspects of ordinary differential equations was always paralleled by a deep interest towards physical aspects of the nonlinear oscillations described by those equations.

Nonlinear Oscillations (1979) was the comprehensive outcome of intense and extended applications of the multiple scale technique to obtain the solution of both archetypal oscillators (representing discrete systems, or simplified reduced models of continuous systems) and some multidegree-of-freedom models.

In the preface, referring to a comprehensive list of books on the same topic and generally with a similar title, it is stated that "the *previously published books emphasized*, and some exclusively treated, systems having a *single degree of freedom*", whereas "the *primary purpose* of this book *is to fill this void.*" Specifically, Chap. 1 (Introduction) "attempts to *abstract the entire book*" and provides an useful and *comprehensive overview* of the addressed topics (according to a criterion later on adopted successfully in the introduction of other books), describing "*only the physical phenomena, leaving all the algebra to the subsequent chapters.*"

Seven chapters are devoted to Single-Degree-of-Freedom Systems (conservative, nonconservative, forced); Parametrically excited systems; Systems having finite degrees of freedom, with the treatment of internal resonance; Continuous systems (beams, string, plates); and Traveling waves. In particular, Chap. 7 "concentrates on the *physical mechanisms* and *effects*, restricting the attention to uniform systems with simple boundary conditions whose linear natural modes can be obtained analytically; then using the method of multiple scales to solve the equations describing the temporal functions"; Chap. 8 refers to "simple *physical examples* to explain nonlinear dispersive and nondispersive waves."

Nonlinear Oscillations (1979) was translated in many languages, and became a fundamental classroom textbook in many academic institutions, summing up to 9402 citations (as of April 28, 2019, according to Google Scholar).

Between the 80s and the 90s, a substantial enlargement of research perspectives in nonlinear dynamics occurred within the community of applied mechanicians, with the attention being increasingly paid to strongly nonlinear dynamic phenomena. Nearly in parallel with such developments, the role played by experimental (physical) techniques for a reliable and exhaustive characterization of the dynamics of (mostly) multi/infinite-dimensional systems became fully apparent.

Although not providing specific foundational contributions to the development of these novel topics, Nayfeh swiftly grasped the importance of properly complementing the treatment of nonlinear oscillations via classical perturbation methods with the intensive use of advanced *geometrical* and *computational* techniques from the dynamical system theory also allowing to deal with strongly nonlinear and complex dynamics, along with the need to validate theoretical/numerical outcomes through *experimental* approaches.

Applied Nonlinear Dynamics (1995) testifies to Nayfeh's great capability to grasp recent advancements and scientific trends, and *timely make them available to people involved with research,* and to practicing engineers working on challenging problems in applied mechanics.

Upon worthily summarizing the overall book contents in the introduction, seven chapters are dedicated to Equilibrium Solutions, Periodic Solutions, Quasiperiodic Solutions, Chaos, Numerical Methods, Tools to Analyze Motions, and Control. Specifically, quoting from ca.wiley.com website:

- "*Analytical* approaches based on perturbation methods and *dynamical systems theory* are presented and illustrated through *applications to a wide range of nonlinear systems*."
- "*Geometrical* concepts, such as Poincaré maps, are treated at length, with a thorough discussion of stability and *local and global bifurcation* analyses for systems of differential equations and algebraic equations conducted with the aid of *examples* and *illustrations*. *Continuation methods* for fixed points and periodic solutions, and *homotopy methods* for determining fixed points, are detailed. Bifurcations of fixed points, limit cycles, tori, and chaos are discussed."
- *Chaos* is explored, with many routes treated at length, by also describing methods for "*controlling bifurcations and chaos*."
- "*Numerical* methods and *tools* (Poincaré sections, Fourier spectra, autocorrelation functions, Lyapunov exponents, dimension calculations)" for the *analysis* and *characterization* of motion in both the analytical and experimental context are presented.

Consistent with the awareness of being substantially a (though smart) user of techniques developed by others and with the willingness to provide an understandable framework to effectively implement them in applications, "proofs are not provided but references that provide them are included, some chapters (2, 3) are not written within a mathematically rigorous framework" [11], and many examples are used to explain the different concepts.

The last two books mark a progressive shift (or, indeed, an extension) of Nayfeh's interests (already present in his earlier activity) towards applied dynamics problems in structural and mechanical engineering.

Nonlinear Interactions (2000) is built on the awareness that "an understanding of dynamic characteristics of a *structural system* is essential for its *design and control.*"

(a) A *variety of nonlinear interactions* based on (2:1, 1:1, 3:1) autoparametric and combination resonances are addressed.

(b) *Another type* of experimentally observed interaction between a *directly excited high-frequency* mode and a *low-frequency* mode, accompanied by a *slow modulation* of amplitude and phase of the former, is discussed. It is a mechanism of *great practical importance*, through which *energy from high-frequency low-amplitude sources* (such as rotating machineries, waves, propeller blades passing the rudder) can be *transferred* to *large-amplitude low-frequency modes* of supporting structures and foundations, resulting in their possible *harmful oscillations*, or may entail *decrease of vibration* of a *main system* and *increase of its fatigue life* to the expense of a *sacrificial subsystem*.

Upon the introduction, seven chapters include treatment of (5) Systems with widely spaced modes, (6) Multiple internal resonances, and (7) Nonlinear normal modes. Among meaningfully addressed topics, the following ones are mentioned.

- *Multidegree-of-freedom* systems, with a marked interest to *distributed parameter* (structural) systems besides discrete (mechanical) ones.
- Modulation equations: equilibrium solutions; stability and bifurcation analysis; dynamic solutions. This delineates a *number of issues to be sequentially faced* for a *comprehensive nonlinear dynamic analysis* of a generic system, according to a scheme later on *adopted in hundreds of papers by other authors*.
- Confronting *direct* versus *discretization* approach in asymptotics.
- *Control strategies* based on internal resonances and ensuing phenomena (e.g., saturation in 2:1 resonance).
- Importance of *higher-order approximations*.
- Comparing methods to obtain *nonlinear normal modes* for discrete and continuous systems (direct multiple scales, method of normal form, real- versus complex-valued invariant manifold approach).

Overall, the book is indeed a *mine of information* about the richness and variety of nonlinear phenomena, along with the theory behind them, and a valuable *blend* of *author's expertise* on *classical perturbation* methods and *nonlinear oscillations* theory—revisited through symbolic algebra—with knowledge and tools from *dynamical system theory* and *experimental nonlinear dynamics*.

Twenty years after the publication of [13], the book can be considered as its *continuation* and *update*, and turns out to be "more *in the genuine author's attitude of mind* than some other of his recent books" [15].

Linear and Nonlinear Structural Mechanics (2004) dwells on how the "nonlinear modeling and dynamic analysis of structures becomes a complex but important step in advancing the design and optimization of modern structural systems (with special attention to composites)" and aims "to close the gap between the practicing engineer and the applied mathematician in the modeling and analysis of geometrically nonlinear structures."

- "*Mathematically consistent* and *systematic* derivations of *comprehensive* and *refined structural theories*" of strings, cables, beams, plates and shells, both *exact* and *approximate*, are presented, also including laminates, integration with piezoelectric materials, thermoelasticity, and microbeams/plates.

- "*Physical meaning* of linear and nonlinear structural mechanics" is detailed.
- "*Ready-to-use governing equations* and *boundary conditions*" are provided, "ranging from simple linear to complex nonlinear."
- Exemplary *nonlinear structural analyses* of refined and/or reduced order models are summarized.
- Some main treatments and outcomes of *linear/nonlinear dynamics* for beams, plates and shells are obtained, referring to refined and/or reduced order models.

1.3.2 Some Characterizing Methodological Aspects

Some main *methodological aspects* common to all Nayfeh's books are highlighted and illustrated with examples in the following. His overall production exhibits four basic characteristics.

1. Topics are addressed and presented based on an *incremental/additive* (and overall *inductive*) perspective, via a *series of case histories*.
2. Specific outcomes (about *analytical techniques*, *different methods* and/or *systems*) are embedded into a *unified, comprehensive*, and *comparative* framework.
3. Concepts are illustrated with *numerous examples* and many *exercises* aimed to reinforce and assess *progress in understanding*.
4. *Extended* and *updated bibliographies* are provided.

The incremental/additive perspective can be recognized in the contents of anyone of Nayfeh's books. Figure 1.2 shows two examples [13] concerned with the forced oscillations of systems with a single or finite number of degrees of freedom. Considering "simple systems that exhibit the essential ideas, instead of treating general systems for which the algebra is involved" produces lists of case studies of progressively increased difficulty. Basic nonlinearities (cubic, or quadratic and cubic) and self-sustained oscillations are addressed in the first example, considering different resonant (primary, subharmonic, superharmonic, combination, simultaneous) or nonresonant excitations. In turn, the second example refers to diverse physical systems and deals with various cases of resonance between excitation and system natural frequencies for those having quadratic nonlinearities. The adopted scheme is certainly repetitive. Yet, while being somehow encyclopedic, the underlying comprehensive perspective allows to easily point out differences and peculiarities of distinct nonlinear aspects of both the analytical treatment and the considered systems/excitations. This results in a meaningful amount of information about the richness and variety of nonlinear phenomena, and the theory behind them.

A similar example (Fig. 1.3) is taken from [9], where the general idea of the method of normal forms to "use a 'local' (i.e., near-identity) coordinate transformation to 'simplify' the equations describing the dynamics of the system under consideration" is applied to parametrically excited systems.

Two more incremental/additive examples (Fig. 1.4) refer to topics addressed by Nayfeh later on in the more engineering oriented stage of his activity, and also high-

4. Forced Oscillations of Systems Having a Single Degree of Freedom

4.1. Systems with Cubic Nonlinearities
- *4.1.1. Primary Resonances, $\Omega \approx \omega_0$*
- *4.1.2. Nonresonant Hard Excitations*
- *4.1.3. Superharmonic Resonances, $\Omega \approx 1/3\ \omega_0$*
- *4.1.4. Subharmonic Resonances, $\Omega \approx 3\omega_0$*
- *4.1.5. Combination Resonances for Two-Term Excitations*
- *4.1.6. Simultaneous Resonances: The Case in Which $\omega_0 \approx 3\omega_1$ and $\omega_0 \approx 1/3\ \Omega_2$*

4.2. Systems with Quadratic and Cubic Nonlinearities
- *4.2.1. Primary Resonances*
- *4.2.2. Superharmonic Resonances*
- *4.2.3. Subharmonic Resonances*
- *4.2.4. Combination Resonances*

4.3. Systems with Self-Sustained Oscillations
- *4.3.1. Primary Resonances*
- *4.3.2. Nonresonant Excitations*
- *4.3.3. Superharmonic Resonances*
- *4.3.4. Subharmonic Resonances*
- *4.3.5. Combination Resonances*

6. Systems Having Finite Degrees of Freedom

6.1. Examples
- *6.1.1. The Spherical Pendulum*
- *6.1.2. The Spring Pendulum*
- *6.1.3. A Restricted Ship Motion*
- *6.1.4. Self-sustaining Oscillators*
- *6.1.5. The Stability of the Triangular Points in the Restricted Problem of Three Bodies*

6.5. Forced Oscillations of Systems Having Quadratic Nonlinearities
- *6.5.1. The Case of Ω Near ω_2*
- *6.5.2. The Case of Ω Near ω_1*
- *6.5.3. The Case of Nonresonant Excitations*
- *6.5.4. The Case of 2Ω Near ω_1*
- *6.5.5. The Case of Ω Near $\omega_1 + \omega_2$*

Fig. 1.2 Incremental/additive approach in the contents of [13]

5. Parametrically Excited Systems

5.1. The Mathieu Equation
- 5.1.1. Fundamental Parametric Resonance
- 5.1.2. Principal Parametric Resonance

5.2. Multi-Degree-of-Freedom Systems
- 5.2.1. The Case of Ω Near $\omega_2 + \omega_1$
- 5.2.2. The Case of Ω Near $\omega_2 - \omega_1$
- 5.2.3. The Case of Ω Near $\omega_2 + \omega_1$ and $\omega_3 - \omega_2$
- 5.2.4. The Case of Ω Near $2\omega_3$ and $\omega_2 + \omega_1$

5.3. Linear Systems Having Repeated Frequencies
- 5.3.1. The Case of Ω Near $2\omega_1$
- 5.3.2. The Case of Ω Near $\omega_3 + \omega_1$
- 5.3.3. The Case of Ω Near $\omega_3 - \omega_1$
- 5.3.4. The Case of Ω Near ω_1

5.4. Gyroscopic Systems
- 5.4.1. The Case of Ω Near $2\omega_1$
- 5.4.2. The Case of Ω Near $\omega_2 - \omega_1$

5.5. A Nonlinear Single-Degree-of-Freedom System
- 5.5.1. The Case of Ω Away from 2ω
- 5.5.2. The Case of Ω Near 2ω

Fig. 1.3 Incremental/additive approach in the contents of [9]

light the *inductive* character of his approach: the same form of modulation equations is recognized for systems belonging to a certain group of symmetry [10], and methods of linear/nonlinear structural mechanics can be readily extended to different and/or more complex structures [14].

Overall, as explicitly declared in the prefaces of all Nayfeh's books (starting with the more theoretical one [6]), the material is not presented "within a mathematically rigorous framework." This originated criticisms from more dynamical system-oriented (and generally more rigorous) scientists. But this characterizing

Nonlinear Interactions **(2000):**

- *2:1, 1:1, or 3:1 Internal Resonances*: primary resonance, principal parametric resonance (of first mode, of second mode), fundamental parametric resonance
  Combination Resonances: parametric, external, internal; of the additive or difference type
- *Nonlinear Normal Modes*: a symmetric 2-dof system; an asymmetric 2-dof system; cubic and quintic nonlinearities; quadratic and cubic nonlinearities; multi-dof systems; systems with internal resonances; continuous systems

Inductive: for systems with 1:1 internal resonance:
"the modulation equations have the same form for all systems with O_2 symmetry, irrespective of their physical origin and/or the source of the nonlinearity"

Linear and Nonlinear Structural Mechanics **(2004):**

- *Plates*: linear classical; linear shear-deformable; nonlinear classical; general nonlinear classical; nonlinear shear-deformable; nonlinear layerwise shear-deformable

Incremental/Inductive:
"as a result, the reader can readily extend the methods to formulate and analyze different and/or more complex structures"

Fig. 1.4 Incremental/inductive approach in [10, 14]

feature of Nayfeh's scientific personality ensued from his being basically a scholar in engineering sciences capable of exploiting a strongly founded knowledge of fundamentals of applied mathematics to understand the nonlinear behavior of involved mechanical/structural systems; this being the actual main focus of his research interests. The inductive approach adopted by Nayfeh in his presentations of scientific material was likely brought to its extreme consequences. Yet, it was somehow in line with the perspective adopted by most of the earlier scientists active in nonlinear dynamics (including mathematicians), who "were not led to their discoveries by a process of deduction from general postulates or principles, but rather by a thorough examination of properly chosen particular cases", with the generalization coming later "because it is far easier to generalize an established result than to discover a new line of argument" [5]. In this sense, Nayfeh's approach was not too far from the purpose of studying *concrete nonlinear systems* with their "natural effects", which also inspired earlier theoretical research in nonlinear dynamics at Moscow [1] and Kiev [3] schools.

It is anyway important to notice that such a 'list-looking' aspect of his presentations was constantly paralleled in all books by the successful attempt to achieve a *unified, comprehensive,* and *comparative* framework into which presenting the various case-studies and highlighting the relevant differences and peculiarities. Sentences from the prefaces of some Nayfeh's books quoted in Fig. 1.5 witness the care he always paid to the matter.

Two more aspects of Nayfeh's fully effective educational and professional perspective as a book author are to be mentioned. Concepts were always illustrated through worked-out examples and exercises useful to reinforce understanding and

- *Perturbation Methods* (1973):
  "Presents in ***a unified*** way an account of most of the ***perturbation*** techniques, pointing out their ***similarities, differences, and advantages,*** as well as their ***limitations***"
- *Applied Nonlinear Dynamics* (1995):
  "Unlike most other texts, which emphasize either classical methods, experiments and physics, geometrical methods, computational methods, or applied mathematics, provides **a** ***coherent and unified*** treatment of ***analytical, computational, and experimental*** methods and concepts of ***nonlinear dynamics***"
- *Nonlinear Interactions* (2000):
  "Provides a ***coherent and unified*** treatment of ***analytical, computational, and experimental*** methods and concepts of ***modal interactions***"
  As an obvious extension of *Applied Nonlinear Dynamics* (1995), the relevant "methods are used to explore and unfold in a ***unified*** manner the ***fascinating complexities*** in nonlinear dynamical systems"
- *Linear and Nonlinear Structural Mechanics* (2004):
  "A ***unique unified*** approach, more general than those found in most structural mechanics books, is used to model ***geometric nonlinearities*** of structures"

Fig. 1.5 A unified framework

assess progress of students. Moreover, very rich and (for their time) updated bibliographies were provided at the end of each book. They are still invaluable sources of information/knowledge about 'classical' and more recent books and journal/conference papers, for scholars in nonlinear dynamics. This is even more important in the current time in which scientific problems are sometime 'rediscovered', in a context of overall minor care paid to the historical evolution and advancement of science by some young scientist mostly aimed at attaining specific, and also possibly limited, goals.

1.4 Complementing Mathematical and Engineering Approaches to Nonlinear Systems

A fundamental part of Ali Nayfeh's legacy consists of the conceptual, and mostly operational, contributions to the method of multiple time scales provided at the beginning of his scientific activity, and later on expanded in the direction of frequently needed higher-order expansions and in terms of popularization. As a matter of fact, multiple time scales are continuously being used by generations of scientists for solving nonlinear dynamics problems. This holds irrespective of mathematical warnings raised about pitfalls possibly occurring in the asymptotics of the MMS owed mostly to the anticipated choice of timescales, a feature which does not occur in the averaging method where they emerge by nonsecularity conditions without any a priori assumption [17]. But Nayfeh was most likely aware of those pitfalls. This is witnessed both by the reference constantly made to the need to *properly select timescales* (including fractional ones) depending on the dynamic problem at hand, and by his many performed *comparisons of equivalent approximations* provided by multiple scales and averaging. It is however to be noticed that Nayfeh's *enormous skill and experience* about how *properly selecting* a priori the timescales necessary for a reliable asymptotic solution of a given problem ensue from exceptional *personal* features in terms of *intuition* and *overall understanding* which are not in the patrimony of all scientists.

As a matter of fact, looking at his outstanding scientific activity on a huge variety of themes [16], Nayfeh's main nature of a *scholar in engineering sciences* could be further highlighted. While being well-acquainted with applied mathematics, he was fundamentally interested in using his skill and expertise to catch *the nonlinear behavior* of *mechanical and structural systems*. Indeed, moving from his earlier achievements on the nonlinear oscillations of simple models, in the second part of Nayfeh's academic life further *distinguishing* and highly *influential features* in the realm of *applied mechanics* and *engineering* emerged clearly. Therein, with an unrivaled capability to *grasp novel advancements* and *research trends*, he succeeded in remarkably complementing a profound *expertise* on *asymptotic methods* and *nonlinear oscillations* theory, revisited through modern symbolic algebra, with the effective *interpretation* and *organization* of *experimental outcomes*, both in-house and from the literature, along with the *smart use* of *knowledge and tools* from *modern dynamical system* theory.

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