Research in Mathematics Education

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Designing, Conducting, and Publishing Quality Research in Mathematics Education

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Foreword

This volume is about designing, conducting, and publishing quality research in mathematics education. The term *quality* is loaded in our field. As editors of this series and of journals and other volumes in mathematics education, we hear authors lament the "good old days" when quality was easier to judge. This lament is not a judgment on the lack of quality in mathematics education research but rather a commentary on the great variety of methods and theoretical perspectives available to the researchers today and the different audiences for which research and scholarly activity is targeted. Young scholars, in particular, often find it difficult to navigate the boundaries between quantitative and qualitative methods and design-based research. We have all struggled with establishing a coherent research program while responding to the sometimes competing needs and desires of the field and to the vagaries of funding.

This volume is geared toward young mathematics education researchers, those who are new to our now rather well-established community. It takes fundamental issues that confront all research programs and addresses them from their very beginnings, describing how one establishes a theoretical framework, situates it in the relevant literature, constructs an argument regarding the importance and tractability of a problem, and then considers issues of method—tailored both toward appropriateness for the question of study and toward analytic rigor. Furthermore, this volume deals explicitly with how to write well, to communicate complex and sometimes controversial ideas to intended audiences, *and to work to get it published* so that the fields of scholarship and practice are changed by the work that we do. The volume does an exemplary job of explaining the different stages of the larger research picture, and so it would be ideal for a graduate course designed to support students in designing, conducting, and publishing their independent research.

The authors were carefully curated to be representative of research foci, epistemological tradition, methodological expertise, and experience as reviewers and editors of *quality* research in our field. Their own work represents a sizeable chunk of the body of evidence we have today on teaching, learning, curriculum, teacher education, and policy in mathematics education.

We are thankful for the volume editor, Keith Leatham, for accepting our invitation to put this quality volume together. We are also thankful that the authors take their own advice and speak directly and forthrightly to the audience. Each of the chapters, in our estimation, allows the reader to gain insight into the issues with which they are struggling, query the different approaches they should consider, and get a bit of mentoring from some of the best scholars in the world.

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Preface

The purpose of this book is to disseminate collective wisdom with respect to designing, conducting, and publishing quality research in mathematics education. This wisdom has been gleaned from among those who, over the past several decades, have been instrumental in guiding the field in the pursuit of excellence in mathematics education research—insightful editors, educative reviewers, prolific writers, and caring mentors. Each chapter is written to the novice researcher with the intent of aiding them in avoiding common pitfalls, navigating difficult intellectual terrain, and understanding that they are not alone in experiencing rejection, frustration, confusion, and doubt. The authors were asked to write chapters with this prompt in mind: Imagine you were meeting with a mathematics education graduate student or assistant professor and they were struggling with an issue related to designing, conducting, or publishing quality research in mathematics education. Write the chapter you would want to recommend as part of your mentoring.

Provo, UT, USA | Keith R. Leatham

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Keith R. Leatham

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Part I
Designing Quality Research in Mathematics Education

Chapter 1
The Role of Theoretical Frameworks in Mathematics Education Research

Denise A. Spangler and Steven R. Williams

Abstract Although both novice and experienced researchers can struggle with what theoretical frameworks are and why they are necessary, it is nevertheless clear that they are an important part of designing, carrying out, and reporting on research in our field. In this chapter, we attempt to clarify the roles played by theoretical frameworks in mathematics education and to explain why they are important and useful, both to individual researchers and to the field as a whole. Finally, we provide some examples of how different frameworks might be applied to a typical set of data: videotaped whole-class, teacher-student, and student-student interactions in a mathematics classroom.

Keywords Theoretical frameworks · Nature of theory · Implicit theories · Explicit theories

The questions of what theoretical frameworks are, why they are important, and how they are used in mathematics education research are by no means settled. Textbooks designed to help students plan and execute qualitative research, and thus answer questions such as these, run the gamut, from those where theoretical frameworks take a prominent place to those where theory is barely mentioned (see Anfara & Mertz, 2015, for an overview). Definitions or descriptions of theoretical frameworks are similarly diverse, and to further muddy the waters, there are myriad related terms that may stand in for, or overlap in meaning with, theoretical framework (e.g.,

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paradigm, conceptual framework, model, theoretical orientation).[1] It is not surprising, then, that both novice and experienced researchers can struggle with theoretical frameworks (Anfara & Mertz, 2015; Merriam & Tisdell, 2016; Mewborn, 2005; Ravitch & Riggan, 2017).

At the same time, it is clear to anyone who has written a research report or proposal in our field that a theoretical framework is usually expected (in a dissertation, its presence is nonnegotiable). Some academic journals and professional meetings require explicit attention to theoretical frameworks in submissions, and even when such explicit attention is not required, a weak or missing theoretical framework is a common reason for proposals and papers to be rejected. In this chapter we hope to clarify what is usually meant by theoretical frameworks in mathematics education, explain why they are both important and useful, and provide some examples of how different frameworks might be applied to a fairly typical set of data: videotaped whole-class, teacher-student, and student-student interactions in a mathematics classroom.

1.1 Why All the Confusion?

Part of the confusion over theoretical frameworks is due to the word *theory* having various levels of meaning, from informal, everyday hunches and intuitions to formal, highly structured propositions for explaining how some aspect of the world works (Flinders & Mills, 1993; Ravitch & Riggan, 2017). We find it useful to think of theory as varying on a continuum from implicit to explicit. What we call implicit theories include tacit assumptions, preconceptions, or ways of thinking about the world that are present in any attempt to bring order and meaning to our perceptions. It has long been accepted that we understand new phenomena in terms of the understandings we already possess (Minsky, 1985; Rumelhart, 1980; Schank & Abelson, 1977). These preunderstandings go by many names (e.g., frames, schemas, scripts), but in general, they describe the implicit theories that are both always with us and necessary for us to make sense of the world.

That implicit theory is necessary for any understanding has some important consequences for research. First, as Schwandt (1993) put it, “atheoretical research is impossible” (p. 7). No attempt to “let the data speak for themselves” can completely succeed because the researcher’s implicit theories (and those of readers, including reviewers) will always affect how the data are interpreted (and even what should count as data). Schwandt further asserts that researchers cannot observe “the way things really are, free of any prior conceptual scheme or theory … without some theory of what is relevant to observe, how what is to be observed is to be named, and so on” (p. 8). Thus, even if we claim to enter the field without any theoretical sup-

[1] We agree with Ravitch and Riggan (2017) that “this overall confusion and lack of coherence will not be resolved by renaming things (or, worse yet, arguing about which things should get which names)” (p. 33).

positions, we are in fact relying on the implicit theories we bring with us. Moreover, as the name suggests, implicit theories are often invisible to us, and we can use them without reflection or even awareness. Because we cannot avoid implicit theories, the best we can do is to make them as explicit as possible by forthright and critical examination of our own preconceptions, assumptions, and biases. This is why scholars who rely on qualitative methods, particularly those who do identity work or critical theory of various kinds, are so adamant in emphasizing how the backgrounds, beliefs, characteristics, and histories of the researcher—all of which are intimately connected to implicit theories—necessarily affect the research process. Thus, such scholars expect to see a statement about the researcher's positionality and potential biases in the methods section.

By contrast, explicit theories are generally more public—either codified in scholarly literature or explicated by the researcher in some way. Importantly, they can be examined, modified, and tested by the scholarly community. Explicit theories also vary widely in their scope. Some focus on a broad swath of phenomena (e.g., grand theories; cf. Mills, 1959); these theories are often more abstract and general and might be described as perspectives or orientations to the world. Examples include constructivism, structuralism, or Marxism. Other explicit theories are more local or specific to a narrower context (e.g., middle-range theories; cf. Merton, 1957). Examples of these theories (which we take up again later in the chapter) include the framework used to classify teachers' invitation and support moves by Franke et al. (2015) and the five practices for orchestrating mathematical discourse set forth by Smith and Stein (2018). Yet other explicit theories may have a quite narrow focus, as those that emerge from data from a particular case, as in the grounded theory tradition (Glaser & Strauss, 1967).

The rest of this chapter focuses on explicit theories—frameworks we deliberately choose to help us organize, think about, and understand what we observe in our research. We focus in particular on more local (as opposed to *grand*) theories because they are more typical of the theoretical frameworks associated with most of the research in our field. We begin by highlighting some critical characteristics of theoretical frameworks.

1.2 The Nature and Effects of Theoretical Frameworks

Recognizing that there are other terms that are often used for what we are calling theoretical frameworks, we choose that particular term to emphasize the two aspects foregrounded by the individual words it comprises.[2] First, we see theoretical frameworks as *theoretical* because they are concerned with explaining or understanding why things are as they are. They are "characterized by the pursuit of insight into the nature of things" (Theoretical, 2019) and focus on getting at essential properties and

[2] We will, however, occasionally shorten the term to either framework or theory as seems reasonable.

relationships that characterize a phenomenon. Thus, theoretical frameworks go beyond description to provide insight into causes, meanings, and connections; they aim at giving us understanding and insight.

Second, we see theoretical frameworks as *frameworks* (or frames) because they lend both focus and structure to our efforts to explain phenomena. This might best be communicated by considering the everyday uses of a *frame*—a picture frame, a bed frame, or the frame for a house under construction. A picture frame serves to demarcate an image and set it off so that it will be noticed by others and can be easily distinguished from the wall on which it is hung. A bed frame serves as a base upon which a mattress can be placed without fear of the mattress warping or sagging with time. A house frame provides an underlying structure to support the drywall, floor joists, and roof that make up the house. Without the framing, the house would collapse upon itself. Theoretical frameworks are similar. A theoretical framework can "set off" ideas or concepts to draw attention to them, giving them names and robust definitions. It can support the building up and deepening of an idea, or it can provide a structure on which to hang new ideas (Mewborn, 2005). Theoretical frameworks organize and give structure to our attempt to understand and explain.

A consequence of the *framing* aspect of a theoretical framework is that it determines what can be seen and understood in a research setting. A window frame focuses our attention on certain aspects of things on the other side of the wall, but it also limits what can be seen as we look through. As Eisner (1985) put it, "When you provide a window for looking at something, you also, if I can use the analogy, provide something in the way of a wall" (pp. 84–85). Thus, theoretical frameworks both focus our attention on some things and obscure or hide others. The choice of a theoretical framework has consequences for what data will be gathered, how they will be interpreted, and what can be concluded from them.

Much as physical frameworks can be purchased premade or can be custom-built to fit circumstances, theoretical frameworks vary in the degree to which they are borrowed ready-made from other scholars or other areas of scholarship or are assembled to suit a particular research need. It is common to hear researchers speak about the need to "find" a theoretical framework, as if the task at hand is to peruse a shelf full of possible frameworks and pick one that seems to fit. Indeed, some authors of research method texts seem to support this view. Anfara and Mertz (2015), for example, describe a theoretical framework as "any empirical or quasi-empirical theory of social and/or psychological processes, at a variety of levels (e.g., grand, mid-range, and explanatory), that can be applied to the understanding of phenomena" (p. xxvii). Certainly, there are some reasons for adopting a framework from the literature. In a well-researched area with established frameworks, adopting an existing framework from the literature might better allow for the accumulation of research results, a point we discuss in a later section. Replication studies whose purpose is to confirm results could also reasonably adopt the theoretical framework of the original study.

At the same time, adopting an existing framework from the literature may be difficult or may present undesirable challenges. For example, preexisting frameworks tend to have an unwarranted epistemological status, increasing the chances that data will be forced to “fit” theory and thus warping our analysis and results. They can similarly cause us to miss the perspectives and insights of those who are participants in our study, increasing the risk of marginalizing or trivializing their viewpoints. In an underresearched area, or when breaking new conceptual ground, it may make better sense to construct a framework from the researcher’s preliminary concepts and the relationships among them, gleaned as appropriate from literature, previous experiences, thought experiments, and preliminary results from pilot studies (Maxwell, 2013; Ravitch & Riggan, 2017).

In summary, we see theoretical frameworks as created for a given study through a process that certainly involves consideration of other frameworks from the literature but also takes into account the researcher’s goals, beliefs, and research purposes. The issue is not so much one of choosing versus custom making a framework but of arriving at a framework that is coherent and compatible with the goals for the study.

1.3 Advantages of Theoretical Frameworks to Individual Researchers

In this section we describe how theoretical frameworks support mathematics education researchers as they contemplate, design, and carry out a study. As noted above, it is common for beginning researchers to struggle with conceptualizing just what a theoretical framework is. It is likely that many doctoral students see it as something that needs to appear in Chap. 2 of their dissertation to satisfy their committee (cf. Mewborn, 2005). Viewed in this way, a theoretical framework cannot really inform, or be informed by, other aspects of research. We see this “insulation” of theoretical frameworks as a common and significant flaw in papers submitted for publication in our field. Authors usually describe a theoretical basis for their work, making sure that it is prominent early in their paper, but theory is often then abandoned for the rest of the report. In reports of research, theory should wind and weave throughout a paper and be used to “tie it all up” in a neat package at the end. (Maher and Williamson, (2019) discusses further the issue of framing our work at the dissemination stage.) Ideally, theory ought to be the foundational element that guides an entire research project from the research purposes and questions, through data gathering and analysis, all the way to the conclusions.

As noted above, theoretical frameworks need to be compatible with the goals of the research. More than that, however, they can both inform and justify decisions about what is to be studied or observed (Miles & Huberman, 1994; Schram, 2003). Theoretical frameworks can provide ways of describing the phenomena of interest, allowing us both to narrow down the scope of the research to focus on particular aspects of the situation and to recognize what we are interested in when we see it. They can provide a language for describing the phenomena of interest and thus for stating research questions. And by connecting these phenomena to important related concepts, they can help us decide what other phenomena maybe of interest, what data to gather, and what parts of our data may act mainly as “noise.”

Theoretical frameworks help researchers make sense of their data. They often give us a starting place for our analysis by providing initial categories or codes by which to organize our data, as well as a hint about the relationships we might find. One way they deepen our analysis beyond these preliminary steps is by what Eisenhart (1991) and others have called *sensitizing*. Eisenhart noted that frameworks cause the researcher to "tack between the concepts advanced or assumed and the meanings given or enacted in context" (p. 211). Thus, a framework forces a researcher to constantly compare and contrast what the framework is saying with what the data are saying. Eisenhart suggested that this tacking between the framework and data helps guard against poorly warranted conclusions. Furthermore, allowing a researcher to question the theory—what Elbow (1973) calls the *doubting game*—provides a way to refine the theory itself based on the data of the study. Note, however, that in order for a framework to accomplish either of these purposes—allowing theory to question data or allowing data to question theory—it must be actively used as a research tool throughout the study. The researcher must be tacking back and forth between the framework and the data during data collection and analysis.

1.4 Advantages of Theoretical Frameworks to the Field

In addition to the strength that frameworks lend to individual studies, they also serve a purpose at the level of the field. First, when frameworks span a number of studies, they begin to have a cumulative effect (diSessa, 1991) that leads to predictive power. When frameworks have predictive power, they also afford us greater credibility for making links to practice. The nearly miraculous technology of our century could not exist without theory. As diSessa pointed out, it may not take a powerful theory to drive from work to your home, but it does take a powerful theory to build machines that fly, safely and reliably, from Georgia to Utah. Similarly, it may not take powerful theory to teach about quadratic equations to a family member; we might get by on intuition, our relationships, or a "gut feeling" of how to proceed in such a case. But it may take very powerful theory indeed to develop an effective curriculum for hundreds of thousands of students.

Second, there is the issue of communicability. We do not have the luxury of educating teachers, for example, only by an apprenticeship model. While there is much to be gained by such learning, and while there may be things that are best or exclusively learned through practice, there is value in the compactness and portability of a theory. Theories can give us important ways of thinking about teaching that apply across individual instances, as well as ways of communicating our collective knowledge.

Thus far we have distinguished among implicit and explicit theoretical frameworks and among various levels of generality and applicability in frameworks. Narrowing our discussion to the explicit, more focused frameworks typical in mathematics education, we attempted to clarify the nature and value of such theoretical frameworks both to researchers in mathematics education and to the field as a whole. We also argued that theoretical frameworks should be an important consideration at each stage of a research project, from inception to dissemination of results. In the next section, we provide examples we hope will further clarify the importance of frameworks to our collective work.

1.5 Examples and Applications

In this section of the paper, we apply the ideas from the prior sections in a practical manner by considering how researchers employing different theoretical frameworks might view the same data set differently. In light of the message of these foregoing sections, however, this approach may seem contradictory. Indeed, as already suggested, when designing a study from the beginning, a choice of theoretical framework ideally should be made *before* data are collected to ensure that the data will allow for the analyses needed to use the framework. For instance, the research questions and the choice of framework might influence a decision to collect classroom observation data 1 day per week for a semester versus 15 consecutive days.

There are, however, legitimate reasons to "retrofit" a theoretical framework to an existing dataset to analyze the data for new insights. Given the expense of collecting good data and the existence of some rich data sets (e.g., TIMMS, videomosaic.org—see Maher & Wilkinson, 2019, or databrary.org), it can be advantageous for a researcher to apply a new framework to an existing data set. Furthermore, even in a study where the framework is identified in advance of data collection, it is possible that during data collection or data analysis a new direction for the research becomes apparent due to an unanticipated characteristic of the data set, and this new direction might require an entirely different theoretical framework than the one originally chosen.

When deciding whether to use a framework to analyze an existing data set, it is important for the researcher to determine whether the existing data set is sufficient for answering the research questions with the intended analysis. For example, if the framework relates to classroom discussions, it is important that the data set allows for the analysis of such discussions. If the framework relates to students' mathematical thinking, it would be important to ensure that the data set contains clear audio of student voices, video of students' written work and board work, and perhaps a capture of work done on devices.

For illustrative purposes in this chapter, we are going to assume that a high-quality data set already exists. Our hypothetical data set includes classroom observation data from multiple classrooms consisting of video records and field notes. The data were collected for the purpose of studying how teachers orchestrate classroom discourse and include whole-class and small-group and teacher-student and student-student interactions. The data allow for good access to spoken conversations but not to teachers' thinking or decision making.

To illustrate how a choice of theoretical framework influences how researchers focus their attention and the claims they can make, we consider analysis of our data set through the lens of two extant frameworks. The first example employs a framework that is a good fit for the existing data set and allows for both replication of part or all of the original study, as well as an alternative approach to the data. This framework is also an example of one that is built to account for multiple constructs, each with its own framework. The second example employs a framework that does not fit the existing data set as well. We discuss the ways that parts of this framework might be used with these data and the limitations of using the framework on this particular data set.

1.5.1 *Example 1: Employing the Framework from Franke et al. (2015)*

1.5.1.1 The Framework

Suppose a research team is interested in how teachers get students to engage in classroom discussions and the team members want to use the theoretical framing from a study conducted by Franke et al. (2015). The framing used in the Franke et al. study involves a carefully coordinated patchwork of three constructs, each of which has a framework. The three constructs are invitation moves, student engagement, and supporting moves. Through analysis of the empirical data collected during the study, Franke et al. identified six invitation moves that teachers use initially to engage students with one another's ideas in a discussion. Teachers asked students to:

> Explain someone else's solution.
> Discuss differences between solutions.
> Make a suggestion to another student about his or her idea.
> Connect their ideas to other students' ideas.
> Create a solution together with other students.
> Use a solution that was shared by another student. (p. 133)

The researchers also classified student engagement following these moves as high, medium, or low (which required them to define these levels), and they identified additional teacher moves, called supporting moves, that teachers used when students did not respond to invitation moves with a high level of engagement. The supporting moves of probing, scaffolding, and positioning are established in prior literature, and each was defined and described by Franke et al. using that literature (for instance, see Kazemi & Stipek, 2001, or Brodie, 2010, for probing; Baxter & Williams, 2010, for scaffolding; and Cohen, 1994, or Turner, Dominguez, Maldonado, & Empson, 2013, for positioning).

Note that the framework for the invitation moves was developed as part of the study; that is, it emerged from the analysis of the classroom data. The framework for student engagement is quite simple and consists only of three levels and the descriptions of those levels. The final framework of supporting moves came from an analysis of the data collected for the study, but unlike the invitation moves, these are moves already identified in the literature. Thus, Franke et al. (2015) provides a good example of several ways a framework can be constructed.

1.5.1.2 Applying the Framework

Let us now consider how this framework—in whole or in part—might be used to analyze the hypothetical classroom observation data set, what its use might illuminate or obscure, and what contributions might be made to the field. The researchers might decide to use only the invitation move framework, watching the classroom video to identify instances of the six invitation moves to see if all six moves are found, how their distribution compares to what Franke et al. (2015) found, and whether any additional invitation moves are present in the data, thus testing the sufficiency of Franke et al.'s invitation move framework. Using the invitation move framework helps the researchers narrow and focus their attention on only the portions of the classroom video that show an invitation move. What follows the invitation move (i.e., student responses, supporting moves) would not be of interest in this case. Thus, the mathematical ideas that are central to the discussion might not be relevant to the findings except as a piece of the context for the use of invitation moves. Valuable contributions to the field from an analysis of this nature might include validating Franke et al.'s moves in different classrooms (different grade levels, different student populations, different levels of teacher commitment to inquiry—a construct that would require defining and framing), identifying new invitation moves, or identifying more and less frequently used invitation moves among the six.

Alternatively, the research team might choose to use the invitation move framework and the student engagement construct in tandem. For instance, in addition to identifying invitation moves as above, they might also rate levels of student engagement and then attempt to correlate invitation moves with different levels of student engagement to determine which moves seem to lead to low, medium, and high levels of engagement, which is, again, a replication of Franke et al.'s (2015) original study. This approach would again focus the researchers' attention on the teacher initially but would also widen the lens to include students' responses. Another option would be for the researchers to reverse the process and watch classroom video to identify instances of high levels of student engagement and then backtrack to see what the teacher did immediately before the student engagement to see if the teacher moves map onto those found by Franke et al. This approach initially would focus the researchers on student engagement and consider the teacher secondarily. This approach could offer new contributions to the field by potentially identifying new teacher invitation moves or teacher moves that would not necessarily be classified as invitation moves. Using the two framework pieces together allows the researchers to begin to identify essential properties and relationships that characterize classroom discussions and provide insight into connections between teachers' actions and students' engagement.

The researchers might choose to use all three parts of the framework, just as Franke et al. (2015) did. As with the use of two parts of the framework, the researchers might choose to begin their identification of data to analyze by looking for instances in which the teacher makes an invitation move, instances in which there is a high level of student engagement, or instances in which there is sustained interaction between the teacher and a student where follow-up moves are occurring. This approach still helps the researchers narrow their focus to particular portions of the video but would require the researchers to attend to longer exchanges between the teacher and students. The researchers might find new supporting moves and additional relationships among the three constructs, which are contributions to the field.

We have noted that by helping researchers focus on particular constructs, frameworks obscure other parts of the data, so let us now consider what might not be attended to in this data set with the use of this framework. Because this framework is intentionally focused on the invitation moves that teachers use to get a discussion off the ground, researchers are not likely to attend to how the discussion evolves after students become engaged. That is, matters such as how a mathematical idea is developed, who is participating in the development of the idea, and the quality of the mathematical conversation are not likely to be the focus of analysis. Researchers are also more likely to pay attention to the teacher's interactions with individual students via invitation and supporting moves rather than their interactions with groups of students or the entire class. Thus, the choice of research question and theoretical framework sets the stage for what the researchers will see and not see and for the claims that they can make from the data. This is an important point for readers and reviewers of research reports to remember, as reviewers often express disappointment that the researchers did not attend to particular topics in their analysis (often the topics that are important to that reviewer). For instance, a reviewer reading a research report from our hypothetical study might critique the report because there is little or no attention to issues of equity, such as which students are being invited to participate, which kinds of invitation moves are being used to invite which students to engage, and which students are supported to engage and which are not when the initial invitation move is not successful. While equity issues are critically important to the advancement of our field, and some would argue that they should be attended to in all reports of research, in this particular study, equity issues are not the primary focus. The researchers could certainly do a secondary analysis and pay attention to matters of how students are positioned, but it is not the primary focus of the analysis using the invitation move framework.

1.5.2 Example 2: Employing the Framework from Smith and Stein (2018)

1.5.2.1 The Framework

Now suppose a different research team wanted to investigate the same hypothetical classroom observation data set, again with a goal of understanding how teachers facilitate classroom discourse, but this time using the *5 Practices for Orchestrating Productive Mathematics Discussion* (Smith & Stein, 2018) as a framework. The five practices are as follows:

- Anticipating students' mathematical thinking on the task at hand
- Monitoring students as they work on the task
- Selecting particular students' strategies to share with the whole class
- Sequencing those strategies intentionally in some way (such as least to most complex)
- Connecting the strategies to help students see the underlying mathematical ideas and how they are used in each strategy presented

Each of these practices is focused on the thoughts or actions of the teacher, so the nature of the data set (classroom observations but no interviews with teachers) does not lend itself well to using this framework in its entirety for analysis because the researchers would need to infer a great deal about teachers' intentions. For instance, data from classroom observations would not allow the research team to investigate the first practice, as knowing what students' mathematical thinking a teacher has anticipated would require an interview during lesson planning or a stimulated recall interview after the lesson. While the teacher might say something overt such as "I was waiting for someone to share that idea!" or "Good. That's just what I wanted to hear" or something similar during the lesson, and it might be possible to infer that the teacher had anticipated a particular solution based on nonverbal cues (such as nodding) or the teacher drawing a representation on the board that gets slightly ahead of what the student is saying, conclusions would be highly inferential and the analysis would likely be thin.

The four remaining practices could be observed from a classroom video; however, it would be difficult to do a meaningful analysis of the second practice, monitoring students as they work on the task, without access to the teacher's thinking during this portion of the lesson. A researcher could observe, for instance, that the teacher made a point to visit every group or that the teacher redirected a group whose strategy was headed off in an unproductive direction, but it would be difficult to know what sense the teacher was making of what they observed unless the teacher remarked aloud about various students' strategies.

The final three practices (selecting, sequencing, and connecting) could be observed more readily from a classroom video than the first two practices; however, the researchers would still need to engage in a high degree of inference to attach meaning to the teacher's actions. For instance, it would be easy to identify the student solutions that were selected to be shared with the whole class and to state the sequence of those solutions, but *why* the teacher chose those solutions and in that order would be a matter of inference unless the teacher said something such as "Emma's solution builds nicely on this one, so I want Emma to go next" or "We've seen a graphical solution; now I want us to look at a solution that uses an equation because I think that's a little bit more abstract." The connections that the teacher intends the students to make would likely be more evident from their questions (e.g., "Where do you see the 4 from Ryan's equation in Lucy's picture?" "What do Jared and Marcus' solutions have in common?").

Thus, the 5 Practices framework is probably not the best framework for this data set. Using this framework when it does not align well with the nature of the data could lead to making unsubstantiated claims. For example, the researchers might be tempted to make claims about why the teacher selected particular solutions to be shared during a whole-class discussion, but these claims would be based solely on the researcher's inferences rather than on direct evidence about the teacher's reasoning. The research team might want to use a portion of this framework (likely the last two or three practices) along with one or more additional frameworks to better tap into the richness of this particular data set. Frameworks themselves are not inherently good or bad. Rather, it is their fit with research questions and data that makes them more or less viable as tools.

1.6 Conclusion

In this chapter we have argued that theoretical frameworks are much more than a "necessary evil" in our work. They provide a foundation that both strengthens individual research projects and allows for collective advancement of the field. We advise researchers, novice and experienced alike, to give more attention to theoretical frameworks in designing, carrying out, and reporting on their research. Specifically, we call for researchers to write reports of research in a manner to "allow others to see the details of how our frameworks were used in data collection and analysis" (Mewborn, 2005, p. 8). Doing so will help to demystify the role and use of theoretical frameworks, lead to the production of more robust frameworks, and build better collective knowledge in our field.[3]

[3] Portions of this paper are based on an earlier uncopyrighted paper cited as Mewborn (2005) in the reference list.

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Chapter 2
Conducting a Timely Literature Search

Daniel K. Siebert

Abstract Literature searches play an essential role in designing, conducting, and publishing high-quality mathematics education research. In this chapter, I suggest three stages for completing an efficient and thorough literature search, including specific strategies and tools for each stage. In Stage 1, researchers identify promising sources and compile a prioritized reading list before engaging in extensive reading. In Stage 2, researchers use and regularly update their prioritized reading list so they can read and extract important information from the most pertinent sources. In Stage 3, researchers identify specific needs in their writing and update and use their prioritized reading list to read and extract the information necessary to address these specific needs. By following these stages and using the recommended strategies, researchers can efficiently conduct the literature searches they need to support their research work.

Keywords Conducting literature searches · Reviewing the literature · Literature search stages · Prioritized reading list

One of the essential tasks when beginning a new line of research is to conduct a literature search. I use the term *literature search* to refer to the process of identifying, reading, and drawing upon pertinent sources to achieve purposes related to planning, conducting, and reporting empirical research. Researchers need to consult relevant literature to achieve a variety of purposes as they engage in empirical research, including the following:

- To identify a *research topic*, *research problem*, and *research questions;*
- To build a *rationale*, i.e., an argument for why the research topic and problem are worth studying;

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- To create a *review of the literature* that not only synthesizes important findings relevant to the research problem but also identifies what is yet unknown about the problem;
- To identify or construct a *theoretical framework* that can be used as a lens to identify and investigate the phenomenon or problem being studied;
- To choose and justify the *methods* for conducting the study, including the selection of setting and participants, types of data gathered, and processes for collecting, managing, and analyzing data;
- To interpret the results so as to identify the *contributions* and *implications* of the research.

Because of the many ways that a literature search is used to inform an empirical study, literature searches can feel complex and overwhelming. There is also the very real danger of venturing into the literature and floundering, perhaps wasting weeks and months of reading without finding direction or insight. The purpose of this chapter is to identify particular activities and strategies that researchers can use to manage a literature search and reduce the amount of time spent collecting and reading unhelpful sources.

Before proceeding further, however, it is useful to distinguish my use of the term *literature search* from two other terms that are used by researchers to refer to the activity of drawing upon the literature while conducting research, namely *literature review* and *systematic literature review*. The term *literature review* is often used by researchers to refer both to what I have defined as a *literature search* and to the more specific activity of drawing upon the literature to write the literature review section in research proposals and papers. For purposes of clarity, I use the term *literature review* in this chapter to mean only the latter activity. A second common term is *systematic literature review*, which is often used to refer to the process of systematically surveying the literature with the sole purpose of writing a comprehensive and unbiased review of the literature as a standalone, end product (Boland, Cherry, & Dickson, 2017; Jesson, Matheson, & Lacey, 2011). Because the purposes of a literature search and a systematic literature review differ greatly, they require different approaches to identifying and drawing upon relevant literature. This chapter specifically addresses the strategies used for literature searches. Researchers who wish to conduct a systematic literature review will benefit from consulting sources that address this particular type of literature survey (e.g., Boland et al., 2017).

2.1 Stages in a Literature Search

To help make the literature search more understandable and manageable, I have broken the process down into three stages. These three stages can be used to conduct a literature search in support of any of the purposes listed at the beginning of this chapter. A separation into three stages helps make clear some of the different processes that you will engage in and products you will create during your literature search, as well as the order in which you might proceed. However, the stages are an unavoidable oversimplification of the process of conducting a literature search. You will likely have multiple foci in your literature search (e.g., determining how you will define the phenomenon you are studying, synthesizing research findings related to your research question, deciding on and understanding the particular data analysis methods you are going to use), so it is possible that you will be engaged in multiple stages at the same time throughout most of your literature search. Moreover, what you discover in a later stage may cause you to return and reengage in an earlier stage. Thus, you should think of the stages as descriptions of the different kinds of work you will do while engaged in your literature search rather than as a series of steps that you must follow exactly.

2.1.1 Stage 1: Gathering and Organizing Materials

While it is tempting at the beginning of a literature search to do a quick web search and immediately begin reading the papers from the search results, such an approach often results in weeks wasted reading papers that provide you with only a limited understanding of important issues, theories, and findings related to your topic. A more prudent approach is to first scout out the literature to identify some of the seminal works, prominent researchers, key issues and theories, and important findings pertaining to your topic. You can think of this stage as constructing a rough map of the research terrain surrounding your research topic. Spending time at the beginning of your literature search to gain a sense for the research terrain will enable you to use your reading time more judiciously; you will be able to start your reading with the key sources related to your topic. By reading important works first, you will be able to narrow your research focus and identify your research problem and questions more quickly, which in turn will reduce the number of papers you will then need to read. Simply put, if you start your literature search by first compiling a prioritized list of what to read and then regularly modify and reprioritize that list as you read, you can avoid wasting weeks and even months of reading unrelated or unhelpful material and thus shorten the time it takes to complete your research.

2.1.1.1 Strategies for Compiling Sources

Surprisingly, you can often compile a useful initial prioritized reading list without actually doing much reading, even if you initially know very little about your topic. Some of the common methods for compiling such a list are provided below. I have introduced them in the order in which I personally have found them useful in conducting many of my own literature searches.

Ask Experts

Knowledgeable colleagues, mentors, and advisors can be valuable assets in creating a prioritized reading list (Ridley, 2008). Ask them to identify seminal papers, important theories, crucial issues, and prominent researchers related to your research area. If you feel nervous about approaching an expert right from the start, first try some of the methods below to compile an initial reading list and then solicit the expert's feedback concerning which sources he or she thinks are worth reading, which are not, and which you should read first. Also ask him or her to recommend papers you should read that are not currently on your reading list. Note that a few minutes of expert guidance can literally save you weeks of work, so consider consulting with experts even if it feels intimidating to do so. On the other hand, keep in mind that no expert is perfect, so it is wise to use some of the strategies below in addition to consulting experts.

Consult Handbook Chapters

Browsing handbook chapters related to your research topic can often help you quickly construct a rough map of the research terrain surrounding your topic because they are purposely written to summarize the important issues, research directions, and research findings in particular areas of study. These chapters also contain a comprehensive list of sources that are potentially pertinent to your research topic. Papers that are cited in the sections of the chapter that seem most closely related to your research focus may be particularly good candidates for your reading list.

Search Prominent Journals

You can increase the likelihood of finding helpful papers if you search for them in the premiere journals in the field. Editors of premier journals strive to publish only papers that make substantial contributions to the field. Thus, starting with papers from these journals can often be advantageous when doing a literature search. To find these papers, consider perusing several years of the table of contents from prominent mathematics education journals for articles related to your research topic. You may wish to start with some of the mathematics education journals that Williams and Leatham (2017) identified as being of high quality based on citation- and opinion-based surveys (see Table 2.1 for the seven highest ranked mathematics education journals from this study).

Search the Reference Sections of the Papers You Locate

While you should probably refrain from closely reading articles as soon as you locate them during the first stage of a literature search, you might still want to carefully examine their reference sections. Reference sections can provide additional

Table 2.1 Highly ranked mathematics education journals

Name of Journal
Journal for Research in Mathematics Education
Educational Studies in Mathematics
Journal of Mathematical Behavior
Zentralblatt für Didaktik der Mathematik (The International Journal on Mathematics Education)
Journal of Mathematics Teacher Education
Mathematical Thinking and Learning
For the Learning of Mathematics

Note: Journals ranked according to Williams and Leatham (2017) using the results from an opinion-based survey

sources to add to your reading list and are particularly helpful in identifying pertinent books and book chapters that online searches can miss. Also, as you look through several bibliographies, you will begin to notice oft-cited sources and authors. These repetitions can help you identify the seminal works and prominent researchers in your research area. You may also discover trends concerning which journals or publishers tend to publish research related to your topic, which you can then mine for additional sources.

Conduct Web Searches

Although web searches may not be the most effective method for identifying relevant literature at the beginning of Stage 1, they can be very productive once you have developed a feel for the research terrain. Not only will you have a better sense of the keywords, titles, and authors to use in your searches, but you will also be able to sort through the results more skillfully based upon your knowledge of the key researchers, prominent journals, and important issues. Search engines such as Google Scholar can also be used to identify sources that cite a particular seminal work or key researcher. While you can use this feature to locate new sources, you can also use it to help you determine how well cited a particular paper or author is, which in turn can inform your decisions about what papers to read first. Because technology continues to advance rapidly, you might want to ask a librarian for help in identifying the most recent search tools, databases, and search strategies for conducting efficient and effective web searches on your research topic.

Although the above strategies are typically helpful for identifying relevant sources, researchers who engage in interdisciplinary or critical research may find little benefit in, or even be misdirected by, the above methods because these methods do not challenge systems of power, authority, and privilege. In my own interdisciplinary research, I have found the following two additional strategies particularly useful for identifying pertinent sources.

Cultivate a Relationship with an Ally

Sometimes the gap in the literature between what has been written and what you want to study is so wide that you cannot bridge the gap without the help of an ally. I use the term *ally* to refer to an expert who is open-minded, willing to support you in your exploration of ideas related to your research, and typically in a field other than mathematics education. Allies differ from regular experts in that they do not require you to adopt or pay homage to the dominant perspectives, theories, values, and norms in their field of expertise but instead support and encourage a critical stance toward knowledge and research. They are often interested in issues that lie at the edges of their own field and enjoy extending theories and frameworks to address novel contexts and problems. Allies can help identify sources in their fields of expertise that are related to your research topic. They make recommendations with the understanding that the ideas from the sources they suggest will likely have to be modified to fit your research problem. Allies can also function as sounding boards as you attempt to apply ideas from the readings; they can help you stay as true to the original ideas in the literature as possible as you attempt to apply them to your research problem and can point out limitations in the dominant theories and perspectives in your own field. In my own work with allies, I have often engaged in the following cycle: identify a short list of 1–3 works with the help of my ally, read them carefully, attempt to apply them to my research problem, discuss the results of these attempts with my ally, and then together identify another short list of works to use in the next repetition of the cycle. Long, prioritized reading lists were not possible for me to generate because the selection of sources to read relied heavily upon the outcome of the previous cycle.

Sample Widely

A second strategy for bridging a wide gap in the literature is to sample broadly for potentially relevant ideas, theories, and studies. There are many ways to do this sampling, including conducting broader web searches, attending colloquia or conference sessions outside of mathematics education, talking with scholars across your university, and regularly reading outside of mathematics education. While this method is haphazard and time consuming, it can lead to discoveries that likely could not have been achieved any other way. Once a discovery has been made, the researcher can use some of the initial strategies from above to more systematically explore the discovered ideas, theories, or studies.

2.1.1.2 Products Created During Stage 1

There are three important products that I recommend you create as you engage in this stage of the literature search. First, you should consider creating a physical or electronic collection of the papers and book chapters that you have placed on your reading list. Frequently when you locate a source, you do so through electronic means that gives you direct access to an electronic copy. Downloading a copy of the material at that moment is often much easier than trying to relocate the source later. Many reference management software programs (e.g., Endnote, RefWorks) offer ways to store the electronic copies alongside the bibliographic information, enabling you to quickly access the papers when you are ready to read them. Obtaining copies of sources also grants you access to their reference sections, which can be mined for additional sources. You will also want copies so that you can quickly scan through them to assess their value to your research. Having quick access to the sources on your reading list will save you time and frustration in the long run.

The second important product of this stage is your prioritized reading list. Reading lists can take many forms, such as a pile of papers organized from most to least important or a bibliography in which sources are listed according to priority for reading rather than alphabetically. Often it is useful to organize sources into categories rather than a numbered list of sources to be read from first to last. I usually organize my sources into four categories: *read immediately* (seminal pieces, i.e., pieces on the topic that are highly recommended or most cited), *read eventually* (papers on the topic that are written by prominent researchers, published in premier journals, frequently cited, and/or appear to contain important theories or findings), *possibly read* (papers that are related to the topic but are published in less prominent journals by less prominent researchers and cited infrequently), and *read references only* (papers that seem tangentially related to the topic or of very low quality that may nonetheless contain references to valuable sources). Sorting your sources into categories such as these allows you to make useful distinctions while avoiding the stress and strain of creating an exact reading order. Use your experiences from your paper gathering efforts to decide which papers go in which categories. Limit your reading at this point to a quick read of the abstracts or a quick scan of the paper. If you are not sure which category to assign to a paper, make a quick decision and move on; you will have plenty of opportunities to reorganize your papers in the next stage.

The third important product of this stage is a search diary. Because your activity during Stage 1 will be varied and often extend over a few weeks, it is helpful to keep a record of what you did. Consider recording which journals you search, which reference sections you check, and what web searches you perform. Alongside the record of what you did, write your hypotheses about which works are seminal, which researchers are prominent, and which issues and theories are central to your research topic. These hypotheses can be used to conduct additional searches. While your notes do not need to be extensive at this point, a little bit of record keeping goes a long way toward sustaining your focus and productivity. Consider stopping a few minutes early every time you work on your literature search to write notes. Read over your notes every few sessions to see if the direction you are taking in your search makes sense. Regular reevaluation of your search activities can help you make better decisions about what actions to take next.

2.1.2 Stage 2: Managing Information and Narrowing Your Focus

The second stage of a literature search consists of reading selectively from the sources that you amassed in Stage 1 so as to extract important information related to your research and narrow the focus of your search. There are three important issues you must attend to as you progress through this stage: managing your reading, managing information, and narrowing your research focus.

2.1.2.1 Managing Your Reading

When you begin to read your papers, you want to carefully manage which papers you read, the order in which you read them, and the way you read them. Your initial prioritized reading list from Stage 1 identifies which sources you should read first. Every time you read a few sources from your list, you should take a few minutes to add any new sources that you may have discovered in your reading and reprioritize your list. Insights you gained from your reading should be used to identify the next most promising sources to read. You may also be able to remove some sources from your list because they no longer seem to apply. Frequently revising your prioritized reading list as you read can help you to continue to read the most promising sources throughout your literature search.

Not only will you want to carefully choose which sources you read and in which order, you will also want to purposely decide how you will read each paper. Not all papers are worth reading carefully, nor are some parts of a paper worth reading as closely as other parts. If you want to save time in your reading, you should first scan the paper, attending to particular elements of the paper that can help you quickly assess its content and value. These elements include the abstract, the introduction, titles and headings, topic sentences, figures and tables, and summaries (Manz, 2002). Then make decisions about which parts, if any, to read; how closely to read them; and for what purpose. For example, after scanning a paper, you may decide to skip reading the rationale and quickly skim over the theoretical framework and literature because those sections look similar to papers you have already read. Alternatively, you may decide to carefully read the method section because the authors seem to be analyzing their data similarly to the way you plan to, and you want to see how they described and justified their data analysis. By being selective in what parts of a paper you read and how closely you read them, you can significantly reduce the time spent reading without compromising the results of your reading.

2.1.2.2 Managing Information

One of the challenges of engaging in extensive reading is keeping track of the information you feel is pertinent and valuable to your research. It is not enough to merely read the sources; you must also be able to remember and relocate the important ideas and findings as you write research proposals and papers, preferably without having to do too much rereading. To achieve this purpose, researchers often use a variety of methods to extract information from sources and to mark sources so that key sentences or paragraphs can be quickly relocated.

Note Taking

One of the most common ways of keeping track of important information from sources is to take notes. These notes may vary in format from article summaries in the form of paragraphs to bulleted lists of important points accompanied by the page numbers on which they can be found. Notes can be written on notecards for easy sorting, typed into an electronic document, or attached directly to source records in reference management software. As you create your notes, it is valuable to clearly mark which parts of your notes represent direct quotes so that you can avoid accidental plagiarism. Including page numbers for each direct quote is also important.

Annotating Sources

Researchers often mark up their personal copies of sources as they read them to record in-the-moment insights or thoughts and to identify specific passages they might want to use in the future. While annotating has traditionally been done by marking physical copies, you may want to consider using a PDF annotation app to do your annotating electronically. To shorten the amount of time you spend annotating, consider using symbols such as exclamation points, question marks, stars, and numerals instead of words and phrases to annotate the paper, particularly while skimming. You may also want to draw vertical lines down the sides of text passages you wish to mark instead of using underlining. Using these shortcuts in your annotations will allow you to keep your reading speed up and still enable you to locate important passages in the future.

Extracting Common Data

One of the common purposes for conducting a literature search is to prepare for writing a literature review, which often requires you to compare and contrast a collection of studies. You can prepare for the task of comparing studies better by extracting the same type of information from each source in the collection of studies as you read them. While it is often the case that comparisons focus solely on the findings from different studies, in some literature reviews additional information such as the following may be valuable to compare: research questions; the theoretical lens used to analyze the data; the setting for data collection; the age, sex, and grade levels of participants; the types of data collected; or the recommendations for practice. Before reading your collection of papers, make a list of the common types of information you might want to track from each of the studies. Consider recording the information from each paper in a spreadsheet where a column has been created for each type of information and a row for each source. Storing data in this format will enable you to make quick comparisons across studies. Keep in mind that your initial decision about which types of information to track may need to be modified after reading the first few papers so that you can make the comparisons you want in your literature review.

2.1.2.3 Narrowing Your Research Focus

While reading in this stage, you are looking for broad themes and issues, as well as ideas, questions, or phenomena that pique your interest. Often you will recognize important ideas in the literature by the emotions that you experience while reading about them. If a paper sparks a strong emotional response in you, it is worthy of a more careful reading, particularly in terms of what particular problem, idea, or issue it might suggest to you for further research.

An important technique for narrowing your research focus is writing memos, which are different in nature from the notes you take as part of managing the information from the sources you read. Recall from above that notes serve the purpose of helping you clarify, remember, and later access the ideas in the sources you are reading. In contrast, memos serve the purpose of helping you think critically about what you are reading—how ideas fit together across sources, what issues are important, which research methods are useful for investigating your topic, what holes exist in the literature, etc. While it is true that you can engage in critical thinking without writing memos, the process of writing will help you generate insights that far surpass the insights you might generate from thinking alone. You may also benefit from occasionally augmenting your written memos with graphical organizers of your reflections, such as concept maps (Novak & Cañas, 2007). Although there are multiple ways to store memos and concepts maps, ranging from handwritten notecards to blog entries, it is often helpful to choose one method and then stick with it for your entire literature search.

The activity of writing memos and creating graphic organizers contributes to narrowing the focus of your research in three ways. First, this activity can help you identify the issues, ideas, theories, methods, and/or findings that are not essential to the purpose of your literature search, which allows you to weed them out of your reading list and your research study. For example, you may realize that two of the three constructs you have been reading about are sufficient for conceptualizing the phenomenon you are studying, thus allowing you to drop the third construct from your study and your literature search. Second, this activity can help you identify particular areas that require further reading. For example, you may discover a certain data analysis method that seems promising for studying the phenomenon in which you are interested but realize that you need to read more in order to determine whether the method will work, given your time constraints and the type of data you will be able to collect. Third, writing memos and creating graphic organizers can help you consolidate some of the issues, ideas, theories, research methods, and findings into larger constructs or categories. For example, through writing memos and creating graphic organizers, you may come to realize that there are three main types of research studies that have been done that are pertinent to your research, which allows you to consolidate the collection of empirical studies related to your topic into three main categories.

2.1.2.4 Products Created During Stage 2

As suggested above, the products you will produce in this stage of your literature search include an ever-evolving prioritized reading list, a set of notes for the contents of the sources you have read, a collection of annotated sources, and a collection of research memos and graphic organizers. However, the most important product of this stage is the identification and understanding of the main elements (the issues, ideas, theories, methods, and/or findings) that you plan to use to address your research purpose. Once you have identified these main elements, you are ready to move to Stage 3.

2.1.3 Stage 3: Filling Gaps and Writing

In Stage 3, the nature of your literature search changes from exploration to construction. In particular, you now begin reading with the intent to fill in your understanding of the elements you selected in Stage 2 and the relationships between them. Typically, the purpose of the reading you do in this stage is to improve your understanding so that you can create a particular written product, such as a rationale for your research problem, the literature review section of your research paper, or a list of initial codes for analyzing your data. It is valuable at this point to directly link your literature search to the progress you are making in creating the written product because this progress (or lack thereof) can help you decide what you still need to read and when you have read enough. Of course, in order to link your literature search to the progress you are making in creating a written product, you must actually be writing that product while you work in Stage 3 of your literature search.

To begin working on your written product, you may find it helpful to first create a graphic organizer of the elements you identified from Stage 2 and fill in the relationships or crucial connections between them. You can then use this graphic organizer to begin writing outlines for the major sections of the written product you are creating. Anticipate that this exercise will feel uncomfortable because it is unlikely at this point that you will have read enough of the literature to write complete, detailed outlines. However, by attending to what parts are easy to outline and which are not, you will gain a sense for where you need to focus your reading efforts. Revisit your prioritized reading list and see if you have sufficient sources for filling the gaps in your graphic organizer and outlines. If not, you will need to repeat Stage 1, collecting sources on very specific topics and issues to fill the gaps. You will once again sort and prioritize these sources and then begin reading with specific purposes, such as to refine the definition of the phenomenon you are studying, to address holes in your literature review, or to justify your choice of particular data collection and analysis methods.

As you read in Stage 3, you will continue to regularly evaluate and revise your prioritized reading list and take notes from and annotate the sources you are reading like what you did in Stage 2. Unlike in Stage 2, however, your memo writing here should focus on how the particular ideas from each source you read can be used in specific sections of the written product you are creating. As you read to close the gaps in your concept map and outlines, you should test whether you have read enough by creating more detailed outlines of the major sections and then writing them. Note that Stage 3, and thus your literature search as a whole, is not complete until you have finished writing because you will not know for sure if you have read enough until you are able to complete a satisfactory draft of your written product. This connection between the completion of your literature search and the completion of your writing makes clear that a literature search is not an activity that you can or should complete *before* you start writing. Rather, writing actually drives and determines the completion of your literature search.

2.2 Planning and Maintaining Focus

It is essential that you create a plan for your literature search and regularly evaluate progress if you desire to complete your search in a timely fashion and avoid wasting time on unproductive searching and reading. Although it is not possible to anticipate all of the specific actions you will have to take to complete your literature search or to anticipate exact dates for completion, listing specific actions and deadlines can help you keep your search moving forward. You may want to start with a deadline for the completion of the written product and then work backward, setting intermediate deadlines for the completion of important tasks and stages in your search and your writing. Once you have created your plan, you may wish to share it with others who have experience in conducting literature searches to get feedback on both the actions you will take and the deadlines you have created for completing those actions. Also, consider sharing important deadlines with friends or colleagues because doing so can increase accountability for meeting your deadlines.

Note that even experienced researchers have difficulty maintaining focus during a literature search; it is easy to get lost in the activity of reading interesting papers and to put off the important activities of reevaluating your prioritized reading list, writing memos, constructing concept maps and outlines, and drafting your written product. I find the following guidelines helpful in balancing the activity of identifying and reading sources with other essential activities in my literature searches:

- **Stage 1:** For every 3–5 h of identifying and locating sources, try to write 10–20 min in your search diary about what specific searches you performed during that time; what seminal works, prominent researchers, key issues and theories, and important findings seemed to emerge from those searches; and what your next searches should be.
- **Stage 2:** For every 5–10 h of reading and taking notes, try to spend at least 30 min adding to and reprioritizing your reading list and writing memos about what important ideas have emerged, how those ideas are related to what you have already read, and how they are influencing your decision about what your literature search foci should be.
- **Stage 3:** For every 5–10 h of identifying additional sources, reading, and note taking, try to spend at least 1 h in one or more of the following activities—writing in your search diary about the searches you are performing, constructing and revising graphic organizers and outlines, writing memos about how the new material fits into the written product you are creating, or actually drafting the written product.

Balancing your finding, reading, and note-taking activities throughout the stages of your literature search with these other essential activities can help you regularly evaluate and focus your literature search. This in turn will allow you to make the most of the time you spend identifying, collecting, and reading sources and help you to avoid wasting time on sources that do not contribute to your research.

2.3 Conclusion

Because of the diverse purposes that literature searches fulfill in research and in extending our own personal knowledge and understanding, it is valuable for researchers to become adept at conducting literature searches in efficient ways. The strategies introduced in this chapter represent some of the common approaches that experienced mathematics education researchers use to conduct literature searches. By learning about and using some or all of these strategies, new researchers can maintain focus in their literature searches and avoid wasting weeks or months reading unhelpful sources, enabling them to efficiently complete the thorough literature searches that are necessary for designing, conducting, and publishing quality mathematics education research.

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Chapter 3
Designing and Conducting Quality Research in Mathematics Education: Building a Program

Carolyn A. Maher and Louise C. Wilkinson

Abstract This chapter addresses a significant issue that novice mathematics education researchers confront: how to conceptualize, design, and conduct a quality, comprehensive, research program. Developing, executing, and sharing the results and the process of one's research program is a primary professional commitment for all researchers. For novice mathematics education researchers, a primary challenge may be how best to navigate the intellectual terrain of their focus of inquiry and also to acknowledge that they are not alone in initially experiencing rejection, frustration, confusion, and doubt but that successes will follow. Conceptualizing and launching one's research program, one that is both original and generative, is an issue that all researchers have faced in their careers. We encourage beginning researchers to think broadly and creatively about the opportunities to share and to bring new expertise to their work by expanding their community of researchers. In this chapter, we provide insights and suggestions regarding how novice mathematics researchers may reflect upon, construct, and sustain a research program that is both original and generative. Throughout this chapter, we offer detailed examples from our research program in mathematics education to illustrate this process. We report on decisions made and the conditions that prompted the directions we took.

Keywords Designing a research program · Sustaining a research agenda · Tracing content learning · Attending to research contexts · Selecting methodological tools · Sharing stored data

The purpose of this chapter is to address a significant issue that beginning mathematics education researchers confront: how to conceptualize, design, and conduct a

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quality, comprehensive, research program. For novice mathematics education researchers, a challenge may be how best to navigate the intellectual terrain of their chosen focus of inquiry and also to acknowledge that they are not alone in initially experiencing rejection, frustration, confusion, and doubt but that successes will eventually follow. Conceptualizing and launching one's own research program, one that is both original and generative, is an issue that all researchers have faced in their careers. In this chapter, we provide insights and suggestions regarding how to think about and how to construct your research program, which is both original and generative. To clarify these ideas, throughout this chapter we offer detailed examples of the development of such a research program in mathematics education. We report on decisions made and the conditions that prompted the directions we took. Our work offers the reader the opportunity to join us on reflecting how to build and sustain such a program.

This chapter begins with a description of the first steps for establishing an ongoing research program of high quality. These steps should be prompted by program goals and include such essential elements as deciding on a focus of the research program and the question(s) that will be addressed. Fundamental to establishing any research program is deciding on the conceptual framework that will drive the inquiry. We remind young researchers to make these decisions in light of the knowledge of which they are seeking to gain better understanding. This approach requires having conducted a careful review of the literature in the area that is being studied. After identifying a research focus, it is important to become very familiar with what others have asked and learned about this topic prior to proposing any subsequent studies. For example, one must consider not only empirical findings but also how other researchers defined their work in this domain and focused their research. It is important to recognize and understand the specific questions they endeavored to address, how they went about their inquiry, and what the findings revealed.

One way to begin the pursuit of this knowledge is to review carefully, critically, and comprehensively the extant corpus of research. An important complement to this process is to seek out experts and colleagues who share your passion. Reach out at conferences or through other means of direct communication and begin to establish a dialogue with them. By attending, participating in, and sharing research at professional conferences—locally, statewide, nationally, and worldwide—you will find others whose interest and work might align with yours. Also, you can volunteer to be a reviewer for pertinent journals, conferences, and funding agencies such as the National Science Foundation. By reaching out, you will have begun the important step of connecting to a broader community. Some of these efforts will have been planned out ahead of time; others will come from taking advantage of opportunities that arise and from adjusting to circumstances.

3.1 Building and Sustaining a Research Program

The example of the research program developed in this chapter illustrates, in some ways, the path that we, as researchers, have taken in our journey of inquiry. Reflecting on the evolution of our research interests and the multiple directions our work has taken, we recognized that for each of us, a research agenda—our specific interests in the overall research program—has naturally emerged. We began our early work based on findings from our initial studies. This knowledge informed and redirected our subsequent inquiry. This chapter describes how our research agenda evolved, pointing to decisions that we made along the way. To that end, we describe shifts in focus as we became more aware of the complexity of student learning, proposing research questions that became increasingly more refined.

3.1.1 How Do You Begin to Construct a Research Agenda and Research Program?

The first step is to recognize the topic of your research agenda and potential program. This recognition may be followed by a series of research questions that flow logically from the topic in which you are interested.

For example, in our case initially, we sought to investigate how mathematical ideas developed in learners under conditions that supported collaboration and invited the production of justifications for proposed solutions to problems. As we studied explanations and arguments that were produced by students, we discovered that children, even young children, produced arguments that were "proof like" (Maher, 2005; Maher & Martino, 1996a, 1996b). These findings were unexpected and actually surprising to us, prompting us to refine our work and attend to student arguments. We collected data that enable us to attend to the variety of representations that students used to express their knowledge. These data underscored the importance of gaining an understanding of what ideas were built up in the minds of students and how they expressed and expanded their knowledge, especially with students for whom English was not their first language (Davis & Maher, 1993; Sigley & Wilkinson, 2015).

3.1.2 What Is the Conceptual Framework Guiding Your Research Program?

The second and essential next step is to develop a framework to delineate your research program. In our case, we chose an approach that aligned with what the National Research Council (NRC) has defined as "deeper learning" (NRC, 2012, p. 5)—that is, the transfer process reflecting the dimensions of human competencies that are cognitive, intrapersonal, and interpersonal and where skills and knowledge were intertwined. Among the relevant competencies were critical thinking, information literacy, reasoning and argumentation, creativity, initiative, innovation, metacognition, communication, and collaboration. Such competencies are widely perceived as being essential for success in school, at work, and, more generally, in life. Clearly, this was an ambitious framework, requiring designs that captured the complexity of learning under conditions that might evoke those behaviors. To this end, we chose to videotape the behaviors that were collected and carefully transcribe, code, and analyze the data. There is no shortcut to carrying out this kind of work, and we found, early on, that we had more data than we could analyze in a meaningful way. A choice was either to cut down on the amount of data or to train graduate students to work on the analyses with us. The decision to go with the latter choice became a win-win situation as we provided young researchers the opportunity to work with us.

While our evolving research program was oriented to discovering children's developing mathematical understandings, a joint major focus was on what teachers needed to know and do to optimally support children's construction of mathematical knowledge. Specifically, with regard to mathematics education and our developing research program, we responded to the challenge of the NRC (2012) regarding the need for teachers to participate in ongoing professional learning opportunities, particularly "to develop new understandings of the subjects they teach and how to assess (manifestations of) competencies in these subjects" (p. 12). In this way, our agenda took another turn. We realized that in order for teachers to promote their students' engagement in mathematics as a sense-making activity that involved reasoning and justification, the teachers also needed to experience that mode of mathematical learning for themselves (Maher, Palius, Maher, Hmelo-Silver, & Sigley, 2014). This important finding directed our work toward studying teacher learning. We realized that teachers needed to move beyond their own way of solving particular problems so that they could gain expertise in considering other valid approaches, delve into other strategies, build other representations, and construct their own forms of reasoning to support the arguments used to justify solutions (Maher & Alston, 1990). We learned that recognizing students' justifications for solutions required both knowledge of the variety of forms of reasoning that could arise in students' arguments and their own knowledge and judgment to recognize the validity of an argument that may not, initially, appear in "standard" form (Maher, Davis, & Alston, 1992).

We note the extensive resources garnered to support deeper learning among teachers, such as the *Robert B Davis Institute for Learning's* (RBDIL's) collection of mathematical tasks and videos illustrating learning environments in which children work on those tasks. Viewing and discussing what is observed on the videos functions to make teachers aware of the opportunities to create classroom situations that promote mathematical reasoning for their own students. This collection may be accessed via the Video Mosaic Collaborative (VMC) (https://videomosaic.org). In sum, prior work of the *Institute* over the past quarter century developing, utilizing, and testing tools for research, such as the VMC, provides a resource for teachers' learning about students' mathematical reasoning.

3.1.3 What Content Learning Do You Plan to Examine in Your Research Program?

The next step in building your research program is to be clear about the initial content focus. You cannot study everything on the topic or even for the particular questions you have identified. Focus and specificity are required here. Thus, the content (and grade level) is an important consideration when studying learning (e.g., use the standard curriculum or use a variation that makes use of new tools and affordances available to learners).

For example, for our research, we decided from the onset to introduce content that was not yet included in the curriculum at the particular grade level of our subjects. This decision was based on a desire to explore student learning before students were taught to memorize rules and procedures that they did not understand. We wanted to avoid the possibility of confounding our results because the content was part of the grade-level curriculum. During our early years of research, the predominant classroom approach to teaching mathematics was teacher centered. We deliberately sought to shift the traditional approach to one more likely to engage students. Using video, we were able to observe evidence of what mathematical understandings they could build while engaging in challenging problem tasks that were designed to elicit meaning and thoughtfulness. For all of these tasks, students were invited to produce justifications for their solutions and to convince themselves and others of their validity. This repetitive invitation became a "taken as shared" expectation, as our later work disclosed. We began with counting problems in the lower grades and shifted to more challenging strands involving combinatorial reasoning. Students were also challenged to investigate fraction and rational number ideas, probability, early algebra, and calculus.

3.1.4 How Do You Connect Your Research Program with Students, Their Schools, and Their Families?

In conducting mathematical research with children, youths, and adults, it is important to recognize that learning occurs in a variety of contexts and settings. Consequently, as a researcher you must undertake thoughtful planning and affirmative outreach to those organizations overseeing and governing those contexts—such as school principals and teachers.

For example, we recognized early on that working with teachers required working with schools. Thus, establishing partnerships was essential in carrying out our research studies. We recommend that, early on, researchers offer service to schools. This service establishes trust and provides schools with resources that they may not otherwise be able to afford. Working with teachers in their classrooms, in diverse school districts, paved the way for us to bring in research sites for grant proposals. Also, many of our own students were either school administrators, practicing teachers, or preservice teachers enrolled in teacher preparation programs. Districts, with limited funds available, welcomed our working with their teachers and providing professional, in-service development. The support we offered for teacher education provided a fair exchange for subsequent research initiatives with teachers, comfortable in inviting us to their classrooms and work as partners with them and their students. In addition, administrators were eager to see improved teaching and learning of mathematics.

Teachers were generous in inviting us into their classrooms, even making adjustments in scheduling to accommodate our interventions (expanding the time allotted for mathematics sessions from the usual 30–45 min to 60–90 min during our sessions). They took pride in partnering with us, which enabled them to more comfortably provide challenging mathematics to their students as we worked together. Over the years, we were successful in establishing several partnerships with schools in urban, suburban, and working-class districts and conducting research studies with their students. Without the invitations and relationships to work with teachers and their students, much of our research work might not have evolved.

3.1.5 How Do You Attend Thoughtfully to Research Contexts?

Once you have established your research topic, inquiry questions(s), and content focus, the next step in developing a research program is to identify, specify, and define the particular contexts within which you will conduct your research.

For example, one of our early interests was to investigate, within a variety of settings, how students justified solutions to problems. As a result, we made the decision to expand our work to include multiple settings and contexts. We observed using video data the work of individual students, dyads, small groups, and whole-class discussions. To better understand our data, we followed up our inquiry by conducting task-based, clinical interviews. Our findings motivated us to further extend our work and to design teaching experiments. By capturing on video the thoughtful argumentation of students working in pairs and small groups, we developed further insight into the forms of arguments that students used. Our analyses also provided further insight into obstacles that learners encountered in building valid justifications. It is essential to not only trace the growth of learning but also to gain insights into the obstacles to that learning. These obstacles varied according to students' background and grade level.

We next expanded our work to informal, after-school urban settings with longitudinal studies in several content domains. As our work expanded, the need for funding became essential. Our interest was in capturing, with multiple cameras, individual student work, as well as student collaborations. We chose to extend our work to conduct cross-sectional studies with populations of students from different socioeconomic backgrounds to include both urban and suburban communities. Funding made possible the continued and extended study of student growth in learning, across different populations, and longitudinally with data that enabled tracing individual mathematical learning from first grade to high-school graduation and beyond (Ahluwalia, 2011; Steffero, 2010).

3.1.6 How Do You Select Appropriate Methodological Tools, Even Though They May Be Unconventional or Innovative?

Designing a research program requires that you specify *how* you will collect information. Our best advice is to listen carefully to what you want to know about and then choose tools that optimize the likelihood that you'll get that data and be able to interpret it. Sometimes you may choose a methodological tool that has been rarely used for your type of study, or it may be very expensive to implement. For example, consider video and audio-taping interactions among students and their teachers when discussing mathematical problem solving. When we initiated our research program decades ago, the use of video was rare and expensive; other tools such as self-reports and observational tools such as checklists with on-the-spot coding predominated. We believed that videos allow viewers to have a virtual entrée into the world of the learner activity and thus potentially would inform our research questions optimally. By having access to video data, users could rewatch, review, and reflect on the learning events (Maher, 2008; van Es & Sherin, 2008). For our work, these data allowed our team and graduate students to conduct critical analyses that live classroom observations or clinical interviews do not allow (Derry, 2007). In order to capture the important detail and complexity in students' construction of knowledge, we supplemented video data by collecting written work and observer/researcher notes. This combination of data sources enabled us to transcribe and code data, often revisiting sessions to trace learning over time. Video became an essential tool in our subsequent work (Maher, 2008). Newer cameras enable researchers to capture sound with reduced ambient noise, enabling the use of new tools to produce transcripts of the video data. These cameras were not available when we began our work, and the use of machine transcription was not feasible. By carefully selecting video equipment, the production of transcripts can be more efficiently made.

An additional benefit to collecting video data, of course, is the opportunity for multiple analyses, reanalyses, and shared secondary analyses of the data set. To this day, we analyze stored data, using new lenses and attending to data collected over the years to follow students' long-term learning. We used multiple cameras to capture the talk and inscriptions that children produced while working on cognitively challenging, yet accessible, tasks that allowed them to explore mathematical ideas *before* receiving formal instruction on those topics in their regular school curriculum. We collected video data spanning 25+ years and, as a result, amassed a rich and unique collection that reveals how mathematical ideas and ways of reasoning are built by students. At the outset of this endeavor, we did not plan or even imagine that our research would span a quarter of a century; however, as the research studies generated further questions, we followed those questions and garnered support for our continuing research. Our focus was on what the children had to say and what they did when working with a mathematical problem presented to them. The resulting collection of over 4500 h is digitized and stored at the RBDIL at Rutgers University. With subsequent National Science Foundation and New Jersey State funding, partnering with Rutgers Library, we digitized the data to store on a repository, the previously mentioned VMC, which currently makes available video, metadata, and tools to build video narratives.

3.1.7 How Do You Store and Maintain Your Data?

An essential setup for any research program is to capture, store, sustain, and maintain access to valuable research data. As noted above, this resource allows for subsequent analyses, reanalyses, and shared secondary analyses.

For example, in our case, over 500 h of video and related metadata (e.g., student, grade level, content, task) are accessible worldwide, open source, on the VMC, a searchable, open-source database. Researchers, scholars, and practitioners are invited to join the VMC community to obtain access to the database and the coding tool, the RUanlaytic, which enables a user to create video narratives (VMCAnalytics) from the available video clips (see Agnew, Mills, & Maher, 2010). The video narratives can be published and referenced to other publications (e.g., see Maher & Yankelewitz, 2017). The repository contains well-documented examples of students' mathematical reasoning and has thus made possible detailed examination of students' learning mathematics from a variety of lenses (Maher, 2008). Transcripts of video clips encourage reanalysis of video data from different frameworks and perspectives. This broad availability affords the continued study of the video, making visible the complexity of learning as researchers have identified critical events in student activity. Fine-grained transcriptions of video data that have been preserved enable researchers to build on the work of others as they pursue more refined study of student mathematical learning (e.g., Koschmann, Sigley, Zemel, & Maher, 2018; Sigley & Wilkinson, 2015). The availability of the VMC has made possible new research agendas over the years to emerge, and it enables longitudinal and cross-sectional studies.

It is no surprise that shifts in interest, motivated by attention to new findings, result in new, emerging themes. Studies using the VMC continue to be pursued by graduate student researchers in their course and project work, by other faculty members, and regularly by RBDIL researchers and doctoral students. Advice and guidance from Advisory Board members from research initiatives helped to define new, emerging themes.

3.2 Emerging Themes in a Research Program

Once you've constructed your research program by taking the steps enumerated above, the next step of course is to conduct the research studies. In this section, we highlight the importance of recognizing the value of each individual study as it contributes to your corpus of work. This is what building a research program that is generative means—emerging themes are identified that inform future work. In the following sections, we describe this process by referencing our own research of the past decades.

3.2.1 Theme 1: Proof-Like Reasoning

We began our work by focusing on how mathematical ideas were built by learners. In so doing, we recognized and documented the forms of student reasoning that were used to justify solutions to the tasks we presented. We noticed, early on, that the "form" of reasoning used by students to justify solutions was "proof like." These early findings (Maher & Martino, 1996a, 1996b) triggered continued study wherein we attended to details and contexts and unearthed connections made by students as they discovered relationships among isomorphic tasks. It became apparent early on that students as young as 8 years old, when invited to support their solutions to problem tasks, produced "proof like" justifications, showing direct and indirect reasoning, without having previous instruction in producing valid arguments. The justifications that naturally emerged included students building case arguments, arguments by contradiction, and arguments by upper and lower bounds (Maher & Martino, 1996a, 1996b; Maher, Powell, & Uptegrove, 2010; Maher & Yankelewitz, 2017). These findings led to a focus on reasoning and the conditions that supported the building of valid arguments (Van Ness & Maher, 2019).

3.2.2 Theme 2: Tracing the Emergence of Students' Symbolic Representations over Time

Longitudinal studies enabled us to follow cohort groups of students who did mathematics together over the years, exploring strands of tasks in different content domains. Students, after being introduced to early algebra in grade 6 as a content strand in our research program, began to express fundamental mathematical ideas and images more elaborately, presenting their knowledge with symbolic expressions and providing more elegant justifications for their solutions and generalized mathematical ideas. Before our introduction to early algebra to the sixth graders, the students freely used symbols for coding and creating pictorial representations (including graphs and charts) to express their ideas. However, with the introduction to early algebra tools, we noticed a shift in representations of generalized mathematical ideas that continued throughout the longitudinal study. The students produced more elegant justifications for their solutions, as they made connections to the Binomial Theorem and Pascal's Triangle, for example, in subsequent problem solving. When we conducted our research, early algebra was not available as a component of the regular school curriculum until high school. The eagerness of our young students to engage in algebraic thinking suggested to us the importance of introducing symbolic notation and early algebra ideas earlier in their mathematics learning.

3.2.3 Theme 3: Gaining an Understanding of How Students Reflect on Their Own Learning

From high school and university interview data, we learned from participants how they viewed their mathematical activity in structuring their investigations and justifying their solutions. We partnered with researchers from the Harvard Astrophysics Observatory and, with funds from the Annenberg Foundation, produced the Private Universe Project in Mathematics (PUP-Math) (http://www.learner.org/workshops/pupmath), a professional development program for teachers based on our longitudinal research program. A component of the program included individual interviews of students who shared their reflections on their participation in the longitudinal study, reporting the importance of meaningful mathematical learning, of having their ideas listened to, and of collaborating with others. They referred to the mathematics of our intervention as doing "Rutgers math," in contrast to the regular mathematics that was part of their schooling. Using archived video data, several doctoral dissertations reported detailed case studies that traced student growth in learning and how they viewed their learning (see, for example, Ahluwalia, 2011; Francisco, 2004; Pantozzi, 2009; Steffero, 2010).

3.2.4 Theme 4: Language Development and Mathematics Learning Are Intertwined and Interdependent

An additional focus of our ongoing work is studying mathematics learning of students whose home language is not English—that is, students who are learning English at the same time they are learning, in this case, mathematics. The fact that these students, English-learning (EL) students, are required to speak, read, and write their explanations and problem-solving processes in English for their classroom work and for their performance on tests can be a challenge. The question arises as to whether teachers should teach writing in the mathematics register—that is, whether teachers should make sure that students know how to produce complete, precise, grammatically correct explanations that follow the expected outline. For example, in the case of vocabulary, is there a role for preteaching vocabulary words such as "hypotenuse" in mathematics for ELs? Preteaching may well be a common practice in U.S. schools, yet the efficacy of this practice is unclear. Importantly, preteaching vocabulary should be presented in meaningful contexts for ELs, where the relationship of the vocabulary item to the task at hand is needed to solve a particular problem. In addition, research has established that multiple exposures to key mathematics vocabulary items is critical so that EL students can link their concept to their own representations in both their native language and their second language (Bedore & Peña, 2011).

Students in our studies expressed their solutions to problems using a variety of representations. We never required that they initially present their solutions making use of the fully formed mathematics register. Rather, we tried to model the appropriate use of language, introducing appropriate language when a concept had meaning and, as we interpreted student expressions, inviting students to express ideas so that others could also follow the solutions. The implications of our success with this approach to practice are numerous. Sharing our outcomes with practitioners, as well as researchers, created opportunities for changes in course content and for professional teacher development. As we monitored student progress in conceptual understanding, problem solving, and computation over the elementary years through the district's standardized testing, we found consistent improvement in the areas of conceptual understanding and problem solving and no loss in computation (Maher, 1991).

3.2.5 Theme 5: Our Ongoing Research Suggested That We Expand Our Research Focus

Researchers who have studied U.S. mathematics textbooks have determined that there are English features that are quite complex, including unique and precise vocabulary (such as hypotenuse), the way in which explanations are given (a discourse structure), and the complexity of the grammar, which can result in very long sentences (such as using complex clauses and nouns with many modifiers such as "the large, red, school house"). All of these features of English used in mathematics, taken together, are referred to as the mathematics register. The relationship among mathematics, language, and literacy has emerged as a research focus within the past two decades. We noted that each discipline, whether mathematics, history, geography, or biology, employs its own specialized language—a register—that is quite complex and differs from everyday conversation (Nagy & Townsend, 2012). Each register defines the unique way of cultivating reading, writing, speaking, and ways of reasoning that students must master if they are to be deemed proficient in the discipline and also if they are to be successful with the multiple ways (e.g., tests, texts) of demonstrating that proficiency. Disciplinary language registers can be both oral and written and can share lexical and grammatical features, including (1) increased conciseness in word selection to avoid redundancy, (2) a higher frequency of informational words as the means to achieve a more concise expression, and (3) grammatical constructions that embed complex ideas into fewer words.

The mathematics register refers to both oral and written language. That is, this way of using English includes speaking, listening, reading, and writing. For mathematics, we had to keep in mind that the discipline is constructed via a blending of natural language, technical language, mathematics symbolism, and visual displays, as noted above. Consequently, for mathematics learning, students must be able not only to articulate the natural language that can be highly technical, dense, and precise but also to make connections among the three semiotic systems.

Later on in our research program, we recognized that language development and mathematical learning are intertwined and interdependent. Were recognized that it would help us to have experts in language and literacy (both first and subsequent language) to work with us to share ideas and see what could be gleaned from our data, addressing our research foci beyond what we had already done. In our collaborative research, bringing together specialists in mathematics learning, psychology, and language learning, there were rich rewards. Now we could examine students' problem solving in mathematics through additional lenses (see Sigley & Wilkinson, 2015). Attention to how students used natural language as they explored open-ended mathematics problem solving produced new research on how students use the English language in approaching and solving open-ended mathematical problems.

3.2.6 Theme 6: Research on Teachers Attending to Student Learning

One significant outcome of the decades of our research from an expanding agenda is the shift in attention to teacher learning. Several doctoral dissertations produced detailed studies, backed with video data, of interventions that invited teachers to attend to student learning. Videos of students engaged in a variety of mathematical tasks and in multiple settings introduced teachers to student learning (see, for example, Cipriani, 2017; McGowan, 2016; Sigley, 2016; Van Ness, 2017). From these and other studies, published video narratives have emerged. Some video narratives are linked to publications that reveal the video data that were analyzed and presented in the analyses (see, for example, Sigley & Wilkinson, 2013; Uptegrove, 2015; Van Ness, 2015).

3.3 Concluding Remarks

Sharing the results and the process of one's research program is a primary professional commitment for mathematics education researchers. We encourage beginning researchers to think broadly and creatively about the opportunities to share and to bring new expertise to their work by expanding their community of researchers. Opportunities, interests, resources, and a desire to learn and share knowledge define the journeys we take during our careers. There are no templates to follow; intuition and judgment, as well as a passion for the work, are critical. Searching for other colleagues whose interests and work align can lead to collaborations that enhance the efforts that each, individually, may make.

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Chapter 4
Developing and Enacting a Research Program in the Context of Your Own Classroom

Eva Thanheiser

Abstract In this chapter I reflect on how I have designed and enacted a research program in the context of teaching mathematics content and methods courses for university prospective elementary teachers. In my work, my research and teaching inform each other. I discuss the cyclical nature of such a research program where each research project lays the foundation for the following ones. I also discuss how to build collaborative research programs with other researchers interested in similar research.

Keywords Scholarship of teaching and learning · Research program design · Research collaboration · Negotiating research and teaching · Research to inform teaching · Teaching to inform research

In my experience, I have found that sharing stories is a good way to learn and teach. "We use stories to make sense of our world and to share that understanding with others" (Rose, 2011). In this chapter I share the story of how I developed my first research study (my dissertation) and then how I developed a research program based on that study in the context of my own classroom. I am a mathematics (teacher) educator who has worked in a Graduate School of Education as well as in a math department. This story begins during my time in graduate school when I was getting ready to find a dissertation topic. During my time in graduate school, I had been working with faculty in the context of mathematics content courses for teachers. By sharing my story, I hope others will see how such a research program can be developed and implemented in their own classroom. Sharing stories "is a way of finding common ground and sharing experiences. It can feel very positive when someone has had a similar experience and we feel that they understand where we are coming from" (Making Waves, 2015). As I share my story, I integrate advice to others who

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may want to follow a similar path. At the end, I summarize and reflect on the advice I would give for developing and enacting a research program in the context of your own classroom.

4.1 Initiating a Research Program

4.1.1 Setting the Stage

I regularly give the prompt in Fig. 4.1 to prospective elementary teachers (PTs) in mathematics content courses. When I first began examining students' responses to this prompt, I realized that most people in the United States solved the problem as depicted in Fig. 4.2.

While the PTs were typically able to solve the problem (i.e., carry out the computation to produce the correct answer), many struggled in explaining *why* the algorithm works, specifically the regrouping of digits (Thanheiser, 2009, 2010, 2018a). When asked to explain, PTs would provide answers such as this one given by Claire:

> You have to borrow from the neighbor, which is 5. And you change the 5 to the number below it, which is 4. And you mark a 1, which makes this [2 in the ten's place] 12 … You put a 1 over next to the number and that gives you 10 … The small numbers mean that you borrowed … You are always going to take 1 and add 10 [to the next digit] … they [referring to a mysterious "they"] set rules so you'll be able to [regroup]. … I don't see a relationship [between the one she took and the 10 she added], because they are both two different numbers … I don't get how the 1 can become a 10. One and 10 are two different numbers. How can you subtract 1 from here and then add 10 over here? Where did the other nine come from?

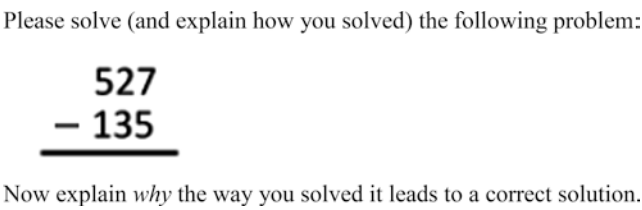
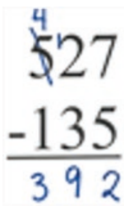


Fig. 4.1 Subtraction task

Fig. 4.2 Solution to subtraction prompt using standard algorithm for subtraction in the United States



When asked why regrouping works in the subtraction algorithm, Claire stated, “I have no idea of why it works. Just principles that you had learned throughout school.” Claire and many other PTs were not able to explain regrouping, and the reliance on “because that is how I learned it in school” is widespread. Few PTs were able to give an explanation like Rebeccah’s:

> Instead of it being like 5 hundreds and 2 tens and 7 ones, you get 4 hundreds, 12 tens, and 7 ones … taking a hundred and making it into groups of ten … 1 hundred, which is 10 groups of ten.

The fact that so many PTs were able to solve the problem in Fig. 4.1 but were not able to explain why it worked led me to the beginning of my research program, which I lay out in this chapter. All productive research programs begin with genuine questions. Once a question is identified, a plan must be made for data collection, data analysis, and forming conclusions. These conclusions may then give rise to the next research question. In the following sections, I examine each of these steps through describing how this pattern played out in my own experience.

4.1.2 Genuine Questions

When working with graduate students or junior faculty, my main piece of advice for selecting a research question is to pick something that is of genuine interest. This choice is essential as this question is what drives the work and keeps the researcher engaged. At the beginning of my career, my genuine question was: why do prospective elementary teachers struggle to explain the algorithms for addition and subtraction?

When first working with students in mathematics content courses for prospective elementary school teachers, I was not yet familiar with this population or with the depth of mathematical content knowledge required to teach elementary school. However, I was fascinated by the difficulties people had when asked to explain why the standard algorithms for addition, subtraction, multiplication, and division work. One of the most fascinating aspects was that almost everyone I talked to could solve the various addition and subtraction problems and get the correct answers, but hardly anyone could explain why the algorithms they applied resulted in those correct answers. I was intrigued by this issue and wanted to focus my dissertation research on how to support PTs in developing an understanding of why the algorithms work.

My research is typically grounded in the context of student learning. How are my students thinking? What do they know? How can they build on what they know to learn something they do not yet know? As I began thinking about activities designed to support the development of PTs’ understanding of why regrouping works, I realized that I did not have a solid understanding of the PTs’ currently held conceptions, nor did the research literature provide any answers to this question. It then became clear that the first step had to be examining my students’ incoming conceptions so I could understand where they are at and explicate what I wanted them to build toward. Thus, my first research interest was born!

4.1.3 Theoretical Framing

Now that my interest was identified, I needed to think about my theoretical framing. To allow me to communicate about my work, I needed to explicate how I thought about learning. I believe that students and PTs learn best when given opportunities to learn (Bransford, Brown, & Cocking, 1999; Cai et al., 2017; National Research Council, 2001). Thus, educators need to be aware of the PTs' current ways of reasoning so they can design tasks that build on those current ways of reasoning and help "students to develop their current ways of reasoning into more sophisticated ways of mathematical reasoning" (Gravemeijer, 2004, p. 106). This means that as a first step, we needed to identify the PTs' current ways of reasoning.

4.1.4 Research Questions

To form my genuine question into a research question, I had to develop a question that would add new information to the field, was answerable through data collection and analysis, and was realistic in scope (for my dissertation). To get an understanding of what such a research question might look like, I examined the research literature on what was already known about PTs' understanding, the understanding of adults in general, and children's understandings of number and algorithms. Through this literature review, I learned that we knew how children (who had not yet learned how to add/subtract/multiply/divide) think about numbers and how their thinking may develop (Fuson et al., 1997; Kamii, 1986, 1994; Kamii, Lewis, & Livingston, 1993). I also learned that PTs could apply algorithms confidently but were not able to explain them (Ball, 1988), which aligned with my own experience. However, my exploration showed that there was no literature (yet) on how PTs, who can apply algorithms confidently, think about number and why they struggled to explain the algorithms. This was a piece of information that was not yet available through the literature and that I (and the field of mathematics education) needed in order to design tasks for PTs to develop their content knowledge. My (dissertation) research question was born: what are PTs' conceptions of multidigit whole numbers?

And because I was particularly interested in PTs' conceptions in the context of the algorithms, I asked two subquestions:

- What are PTs' conceptions of multidigit whole numbers in the context of the standard algorithms for multidigit addition and subtraction?
- What are PTs' conceptions of multidigit whole numbers beyond the context of the standard algorithms for multidigit addition and subtraction?

4.1.5 Methods

Once the research question was established, I needed to figure out a way to find answers to that question. Because I was interested in figuring out how PTs were able to explain numbers and algorithms, I decided to conduct one-on-one interviews with PTs and pose a variety of problems—some in the context of algorithms and some outside the context of algorithms—to examine their explanations. I chose both problem types (inside and outside the context of the algorithms) to examine whether the PTs drew on different kinds of conceptions in different kinds of contexts. Building on what I learned from the literature, I developed interview tasks that I could give to PTs and then tested the tasks by asking PTs in my own classes to respond to them. I refined the tasks based on the PTs' responses. After a few cycles of piloting tasks, I refined and developed a collection of tasks to use with PTs in my study.

The data I collected had to match the research question and analysis plan. To answer my question, I chose to individually interview PTs to understand how they think. Since there was no available research in this area, I chose a grounded theoretical study design (Glaser & Strauss, 1967; Strauss & Corbin, 1990) and utilized the constant comparative method. Despite the fact that the grounded theoretical study implies the collection of data until there is no more variation, in order to assure feasibility, my advisor and I put a cap on the data collection. We decided that I would interview up to 15 PTs even if I had not yet stopped finding additional variation. Feasibility has to be part of a dissertation, and of any study for that matter!

My research design was such that I would interview each PT twice so that I could analyze the first interview and use this analysis to guide my follow-up questions in the second interview. This design was necessary as I wanted to make sure that I had a deep understanding of each PT's conceptions. These interviews were typically about 1–2 weeks apart. Once I finished an interview sequence for a participant, I would perform a thorough analysis and compare and contrast the results with those of the interviews conducted before. This comparing would often require me to reorganize the categories in prior analyses and reanalyze the prior interviews. This proved to be a slow process that spanned multiple semesters of simultaneous data collection and data analysis. An image of the data collection and analysis cycle can be seen in Fig. 4.3.

I began each interview by explaining to each PT that I would be asking them questions beyond what they were able to explain so that I could really understand where they were at, hoping to help them to feel comfortable in the eventuality that they were not able to answer some questions. I would then make sure to ask as many follow-up questions as needed to get a picture of the PT's thinking about a question. For example, in the task at the beginning of this chapter (see Fig. 4.1), I would ask about the regrouped digit, about its value, and for as in-depth explanation as possible for why the regrouping algorithm worked. All interviews were audio recorded and transcribed after the interview. The transcripts were then analyzed line by line to make sense of how the PTs were thinking. More about this process can be found

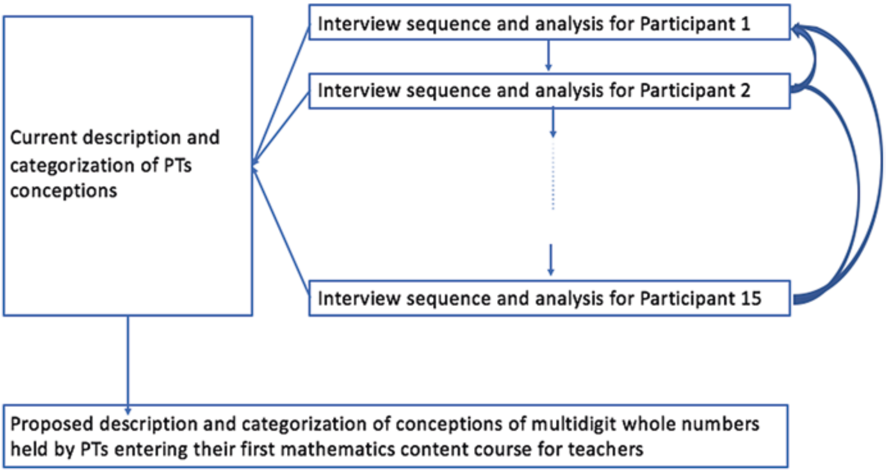


Fig. 4.3 Data collection and analysis for dissertation study

in Thanheiser (2005). As a result of the analysis from my interviews, I was able to develop a framework for PTs' conceptions at the start of the content courses (Thanheiser, 2005, 2009).

4.2 Developing a Research Program

A research program is a collection of related studies, building on and connected to one another. My research began with developing a framework for PTs' conceptions. Two research directions immediately followed:

1. Generalizing the findings from my dissertation work: my dissertation study was limited in that it only had 15 participants, all from the same university. To broaden the scope of this research, in the following years I interviewed most of my incoming PTs in the content and method courses I taught to explore whether the framework would generalize beyond the initial study.
2. Returning to my original plan to build tasks that PTs could engage with to develop their conceptions (now that I had a better understanding of what conceptions they came in with).

In terms of the first research direction, I have replicated the results of my dissertation many times over the years and found that they generalized (Thanheiser, 2010, 2018a). In addition, just recently my work was replicated by Jacobson and Simpson (2018). I have continued to interview my students at the beginning of class over the years. However, interviewing is a time-intensive process and is therefore not practical. Thus, my next study focused on developing and validating a survey to identify PTs' conceptions. More about this study can be found in Thanheiser (2010, 2018a). In the meantime, I also developed a shorter interview to identify PTs' conceptions. This data collection and data analysis developed my understanding of the students in my classroom, and thus I was able to better address their incoming conceptions. I address the second research direction in the next section.

4.2.1 Task Design

The framework I developed for PTs' conceptions of multidigit whole numbers (Thanheiser, 2009) helps mathematics teacher educators (MTEs) understand how PTs are thinking about numbers when entering their courses, thus informing the design of tasks to guide the PTs through an exploration and development of their conceptions. I began to develop tasks designed with the PTs' incoming conceptions in mind and examined the PTs engaging with the tasks. For the next few years, I focused on understanding the development of PTs' conceptions. I examined tasks in teaching experiment settings, as well as in whole-class settings (Thanheiser, 2014; Thanheiser & Melhuish, 2019), continually refining them to better address the PTs' conceptions. I also explored task design in general and started to work with others on understanding the elements of task design (Thanheiser, 2018b; Tobias et al., 2014). This led me to examine content areas beyond whole numbers—for example, fractions (Thanheiser et al., 2016).

Up to this point, my research program had been fairly predictable and linear: start by examining what PTs know, then build on that to examine how they learn. While working along this research trajectory, I kept listening to what my students had to say with the goal of answering my research questions, but I also kept an open ear and mind as to what else I might learn from my students. This focus allowed me to hear things from my students, which changed my research trajectory.

4.2.2 Collaboration with Students/Participants in the Study

I collaborate with my students to understand their thinking. This is a stance I make explicit at the beginning of each research project I take on and each class I teach. Students are part of the research and often find it enjoyable to collaborate in order to make the teaching of such courses better in the future. In my first study, I explained that to develop activities to support their learning, I needed to understand what they knew and did not yet know so I could build on their prior knowledge and conceptions. To accomplish this, I would ask them a lot of questions to understand exactly what they were able to explain and what they were unable to explain. Typically, when I teach, I ask my students to allow me to use anything they did in the context of the course for my research. I regularly have IRB approval ahead of time for my classes, and PTs are able to opt in or out of participating in the research. To protect the PTs, I do not know who opted in or out until I have submitted grades for the class. The majority of students regularly give consent.

4.2.3 Listening to Your Students and to Your Data

One of the best pieces of advice I can give is to listen to your students/participants and to your data. After my dissertation work, I continued to interview PTs at the beginning (and sometimes at the end) of the content courses to learn more about their conceptions and to study the development of their conceptions. In this process, the PTs began to reflect on how the interview affected their learning in the class because it showed them that they did not yet know the things they needed to know. For example, they would say things like the following:

> I think without the interview, I might have gone into class a bit cockier (if possible) and possibly have taken the class less seriously in terms of the amount of work I would have to put forth. As it was, I knew from the relative start I had work to do.

I listened to this reflection (and others like it) and turned it into a new direction for my research, namely, examining how helping PTs understand what they do not yet know motivates them to engage in the content courses (Thanheiser, Philipp, & Fasteen, 2012). I have found benefits in showing the PTs their initial interview at the end of the course, which has led to the PTs recognizing and valuing their own learning in the course. This work is essential because PTs who are not motivated to learn might not take advantage of their university content courses, and research has shown that motivation and engagement lead to increased learning.

In both my research and teaching, I often ask the PTs to reflect on their experiences, and I honor their reflections by reporting back to them what they have said. This reciprocation builds trust, which is essential in both research and teaching. Transparency with the PTs also serves as a model for how their teaching practice could work in the future. They like seeing their instructors listen and adapt their teaching to them, which encourages them to do the same in the future.

4.2.4 Motivation to Engage

As just mentioned, listening to my students led me to study motivation and engagement in mathematics content courses. I examined experiences designed to motivate PTs to learn by piquing their interest (for example, by individually interviewing each PT one on one to help them realize that there is something for them to learn in the course) and by maintaining the PTs' motivation throughout the course (for example, by using authentic tasks such as designing and enacting a family-math-night experience at a local elementary school to connect the university classroom to the K–12 classroom). I have also studied interventions designed to help PTs share their rough draft thinking and actively engage in class discussions (Thanheiser & Jansen, 2016) and connect to what the PTs cared about most (children) via children's mathematical thinking. I incorporate at least one of the following experiences into each of my content courses: (a) bringing artifacts of children's mathematical thinking into the classroom (Thanheiser, Strand, & Mills, 2011), (b) taking my PTs to local schools to work with children, (c) planning and enacting a Family Math Night at a local elementary school (Thanheiser et al., 2012), (d) bringing children into the mathematics content courses to work with PTs, and (e) having PTs read papers published by teachers for teachers (Strand & Thanheiser, 2017). Through these experiences, the PTs gain a more thorough understanding of their content knowledge and the content knowledge of children.

4.2.5 Task Design in Social and Political Contexts

Listening to my students in a larger sense has had me reconsider what the goal of mathematics education is and how to respond to the constant question: *when will we ever need this?* Trying to help my PTs understand and respond to this question resulted in another related research strand in my research program. Instead of trying to find contexts to which we could apply the math we learned in class, I was now thinking about contexts through which we could learn the mathematics. I began to look for meaningful contexts for the PTs.

First, I expanded listening to/noticing children's mathematical thinking to include listening to/noticing culturally based thinking. For example, children may bring in algorithms they learned from their parents that may not match the teacher's algorithms. For teachers to be able to understand those alternative solutions and to connect them to their own methods requires understanding why the algorithms work. Take, for example, the algorithms for addition and subtraction typically used in the United States and in Germany (see Fig. 4.4).

While the addition algorithms differs only in surface features (where we write the regrouped digits) the subtraction algorithm differs in structure. The German

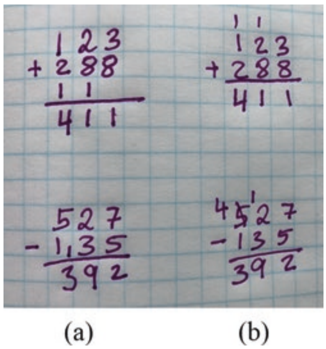


Fig. 4.4 Standard algorithms for addition and subtraction in (**a**) Germany and (**b**) the United States

subtraction algorithm utilizes a comparison model of subtraction (finding the difference between the two numbers), while the US algorithm utilizes a take-away model of subtraction. Both the US and the German subtraction algorithms work within the columns (powers of 10). The US algorithm regroups the minuend so that all the groups (digits) in the columns of the minuend are larger than all the groups (digits) in the columns of the subtrahend. This allows for take-away subtraction within the columns. The German algorithm counts up from the digits (columns) in the subtrahend to the digits (columns) in the minuend (to find the difference). If the digit in the minuend is smaller than the digit in the subtrahend, then 10 is added to the digit in the minuend. The 10, however, is not noted down anywhere in the algorithm. If a 10 was mentally added to the digit in the minuend in the column on the right, then a 1 is noted down by the next digit on the left in the subtrahend. The count in this column now starts with the digit in the subtrahend plus 1.

Preparing oneself to teach in a classroom where children may enter with various algorithms requires understanding the mathematics well enough to understand the newly added algorithm, verify validity or identify where it does not work, and then connect it to other algorithms. Openly welcoming and including a variety of methods that are different from the standard method and including them into the classroom enhances everyone's learning and includes (rather than positioning as "other") students with varied cultural backgrounds and experiences.

To promote inclusive behaviors, I work with my PTs to learn how to make sense of different ways of thinking and intentionally include those ideas. Thus, rather than waiting to see whether alternative ways of thinking show up, we work on actively seeking them out. For example, I invite students to interview friends, parents, or other relatives to find someone who does math differently and then examine that way of doing math. This activity encourages PTs to work on including all student ideas rather than dismiss thinking that differs from their own (Thanheiser & Philipp, 2017).

Second, I began to include social and political contexts into my classes. I did this to various degrees, ranging from simply adding more diverse contexts to typical problems (for example, in a shopping scenario, a child is out with her two mothers) to exploring themes such as income distribution throughout the entirety of a content course. I am currently in the midst of developing tasks, and collecting and analyzing data from this new-ish to me research direction. However, preliminary analysis suggests that most students change or perceive to change their perspective of mathematics, as illustrated by these PT reflections:

- My perspective of math has changed because I now view teaching math as an opportunity to teach real world contexts and to encourage students to stretch their own thinking of strategies.
- The main way my perspective of math has changed is that it can be SO much more than just directions and formulas. It can incorporate real world issues and engage the students on multiple levels.
- I used to think math was a bunch of numbers put together for adding, subtracting, multiplying, and dividing in specific contexts or computations. Now, I think

math is a much more complex process and can be used in meaningful and enriching ways to not only teach critical thinking skills in many contexts or computations, but to bridge gaps and break social barriers such as race and gender.
- I used to think math was fairly politically neutral. Now I think math is never neutral. The teacher must pay close attention to the implications of the math scenarios they place before their students. It takes conscious effort to show every student that math includes them.

4.2.6 Collaborating with Colleagues

Once I started considering how PTs understand whole numbers, I began looking for other MTEs who were interested in PTs' content knowledge. By presenting my work at conferences and reaching out to others who studied this population (via email, attending talks at conferences, contacting people who attended my talks at conferences, etc.) I began talking with people who shared this common interest. I then decided to propose a discussion/working group at the annual conference of PME-NA (the North American chapter of the international group for the Psychology of Mathematics Education) to bring people together who study PTs' content knowledge. This discussion/working group (Thanheiser et al., 2009, 2010) formed the basis for several collaborations, which I describe next.

At the time, there was no comprehensive document in existing literature illustrating PTs' content knowledge, so we tackled that first. We formed subgroups, and each subgroup focused on a specific content area of their choice (whole number, fractions, decimals, and geometry). This work eventually led to a special issue of *The Mathematics Enthusiast* focused on PTs' content knowledge (Thanheiser & Browning, 2014). These articles served as a basis on which to build all of our subsequent research.

In addition, various smaller research groups formed through this working group. These collaborations also eventually led to an ongoing research group on task design among six MTEs at various institutions. This research group proved to be a powerful support system of colleagues who were all teaching similar kinds of courses (content courses for elementary teachers) and were all interested in conducting research in those courses. In our regular Skype meetings, we would discuss anything from teaching issues to designing tasks, collecting data, and jointly analyzing tasks. This work led to several joint publications (Feldman et al., 2014, 2016; Olanoff et al., 2014, 2016; Thanheiser et al., 2013, 2016; Tobias et al., 2014; Welder et al., 2015).

I also began attending conferences specifically designed for my area of interest, such as the ICMI study 23, *Primary Mathematics Study on Whole Number*, in 2015. Attending such conferences allows one to meet people who have joint interests and can often lead to collaborations and publications (e.g., Sun et al., 2018). Attending such conferences can also lead to opportunities to colead such groups at future conferences.

4.2.7 Bridging Research and Teaching

My research and teaching are inextricably intertwined. I research ways to motivate prospective and in-service teachers to learn, as well as to help them develop their conceptions and to help them shift their perspective of mathematics. Then I apply what I am learning in my research to all courses I teach (undergraduate, graduate, and PhD).

Bridging research and teaching responsibilities by conducting research in the context of your own teaching allows you to build up a research program while also continually improving your teaching. It is, in a sense, "the best of both worlds." Collaborating with others who do the same is also deeply rewarding as you can see how your tasks play out in other classrooms. For example, I have struggled a lot (and still am struggling) to help my PTs develop conceptual explanations for the area model of multiplication. When sharing this struggle with several colleagues, they were initially unsure what I was talking about. After trying some of the tasks with her own PTs, a colleague emailed me and said that she had tried the task and found the same thing, stating that she "learned that Eva is not crazy." This email was validating as I realized that others have similar struggles (it is not just me), and these shared struggles can open the door for a collaboration. (After this instance, we began to work on designing tasks that we would all use to work with PTs. This project is currently ongoing.)

4.3 Summary Advice

When asked to write a chapter for this book, the purpose of the book was described in terms of writing chapters that could mentor a graduate student or assistant professor. I selected to discuss *Developing and Enacting a Research Program in the Context of Your Own Classroom* as that is what I felt I could give advice on. Reflecting on the story I have shared, I realized that my main pieces of advice can essentially be boiled down to advice about conducting a research study, developing a research program, and fostering collaborations.

4.3.1 Conducting a Research Study

- Pick something of genuine interest to study. This choice will make you a better scholar because you will be genuinely interested in your work. It will also make your life easier because you enjoy doing your work.
- Be explicit about your underlying theoretical perspectives. In my case, it is important to know how I think about learning and teaching. Only with an explication of my theoretical perspectives does my research program make sense. These perspectives guide both data collection and analysis in any research study.

- Pick something that is doable. (This does not mean it cannot also be ambitious.)
- Match the data collection methods to your research question and data analysis plan.
- Collaborate with your research participants. You will learn from them, and they will learn from you. Mutual respect and genuine interest go a long way.

4.3.2 Developing a Research Program

- One of the best pieces of advice I can give is to listen to your students/participants and your data as you are conducting research. Carefully listening may open up research avenues that you might not have anticipated.

4.3.3 Fostering Collaborations

- Read papers and attend talks of people who are doing work that relates to yours. Talk to them. If you read a paper, send an email to the author. (Everyone loves to know that their papers are being read.) If you attend a talk, follow up in person or via email.
- Be bold! Ask people who you would like to work with whether they may be interested in collaborating. Be prepared to hear both yes and no answers.
- Start a research group, discussion group, or other ways to bring people together around a topic of interest.

4.4 Conclusion

I close this chapter by looking forward. I am currently building on my prior work in order to redesign my content courses to have an underlying social or political theme. I then want to study the effect of this theme on PTs' development of both content knowledge and knowledge of social and political issues. For example, my fraction and statistics course is now being taught in the context of income distribution, which allows PTs to develop an understanding of income distribution in the United States while building a mathematical understanding of fractions and statistics. In addition, I am working on conceptualizing professional knowledge for teaching that includes an aspect about social and political issues. For example, what does a teacher need to know to be able to teach fraction comparison in the context of income distribution? If you enjoyed reading this chapter and would like to engage in further discussion, I would love to hear from you!

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Chapter 5
Research and Curricula

Julie Sarama and Douglas H. Clements

Abstract Connecting curriculum development and research benefits both. Those designing curricula should ensure that their work is scientifically based and evaluated. Those studying existing curricula should understand the ways in which they were developed and validated (or not) and that a comprehensive evaluation program involves more than final outcomes. We use a curriculum research framework to draw implications for research in both development and evaluation projects. For each phase of the framework, we discuss how publishable research and curriculum development (R&D) might occur, as well as what opportunities there may be for evaluation research alone. In all cases, we briefly suggest methods.

Keywords Cognition · Curriculum · Design science · Evaluation · Learning trajectories · Mathematics · Professional development · Research · Scale-up

The development of curriculum materials and educational research are often seen as unconnected activities, where those writing a curriculum have distinct goals and abilities from those conducting research (who also must be separate to be "objective"). Indeed, one reviewer of a description of our own research-and-design model criticized it on the basis that the two were separate and must stay so. Our position is that such isolation harms both (cf. Clements, 2007; Clements & Battista, 2000; Lagemann, 1997; Lloyd, Cai, & Tarr, 2017; Sarama & Clements, 2008). The two are often viewed as separate in that research creates scientific knowledge and curriculum development creates (only) instructional materials. However, the lack of connections between them blocks progress in both (Battista & Clements, 2000; Clements, 2002; Doabler et al., 2014; Lloyd et al., 2017; Schoenfeld, 2016). If you wish to design any type of curriculum or part of a curriculum (an intervention, instructional unit, or simply a short sequence of activities), you should ensure that

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they are research based *and* scientifically[1] evaluated. On the other hand, if you wish to study nascent or existing curricula, interventions, or approaches to teaching, you should understand thoroughly the ways in which they were developed and validated (or not) and understand that studying final outcomes is important but only one research path.

To help guide you through these complex issues and methods, we walk through the categories and phases of our comprehensive framework for curriculum development and research (Clements, 2007; Clements & Sarama, 2013). For each phase, we discuss how publishable research and curriculum development (R&D) might occur, as well as what opportunities there may be for evaluation research alone. In all cases, we will briefly suggest methods. Although a comprehensive R&D program would involve all phases (eventually!), they need not, and usually cannot, be employed in a single project.

5.1 Research and Curricula: A Framework

Most developers and publishers claim that their curricula are *based* on research, although few document these claims. Further, little distinction is made between that which is research based and that which is research *validated.* Thus, there are myriad opportunities for both R&D and research efforts focusing solely on evaluation at all phases of curriculum development. The Curriculum Research Framework (CRF) (Clements, 2007; Clements & Sarama, 2013) we use to organize our discussion includes three broad categories of research and development work, within which there are ten phases. The three categories involve (1) reviewing existing research (a priori foundations), (2) building models of children's thinking and learning in a domain (learning trajectories), and (3) appraising the effectiveness and general worth of the result (evaluation, both formative, leading to revisions, and summative, to determine the effects of the completed curriculum). The categories and phases within them are outlined in Table 5.1. The categories are described in the leftmost column. The questions addressed are provided in the middle column, and the specific methodologies to address these questions within each phase are described in the rightmost column.

One guideline is critical for all categories: equity must be considered from the inception (Aguirre et al., 2017; Confrey & Lachance, 2000). For example, research reviews covering all populations must be considered. Also, considerable thought must be given to the students who are envisioned as end users and those who participate at every phase of R&D; a convenience sample is inappropriate.

[1] To us, the sine qua non of evaluation, but not the only approach; others include aesthetic (Eisner, 1998), narrative (Bruner, 1986), historical (Balfanz, 1999; Kilpatrick, 1992) and other perspectives.

Table 5.1 Categories and phases of the *Curriculum Research Framework* (adapted from Clements, 2007)

Categories	Questions asked	Phases
Research reviews: a priori foundations—in variants of the research-to-practice model, extant research is reviewed and implications for the nascent curriculum development effort drawn	What is already known that can be applied to the anticipated curriculum?	Established review procedures and content analyses are employed to gather knowledge concerning the specific subject matter content, including the role it would play in students' learning (phase 1); general issues concerning psychology, education, and systemic change (phase 2); and pedagogy, including the effectiveness of certain types of activities (phase 3).
Learning trajectories—activities are structured in accordance with empirically based models of children's thinking and learning in the targeted subject matter domain	How might the curriculum be constructed to be consistent with models of students' thinking and learning?	In phase 4, the nature and content of activities is based on models of children's mathematical thinking and learning. Specific learning trajectories are built for each major topic.
Evaluation: formative and summative—in these phases, empirical evidence is collected to evaluate the curriculum, realized in some form. The goal is to evaluate the appeal, usability, and effectiveness of an instantiation of the curriculum	How can market share for the curriculum be maximized?	Phase 5 focuses on marketability, using strategies such as gathering information about mandated educational objectives and surveys of consumers.
	Is the curriculum usable by, and effective with, various student groups and teachers?	Formative phases 6–8 seek to understand the meaning that students and teachers give to the curriculum objects and activities in progressively expanding social contexts so as to improve the curriculum, for example, the usability and effectiveness of specific components and characteristics of the curriculum as implemented by a teacher who is familiar with the materials with individuals or small groups (phase 6) and whole classes (phase 7) and, later, by a diverse group of teachers (phase 8). The curriculum is altered based on empirical results, with the focus expanding to include aspects of support for teachers.
	What is the effectiveness (e.g., in affecting teaching practices and ultimately student learning) of the curriculum, now in its complete form, as it is implemented in realistic contexts?	Summative phases 9 and 10 are intended to assess if the goals of the curriculum have been met. Both use randomized field trials and differ from each other most markedly on the characteristic of scale. They both examine the fidelity or enactment, and sustainability, of the curriculum when implemented on a small (phase 9) or large (phase 10) scale, with phase 10 also investigating the critical contextual and implementation variables that influence its effectiveness. Experimental or carefully planned quasi-experimental designs, incorporating observational measures and surveys, are useful for generating political and public support, as well as for their research advantages. In addition, qualitative approaches continue to be useful for dealing with the complexity and indeterminateness of educational activity.

5.2 Reviewing Research [A Priori Foundations]

This category aligns most closely with the notion of research-*based* curricula. The phases in this category involve reviewing existing research and drawing implications for one's nascent curriculum development effort or checking if curricula being evaluated are consistent with empirical evidence. The questions concern the specific subject matter content, including the role it would play in students' learning (phase 1), cognitive and developmental psychology and education in general (phase 2), and pedagogy, including the effectiveness of certain types of activities (phase 3). For all, one criterion for a successful body of literature must be the inclusion of different populations, such as English learners, different ethnic groups, those with IEPs, and other underrepresented populations.

5.2.1 Phase 1: Subject Foundations

In phase 1, research is used to identify mathematics that is developmentally appropriate and interesting to students in the target population. The ultimate goal is to identify those domains that would make a substantive contribution to students' mathematical development. That is, the domain should play a central role in mathematics per se, and the concepts and procedures of the domain should be generative in students' development of future mathematical understanding. Studies that report what skills predict later mathematical achievement (Nguyen et al., 2016; Rittle-Johnson, Fyfe, & Zippert, 2018; Verdine, Golinkoff, Hirsh-Pasek, & Newcombe, 2017) can be consulted, but with the understanding that only using correlational approaches is "risky business" (Bailey, Duncan, Watts, Clements, & Sarama, 2018). As a simple example, researchers have assessed children only on number tasks and then correlated the results with these children's achievement in number and operation (arithmetic) tasks years later. Such studies have yielded useful information but should not be interpreted as a comprehensive evaluation of the most important topics to teach because the possible contributions of other topics, such as patterning or spatial and geometric thinking, were never measured.

5.2.2 Phase 2: Cognitive Foundations

Phase 2 similarly reviews theories and studies on students' thinking and learning to form a broad theoretical foundation for the curriculum. For example, we created our own theory, Hierarchical Interactionalism, from a synthesis of empiricism, (neo)nativism, and interactionalism theories (Sarama & Clements, 2009) on which to ground our curriculum R&D projects. There has been a burgeoning of research on students' understandings and learning of mathematics (e.g., Cai, 2017), learning opportunities (e.g., Cai et al., 2017; Civil, 2016), and curriculum development and research (e.g., Donegan-Ritter & Zan, 2017; Lloyd et al., 2017; Stein, Remillard, & Smith, 2007) that you should consult as you plan any R&D or evaluation effort.

5.2.3 Phase 3: Pedagogical Foundations

In phase 3, empirical findings on the creation of specific types of instructional activities that are both educationally effective and motivating are reviewed as general guidelines for the creation of instructional tasks and pedagogical strategies (e.g., Franke, Kazemi, & Battey, 2007; Frye et al., 2013; Sakakibara, 2014; Sztajn, Confrey, Wilson, & Edgington, 2012), including their relationships to assessments (Penuel & Shepard, 2016). This phase is general, so it speaks to an overarching structure for teaching, rather than, say, topic-specific teaching. As just one example, because we planned to include educational technology in our curriculum, we reviewed literature on the types and specific characteristics of educational software that are effective in addressing different educational goals, including specific characteristics of that software (e.g., Clements & Sarama, 2016; Foster, Anthony, Clements, Sarama, & Williams, 2018; Kulik & Fletcher, 2016; Moyer-Packenham et al., 2015; Sarama, Clements, & Vukelic, 1996).

As previously stated, equity must be considered in every phase. Here, the funds of knowledge from different cultures should be considered (Civil, 2002; Moll, Amanti, Neff, & Gonzalez, 1992; Presmeg, 2007). For example, if a goal is developing students' spatial abilities and spatial structuring (as used in area and volume measurement), one might introduce the topic through art and design (e.g., tessellations), as well as puzzles (e.g., tangrams), activities that appear almost universally among cultures (Danesi, 2009). Such use of spatial activities also has been shown to be especially helpful for engaging (reengaging) underrepresented populations of students in mathematics, tapping into their spatial funds of knowledge, thereby increasing their mathematical self-efficacy (Casey, Dearing, Vasilyeva, Ganley, & Tine, 2011; Cheng & Mix, 2012).

5.2.4 Designing, Conducting, and Publishing Research Reviews

Research syntheses, when they exist, are good sources for this category, and when they do not yet exist, the topic or issue may present an opportunity for an additional research publication for you to tackle. Next, let us consider the opportunities separately for those who wish to pursue R&D and those who plan to evaluate existing curricula.

5.2.4.1 Research and Development

For developers, goals may emerge from standards such as the CCSSM (NGA/CCSSO, 2010) or from research. As stated, they should be central mathematically and generative of students' learning (for a discussion and examples, see Clements, Sarama, & DiBiase, 2004). Consider also the scientific research strategies that constitute comprehensive content analyses (National Research Council, 2004).

For cognitive and pedagogical foundations, we assume that no theory, research corpus, or method can definitively answer the question "What is best for students?" for numerous reasons, from practical limitations on branching at each stage to recognition that such questions are inherently grounded in goals and values (Clements, 2007; Hiebert, 1999). Further, we believe that theoretical "purity," especially positions that reject theoretical or empirical work done within different paradigms, can be satisfying but is a conceit that curriculum developer-researchers can ill afford. Instead, they must integrate the issues and findings that researchers and teachers from other philosophical positions experience and report. Different theories and research corpuses explain some things well and others less well (Schoenfeld, 2002).

Finally, a fecund and rarely exploited research design includes a separate researcher who is responsible, in this and the other two categories, for taking a perspective of "standing outside," the R&D process, observing and documenting the curriculum development and research team's activities, decisions, and reasons for decisions (Lesh & Kelly, 2000). You could make an invaluable contribution to any existing curriculum development effort by serving in this capacity.

5.2.4.2 Evaluation of Existing Curricula

Existing curricula can be analyzed to either ascertain if the content is consistent with the stated research foundation, or, if none is stated, the foundation(s) could be abstracted from the curriculum's structure and content and critiqued on that basis (see examples and discussions of methods in Lloyd et al., 2017; Stein et al., 2007). The same procedures for content analyses can be used (National Research Council, 2004), as well as other procedures to evaluate curricular coherence (e.g., Schmidt & Houang, 2012).

We argued that curriculum developers should not a priori reject research emerging from theoretical approaches they do not like, and this issue emerges when composing research as well. Of course, a theoretical grounding is sine qua non for most research publications. Therefore, one must adopt, adapt, or create a synthesized theoretical framework such as our Hierarchic Interactionalism (Sarama & Clements, 2009); choose a theoretical lens most valid for the research questions; or critique the materials from the theoretical stance they take (or appear to take). See the excellent topology and discussion of theories in Ernest (1995) and the similar work of others (Cobb, 2007; Silver & Herbst, 2007).

One can also critique approaches to instruction of existing curricula. For example, see how Baroody (1987) critically analyzed often popular approaches/tasks/strategies in terms of the existing empirical evidence, such as contrasting and criticizing both the behavioristic and Piagetian approaches to early instruction (Baroody, 1987; see also Lloyd et al., 2017; Streefland, 1991).

5.3 Learning Trajectories (LTs)

In this major category with a single phase (4), activities are structured and sequenced in accordance with models of children's thinking and learning in the targeted subject matter domain(s). The goal is to construct a curriculum to be consistent with students' thinking and learning. The assumption is that this learning has characteristics and developmental courses shared by most students and that knowledge of these can aid in the designs of effective and engaging curricula.

These LTs ultimately include "the learning goal, the learning activities, and the thinking and learning in which the students might engage" (Simon, 1995, p. 133). Building on that seminal construct,

> we conceptualize learning trajectories as descriptions of children's thinking and learning in a specific mathematical domain, and a related, conjectured route through a set of instructional tasks designed to engender those mental processes or actions hypothesized to move children through a developmental progression of levels of thinking, created with the intent of supporting children's achievement of specific goals in that mathematical domain. (Clements & Sarama, 2004, p. 83)

The importance of the goal to our definition of LTs justifies our emphasis on content in the first category (Subject Foundations). Before we turn next to the other two components of an LT, the development progression and the instruction, here's a brief side note: terms other than LT, such as "learning progressions," are sometimes used ambiguously, often indicating developmental progressions and at other times suggesting a sequence of instructional activities. Although studying either psychological developmental progressions or instructional sequences separately can be valid research goals, and studies of each can and should inform mathematics education, we believe that the power and uniqueness of the LTs construct stems from the inextricable interconnection between these two aspects (Clements & Sarama, 2014b).

5.3.1 Designing, Conducting, and Publishing on LTs

5.3.1.1 Research and Development

For developers, procedures for this category differ depending on the maturity of the research in the domains or topics considered. Begin with a literature search to determine if there are developmental progressions that you can use or build upon (e.g., Clements & Sarama, 2014a; Fuson, 1992; Maloney, Confrey, & Nguyen, 2014; National Research Council, 2009; Sarama & Clements, 2009). If not, a research review may suggest a developmental progression (at least for a given age range of students in a particular culture) and a synthesis of this literature is itself a contribution worthy of publication. Such syntheses can be challenging; in our work, we usually had to integrate separate studies using different tasks with different populations. We began by aligning their results using students' ages as a guide, yielding a rough nascent draft of the progression. Then a series of cross-sectional clinical interviews using tasks designed to elicit pertinent concepts and processes helped us examine students' knowledge of the content domain, including conceptions, strategies, intuitive ideas, and informal strategies used to solve problems. From these we hypothesized mental objects (e.g., concepts) and actions (processes) that define each level of thinking, specification of which allows a degree of precision not achieved by previous theoretical and empirical efforts. Recall that all research, reviewed and conducted, should involve students from diverse communities and should, consistent with our theory and LT approach, be *asset-based* (Celedòn-Pattichis et al., 2018).

These efforts lead to a second draft of the developmental progression. This revised draft is tested and extended with teaching experiments, which again present limited tasks and adult interaction to individual children so as to build models of children's thinking and learning for each level (Steffe, Thompson, & Glaserfeld, 2000; for examples, see Barrett et al., 2011; Confrey & Lachance, 2000; MacDonald, 2015; Van Dooren, De Bock, Hessels, Janssens, & Verschaffel, 2004). The tasks and interaction are limited to emphasize the natural developmental progression but do represent an initial foray into the LT's instruction component.

A related approach is the design experiment, which might involve a single classroom with the teacher collaborating with a researcher and graduate assistants (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003). If the study involves a researcher-developer, two teams (one involved in the development, one not) may be beneficial to examine all aspects of the LT and its theoretical assumptions critically. Like teaching experiments, design experiments include conceptual and relational, or semantic, analysis; are theoretically grounded; and allow researchers to build models of the child's mathematics, of mental actions on objects, of learning, and of teaching interactions. Thus, design experiments are an important component of the CRF. However, we argue that other research and development strategies are necessary to meet the goals of a complete curriculum research and development program. Design experiments cannot control the many variables in their complex settings. The large amount of data collected rarely can be analyzed fully before the next cycle of revision, enactment, and analysis takes place. In addition, different participants may have different data and perspectives so that the ultimate paths and products may be arbitrary to an extent and generalization may be difficult. This is why we include design experiments as an important component, but only one component, of a comprehensive curriculum research framework (Clements, 2008).

Once several iterations of such work indicate substantive stability, it is accepted as a working model. Thus, the developmental progressions' levels of thinking and explication of transitions between models describe in detail the following: (a) what students are able to do, (b) what they are not yet able to do but should be able to learn, and (c) why—that is, how they think at each level and how they learned these level of thinking. This sets LTs' developmental progressions apart from earlier efforts to develop educational sequences, which, for example, often used reductionist techniques to break a goal into subskills, based on an adult's perspective. Research that develops, refines, or validates developmental progressions is a burgeoning area and needs increasing attention from new researchers.

This is our approach to developing LTs, but it is not the only one to consider. For example, LTs have been based on historical development of mathematics and observations of children's informal solution strategies (Gravemeijer, 1994) or emergent mathematical practices of student groups (Cobb & McClain, 2002). The literature provides guides to other fruitful approaches for determining developmental progressions. Some start as much with instruction as they do with student thinking and learning (Confrey, Maloney, Nguyen, & Rupp, 2014; Confrey, Rupp, Maloney, & Nguyen, 2012; Maloney et al., 2014; Simon et al., 2010). We believe in our comprehensive approach grounded in the cognitive sciences (including issues of motivation, culture, and engagement) but also acknowledge that other strategies can be valid and useful complements or replacements to our approach.

In our approach, the developmental progression establishes the initial basis for instruction, the third and final component of an LT. Instruction includes key tasks and pedagogical strategies designed to promote learning of a particular level of thinking. As to developmental progressions, there are well-researched instructional sequences for some topics and grade levels in the literature that provide valuable resources (e.g., Baroody, 2016; Clarke et al., 2002; Clements & Sarama, 2014a; Maloney et al., 2014; Murata, 2004; Sztajn et al., 2012). For all topics, especially those not previously researched, we suggest broad literature reviews, seeking both research evidence and the wisdom and creativity of expert practice, to identify activities and interactions shown as effective in promoting the learning of students to achieve each level, by encouraging children to construct the concepts and processes that define it.

Our next step is to adapt the selected tasks as needed so that they include external objects and actions that mirror the hypothesized mental actions on objects of the target level as closely as possible. For example, objects may be shapes or connecting cubes, and actions might be creating, copying, uniting, disembedding, and hiding both individual units and composite units. Tasks require children to apply, externally and mentally, the actions and objects of the goal level of thinking. These tasks are, of course, sequenced corresponding to the developmental progressions to complete the hypothesized LT (Simon, 1995).

We do not claim that this approach determines the "best" activities. Activities and teaching strategies are always under revision (cf. Cobb, 2001). However, we do claim that they illustrate and embody the *type* of activity hypothesized to be appropriate and efficacious for students at a given level of the development progression. And, again, when these are not available, there are other resources for developing instruction that are alternatives, or complementary, to our own (e.g., Gravemeijer, 1994). Collaborators with the Netherlands developers (McClain, Cobb, Gravemeijer, & Estes, 1999) Cobb and his colleagues have similar philosophical and curriculum development perspectives (Cobb & McClain, 2002) in which instructional design serves as a primary setting for the development of theory (Cobb, 2001). Another approach is what Japanese educators call research lessons or lesson study. In this approach, a group of teachers work together, often with researchers, to design a single lesson to have specific characteristics. Then one teacher implements it with their students while being observed by the other teachers and outside educators. The entire group then analyzes, discusses, and revises the lesson. Finally, after cycles of such enactment and refinement, they ultimately share the lesson widely (Lewis & Tsuchida, 1998; Presmeg & Barrett, 2003).

Please publish! This is R&D, not just development. Plan for specific research results on your LT but also recognize that in well-conduced R&D efforts, results are also applicable to knowledge development in larger domains. For example, retrospective analysis of an experiment contributes to the development of instructional theory. Emerging from analyses of the several cycles of teaching and learning, such results and theoretical contributions can explain the relationships between the two, thereby generating grounded generalizations that are sorely lacking throughout the research and theory literature. Other methodologies can also contribute, as we discuss in the following section.

5.3.1.2 Evaluation of Existing Curricula and LTs

Existing curricula can be analyzed to either ascertain if they follow research-based developmental progressions or the instructional strategies connected to each level of thinking. Also, existing LTs, either embedded in curricula or available separately, can be evaluated in the same manner—the developmental progression and/or the connection to research-based instruction. As an example, Baroody analyzed several instructional strategies as inconsistent with empirical evidence, such as criticizing the conventional wisdom about the causes and cures for numeral reading and writing difficulties (Baroody & Coslick, 1998; see Baroody & Purpura, 2017, pp. 335–342, for a summary).

Of course, a direct approach to this work is to conduct empirical studies of LTs, such as with teaching and design experiments as described in the previous section. However, other types of evaluation have been used to evaluate a developmental progression, such as strategies using assessments with items designed to assess each hypothesized level (Penuel, Confrey, Maloney, & Rupp, 2014, include a discussion of the challenges of designing such assessments). In another approach, we conducted a series of clinical interviews and then used quantitative approaches to test whether a correlation matrix of the levels (each measured by qualitatively verified items) would display a simplex structure indicating that the set of levels form a disjunctive scale (Clements, Wilson, & Sarama, 2004).

A related approach is to use item response theory (IRT) models to determine if the developmental progression is a coherent unidimensional scale and if the levels are developmentally distinct (that is, if confidence intervals around tasks measuring different levels do not overlap). This strategy was used successfully to mostly validate, but also slightly refine, a developmental progression for length measurement (Szilagyi, Sarama, & Clements, 2013). There have been similar studies (Langhorst, Ehlert, & Fritz, 2012), and Wilson (2012) provided an extended discussion of measurement and analytic challenges related to this approach.

Microgenetic studies can examine the instructional component of an LT in fine detail (Siegler, 2006) as they can answer questions about *how* learning occurs. Microgenetic methods have three main properties: (a) observations span the period of rapidly changing competence; (b) within this period, the density of observations is high, relative to the rate of change; and (c) observations are analyzed intensively with the goal of inferring the representations and processes that gave rise to them (p. 469). Thus, you can check if the specific attributes of the instructional tasks and pedagogical strategies do engender the mental actions on objects hypothesized as defining a target level in the LT.

When designing, conducting, and disseminating evaluations of LTs, keep in mind that the LT construct differs from previous approaches. An instructional design using task analysis may use a reduction of the skills of experts, whereas one using LTs is usually based on models of children's thinking and learning. LTs involve continuous, detailed, and simultaneous analyses of goals, pedagogical tasks, teaching strategies, and children's thinking and learning (with cognitive models describing specific processes and concepts involved in the construction of the targeted mathematics goal across several distinct structural levels). Such explication allows the researcher to *test the theory by testing the LT* (Clements & Battista, 2000; Clements & Sarama, 2014b; Cobb et al., 2003) and thus contribute to the literature beyond simply suggesting better instructional approaches (for additional discussion, see Baroody, Cibulskis, Lai, & Li, 2004).

5.4 Evaluation of Enacted Curricula

The third category, *evaluation*, includes phases in which empirical evidence is collected to evaluate the appeal, usability, and effectiveness of an instantiation of the curriculum, realized in some form. Phase 5 focuses firmly on questions of marketability. We do not discuss this phrase in this chapter, except to note that market research is a commercially oriented research about the customer, what the customer wants, and what they will buy. The CRF does not ignore it but keeps it public and shared, contrary to the usual commercial procedures. Phases 6–8 involve formative evaluation, asking whether the curriculum is usable by, and effective with, expanding social contexts, and, especially, ideas for improving the curriculum. These phases move from individuals or small groups to a focus more on teachers in single, then multiple, classrooms. One set of questions for this category involve understanding the meaning that students, then teachers and their students, give to the curriculum within progressively varied social and cultural contexts. Another set of questions concern the usability and effectiveness of the curriculum. All aspects of the curriculum are altered as the formative phases indicate, with the focus expanding to include additional aspects of support for teachers. Formative evaluation phases are critical for R&D, but the research methods can be used for a summative evaluation of existing curricula if they address your questions. Extensive resources are available discussing ways to study enacted curricula (Heck, Chval, Weiss, & Ziebarth, 2012; Stein et al., 2007).

Phases 9 and 10 involve summative research, with the goal of rigorously evaluating the effectiveness (e.g., in affecting teaching practices and ultimately student learning) of the curriculum, now in its complete form, as it is implemented in regular school contexts. Thus, these phases are firmly in the purview of those evaluating curricula. Phase 10 should examine the fidelity and *sustainability* of the curriculum when implemented on a large scale and the critical contextual and implementation variables that influence its effectiveness. Both quantitative and linked qualitative approaches are used to deal with the complexity and indeterminateness of educational activity (Lester Jr. & Wiliam, 2002). Done right, these phases require considerable funding.

5.4.1 Phase 6: Formative Research—Small Group

For R&D, pilot testing with individuals or small groups of students is conducted on components (e.g., a particular activity, game, or software environment) or on small (e.g., a week's activities) or large (e.g., a complete LT for one topic) sections of the curriculum. Early interpretive work evaluates components using a mix of model-testing and model-generation strategies, including group-based teaching experiments (Blanton, Brizuela, Gardiner, Sawrey, & Newman-Owens, 2015; Lamberg & Middleton, 2009; Simon, Placa, & Avitzur, 2016). The goal is to understand the *meaning* that students give to the curriculum objects and activities. Evaluating sections of the curriculum focuses on consonance between the actions of the students and the LT. If there are discrepancies, either the hypothesized mental actions on objects or the way in which they are intended to be instantiated in the curriculum activities is altered. The developer-researcher also records elements of the teaching and learning environment that contributed to student learning. Similar to microgenetic studies, but involving more complex social dynamics, the goal is to connect the curriculum's environment/tasks/teaching with evidence of student learning. Usually, this is the most iterative research-design phase; evaluation and redesign cycle in quick succession, possibly as often as every 24 h. Activities may be completely reconstituted, with edited or newly created tasks tried the next day with the same or different students. Field notes are essential and may be complemented with video or audio recordings.

5.4.2 Phase 7: Formative Research—Single Classroom

Although teachers are ideally involved in *all* phases, a special emphasis here is the process of curricular enactment. For example, a goal of the curriculum may be to help teachers interpret students' thinking about the activities and the content they are designed to teach; support teachers' learning of that content, especially that which is probably new to teachers; and provide guidance regarding the external representations of content that the materials use. Thus, there are two research foci. First, classroom-based teaching experiments help track and evaluate student learning, with the goal of making sense of the curricular activities as they are experienced by individual students (Blanton et al., 2015; Lamberg & Middleton, 2009; Simon et al., 2016; Stephan, Cobb, Gravemeijer, & Estes, 2001). Extensive field notes and possibly video recordings allow developer-researchers to examine students' performance for evidence of their interpretations and learning. Second and simultaneously, the entire class is observed for information concerning the usability and effectiveness of the curriculum. Ethnographic participant observation (Spradley, 1980) is used because we wish to research the teacher and students as they construct new types of classroom cultures and interactions together. The focus is on how the materials are used and how the teacher guides students through the activities.

5.4.3 Phase 8: Formative Research—Multiple Classrooms

Multiple classrooms from diverse communities are observed to ascertain the effectiveness of the curriculum, with more emphasis placed on its usability (cf. Drake, Land, & Tyminski, 2014). Because teachers may agree with the curriculum's goals and approach but their *implementation* may not be aligned with the developer-researchers' vision (Sarama, Clements, & Henry, 1998), methods like those of phase 7 are used to determine the meanings that the various curricular materials have for both teachers and students. Observational instruments, whether general or specific fidelity measures, are often used to focus and direct these observations. In our view, fidelity instruments should go beyond simple compliance fidelity to include fidelity to the *vision* of the curriculum (Clements & Sarama, 2000/2018; Clements, Sarama, Wolfe, & Spitler, 2015). As a complement to this data collection, a wide variety of questions can be addressed through teacher questionnaires and interviews. For example, do the materials help teachers maintain a high cognitive demand (Stein et al., 2007)? How do teachers interpret and use the texts (Lloyd et al., 2017; Remillard, 2005)? Do they make productive adaptations and avoid lethal mutations (Brown & Campione, 1996; Remillard, 2005)? Also, do different teacher characteristics, community characteristics, or contexts link to different enactments (Lloyd et al., 2017)? Is the curriculum design consistent with patterns of curriculum use (and other issues of curriculum ergonomics, Choppin, Roth

McDuffie, Drake, & Davis, 2018)? See the resources cited for more perspectives on curricular enactment.

Materials for professional development can be based on this research. Well-documented studies in phases 7 and 8 can make substantial contributions to the literature (see many examples in Lloyd et al., 2017, including identified weaknesses in the literature).

5.4.4 Phase 9: Summative Research—Small Scale

This phase evaluates what can actually be achieved with typical teachers under realistic circumstances (cf. Burkhardt, 2006). Again in multiple diverse classrooms (about 4–10), pre- and posttest (standardized instruments), experimental or quasi-experimental designs using measures of learning are often implemented, in conjunction with, and to complement, methodologies previously described (e.g., Clements and Sarama, 2007a; Doabler et al., 2014). Surveys of teacher participants also may be used to compare data collected before and after they have enacted the curriculum. The combined interpretive and survey data address whether such supports are viewed as helpful by teachers and other caretakers and whether their teaching practices have been influenced. These data help to answer questions such as these: do before-and-after comparisons indicate that they have learned about children's thinking in specific mathematical domains and adopted new teaching practices accordingly? Have they changed previous approaches to teaching and assessing mathematics? Are they able to situate their students' learning in the context of LTs?

Such research is more similar to, but still differs from, traditional summative evaluations. A theoretical framework is essential. Comparison of scores outside of such a framework, used in some traditional curriculum evaluation, is inadequate for our vision (and for publication, a topic to which we shall turn). That is, unless we understand *how* and *why* one curriculum led to different outcomes than another, we fail to make adequate contributions to the research corpus. The selection of assessments is critical to capture outcomes that are desired. As an example, students using some standard-based curricula score equally with traditional curricula on computation but better on conceptual knowledge and problem solving. Further, they may develop different beliefs and understandings of the nature of mathematics (Stein et al., 2007). These findings are not possible if assessments are selected only for reasons of history or convenience.

5.4.5 Phase 10: Summative Research—Large Scale

The CRF is not complete until evaluations are conducted on a large scale. Such research should use a broad set of instruments to assess the impact of the implementation on participating children, teachers, program administrators, and parents and relate this to the fidelity of teachers' implementation across diverse contexts (cf. Clements, Sarama, Spitler, Lange, & Wolfe, 2011; Larson, Dearing, & Backer, 2017; May et al., 2015; Sarama, Clements, Starkey, Klein, & Wakeley, 2008; Sarama, Clements, Wolfe, & Spitler, 2012; Stein & Kaufman, 2010). The goal should be to measure and analyze the critical variables, including (a) contextual variables, such as urban/suburban/rural, type of program, class size, teacher characteristics, student/family characteristics, and (b) implementation variables, such as engagement in professional development opportunities; fidelity of implementation; leadership, such as principal leadership, as well as support and availability of resources, funds, and time; peer relations at the school; "convergent perspectives" of the developer-researchers, school administrators, and teachers in a cohort; and incentives used. A randomized experiment provides an assessment of the average impact of being exposed to a curriculum. However, not all intervention classrooms will implement the program with equal veracity; therefore, a separate series of analyses would relate outcome measures with a set of target contextual and implementation variables.

Simultaneously, qualitative methods are used within that structure. The combination is critical. Quantitative methodologies provide experimental results, garnered under conditions distant from the developer-researchers, that are useful in and of themselves and in that they can generate political and public support. Such methodologies also can uncover unexpected and subtle interactions not revealed by qualitative investigations. Qualitative methodologies, however, are just as important. Curriculum research includes interpreting the interactions within diverse groups of individuals. It seeks to understand individual students' understanding and learning and how these change in the context of, and as a result of, the interactions among teachers and students around a specific curriculum. Further, qualitative research in a triangulation context may serve to validate or invalidate quantitative results, more so than the inverse (Russek & Weinberg, 1993).

From the wide breadth of data produced, researchers conduct iterative analyses to determine the significant meanings, relationships, and critical variables that affect implementation and effectiveness. Such studies are sine qua non for a complete CRF evaluation. They are expensive and difficult, however, and publications may not be produced for years after the beginning of the project (which may be years after starting to write proposals for the necessary funding). Therefore, new researchers may wish to join large-scale evaluations already underway but plan to conduct such challenging research projects themselves only after they have established their careers with less complex studies.

5.5 Disseminating and Publishing

Another potentially frustrating aspect of the type of work we describe is that it can be difficult to disseminate the *research* (more so than the curriculum per se). Even the more traditional methods, as in the summative research of phases 9 and 10, have often been viewed as atheoretical "races" contributing minimally to the literature.

However, there are more publication outlets than ever before for well-conducted studies, even those from phases before phase 9. Many of the journals we have cited throughout this chapter are viable options, and consider especially *Cognition and Instruction*, *Curriculum Studies*, *Early Childhood Research Quarterly*, *The Elementary School Journal*, *Journal for Research in Mathematics Education*, *Journal of Curriculum Studies*, *Journal of Education and Practice*, *The Journal of Mathematical Behavior*, *Mathematical Thinking and Learning*, and *ZDM*. As always, it is important to read the journals to which you may wish to submit manuscripts to understand their goals and to see if they feature the types of questions and methods you have chosen. Further, using the curriculum research methods comprehensively is important not just to ascertain if the design is successful but also to trace whether that success can be attributed to the posited theory-design connections—essential to contributing to the literature. (Too common, in the rush to get the next iteration of a curriculum into the field, is the neglect of *what*, *how*, and *why* changes were made.) For example, in our *Building Blocks* project, we initially planned to develop preschoolers' number sense with a series of estimation tasks. Considerable work at phases 4 and 6 convinced us that even with engaging activities, there were no measurable learning gains. Analyzing their responses showed us that most children needed to develop benchmarks through subitizing and counting *before* they could meaningfully development competence in estimation (Brade, 2003), so we moved the latter activities toward the end of the curriculum (Clements & Sarama, 2007b/2013). As a different example, a series of studies documented how a workbook used people getting on and off a bus to teach arithmetic. Researchers first examined the goals, next the different ways teachers understood and used it, then how children understood it, and finally how results compared to traditional teaching (Van den Brink, 1991).

5.6 Final Words

We admit that the CRF asks developers and researchers to accept considerable responsibilities, such as expanding their knowledge, including the literature of the subject matter, psychology/cognitive science, instruction, and their integration into LTs (Maloney et al., 2014), as well as implementation and scaling up. Knowledge of scientific research procedures and ideas is also essential as a variety of methods are relevant to research in the service of curriculum development and evaluation. Even if multiple methods are used, if they are all from a priori foundation phases, they are inadequate. This produces, at *best*, research-"based" rather than research-*validated* curricula. Subtle differences in activities can enhance or sabotage effectiveness. Effectiveness must be checked at every phase, maintaining links to the hypothesized theories and models. Thus, research must be conducted throughout the development process. This brief chapter could but introduce the landscape. You will need to read the references provided—and beyond—to be prepared for engagement with development and/or research from any phase(s).

We end by emphasizing two reasons that curriculum research is so beneficial compared to other forms of research. First, it can produce direct contributions to practice as curricula remain substantial confluence on practice (Lloyd et al., 2017; Whitehurst, 2009). Second, across different methods, and within them, there are iterative cycles, which must "work" to proceed and reveal weaknesses if they do not work (avoiding confirmation bias, Greenwald, Pratkanis, Leippe, & Baumgardner, 1986) and thus offer tests of construct validity that are both more frequent and more trustworthy than tests in most other approaches to research.

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Chapter 6
Securing Funding: Getting Started

Paola Sztajn

Abstract In this chapter I provide insights on grant writing. The chapter is written for junior faculty members who want to become more competitive when preparing proposals for external funding. It is guided by the premise that junior faculty members should find funding opportunities to support the work they want to do. I first address the grant review process and invite readers to think about who their audience is. Then I discuss the grant-writing process, helping readers understand that grants are not journal articles; these are different genres that require different kinds of rhetoric.

Keywords External funding · Research support · Grant writing · Grant review process

Many mathematics education doctoral students are funded through grants that their professors secured, and when asked why they apply for grants, several mathematics education faculty members stated that a main reason is to fund doctoral students. Yet grant writing is seldom addressed as part of the preparation of the next generation of academics. Although some doctoral programs in mathematics education have worked to provide doctoral students first-hand practice with the grant proposal process (Reyes & Reyes, 2017), training for grant writing in graduate programs is, at best, unsystematic. Typically, only students who happen to become involved in research teams that write grants have the opportunity to learn the grant-writing craft. Therefore, in mathematics education, many junior faculty members have not considered or practiced grant writing.

The concept that tenure-track assistant professors need to focus their early writing on preparing papers for publication and can defer grant writing until later years is quickly fading away. Similar to the hard sciences, mathematics education faculty members are increasingly expected to obtain internal and external funding to

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support their research in the early years of their careers, and promotion and tenure committees are looking at funding as part of faculty evaluation. Gallup and Svare (2016) suggested that getting grants is quickly replacing the old “publish or perish” mantra. They warned that the ever-growing push for more funding takes a significant amount of faculty time and can have unintended consequences such as suppressing creativity and risk taking. Still, being able to support one’s own research is an increasing necessity in academia.

It is important to note, however, that faculty members do not, in general, simply replace the writing of papers with the writing of grants. On the contrary, oftentimes faculty members write grants so that they have interesting results to report in their papers. For junior faculty members wanting to publish in top-tier journals, the important implication is that, increasingly, the interesting research that gets published in top journals is complex and requires funding to be carried out. For example, a quick review of articles published in the most recent full volume of the Journal for Research in Mathematics Education (volume 48, 2017) revealed that over half of the research reports acknowledged the support of grant funding for the work reported. Therefore, those who want to publish in top-tier journals need to think about the resources needed to support the research that is getting published in those venues. To that regard, securing grants can foster faculty publication.

Returning to my conversation with mathematics education faculty members regarding why they apply for grants, several replied that they do so because grants allow them to do the research they want to do. Overall, grants are about the depth and meaning of the work that members of faculty get to do and the breadth of the research questions they get to ask. In conversations with faculty members who are successful in obtaining grants, they indicate that this funding gives them intellectual freedom, opportunities to explore a wider range of issues, and support for collaboration with colleagues and across institutions. Grants help pay for researchers’ and participants’ time, equipment, and travel for data collection and dissemination—among other things. Thus, learning to write grants and secure funding is about seeking opportunities to engage in meaningful, important, and impactful research.

The goal of this chapter is to provide insights on grant writing to junior faculty members who want to become more competitive when preparing proposals that will support the work they want to do to build their research programs. The chapter is guided by the premise that available funding should not define one’s curiosity—rather, given their intellectual curiosity, scholars search for grants that fund their research, or parts of their research program, adding to a trajectory of funded studies that comes to define their contributions to the field. This means that junior members of faculty should be encouraged to find the funding opportunities that work for them instead of them working “for” funding agencies.

The chapter starts at the point when junior faculty members have defined the work they want to do and have identified a funding source that can support that work. It assumes that there is a request for proposals (sometimes also called a request for application or a solicitation) related to the intended work and that one is considering the preparation of a proposal. With this in mind, the chapter first addresses the grant review and then the proposal writing processes. This backward order invites readers to first think about who their audience is. Meant to engage junior faculty in reflections about securing funding, the chapter is based on several conversations had with members of faculty over the years and is constructed as a dialogue between the readers (you) and the writer (me).

6.1 The Grant Review Process

When discussing grants with junior faculty members, the question of what happens during the review process often emerges. This is an important question not only about who will be reviewing the grant proposal but also about the conditions under which the review might happen.

6.1.1 Who Reads My Grant?

For every section of your proposal, you should assume that one or more experts in the areas involved will be part of the review process. This means that for every area of work in research and development included in your grant, you should assume than at least one expert will read that particular part of the proposal with a critical eye. For example, for a proposal about mathematics teacher education, my area of expertise, I usually think that the group of reviewers will include a mathematics teacher educator, a mathematician, a teacher educator who is a generalist, a research methodologist, an evaluator, a teacher, and perhaps a school administrator or a district-level specialist. Each of these experts may focus on different parts of the proposal, such as discussions of mathematics content, research, or the feasibility of the proposal for teachers and schools. If the grant involves the use of technology, an expert in instructional technology will read the proposal. And if the grant has a component of curriculum development, a curriculum expert will likely review it. The consequence of having all these readers is that every part of your proposal has to speak to its respective expert. As demanding as this idea may sound, my goal is to make you aware that there is no space for any weak section in a grant. A proposal, like a chain, will only be as strong as its weakest link.

Because experts in one area, who may have a narrow area of expertise, are reading the whole proposal, another important aspect of the review process is that every part of the grant has to also be written in a way that nonexperts in that area can easily read it and understand your work. This means that, given the composition of the review panel, you have to communicate your ideas to nonexperts and experts in all aspects of the work at the same time! The writing has to have the depth that experts will expect and yet be explained in a clear, concise way that nonexperts can read and follow. You have to communicate important ideas without jargon. You have to organize your argument in a logical fashion that anyone can follow while also showing experts that you know what you are talking about.

Including experts and nonexperts, different funding agencies practice different review processes. For example, in some funding agencies, program managers (who are often well-read, highly informed education scholars but not necessarily experts in any of the components of the proposed work in a grant) read all proposals first. If the proposal as a whole does not make sense to them, it does not go to a second round of review by experts. For this member of your audience, elimination of jargon is a must as these program managers are mostly looking for the timeliness and importance of the ideas, feasibility of the work, and clarity in the writing. In other funding agencies, review panels are constituted to read a particular set of proposals and the expertise of the members are tailored to the set of proposals at hand. These panels can be quite diverse based on the proposals in the set, and additional review by experts can be solicited as needed. Agencies can also have standing panels of experts in a field; proposals are then fielded to the panel that best fits their areas of proposed work. No matter the constitution of the panel or how the process is organized, a proposal has to be clear enough for the nonexpert to follow and deep enough for the expert to see value in it.

6.1.2 How Do Reviewers Operate?

Although review panel constitution and processes vary across funding agencies, one thing is for sure: reviewers are reading a lot of proposals with little time to do so. In an exaggerated fashion, I like to think that reviewers are reading the proposals in the plane on their way to the panel meeting, or, for more recent virtual review panels, reviewers are reading proposals the night before the meeting. With a pile of 20 or so proposals in front of them, they are trying to make quick decisions about whether they like the work, whether they want to discuss it further, and whether the work should be funded. The question then is this: how will you make your proposal stand out in this pile to someone who is reading quickly? You want to make sure reviewers can easily understand your ideas, follow the case you are making for your work, remember what you want to do, and get excited about the work you are proposing. To me, three main ideas come to mind when trying to achieve these goals.

First, it is important to address big ideas or topics. If the work proposed is significant and of quality, and if it addresses really key current issues of challenges, reviewers will remember your work was important and addressed an area in which work is needed. Of course, if you have a great idea and the proposed work has considerable flaws, reviewers see it too. But well-presented, correct work about ideas that are not that important get forgotten in the pile of proposals. The main point: you need to awe reviewers with the significance of your work and its contributions!

Second, it is important to be consistent from the beginning to the end of the proposal. If you have three research questions, you have three research questions—don't add to or take away from them during the proposal! There is nothing more confusing than when, halfway through the reading of a proposal, the proposed work seems to shift or new work seems to be added to what was said earlier or a different question is asked. When that happens, in the little time they have, reviewers must compare what was said earlier with what is being said later and try to figure out how it all comes together. This takes time, which reviewers may not have. The main point: you do not want to confuse the reviewer!

Third, reviewers need you to help them remember the work you are proposing. This means that you need to be connecting main ideas for them, highlighting significant parts, and synthesizing what was said along the way. A good proposal connects the dots and makes main points explicit. It comes back to those points several times, reminding readers how it all comes together and repeating main ideas and terms to emphasize significant points—again and again. In this way, after reading your proposal, a reviewer should be able to easily summarize the main proposed work in a few statements that define the gist of the proposal. And further, reviewers need to quickly remember what your proposal is about by glancing at your text. The main point: you want to highlight and repeat the main points of your proposal so that reviewers can quickly see and later recall them.

Once your proposal awes reviewers, consistently and clearly presents the work, and can be easily remembered, it also needs to convince at least one reviewer that it is amazing work and must be funded. This means that you have to appeal to reviewers' passions. By the time a group of reviewers is discussing a large number of proposals, they have made decisions about proposals they think should definitely not be funded, could be funded, or should definitely be funded—that is, they have created basically three main piles from the proposals they read: no, maybe, yes. The hope is that there will be at least one person who, after reading your work, thinks it must be funded and is (voluntarily) willing to step up and explain the work, answer questions other reviewers raise, and remind others of the contribution of the work. This person can help other reviewers move your proposal from their "maybe" piles to their "yes" piles. Without such a person, your proposal is likely to stay in the pile of very good proposals that do not get funded.

It is also the case that a reviewer with a criticism of the work can carry the panel in the other direction. Any proposal that has a reviewer with a very strong and well-articulated argument against any part of the proposed work, and who is willing to step up and highlight the problem(s) with the proposal, runs that risk of having that reviewer carry the discussion and convince other reviewers that the proposal needs to move from the "maybe" pile or even the "yes" pile to the "no" pile. For example, I have seen a proposal in teacher education be discarded because a teacher among the reviewers was extremely convincing in arguing that no teacher would ever engage with the work as proposed and that even though the research methodology was perhaps exemplary, the work was not feasible in today's schools.

In summary, writing to experts and nonexperts who are reading a large number of proposals in a small amount of time is actually the art of convincing them that your proposal addresses an important issue or challenge that needs to be solved and that your plan for doing so is clear, reasonable, and well thought out. Your goals and the proposed work need to be easily followed and remembered, and your ideas should communicate the passion and the importance of the proposed work in a way that at least one reviewer will argue that your proposal is outstanding and has to be funded.

6.2 The Grant Writing Process

Because of the unique nature of the grant review process, constructing a competitive grant requires a specific writing style that is unique and specific to this medium. Skill in academic writing or in other genres does not necessarily translate into grant writing effectiveness. Thus, when writing a grant, it is important to realize that a grant is not a journal article to be published. It requires a different kind of rhetoric. (Reading several successful and unsuccessful grant proposals and analyzing the construction of their arguments can help you begin to think about how to structure grants and how to write them.) In what follows, I offer a few ideas on how to get started with your writing.

6.2.1 What Is Unique About Writing Grants?

The first suggestion and feedback I typically give to my colleagues when they start writing grants is the following: journal articles are gray; grant proposals are black and white with sparkles on top! Most of our academic writing for publication dwells on the details and the intricacies of the study. Grant proposals, on the other hand, are there to convince reviewers that there is a big problem that needs to be addressed and that you clearly understand what the problem is and have a solution to it. Grants are written to persuade reviewers to allow you to do the work you want to do. Thus, they are written with certainty (black and white). Furthermore, a grant proposal needs to be exciting and needs to convince the reader that the work you are proposing is novel and simply must be done—this is what I mean when I say it needs sparkles!

In a paper titled "Why academics have a hard time writing good grant proposals," Porter (2007) suggested that grant writing should be energetic, direct, concise, easy to understand, and intriguing. He suggested several contrasting categories between writing grants and papers. For example, papers pursue scholarly goals, describe past work, use expository rhetoric, are impersonal, gain from verbosity, and use specialized terminology. Proposals, on the other hand, address the sponsor's goals, describe work to be done, use persuasive rhetoric, are personal and passionate, gain from brevity, and use easily understood language. Porter concluded that "success in grant writing is a matter of style and format as much as content" (p. 38).

Discussing the need to use persuasive rhetoric in grants, Porter (2007) indicated that the language in the grant needs to sell ideas because one is trying to procure funds for a project that is not yet in existence. Thus, a good proposal is written to build a winning argument, a compelling case for work that deserves part of scarce resources. This type of rhetoric requires the personal and passionate tone of one who is seeking endorsement from reviewers. Proposals need to be written in active voice, have energetic phrasing, and have direct references to the work of the research team. Porter provides the following statement as an example of grant writing that would be unacceptable in scholarly journals: "This project will provide your grant program with a powerful combination of cutting edge nanoscale science and frontier research in applied geochemistry" (p. 40). Such a statement, black and white with sparkles on top, does not leave space for doubt (usually a faux-pas in academic writing) and asserts the importance of the work in solving a problem.

Successful grant writers spend a significant amount of time studying the request for proposals and then finding ways to present their expertise in ways that match the description of the sponsor's goals. One way to communicate this connection is to feed back to the reviewers and to the funding agency some of their own wording. In studying a solicitation, you can look for the words and expressions used to talk about aspects of the work you want to do and then use those same words in your proposal. You want to explicitly connect your work to the stated goals of the solicitation, and when certain areas of work are listed as required in the solicitation, you want those areas to be clear headings in the proposal. What you are trying to do is help reviewers realize how well connected your work is to the request for proposals. You are guiding the reviewers' readings and explicitly showing how your work answers the request from the funding agency. In grant writing, it is both acceptable and even desirable to use the same expressions or terms again and again and again. This repetition is part of clearly aligning your work with the solicitation.

In my opinion, perhaps the most difficult thing for mathematics education researchers to write in a grant is the assertion of the problem that the project will help to solve. Most of our research addresses a small part of a bigger problem, and we are all too aware of all the nuances and details of that problem, as well as how hard it is to solve some of the larger societal problems related to mathematics teaching and learning. And yet, in a proposal, we must connect the research we want to do to these larger problems in ways that propose solutions. Proposals are not funded because they answer a specific, narrow, research question properly; they are funded because beyond properly answering research questions, a case is being made for the importance of the question and how it fits into a larger national or international issue that needs a solution.

6.2.2 How Do I Start Writing a Grant Proposal?

Thinking that reviewers will be reading proposals quickly, and thus making decisions quickly, I strongly believe that those writing proposals have about two pages to convince reviewers to fund the work. Reviewers make important judgments about the quality of the proposal within the first two pages, and thus these pages determine the level of attention that reviewers devote to the remainder of proposal. Within these initial pages, reviewers should already be awed by the problem that the proposal addresses, be convinced that the research questions can be answered with the proposed methods, and be persuaded that the researchers know what they are talking about and can conduct the work.

I suggest spending a considerable amount of time crafting these initial pages. They should present your goals, plans, and a brief overview of everything you are proposing to do. The remainder of the allowed pages can then be used to explain the details, review the literature, and show that you know what you are talking about and can carry out the work. In a recent workshop for doctoral students at North Carolina State University, I asked participants to read the first two pages of several successful proposals from different funding agencies and to consider what was typically included in those initial pages. At the conclusion of our discussion, the group agreed that the first two pages of all the proposals included most of the following ideas:

- A discussion about a current, important problem, with a clear explanation about why this problem matters;
- A statement about how the work being proposed addresses this major problem;
- A list of specific goals addressed in the proposed work, the outcomes of the work, and the impact of those outcomes on the problem being addressed;
- The project's research questions (when appropriate) and a general indication of how they will be answered;
- Information about project participants, such as the demographics of the population that the project will work with or serve;
- Information about the research group members and what qualifies them to solved the proposed problem.

This list is quite telling in that it shows that the first part of the proposal is, in fact, about letting reviewers know upfront what the big problem you are addressing is, why they should care about it, how it relates to the funding agency's mission or goal, what should be done about this problem, and why you and your team are best positioned to solve this problem through the proposed work. Being able to clearly and concisely convey all these points requires you, as the writer, to develop clarity with respect to the work to be done, its outcomes, and how it will be carried out. Usually, for me, as I am conceptualizing the work, I write and rewrite the first two pages of my proposal. And even as I continue to draft the rest of the proposal, I constantly revisit the first two pages for their accurate description of the work. When I am satisfied with these initial pages, it means that I have sufficient understanding of the work I want to do that I am ready to write the rest of the proposal.

There are other reasons why writing these first two pages is important. First, these pages serve to clarify and determine the language to be used throughout the proposal. For me, these pages serve as kind of glossary of terms that I continue to use. These choices in naming certain facets of the work can then guide the rest of the proposal writing. Further, in these pages you specify the questions, objectives, and outcomes central to your project, and these should also remain consistent for the rest of the proposal. Finally, these first two pages are important because you can use them to discuss the overall project ideas with other colleagues, invite partners, and, most importantly, talk to a program manager. Most funding agencies welcome questions to their program managers, and starting a meeting with a good two-pager (preferably sent in advance) allows the program manager to ask specific questions and provide useful feedback.

Although these first two pages help reviewers initially decide to fund your project, the remaining pages of the proposal need to sustain the reviewers' decision! Thus, it is also important that your grant have a short but excellent review of the literature that demonstrates your knowledge of the field. Your proposal must also specify the details of the plan of work. How will you carry out the work? What will you do with whom, when, and how? Timelines and details of data collection and analysis are also fundamental elements that demonstrate to reviewers that you know what you are going to do and that you will be able to answer your questions and produce the promised outcomes of the project. In my opinion, detailed work plans not only help reviewers know that you understand what the work entails; they also help funded researchers get started with and manage the work they are promising to do.

6.3 Concluding Remarks

Writing grant proposals is an art, and for those just starting their practice in this genre, it is important to understand its unique features. Practice and persistence are key in the development of your proposal writing skills. Addressing a major problem, presenting your ideas in clear ways with little jargon, explaining your work with consistency, and showcasing the expertise of your team are a fundamental part of this process. Remember your audience and write in a way that will engage them with the problems you are promising to solve.

For junior researchers it is useful to recognize that many academics struggle with writing grants and, from that perspective, you are not necessarily at a disadvantage. Learning about the proposal writing craft early in your career will support your academic success. The proposal development process can offer great opportunities to propose interesting collaborative work with several partners across institutions. Most importantly, writing grants is about being able to support the work you really want to do and communicating to others the importance of your research.

For mathematics education as a field, it is time we recognized the need for doctoral programs to consider how they are preparing future mathematics education researchers for the current academic environment in which securing funding is becoming more and more prevalent. A systematic approach to learning about the grant writing process is needed in our programs and can benefit those going into tenure-track positions in research intensive institutions, as well as graduates taking positions in other types of institution. For many areas of inquiry in our field, the time of a single scholar conducting educational research in the office is passé; the more complex, interdisciplinary, and interinstitutional research programs that are now required necessitate resources and funding for their implementation.

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Part II
Conducting Quality Research in Mathematics Education

Chapter 7
Putting the Quantitative Pieces Together to Maximize the Possibilities for a Successful Project

Robert M. Capraro, Ali Bicer, Yujin Lee, and Katherine Vela

Abstract In this chapter, the authors outline key steps and considerations to take into account when entering into the realm of quantitative research. The primary question guiding this chapter is the following: "How does one conduct quality quantitative research as a novice researcher?" Several equally important questions are subsumed within this general question, such as "What are the first steps to conducting quantitative research?" and "What are the best statistical methods to use?" The authors address each of these questions based on their experience with quantitative research in academia. The authors include senior and junior professors as well as senior doctoral students, each of whom began their research career with as many questions and concerns as any novice beginning their research journey is likely to have. That said, the authors aim to help novice researchers explore how quantitative research in mathematics education requires several sequential and coordinated steps and to outline tips and precautions to take as well.

Keywords Quantitative research design · Quantitative statistical analysis · Novice researchers · Mathematics educational research

To begin one's research is to embark on a long, multistep journey that is best undertaken with an ample amount of determination, intellectual interest, and advice from those who have experience with processes involved in conducting research. That said, the purpose of this chapter is to help novice researchers consider both the steps required to successfully conduct research and the intricacies related to these steps.

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In particular, the authors aim to provide novices with guidelines for selecting the design of and analytic methods for quantitative studies. The conceptual and methodological guidelines in this chapter are meant to be an overview of the components of quantitative research that beginners may overlook as well as the common errors they should avoid. With these aims in mind, the authors introduce how to (a) design a research agenda, (b) select team members, (c) formulate research questions, (d) construct a literature review, (e) create an intervention, and (f) select and run one's data analysis when designing and conducting a quantitative study.

7.1 Designing a Research Agenda

Perhaps the most asked question from doctoral students, postdocs, and junior faculty members alike, after those of logistics (locations of office, bathrooms, and vending machines), involves some form of "How do I develop a research agenda?" Research agendas look different today than they did two or three decades ago. Although researchers in previous decades may have had one or two related research areas subsumed within a central research agenda, it is rare for a researcher today to believe that he or she will be likely to have a single, centralized research agenda. Setting a research agenda in today's climate is dramatically impacted by institutional factors and other external pressures (Terjesen, Hessels, & Li, 2016). Institutions are looking for ways to deliver lower-cost degree options, increase their budget through extramural funding given cutbacks in state funding, and fulfill pressures to hire faculty with external verification of excellence like National Academy of Education fellows and the like.

The pressures both real and imagined at universities have reduced emphasis on a single line of research while increasing the demands on extramural funding (Padilla & Thompson, 2016). Because there are many types of funding and each funding agency has its own interests, a new faculty member interested in obtaining that funding must adjust the scope and focus on her or his research agenda. It is not uncommon to find faculty who have a number of publications in a particular topic for several years and then that topic shifts. This is a reflection of the times and funding. The research agenda must evolve or it will shrivel up and die.

The new landscape for research agendas is messier because earning national impact is difficult in the best cases and nearly impossible when research agendas become broader. The new "messy" refers to the lack of apparent focus that is becoming more evident on scholars' packets when going up for third-year review or promotion and tenure. Because one's research agenda will likely evolve due to both internal and external pressures and the pursuit of funding to carry out worthwhile work, it is important to explain one's research agenda so that one's expertise and skill set are comprehensively and clearly presented. Then, when the time comes, it will be clear what does and does not fit into the researcher's skillset. To a novice researcher, entering into an academic environment in which one is expected to create an evolving research agenda may feel overwhelming; however, this state of evolving agendas has some benefits and is, in many ways, making it a dynamic and exciting time to be an educational researcher. Key among these benefits is the ability to develop a research agenda that affords one the latitude to pursue his or her interests and potentially rewarding collaborations. Researchers should embrace the new opportunities afforded through an evolving research agenda while also using discernment to guide the agenda's direction.

7.2 Selecting Team Members

Now more than ever, it is common to collaborate on research projects and create joint research agendas. Therefore, the selection of team members for the research agenda and/or research project could make or break research outcomes. It is important to ensure the selected members will be a great fit for the rest of the team. A key element of this process is determining what needs must be met (e.g., quantitative data analysts, editor, academic specialists, etc.) to achieve success during the research process prior to identifying and selecting team members who fulfill those needs. Selecting individuals with different skills or perspectives to work together is crucial (Kichuk & Wiesner, 1998). When preparing to conduct a quantitative study, the authors recommend that novice researchers select team members who specialize in (a) running and analyzing specific quantitative methods, (b) research areas that pertain to the selected research agenda or topic, and/or (c) qualitative research that relates to the selected research topic. Furthermore, novice researchers should consider reaching out to scholars outside of their department or college who may be assets to their research team. In addition, one should consider each candidate's personality because personality has been known to predict team performance; a positive and upbeat personality will promote and have a direct impact on the team's experience and performance (Kichuk & Wiesner, 1998; Silvia, 2018). Lastly, one should abstain from using social cues, such as looks and demeanor, when selecting team members. Instead, get to know potential team members' strengths and weaknesses to determine if they would add value to the research team (Neu, 2015). Once the research agenda and team members have been selected, it is time to formulate a research question that will function as a guide throughout the entirety of the research process for the quantitative study.

7.3 Formulating Research Questions

Quality research should perpetuate a continuum of discovery in which each investigation begins with questions that reflect existing issues or problems in today's world and each answer catalyzes the generation of new questions. That said, to formulate a specific and relevant research question, researchers must first determine their general topic of interest and then take steps to identify gaps in the literature related to that topic.

7.3.1 Identifying Gaps in the Literature

In the process of identifying what questions of interest have been unanswered, it is imperative as a novice researcher to commit significant time and effort to determine whether the topic associated with one's question or questions merit(s) investigation (Creswell, 2014). This decision process can be accomplished by conducting a general "systematic review," which is the screening of previous researchers' findings and suggestions related to a particular topic that allow one to categorize, analyze, and draw comprehensive conclusions from their work. This is a particularly crucial step for novice researchers in mathematics education because they need to gain insight into how their topic has been analyzed and discussed in various fields and identify the core elements of the intended topic.

7.3.1.1 Conducting a Systematic Review

The first step to conducting a systematic review is to be able to succinctly summarize a broad version of the intended topic for the quantitative study. For example, a novice researcher in mathematics education may say, "I want to investigate the effects of writing on students' mathematics achievement." If there is difficulty in summarizing the topic, the researcher can (a) use general templates to guide his or her summarization, such as "My topic investigates the relationship between X and Y," or (b) develop a summative or general research question that encompasses the topic, such as "What is the effect of X on Y?" After articulating what broad question or issue to examine through the intended topic, the researcher may then conduct the quick and general systematic review of previous literature related to the topic. Gathering and analyzing the findings from prior work through a systematic review will allow the novice to develop a better understanding of what has been examined and what requires further investigation.

Once the researcher has identified areas related to his or her selected topic that merit further investigation, he or she must address the following question: "What is the gap in the literature that I plan to address?" Answering this question will help the researcher to narrow down and improve the precision of the research topic. If there are existing studies that are closely related to the selected topic, there is still potential for innovative modifications to the design and scope through the incorporation of new elements. For example, a researcher who is interested in mathematical affect of Korean students may gather various published studies about this topic and analyze them to determine if aspects of the topic merit further investigation. In this scenario, it is possible that the majority of previously published studies on mathematical affect were limited to US participants. After identifying this gap in the demographic of participants, the researcher may wish to alter the demographic group studied in this research area by investigating the mathematical affect of Korean students or by conducting a comparative study of Korean and US students' mathematical affect. Once researchers have conducted and analyzed the information from their systematic review and have identified the gap in existing literature that they will address, they have sufficient information with which to construct a research question(s).

7.3.2 Formulating a Quantitative Research Question

Quantitative research questions are generally used to describe, compare, and reveal the relationship between variables. There are two general types of quantitative research questions: descriptive and comparative. Descriptive quantitative research questions are formulated in order to investigate aspects of specific variables. In general, research questions in this category are designed to quantify a single variable. The following are example questions that adhere to the single variable design: "How often do 8th grade mathematics teachers allow their students to use technology (e.g., calculators, computers, etc.) each week?"; "How often do 8th grade students receive problem-posing instruction in their mathematics classroom each week?" While less common than single variable descriptive research questions, it is also possible to formulate a descriptive research question that allows one to quantify several variables by designing a more comprehensive descriptive research question. An example of this type of multivariable question would be the following: "Are gender, ethnicity, and socioeconomic status (SES) statistically significantly associated with mathematical affect among 8th grade students?"

Unlike descriptive quantitative research questions, comparative quantitative research questions are designed and used to reveal the differences between groups or variables. The following are examples of common quantitative research questions in mathematics education: "How does 8th grade students' mathematics achievement differ by gender?"; "How do 8th grade students' mathematics scores change from pretest to posttest?"; "What is the mathematics achievement gap between 8th grade students who come from low-SES and high-SES backgrounds?"

It is also possible to ask several comparative research questions in a single research question. In this case, the comparative research question can be considered as a relationship-based research question. The following question is an example of a relationship-based research question: "How does students' mathematics achievement change after they receive a problem-posing intervention by their gender, ethnicity, and SES?" A researcher who is asking this question is interested in understanding if there are any differences on students' mathematics achievement by their demographic characteristics.

7.4 Constructing a Literature Review

After carefully identifying and refining one's research question(s), it is time for the researcher to search for related published quantitative studies from the literature to incorporate in his or her study's literature review. While the systematic review discussed in Sec. 7.3.1.1 allows the researcher to generate ideas based on previous work and narrow his or her topic, the purpose of a literature review is to gather and share the results of previous research that are closely aligned to the selected topic. Doing so allows the researcher to enter into an ongoing dialogue among fellow researchers in which the literature review will identify and highlight the gap in the literature, outline the extension of existing ideas, and provide the framework for establishing the importance of the selected topic.

To conduct a literature review, researchers must first identify keywords or phrases related to their research question. For quantitative studies, the authors recommend including one's independent and dependent variables as keywords during the search process. The selected keywords will be used to search computerized databases (e.g., ERIC, EBSCO, and PsycINFO) for research articles related to the topic. Occasionally, keywords that are not considered during the first stage of the search may emerge after initial reading of published work. For example, a researcher who is interested in studying the topic titled "The Effects of Writing on Mathematics Achievement" may decide to begin the search for prior research using keywords such as "writing in mathematics," "mathematics and writing," and "mathematical writing." However, after reading several scholarly works about the topic, the researcher may determine that incorporating a new keyword in the search, like "academic writing in mathematics," is necessary to identify additional resources and ensure that he or she has developed a comprehensive collection of related previously published work.

The authors recommend that novice researchers aim to locate 20 resources from journal articles, books, book chapters, and conference proceedings to start writing their literature review. This not a magic number, but it gives researchers the opportunity to decide if they have the "Three Bears" scenario: too little, too much, or just right. During this search and selection process, novice researchers must continually use discernment to decide if the resources they gather are useful to their understanding of the selected topic.

Once the previous literature has been examined and collected, one should consider how to organize the quantitative study's written literature review. For quantitative studies, the authors recommend organizing the literature review to convey findings from previous studies related to the independent and dependent variables and how these variables relate to each other. However, it is also imperative to note that these suggestions for conducting and writing a literature review are not set in stone. For example, arranging the literature based on either themes or factors associated with central phenomenon being studied may work regardless of the selected research method (e.g., quantitative, qualitative, etc.). Taking these considerations into account while conducting and organizing a literature review will help novice researchers successfully summarize important findings from prior studies, identify and contribute to major discussions related to the topic, reveal the gap in the literature, and propose arguments showing how and why the current study will fill the gap.

7.5 Designing a Methodology

A methodology is a map of the research model and the associated tools, procedures, techniques, etc. one selects to conduct his or her research (Jonker & Pennink, 2010; Wahyuni, 2012). The researcher selects his or her methodology based on a set of research-related beliefs that guide him or her to choose steps in his or her research process. In the following sections, the authors explain some important components included in methodology that researchers may overlook or apply erroneously: experimental and quasi-experimental design, sampling, intervention, and validity and reliability. Designing an appropriate methodology plays a critical role in enabling a researcher to examine and identify phenomena that are well connected with his or her research questions.

7.5.1 Primary Types of Quantitative Study Designs: Experimental and Quasi-Experimental Design

An experimental design study should include three components, all meant to assure the validity and reliability of the results. First, there should be a clear description of how participants were assigned to groups and how the sample reflects the population. A common strategy referred to as *completely randomized* is a process by which participants are assigned to groups at random. Second, there needs to be clear alignment between the research participants and the method used to analyze the data. One prominent example is that of randomized block design. Randomized block design is one by which the research team assigns subjects into subgroups called blocks, such that the variability within blocks is less than the variability between blocks. Then, subjects within each block are randomly assigned to study groups. Finally, there should be an explicit description of how confounding variables were managed or eliminated because those confounds can introduce threats that jeopardize the reliability and validity. It is impossible to control every confound in research within school districts and by relationship schools. However, just because the research is not being done in a laboratory, this does not mean one should overlook critical steps related to confounds; researchers should still attempt to control for as many confounds as possible, clearly identify the most obvious ones, and ensure that they use quality quantitative methods. Carefully addressing these three points will help to reduce initial variability to make it clear that the differences obtained at the end of the study are attributable to the study conditions and improve the quality of the inferences.

When researchers cannot randomly assign participants to treatment levels for their study, they may choose to use quasi-experimental design. A quasi-experimental design is similar to an experimental design; however, it lacks the component of being randomized. Two common quasi-experimental designs are (a) pre-/post-tests with a control and treatment group and (b) multiple pre-/post-tests with control and treatment groups (repeated measures). The first design includes administering pretests and posttests to two groups, typically intact groups (i.e., classrooms of comparable groups), in which one group serves as the control group and the other serves as the treatment group. This design is often referred to as nonequivalent group design because you cannot be sure the groups are comparable as they were not randomized. This uncertainty in the comparability of the groups conveys one potential limitation of the design: this design is susceptible to selection as an internal validity threat because the groups may be different in some respect prior to the study, and these differences could impact the results. Therefore, when conducting your analyses, you must correct for this potential difference in order to obtain more reliable results. The second quasi-experimental design, multiple pre-/post-test with control and treatment groups, is similar to the first design, except tests are administered multiple times throughout the intervention to assess changes over time. Although this design is particularly useful for and commonly used in longitudinal studies, this design, like the first design, does not randomize participants and is thus subject to the same internal validity threat. Nonetheless, quasi-experimental designs are more efficient than experimental designs because intact groups are typically used for these designs, but the lack of randomization produces a number of issues that will need to be corrected or discussed in the methodology section to explain why the study and results are still valid.

7.5.2 Types of Sampling: Non-probabilistic and Probabilistic

Sampling is the way in which a researcher gathers participants for a research study. There are two basic types of sampling: non-probabilistic and probabilistic. A core tenet of non-probabilistic sampling is that participants are chosen based on a researcher's subjective judgement rather than a random process. One form of non-probabilistic sampling is convenience sampling, which researchers use to select participants based on their accessibility and likelihood of participation. In contrast, probability sampling is a technique researcher may use to ensure that participants are gathered through a process that gives all the individuals in a well-defined population an equal chance of being selected. Of course, the goal for probabilistic sampling is to ensure that one may make reasonable generalizations. Unfortunately, while a researcher may wish to generalize his or her findings derived from the use of non-probability sampling or convenience sampling, this form of sampling does not allow for generalizations.

7.5.3 Creating an Intervention

When considering an intervention, regardless of the type, it is important to consider its fundamental and necessary characteristics. To do so, one must first note that there is much misunderstanding of the term intervention because it has been historically linked to response to intervention (RTI) and has been considered analogous to quantitative experimental studies. Various components of a study may qualify as an intervention. For instance, even if a study is limited to interviewing a couple of students who exhibit an interesting computational strategy, that interview is an intervention because the students are likely to be influenced by or learn from the interview process. Likewise, location of the study can also influence the results. Determining the location of the study is vital because it constrains the sample. For instance, few people would select Las Vegas as the location for their study and then recruit participants from Hawaii. There should be alignment between location and addressing local problems through the selection of samples that reflect that paradigmatic choice. Designing solid studies that contribute incrementally to the existing research base and build in new locales to examine fit, effect, and replicability is essential.

7.5.4 Striving for Optimum Validity and Reliability

The most general principle of quantitative study design is the consideration of and plan to control for issues related to validity and reliability. Whereas there are many threats to validity and managing those threats often results in violating some other threat to validity, it is important to make decisions regarding one's study design that best align with the anticipated outcomes. Furthermore, one should ensure that the trade-offs of validity and reliability in one's study are understood and communicated to any consumers of that research. In particular, reliability is often misunderstood and communicated as if some instrument automatically imbues reliability to the data. However, reliability is a characteristic of a particular sample on some clearly identified and communicated construct through a data collection instrument (Thompson, 2002). What this means is that a test-retest or internal consistency estimate of reliability for an instrument used with one sample might be very different from the obtained estimate on another sample. Imagine, for a moment, that you have decided to use an instrument to measure computational fluency on two-digit numbers. If you give this test to second or third graders, reliability might be suitably high; however, the same instrument administered to high school students taking algebra will result in very low reliability. Reliability is a measure of the "spread-outness" of the data. If the participants get all of the questions correct (or incorrect), reliability will be inadmissibly low.

7.6 Selecting Data Analysis Techniques

The main purpose of a quantitative study is to conduct quality research with precision and accuracy that allow one to meaningfully contribute one's findings to an ongoing discussion and investigation of relevant issues and questions. That said, researchers must take care both in selecting the appropriate analytic technique for their study and in interpreting and presenting the results of their analysis. Common quantitative statistical methods of analysis include the following: descriptive statistics, effect sizes, confidence intervals, *t*-tests, ANOVA, MANOVA, and regression and correlational analyses. The authors have noticed, however, that novice researchers often have difficulties reporting effect sizes and confidence intervals. Therefore, the authors have included guidelines for successfully utilizing these two statistical methods and have emphasized the benefits of their use below. Furthermore, many novice researchers tend to avoid propensity score matching (PSM), structural equation modeling (SEM), and hierarchical linear modeling (HLM) because they may not recognize the potential and accessibility of these statistical methods. Therefore, in the sections below, the authors will also introduce an overview of these statistical methods for data analysis.

7.6.1 Reporting Effect Sizes and Confidence Intervals

When researchers use null hypothesis statistical significance tests (NHSST) in education, it often seems that they think the *p*-value should be reported. This belief leads the researchers to have misconceptions and misinterpretations of NHSST that could cause them to arrive at erroneous conclusions in their research findings. Many researchers simply make a decision on whether to reject or not reject the null hypothesis based on the reported *p*-value. However, adopting a fixed level of significance without consideration of other factors, such as sample size or prior studies, can create a very different and imprecise result. For example, let us assume that there is a study in which researchers investigate the impact of one type of instructional intervention on students' mathematics achievement. The result of the analysis in this study may yield a *p*-value of 0.06, and, based on this value, a researcher may make the decision not to reject the null hypothesis because the *p*-value is larger than 0.05. On the other hand, if this study had a larger sample size and the other conditions remained the same, the *p*-value would be smaller than 0.05 because sample size impacts *p*-value. In this case, the researcher may reject the null hypothesis. Use of this scenario demonstrates that identical treatment effects can lead researchers to make different decisions about their results if they assign more meaning to the *p*-value than actually exists. A study's statistically significant results are not always practically important (Kirk, 1996); therefore, researchers should report a different form of statistical index, such as effect size, to inform readers about the practical importance of their findings.

7.6.1.1 Effect Size

An effect size is defined as the statistically quantified degree to which sample statistics diverge from the expectations identified in the null hypothesis (Cohen, 1994). Zero effect size signifies that there is no divergence between sample statistics and the expectations. If the sample statistics are increasingly divergent from expectations, the effect size will increasingly diverge from zero. Using the effect sizes, which are standardized calculations, allows the researchers to compare the construct of apples to apples, even with different measures of construct (Thompson, 2007). It is also true that an effect size is more robust to sample size than is a calculated *p*-value.

Researchers also need to interpret their effect sizes because "the manuscript is incomplete unless these effects are evaluated in the context of the study and in the larger context of knowledge" (Trusty, Thompson, & Petrocelli, 2004, p. 109). In such cases, some researchers benchmark the criteria such as Cohen's conventional value, but the interpretation should depend on the situations the researchers are dealing with. Thompson (2001) noted that "if people interpreted effect size [using fixed benchmarks] with the same rigidity that $\alpha = .05$ has been used in statistical testing, we would be merely being stupid in another metric" (pp. 82–83). Even Cohen, who suggested that $d = 0.2$, 0.5, and 0.8 were considered "small," "medium," and "large," respectively, warned that these were only "rules of thumb" (Cohen, 1992). An effect size of 0.1 could be "large" in one context, and an effect size of 0.9 could be "small" in others.

7.6.1.2 Confidence Interval for Effect Size

Confidence intervals (CIs) should be considered when we report effect sizes in a study. CIs give us "the degree of precision in our computation of a point estimate" (Thompson, 2001, p. 90). Reporting effect sizes without CIs could impair readers' understanding of the results of a study. For example, if the readers compare two studies that appear to demonstrate the same effect size, $d = 0.8$, without 95% CIs, they might think the effects of these studies are the same. On the other hand, if the readers were provided the effect size, $d = 0.8$, along with 95% CIs = [−0.1, 0.9], they would interpret the result to mean that the effect may not be statistically significant because the CIs include zero. In addition, $d = 0.8$ with 95% CIs = [0.7, 0.9] is likely to be more accurate than $d = 0.8$ with 95% CIs = [0.2, 1.3] because the range of the second set of CIs is larger than that of the first set. Narrow CIs that do not include zero provide precise statistical inference and power.

Although reporting CIs for effect sizes is considered an important key in inference studies for these characteristics (Thompson, 2002), CIs for effect sizes have not been reported in many studies (Byrd, 2007). This lack of CI reporting among researchers has been traced to researchers either not fully understanding or having misconceptions about CIs (Belia, Fidler, Williams, & Cumming, 2005). Furthermore, even when CIs are reported, this misunderstanding can lead to misinterpretation of the meaning of CIs, such as in the example referenced in the previous paragraph. By understanding the meaning and purposes of CIs when used with effect sizes, researchers can correctly report and interpret them.

7.6.2 Propensity Score Matching (PSM) for Sampling

For observational studies, many researchers use random sampling to select the sample for their study. However, random sampling could in fact cause selection bias because of the lack of randomization in assigning individuals to either treatment or control groups. For example, suppose researchers are investigating the difference in mathematics achievement between students in STEM versus non-STEM schools. They could conduct random sampling to select the same sample size in both STEM and non-STEM schools to use for data analysis. Although the total number of students from each group will be the same, the sample is likely to have a bias because students in this instance are not randomly assigned to either the treatment group (STEM schools) or the control group (non-STEM schools). The estimation of the effects of intervention may be biased by the existence of confounding factors. In the previous example, percentages of students' ethnicity or SES might be covariate variables for students' mathematics achievement. Therefore, to reduce the selection bias, it is better to control for covariate variables to obtain more significant and precise results. Use of propensity score matching (PSM) can help to reduce selection bias and thus strengthen causal arguments of predictors (independent variables; e.g., STEM schools and non-STEM schools) and outcomes (dependent variable; e.g., mathematics achievement).

PSM is a statistical technique in which the treatment case is paired with one or more control cases using values on the propensity score. The propensity score demonstrates the probability of being affected by the covariate variables, such as ethnicity or SES. If we have two participants, one in the treatment group and the other in the control group, and both share the same or a similar propensity score, we can reasonably examine these participants as if they were randomly assigned to their respective group. By accounting for covariate variables, PSM finds the best matches of treatment and control groups and eliminates unmatched cases. This process can minimize any confusion or uncertainty that can arise because of the covariate variables and reinforce the robustness of the estimates.

7.6.3 Structural Equation Modeling (SEM)

When researchers collect empirical data, they might be interested in observing relationships among multiple variables. In many cases, however, it is not easy to see the direct relationships among variables. For example, we can observe students' mathematics and science test scores, but these students' as emotions, attitudes, and values cannot be observed. Therefore, examining the relationships among these observable and unobservable variables is difficult. In this case, we can assign these unobservable variables as latent variables and construct a hypothesized model, in which we guess latent variables (e.g., emotion, attitude, value) through the examination of observable variables (e.g., mathematics and science test scores). We can test the consistency of our hypothesis about the latent construct of variables through a process called structural equation modeling (SEM). SEM is a statistical modeling tool that allows one to assess the validity of a structural model with empirical data by testing the hypothesis about the causal relationships among multiple variables (Lei & Wu, 2007). SEM can be conducted through specialized programs such as Mplus, LISREL, IBM SPSS Amos, and EQS.

7.6.4 Hierarchical Linear Modeling (HLM)

Samples in studies exist within a hierarchical social structure. For example, students are demographically situated within particular categories involving ethnicity, family, gender, grade, school, state, and country. Hierarchical linear modeling (HLM) is one technique researchers can use to investigate these hierarchical ordered systems. The use of HLM allows researchers to investigate simultaneously the relationships within and across hierarchical levels (Hofmann, 1997). In particular, one modeling will show the relationship within each of the lower level units, and the other level modelings will show how these relationships within units vary across units. For example, Bicer, Capraro, and Capraro (2017) applied a three-level HLM to examine Hispanic students' mathematics achievement in the context of their high school types, which were either STEM or non-STEM schools. In their study, Level-1 was the repeated measures of students' mathematics scores, which were nested within students. Level-2 was the students, who were further nested within schools (gender and ethnicity). Lastly, Level-3 was the school types as STEM and non-STEM schools. HLM software is most frequently used for the analysis. The current version, HLM 7, is available for up to four-level model analysis.

7.7 Conclusion

The essential components of this chapter can be consolidated into three broad categories: preparation, design, and execution. First, understanding the knowledge base, having team members committed to the project, and ensuring that the team possesses all of the skills necessary to take the project from conception to publication constitute the components of being prepared. Second, researchers require the skills to develop a research design that reflects the rich history of what is known about the topic, how it has been explored, and how the topic can be explored in new ways. The design can determine whether one's work appears in the highest-quality journals. Finally, execution! While learning is an essential component for us in academia, it is just as important to communicate that knowledge to others. It is only through execution that our planning and design prowess will evolve and our skills develop. Being a researcher is a path, not a destination, and the best researchers leave a legacy of continuous improvement, hard work, and a line of research their graduate students can use to foster their own success. Enjoy the journey.

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Chapter 8
Analyzing Qualitative Data in Mathematics Education

Martin A. Simon

Abstract The weak link in many qualitative research studies is the methodology for analyzing data. I define qualitative data analysis as *a process of working with data, so that more can be gleaned from the data than would be available from merely reading, viewing, or listening carefully to the data multiple times.* I describe and exemplify a recursive analysis process, which we use to analyze teaching-experiment data.

Keywords Qualitative data analysis · Teaching experiment methodology · Abduction in research · Empirically based mathematics education constructs

Arguably the most difficult aspect of qualitative research in mathematics education is the analysis of data. With improvements in technology, design of mathematical tasks, and interview techniques, mathematics education researchers are able to generate and record rich sets of qualitative data. However, the weak link in many mathematics education research studies is the methodology for analyzing the data (Thomas & James, 2006). In the first part of this chapter, I give some background to analyzing qualitative data. In the second part of this chapter, I elaborate and exemplify a three-level analysis process for data from teaching experiments (Simon, 2018; Simon et al., 2010; Steffe & Thompson, 2000).

This chapter was written to address three problems I have observed in qualitative studies in mathematics education research:

1. Many manuscripts lack evidence of any real analysis of data. (In the next section, I explain what I mean by "analysis of data.")

This chapter is partially based on Simon (2013).

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2. Many manuscripts and grant proposals do not specify the data analysis process. (The example of data analysis below illustrates specification of the analysis process.)
3. Where data analysis is specified, the methods used are often inappropriate for addressing the research problem. (I treat this problem only briefly by arguing that open coding is often used in a way that is not consistent with the goals of the research.)

8.1 What Does It Mean to Analyze Qualitative Data?

Qualitative research in mathematics education is not mainly observational in nature. Describing the phenomenon observed is what journalists do; qualitative research must go further. The contributions of qualitative research in mathematics education require complex analyses of, often, implicit phenomena (e.g., thinking, classroom norms, learning, affect). I define qualitative data analysis as *a process of working with data, so that more can be gleaned from the data than would be available from merely reading, viewing, or listening carefully to the data multiple times.* I use the following analogy from mathematics to demonstrate the meaning of this statement.

Consider the function table shown in Fig. 8.1. Most of us could probably look at it for a long time without being able to specify a function that would generate these data points. However, we might use the “data analysis” strategy of taking the first difference. The first differences might allow us to determine a relationship that we could not determine with the original data. We might also need to find second and perhaps third differences before we would be able to determine a relationship. Analysis of qualitative data is similar. *It is the application of levels of analysis that allows researchers to make sense of the data in ways that were impossible when working with the original data.* Note that this analogy is limited; unlike with this mathematical problem, qualitative data analysis generally does not lend itself to a routinized process.

Fig. 8.1 Collection of data points

1	-5
2	6
3	35
4	88
5	171
6	290

8.2 The Products of Qualitative Research

I am convinced from many years of working with doctoral students and novice researchers that one source of difficulty with regard to conducting quality research in mathematics education is the inability to conceptualize the potential products of their research. What kind of contribution could the research study make? What would constitute answers to the research questions? How might detailed research on a particular situation produce contributions to the research domain that are useful beyond that situation? Related to this last question, the researcher could be doing an in-depth case study of a particular teacher or student. However, no one other than the researcher is interested in that teacher or that student. What should be the product of that inquiry?

In qualitative research, particularly the methodologies with which I have the most experience—teaching experiments and interview studies—the product is often an explanatory construct that contributes to our collective understanding of some aspect of mathematics education. For example, the product might be a way of characterizing learning, teacher thinking, classroom practices, or a stage in students' development of a particular concept. Some well-known constructs that continue to be useful in the field are Yackel and Cobb's (1996) *sociomathematical norms*, Thompson's (1993) distinction between *quantitative and numerical reasoning*, and Van Hiele's (1986) *levels* of thinking in geometry. What is important is that these explanatory constructs cannot be *seen* in the data. One can be very familiar with the data but unable to generate useful explanatory constructs. So, if these explanatory constructs are not in the data, where do they come from? Clement (2000) elaborated on this point:

> Scientists often think in terms of theoretical explanatory models, such as molecules, waves, fields, and black holes, that are a separate kind of hypothesis from empirical laws. *Such models are not merely condensed summaries of empirical observations but, rather, are inventions that contribute new mechanisms and concepts that are part of the scientist's view of the world and that are not "given" in the data.* (p. 549, emphasis added)
>
> Essentially, the scientist aims to construct or piece together a theoretical model in the form of a conjectured story or a picture of a hidden structure or process that explains why the phenomenon occurred. Peirce (1958) and Hanson (1958) used the term *abduction* to describe the process of formulating a hypothesis that, if it were true, would account for the phenomenon in question. The initial hypothesis for a hidden mechanism is often more like an abduction than a recognized pattern or induction. It is a creative invention that must account for the results of the analysis. (p. 554)

Understanding the abductive nature of theoretical constructs in mathematics education is important to understanding qualitative research. Theoretical constructs are not the direct result of the two forms of reasoning most commonly associated with research: empirical reasoning (pattern noticing) and logical reasoning. Rather a third type of reasoning, abduction, is required. Thompson (1982) summarized what we might call an "abductive orientation" in his discussion of analyzing teaching experiment data:

> As [the researcher] watches a student ease through some problems and stumble over others, or successively ease and blunder through parts of a problem, the researcher asks himself, "What can this person be thinking so that his actions make sense from his perspective? …" This is the ground floor of modeling a student's understanding. The researcher puts himself into the position of the student and attempts to examine the operations that he (the researcher) would need and the constraints he would have to operate under in order to (logically) behave as the student did. (p. 161)[1]

The abduction needed to postulate theoretical constructs based on data is challenging to carry out and challenging to learn to do. It involves a creative act, the invention of a previously nonexistent explanatory mechanism to explain a corpus of data. It is a somewhat mysterious process. In the discussion (below) of the analysis process, I will indicate how researchers can create a foundation for postulation of new theoretical constructs.

8.3 The Lure of the Coding Process ("Grounded Theory")

"Despite much critique, [grounded theory] continues to enjoy great kudos amongst educators, to the extent that its use can still seemingly validate the publication of a study's findings" (Thomas & James, 2006, p. 768). In this section, my intention is not to review grounded theory or the coding process that is central to it. Rather, based on my experience as a reviewer of research articles, I describe a common and often inadequate analysis process done in the name of grounded theory and open coding.

Corbin and Strauss (2008) described open coding as "the process of breaking down, examining, comparing, conceptualizing and categorizing data" (p. 61). Although open coding is only the first step of analysis in grounded theory, and although grounded theory is a complex process, some researchers have seized on open coding as an easy-to-use and generally applicable methodology. As such, they divide their data into excerpts and develop an initial set of codes to characterize their data. They then re-examine their data to refine their codes and present the refined codes as their findings. One shortcoming of this approach is that a set of categories is often *not* an appropriate answer to the research question. A second shortcoming is that categorization is generally an inductive process (sorting the data as observed), not the multilevel process needed to gain new insights and work toward abduction of new theoretical constructs. Glaser (1992) wrote, "In grounded theory the analyst humbly allows the data to control him as much as humanly possible, by writing a theory for only what emerges through his skilled *induction*" (p. 87, emphasis added). If coding is only a sorting of the data, an opportunity may be missed.

In this section, I endeavored to set up a contrast between the analytical process exemplified (below) and a common use of "open coding." It is not meant to be a critique of grounded theory. Indeed, some might argue that the work described below falls into a broad definition of grounded theory; certainly it is theory generating.

[1] Cited in Steffe and Thompson (2000).

8.4 Before Considering Data Analysis

Data analysis is part of a research project. As such, it is dependent on what comes before. I view research as the assembly of an empirically based, coherent argument relative to a research problem. The up-front part of research is critical to the success of the project: articulation of the research problem/question, grounding in the literature, and specification of theoretical and conceptual frameworks. The methodology is made up of two parts: data collection and data analysis. The key is that each part of the research relates to each other part to create a coherent argument. Thus, the up-front parts must be done effectively prior to specifying a research methodology, and the methodology selected must fit well with those up-front parts of the work. In this chapter, I focus only on the data analysis part of the research methodology.

8.5 Analysis of Data

In my more than three decades of research, I have never found it appropriate to take a research methodology off the shelf and apply it as is. Whereas in quantitative research doing so is the rule rather than the exception, in qualitative research every study seems to be somewhat idiosyncratic in nature. Every researcher engaged in qualitative research must ask, "Given my research question(s), my theoretical and conceptual frameworks, and the current state of the literature, how would it make sense to collect and analyze the data?" In most cases existing methodologies can be adapted, but that adaptation is critical. The reader is referred to Simon et al. (2010) and Simon (2018) for explication of our particular adaptation of teaching experiment methodology to address our research problem.

Earlier, I defined data analysis as *a process of working with data, so that more can be gleaned from the data than would be available from reading, viewing, or listening carefully to the data multiple times.* In this section, I use examples from my teaching experiments to demonstrate how data analysis can afford opportunities not afforded by just viewing the data. Teaching experiments are characterized by two levels of data analysis, *ongoing analysis* and *retrospective analysis*. The ongoing analysis is done between teaching sessions and focuses on assessing results of the previous intervention and designing the next intervention. The retrospective analysis is generally done once the data collection phase has been completed and focuses on developing models of student thinking, analyzing the impact of the instructional intervention, and/or understanding aspects of the social interactions. In the examples that follow, I focus on retrospective analysis for two reasons: it is the more rigorous analysis and, given that it occurs post data collection, it is more relatable to data analysis in other methodologies.

Much has been written about the structure of teaching experiments and the theoretical basis for the methodology. However, little has been written about the specifics of retrospective analysis. One of the richest sources of discussions of teaching-experiment methodology is Kelly and Lesh (2000), which contains five chapters in a section devoted to the subject. Three of the chapters (including my own) have no real explication of the data analysis process. In one of the other two, Confrey and Lachance (2000) mentioned well-known methodologies such as matrix organization systems and grounded theory. Providing an example from their own research, they described a coding process they used for analysis of videotaped data.

My goal here is to demonstrate in greater detail a way of analyzing videotape and transcript data, consistent with the orientation of Thompson (1982, quoted earlier), which my colleagues and I use to understand the development of students' thinking/understanding. The analysis methodology is one that has evolved through four research projects.

In discussing our approach to data analysis, I will describe three levels of analysis.[2] Whereas distinguishing these three levels is useful in the articulation of the methodology, in practice, the levels are not always distinct in time. The researchers might engage in some second-level analysis before the first level is complete. A brief example of each level is provided.

8.5.1 First Level of Analysis

In this first level, we engage in line-by-line analysis. Because we are interested in understanding students' conceptions and the evolution of those conceptions, our line-by-line analysis must move us closer to this goal. The example that follows comes from a one-on-one teaching experiment aimed at understanding how an upper elementary school student, Kylie, is developing new mathematical abstractions through her engagement with a designed sequence of tasks related to fractions. Our line-by-line analysis seeks to infer answers to the following overlapping questions:

1. What was Kylie doing and why was she doing it at this point in the session?
2. What did Kylie mean by what she said at this point?
3. What was Kylie thinking at this point?
4. What might this show about Kylie's understanding at this point?

Several comments are in order here. First, this is an inferential process, but the inferences must stay closely tied to the data. Note that the questions above all relate to the specific segment of the data and do not call for broader inferences. Question 1 should not be interpreted as merely descriptive. As can be seen from the example below, what the student says and does are only clues to her mental activity. Second, because the researchers are only making local inferences, the research questions are

[2] Prior to analyzing the data in an extended study of multiple concepts, we sometimes attach codes to the data to keep track of which data are related to which research question. This coding is used to organize and manage the data, not to analyze it. The codes are not emergent but rather are related to our intended analyses.

not answered at this point. (Generally, all three levels of analysis are needed for answers to emerge.) Third, not all data segments will be successfully interpreted by the researchers at this point; sometimes we just need to record our inability to interpret a segment. Fourth, our analysis of segments analyzed later in this initial level benefit from what was gleaned in the analysis of earlier segments. Therefore, there is somewhat of a level raising even within the first level of analysis.

The following example demonstrates the analysis process of the first level. It is based on a small chunk of data in which Kylie verbalizes her thinking about one of an extended series of tasks in our division of fractions trajectory. The research question for this study was, *How can a student abstract (reinvent) a common-denominator algorithm for fraction division, and how can such an abstraction be fostered?*

Kylie had no algorithm for solving division of fraction tasks. She was given the task "2/5 ÷ 7/5=." The following conversation ensued:

K: Two sevenths. I think it was two sevenths [tone suggests she is not sure].
R: Can you explain?
K: I don't know (pause). Because there's seven pieces, I mean there's two fifths and you want to divide it by seven fifths. … There's seven pieces, which are fifths, but, they're kind of like sevenths but they're fifths. There are seven pieces. So, if you pulled out two, that would be two sevenths. But they're seven fifths. That would be two fifths, which would be two sevenths. The answer is two-sevenths [tone suggests she is now sure], two sevenths.

The reader likely finds the conversation somewhat difficult to understand. This is why the line-by-line analysis is necessary and why simply sorting this segment into an emerging system of categories is inadequate. Following is a brief summary of the local analysis done on this segment.

Kylie's verbalization of her thinking focused on the seven-fifths. She reported thinking about them as seven pieces, which she could treat as sevenths, even though she knew they were fifths of a unit. She then talked about "pulling out two." Why does she think about division in this way and what does she mean by it?

To answer this question, we needed to go back to how Kylie developed her concept of division of fractions. We would not have been able to do more than speculate about Kylie's thinking here without this information. Division of fractions was developed as the inverse of multiplication of fractions. Multiplication in general and multiplication of fractions were developed in the JavaBars Microworld (Biddlecomb & Olive, 2000). Kylie was told that that there is a (fictitious) function in JavaBars, the MULTIPLY button, which when you put in an input "*n*" and apply it to a bar (rectangle), it creates a new bar that is *n* of the original bars long. Kylie progressively extended her anticipation of the multiplication button from whole-number values of *n* to mixed-number values and finally to fractional values (See Simon, Kara, Norton, & Placa, 2018) . This was the basis of Kylie's notion of multiplication of fractions. Division was developed as a process of determining the input *n* needed to make the original bar into the new bar.

I offer one distinction about the analysis described in the last paragraph. Although our attempt was to analyze single lines of data, we use what we "know" about the student in making our local inferences. We considered that Kylie was thinking about division as finding the input for the MULTIPLY button. She would be thinking that the MULTIPLY button was making 2/5 out of 7/5. She seemed to think about the task as what part of 7/5 is 2/5. To do this, she indicated that she could think about the 7/5 bar as a whole with seven parts ("There's seven pieces, which are fifths, but, they're kind of like sevenths.") and that, if the result was 2 of the parts, she would be taking 2/7 of that bar ("So if you pulled out two, that would be two sevenths."). At this point, having thought through the task and reasoned with ideas that were clear to her, she was sure of her answer.

The example demonstrates several points made about the first level of analysis:

1. The first level of analysis can be performed on the raw data and can provide a basis for a more advanced level of analysis. By creating interpretations of each line, the second level of analysis can begin to look across these interpretations.
2. The first level of analysis does not in itself answer the research questions. No claims were made at this point about Kylie's understanding of fraction division, how it developed, or how the development was fostered.
3. The first level of analysis stays close to the data and works locally (i.e., on small chunks of data). Each interpretation was related to an individual line. Thus, there was no attempt by the researchers to give an interpretation of larger chunks of data. The trustworthiness of the analysis is increased by avoiding inferential leaps. Thus, each inferential step is meant to be modest and closely tied to the data in question.

8.5.2 Second Level of Analysis

Once the first level of analysis is completed, the second level of analysis is possible. I use "second level," not to indicate the number of passes through the data but to distinguish two different types of analysis. The second level might require multiple passes, each building on the results of the last. The second level extends the work of the first level recursively. That is, *we use the results of the first level as the "data" for the second level*, making inferences for chunks of these new data.[3] In doing so, we sometimes need to return to the original data.

In this second level of analysis, we are looking to answer the following questions:

1. What understandings did the subject exhibit? (Whereas in the first level, understandings were inferred for particular lines of data, there is now an attempt to use these local inferences to specify understandings represented by larger segments of data.)
2. How can we characterize the subject's thinking as she progressed from one understanding to the next?

[3] Note that in contrast to open coding, this is an interpretive process, not a categorizing process. The issue is what these data show, not how the data cluster.

I illustrate the result of this level of analysis with an example from Simon et al. (2010). To review the work that led to the result exemplified, the reader is referred to the original article.

Erin was a prospective elementary teacher with whom I worked in a one-on-one teaching experiment on division of fractions. In the session under study, Erin reinvented a common-denominator algorithm for division of fractions. Erin began the session by solving division of fractions tasks (that had common denominators) with diagrams, moved to mental runs[4] of her diagram solution, and finally knew the answer instantly when the task 7/103 ÷ 2/103 followed her mental-run solution to 7/167 ÷ 2/167. After this, she was able to solve tasks with common denominators by dividing the numerators. She could also make common denominators when necessary.

Following is an excerpt from Simon et al. (2010) that illustrates a result of the second level of analysis.[5] This second-level analysis was developed from a first-level analysis of Erin's thinking in her diagram solutions, her mental-run solutions, and finally her instant reporting of the answer to subsequent tasks. The second-level analysis specified differences in Erin's thinking and focus as she proceeded through the sequence of tasks. The "stages" in this excerpt refer to our earlier specification of the steps in Erin's activity as she solved tasks using diagrams.

> When she drew the diagram, her major focus was creating a diagram of the dividend in which the divisor-sized groups could be counted. [Drawing the diagram required] her concentration. Stage 2, identifying and counting the groups, was trivial for her, because it only involved counting by ones (twice). In contrast, when she was doing the mental runs, although she reported what size parts [the dividend] she would draw (Stage 1), she did not need to concentrate on the creation of the dividend, because she had no responsibility to actually draw it. Her concentration focused on the number of parts available and the number of parts in a group, the numbers she would need to work with to determine the number of groups (numbers that were to be related using multiplication)…. Erin went through the mental runs knowing that the step-by-step approach would result in a correct solution. We infer that in doing the mental runs, Erin developed a tendency to foreground the numbers on which she needed to act. However, when she began to foreground the numbers in this third problem (7/103 ÷ 2/103), she recognized that she was poised to carry out the same operation with the same numbers. (pp. 104–105)

Although, the reader does not have sufficient information about the study to evaluate this interpretation (without referring to the original paper), two points can be made. First, the excerpt demonstrates the attempt to characterize Erin's thinking as it evolved across tasks. Second, this level of analysis could not have been done without careful analysis, at the initial level, of Erin's work and verbalizations on each of the diagram drawing tasks and each of the mental run tasks.

[4] The mental runs were the student's narration of what she *would do* if she were to draw a diagram to solve the task.

[5] The data on which this analysis is based are too extensive to share in this chapter; the second level of analysis is based on multiple first-level inferences, each representing a chunk of data. The original article gives more detail.

8.5.3 Third Level of Analysis

At this third level of analysis, we work directly on the development of explanatory constructs. The explanatory construct is important for two reasons. First, it offers an explanation for the phenomenon revealed by the analysis. Second, the explanatory construct offers an idea or the extension of an idea that may be useful to the field (whereas attention in the first two levels was primarily on the data). In Simon et al. (2010), from which the example for the second level was drawn, we explicated several constructs and theoretical distinctions based on our analyses of the data. Two are particularly relevant to the segment discussed in the prior section. The first is a characterization of Erin's learning as reflection on activity. The second is discussion of the importance of a particular type of focus of attention. I use the following excerpt from Simon et al. (2010) to illustrate the former.

> Earlier we characterized Erin's quick response to 7/103 ÷ 2/103 as the development of an anticipation. We argue that her new conception was based on an anticipated commonality in her activity. The reason for this claim is the following. Erin's quick response the first time she saw problems with the same pair of numerators suggests that she anticipated something being the same in both problems, that is, that the second of those two problems was the same as the first in some way. The issue then is a commonality in what. One possibility is a perceived commonality in numbers in the problem (e.g., numerators). On the pretest, Erin was given two division-of-fractions examples, A. 17/123 ÷ 13/123 and B. 17/79 ÷ 13/79 and told not to compute the answers. She was asked whether the answer to A would be larger, smaller, or the same as the answer to B. She was unable to answer the question. One can assume that in comparing the two expressions that she saw that the numerators were the same. Thus, one can argue that seeing a commonality in the numerators was not sufficient for her to anticipate the equality of the expressions. A second possibility is a perceived commonality in the quotients. However, Erin never saw a pair of problems yielding the same quotient prior to her anticipation of the quotient of 7/103 ÷ 2/103. Therefore, we argue that the objects of Erin's perception of commonality were her activity of solving the previous problem (7/167 ÷ 2/167) and the activity she was poised to enact on this one (7/103 ÷ 2/103). Using the definition of reflection that we presented earlier, we suggest that reflection, the innate ability to recognize commonalities in her experience, resulted in her seeing commonality in her solution activity—the one she had just carried out and the one she was poised to carry out. (pp. 105–106)

In this example, the third level of analysis is demonstrated. Using a theoretical construct that we defined earlier in the paper, reflection, we provided an argument for viewing the data and analysis as a case of a theoretically important phenomenon, reflective abstraction (discussed elsewhere in the article), and offered a particular elaboration of the construct, reflection on commonality in one's activity. Note that although we were engaged in third-level analysis, the generation of explanatory constructs, the analysis still stayed close to the second-level analysis on which it was based.

8.6 Conclusion

Qualitative data generally do not tell us much directly. Even categorizing what is observed in the data often gives us a limited set of insights. In this paper, I have used a particular methodology to illustrate how qualitative analysis can lead to insights not available through merely viewing or categorizing the data. I discussed three levels of data analysis that together provide a process for raising the level of conclusions that can be drawn from the data. By *level raising*, I mean that the insights are at a level that was not available when the data were originally considered.

Earlier, I referred to the mystery of the abductive process—the process by which a researcher generates an explanatory mechanism. Whereas the mystery of the abductive process is likely to remain, I find that the first two levels of analysis described support the generation of explanatory mechanisms (third level). That is, by building up one's inference progressively, the demand on the final abductive step is reduced.

Although the examples of data analysis I have used derive from teaching experiments, the ways of thinking about qualitative data analysis are potentially useful for other methodologies as well.

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Chapter 9
Leading a Design-Based Research Team Using Agile Methodologies to Build Learner-Centered Software

Jere Confrey

Abstract Using an autobiographical approach, the author reports on over 30 years of designing software to promote learner-centered instruction. She describes what she learned about organizing her teams, first to emulate a scientific lab and, later, to leverage agile software development approaches. She recommends an agile approach as a means to plan and coordinate complex design activities and to emphasize active involvement by all members in the research process. The approach can improve team creativity and productivity.

Keywords Design-based research · Agile methodologies · Learner-centered software

The question addressed in this chapter is "How does one organize an active research program and team in which the *design of learner-centered software* is of primary importance?" Over the last 30 years of my career, I have designed and built numerous pieces of software: Function Finder, Function Probe, Interactive Diagrams for Precalculus, Multimedia Precalculus, GraphsNGlyphs, LPPSync, TurnOncCCMath, and Math-Mapper. Each was designed to promote research on how students think about a variety of mathematical topics. The complexity of the software systems built by my research teams and the technology available to do this work have changed dramatically over time, but the fundamental activity of design has always driven my work, stimulated my creativity, and allowed me to discover a great deal about how learners think about mathematical ideas. Throughout this work, I also learned a fair amount about how to organize a team to thrive in the academic context and how to work toward providing sufficient funding to ensure a team's continuity. I worked in academic settings at multiple universities and in two primarily commercial ventures—with a corporation and with my own company. Through these

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various ventures—some more successful, some less so—my career of research and design has been deeply fulfilling.

Through this chapter I hope to share with a next generation of scholars some of the lessons I have learned from those experiences. I have organized the chapter around three phases of my career: an early phase during which I learned to design and build software, an intermediate phase in which I worked to develop products in a corporate setting, and a later phase which has found me bringing the techniques of agile design back into an academic setting.

9.1 Phase 1: Teamwork and Connections to Practice

My earliest software application was Function Finder (Confrey, 1991), a simple “Guess My Rule” tool for finding a linear function’s slope and intercept from a set of points. Characteristic of all my software development, I interviewed students and designed the tool to leverage their strategies. These results led to a tool in which students could propose x-coordinates that returned y-coordinates, from which they guessed the m and b of the equation $y = mx + b$. Students had to use increasingly sophisticated strategies, because the tool blocked their reliance on the easiest strategies—for instance, disallowing the use of zero, consecutive values, and symmetric values around zero.

To design for learner-centeredness, one must study learners directly and commit to fostering *their* ways of thinking. The challenge is to recognize the innovation in their ideas. Over the years, I have developed a deep conviction that, in the vast majority of the time, if I decenter to their way of thinking, students make sense. Using Function Finder, one young woman student entered the values of 1, 10, 100, and 1000, instead of the values (such as low numbers or zero) that are chosen more commonly. Her choice, for instance, for the linear function $y = 3x + 4$, produced 7, 34, 304, and 3004. Her strategy separated the effect of the multiplier from the adder, quickly making apparent the sought-after parameters in the equation. I tell this story to illustrate that one only needs an idea about interesting ways to elicit student ideas and a programmer to get started in a career of learner-centered software design.

Identifying good software engineers is essential because their role is critical to the success of your team. Building Function Finder was my first foray into learning to identify good software engineers for such a learner-centered research team. Essential qualities in a programmer for this work, beyond coding expertise, are flexibility and the ability to listen. Many programmers have a measure of competence in mathematics and share an appreciation for its structure and logic. However, that competence can obscure a designer’s goal of designing to encourage students to approach the topic from their own points of view. Over the years, I have worked (albeit briefly) with the occasional engineer who would listen to me and then build what *he* (and most, but not all, were male) thought was best, typically sacrificing student-centeredness for perceived “efficiency.” It is of the utmost importance to foster an engineering team with more than a passing interest in serving kids. As the designer, you must welcome your engineering team's feedback but persist in your vision and hold design accountable to its ultimate users—students and/or teachers.

At the same time, as the designer, I had to learn to listen too. The engineers know the tool, its code, and its possibilities. They are continually solving problems of which you are not aware. A talented engineer will warn you of an approaching critical juncture, perhaps by responding to your feature request with "well, everything is possible, but how long can you wait and how much of a priority is this?" As the designer and principal investigator (PI) on a grant with limited funding, you must take this question very seriously. Your budget is never excessive, and, as every software engineer and project manager will remind you, the last 10% of the project takes up to 90% of the time—warning that finishing take time. In addition, and especially in education, you are building with real deadlines. For example, the beginning of the school year does not wait for an incomplete product.

The second application I designed and built, with support from Apple Classrooms of Tomorrow, was "Function Probe," a multi-representational software tool with a grapher, table, and calculator, built in the late 1980s for a precalculus course organized around families of functions (Confrey & Maloney, 2008). I had two primary commitments for the application's design: (1) to let each representation incorporate affordances appropriate to its structure and form (e.g., to let graphical actions be visually driven, rather than algebraically driven) and (2) to both avoid overautomating and allow the learner to proceed as he or she wanted. The graph provides an example of the first commitment: stretch transformations, for instance, required graph action, such as first selecting and placing an anchor line from which the dilation would originate and then stretching by "grabbing the curve" and extending out the iconic spring of the "stretch."

Detailed awareness of the decisions underlying a well-designed tool is critical not only for clarifying requests to the engineering team but for revealing situations in which the designer's *own* understanding requires additional clarification. Much of Function Probe's design was undertaken with painstaking use of a design mockup tool called HyperCard. Each individual screen and button state was diagramed and annotated. Fortunately, for current design, this tedious process has been replaced by building wireframes using user-interface tools such as InVision Studio (InVisionApp, n.d.).

9.1.1 Team Building

During creation of Function Probe, I learned my first lessons from outside education about building teams and running a research group. My husband was a post-doctoral fellow in a science department. I watched how his lab director organized his team: making primary decisions about what system (i.e., organism(s)) and central questions they would work on, assigning scientists in the lab to work on sub-questions, and generating an overall research design for the whole system. This research model is designed to *accumulate* knowledge, something that is too often missing in educational research where novice scholars may shift too frequently to follow their perceptions of fundable research rather than building for continuity.

I imitated the science lab approach. Graduate students and post-docs were required to work on the broad area of precalculus instruction that would be facilitated by software, using a multi-representational approach, grounded in contextual problems. Requiring students to work in my area of expertise was sometimes looked at askance by faculty colleagues as a form of intellectual narrowness. To my mind, there are three compelling reasons to require grant-supported students to work within a shared perspective: (1) the PI is obliged, by accepting funding, to try to produce dissertations in the topic; (2) students benefit from working in their chairperson's primary sphere of expertise; and (3) as a PI, one becomes more productive and efficient in one's use of time. In science, this is expected. It is a key element of programmatic research. If more education faculty followed this model, I would predict that a more focused field, and a generation of generally higher-quality dissertations, would result.

Two other operational commitments followed from my decision to organize like a scientific lab. First, I required research space to conduct interviews and design studies, where my team could be co-located. I cannot overemphasize the advantages, in creativity *and* productivity, which come from the entire research team being in close proximity to each other. So much occurs through direct communication, through sitting around a conference table working through a problem together, or through designing sketches on whiteboard walls. (A center or institute that is supportive of active research teams can provide other desirable options for community and productive work, both within and between teams.)

But co-locating a research team is surprisingly atypical in education, and procuring adequate, long-term dedicated space in education can require determination and sacrifice. The best approach is to negotiate research space during one's hiring process, which can be done upon arrival, with or without bringing external funding. Most scientists are expected to request, and receive, lab space and start-up funds to support materials, lab equipment, and even technical staff. Start-up costs for outstanding faculty conducting empirical scientific research (vs. theoretical) can run to the hundreds of thousands of dollars. Educators should make similar, albeit less costly, demands.

To afford my research groups adequate co-located research space, over the years they have been located in a Quonset hut (Cornell), an old university-owned real estate office (St. Louis), and renovated off-campus office space (currently at North Carolina State). One current approach was suggested by the previous Vice Chancellor for Research. She pointed out that I could charge a lower indirect-cost rate (26%) to my grants by locating off-campus and build rent expenses into the grant. Indirect costs are a percentage of the budget that goes to the university and college for general expenses and can run, on average, near 50% of the other budget costs. Some simple math reveals that, depending on the size of one or more grants, more of one's funds can go to research by paying the lower indirect rate and rent. It may not make your administrators completely happy with you, but over time, such practices may encourage education deans to be more aggressive about securing faculty research space; typically, they want the faculty to bring grants in but do not secure adequate space.

There can be downsides to this approach, however. You are likely to be located at a distance from your colleagues. Being "out of sight" can indeed make you "out of mind" relative to your colleagues and administrators—a risky proposition, especially prior to tenure review. However, one should keep in mind that the best insurance for your tenure is to conduct a robust program of research, so one should prioritize to organize for success.

9.1.2 Design Research: Research Foreshadowing Agile Methodologies

During this first phase of my work, another signature commitment emerged: staying close to active and ongoing classroom practice. Over the years, I have always had an educational practice nearby under study. Originally, this was the college precalculus course I taught. Then it was a nearby elementary school, at which I conducted a 3-year teaching experiment. At the University of Texas, it was a systemic partnership with a local high school. Since I have been in North Carolina, my labs are designed to run in-house design studies, and my team conducts design-based implementation studies using our software in schools in North Carolina, New Jersey, and Colorado.

Ongoing and meaningful ties to practice are very important. While they predated my introduction to agile methodologies in software design, they prepared us for the agile approach. The example I will use for this part of my professional experience was with a software called LPPSync (Learning Progress Profiles Synchronized through networking, supported by NSF and Qualcomm; Confrey & Maloney, 2015). LPPSync delivered diagnostic assessments to students using iPads. Conducted in 2009–2010, the project took place as the use of technologies began to require increased networking capability. It involved a summer program with a group of academically high-needs youngsters in grades 2–4. Our goals were to teach them the ideas of equipartitioning and to involve them in their own learning by sharing with them the results of periodic diagnostic assessments taken through their direct action on virtual manipulatives on the networked software.

The first lesson about staying close to an ongoing classroom practice is that "it takes a village" to conduct design study research (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003). Often there are nearly as many staff members as children during a design study; this was certainly the case in studying LPPSync (Confrey & Maloney, 2015). Each day, challenges arose around the newly created curriculum materials, the software (specifically), the technology (in general), and the academic and social needs of our young energetic urban students.

When working with young children, the tolerance for error is low. The networked application was new, and we needed to constantly adjust the bandwidth to avoid slowdowns or hang-ups with the software. We had to respond rapidly to the needs of the children, who came from impoverished-school settings and, as with all children, brought with them their own strengths, weaknesses, expectations, and tolerances for ambiguity. The kids loved using the iPads, but with little experience with such devices, they had to learn quickly to treat them with care. We observed that the children would return each day eager to learn new things, but the teaching team noticed that the students often exhibited difficulty retaining the previous day's ideas. We needed to quickly convey new norms and expectations about learning and to find ways to help them hold on to the prior day's discoveries. That challenge to our team was urgent, and somewhat unexpected.

A central characteristic of design studies is that your conjectures are continually being tested, not simply being carried out as an intervention to gauge effects (Prediger, Gravemeijer, & Confrey, 2015). A major management challenge was to balance the team's attention to those different elements, especially in light of the fact that the next morning—and the whole passel of energetic, boisterous, exuberant kids—would arrive promptly every day. Therefore, when the children departed at lunchtime, the team engaged in debriefing sessions that systematically considered (a) curriculum, (b) technology, (c) instruction and planning, (d) practical concerns, and (e) data gathering. Afternoons would find us rushing to work with engineers to solve technical problems, gathering, digitizing, and storing the artifacts of student work, devising or developing a new activity to instructionally test an emerging conjecture the next day, or planning an interview to investigate an unexplained phenomenon.

Data-gathering periods are intense and demanding. They require daily planning and logistical coordination, as well as delegation of team members' responsibilities—and a daily practical attention to detail that, for younger research staff members, goes beyond what tends to be covered in research methodology courses. It is imperative to review the quality of the daily data to ascertain whether it is complete and accurate. More than once we found that although a graduate student or staff member had carefully observed the session firsthand, he or she had failed to check camera batteries, memory, or recording-sound quality. The videographers needed to learn that if they failed to focus on optimal camera placement and sound monitoring, and to make data gathering their priority, good data would be lost at a high price to the work. Knowing how to discern what merited attention, especially during periods of small group work, required both understanding of the research and technical skill with the camera. One experienced teacher on the team would typically be devoted to the needs of the children: for supervision on arrival, breaks, and departure, monitoring on task behavior, taking care of the students' basic needs (including snacks), communicating to parents, and planning for the culminating activity for parents, families, and friends. The rest of the research team had to quickly process the day's events, attend to reflecting on and interpreting what had transpired, thoughtfully consider alternative descriptions and explanations, and leave sufficient time for planning and anticipating the needs for the next day.

There are a number of research papers on the conduct of teaching experiments and design studies (Brown, 1992; Cobb et al., 2003; Confrey, 2006; Gravemeijer &

Cobb, 2006), so this is not the place for a deep discussion of method. The purpose of conveying this story is to emphasize that even when prototyping and designing the software to be effectively learner centered, one must have means of engaging with ongoing practice. These episodes prove critically important, not just during the retrospective analysis of the design studies themselves but for their value in creating among the team over time, a shared practice that guides our theoretical ideas. Years after a design study, team members will recall a moment or an exchange between students, to remind us both of what happened and what it means to be genuinely learner centered. During my first phase of undertaking software design, I developed my commitments to listening to students and staying close to practice. I also learned many lessons about how to organize and run a team to undertake programmatic work.

Doing such work within the context of regular academic life amounts to conducting a juggling act of teaching, designing, research, and writing while keeping up one's obligations to service and public presentations. When an opportunity came to apply these lessons in a corporate setting, freed from many of the academic obligations, I chose to accept it.

9.2 Phase 2: Agile Design in a Corporate Setting

Beginning in 2012, I spent 2 years as the Chief Mathematics Officer of a major commercial curricular design project, embarking on major adjustments to a completely different work context. This section explores two intertwined themes of my experience during my 2-year sojourn at an educational technology corporation. This and the following (Phase 3) section draw comparisons between my experience in the corporate and university settings as well.

There were certainly experiences that I can only describe as "ironic." Imagine arriving at a high-tech office to head a team with boxes and boxes of books and looking around to see virtually no books except a few software manuals. Books are disappearing these days from even academic digitally based work environments, but at that time it was a jarring contrast with the academy. I surreptitiously hid my books behind a row of desks and later placed them more kindly in an office library where I would spend time enjoying their smell as I worked out design ideas.

There are major differences in how work is conducted between a conventional education/academic setting and corporate settings, across the various teams of designers, engineers, marketers, etc. For instance, a first lesson was to distinguish between a product manager and a project manager. The product manager is hugely important for delineating what the product should be to serve its purpose, while the project manager ensures that the trains run on time, overseeing the "how" of product development. We could use such roles in the academy—though typical budgets probably preclude their use.

Transitioning back to the academy also revealed some interesting contrasts. For example, shortly after I returned to the university, in a request for Institutional

Review Board (IRB) project review, I used the term called "playtesting." Playtesting is an industry term for early software trials with users to see whether a design concept itself is viable—and it is viewed as an essential part of software development. But the University IRB office understood the term as a free-for-all for the kids, and before they would approve the request, they challenged me to convince them that "playtesting" was a legitimate methodology.

Fundamental to the message in this chapter is the use of what is called *agile*, rather than *waterfall*, methodology. There is room in this brief chapter only to provide an overview and a few references, but I suggest that understanding this concept would dramatically influence how design-based research is conducted. The *agility concept* in software design came about as software became increasingly complex. Traditionally, software managers assumed that software building was a process similar to assembly-line production: one would specify goals, create a complete and well-documented set of requirements and specifications, specify the architecture, and hand off the completed plans to a group of engineers, each tasked to build a particular component. This older, conventional method of software development was called *waterfall*, because it sprang from the top, with expertise and decision-making assumed to cascade only downwards from supervisors to coders. During the 1990s, it became evident that the waterfall model was not sufficient to adapt to the rapid evolution of technologies. Furthermore, it assumed that the designer completely and perfectly understood the client and that clients' needs remained stable over the course of development. Numerous failures (in time, in cost, and in final products themselves) made it evident that smarter ways to craft code were required to make better use of the human capacity, the ingenuity of the coders, and the morphing relationships among designers, engineers, technology, and the needs of customers, clients, and markets.

In 2001, 17 software developers, convening in Snowbird, UT, brought together ideas that had surfaced over the prior decade, coalescing them into a methodology labeled "agile software development." They authored an "agile manifesto" consisting of 12 general principles (Agile Alliance, 2001):

1. Satisfy the client and continually develop software.
2. Embrace changing requirements for the client's competitive advantage.
3. Deliver working software frequently, on the shortest practicable time span.
4. Developers and businesspeople must work together throughout the entire project.
5. Build projects around motivated individuals. Give them the environment and support they need, and trust them to get the job done.
6. Face-to-face communication is the best way to transfer information to and within a team.
7. Working software is the primary measurement of progress.
8. Agile processes will promote development that is sustainable. Sponsors, developers, and users should be able to maintain an indefinite, constant pace.
9. Constant attention to technical excellence and good design enhances agility.
10. Simplicity—the art of maximizing the amount of work not done—is essential.

11. Self-organizing teams usually create the best architecture, requirements, and designs.
12. At regular intervals, the team reflects on how to become more effective, then tunes and adjusts its behavior accordingly.

Agile methodology is a light, nimble form of development that keeps the developers' focus forward to the value they are creating in a product and helps the developers to avoid becoming mired in a priori—and potentially obsolescing—expectations and plans. It allows the team to respond to opportunities that arise during software development. By iteratively planning and building using continuous feedback, agile development allows one to make essential modifications as a product is built. The idea is to optimize the software throughout the development and release cycle by continuously updating and improving it, through a collaborative and self-organizing approach. It is designed to foster a high degree of creativity in the design team, all members making positive and active contributions to design and development.

Working in a corporate setting using an agile approach was both exciting and disorienting. The promise we aimed to realize was to break out of some of the typical tiresome, dull regularities of what is defined as "doing mathematics." Our excitement derived in part from having significantly more resources to approach a design challenge. My mathematics curriculum team worked with an array of different sorts of designers; they debated among themselves about the look and feel of the planned product during dizzying design sessions replete with sketches and colored sticky notes all over the whiteboard walls.

The results could be breathtaking artistry or moments of hilarious comedy. For instance, in creating a video for a problem in which the student was asked to predict if a fully wound up walking toy monkey would stop short of, or fall off, the end of a table (by mathematically predicting the distance it would cover as a function of the number of steps), the designers surprised us by ending the scene with the windup toy marching off the table and falling into a bevy of windup crocodiles, all with their jaws clacking.

And it could be disorienting to have to be constantly vigilant about the mathematics. Not atypically, the correctness of the mathematics, or the opportunities that were planned for the students' mathematical thinking, suffered from a (well-intended) designer's overly scripted scenario. For instance, we developed a problem about a basketball team going for pizza in a restaurant. The problem was intended to produce a variety of configurations of table sizes and players while still maintaining fair sharing based on how the players and pizza would be distributed. Not understanding the mathematical intent, the artist produced drawings with only one table size. An agile approach with large disparate teams supports autonomous contributions by different groups, but commitment to learner-centered mathematics also requires everyone to recognize the need for careful coordination and communication around the details of the mathematical and pedagogical intent.

Agile designs often begin from user stories. User stories are descriptions written in the form, "The student or teacher wants to…." As a researcher, I frequently designed based on a single objective at a time. But experienced agile designers generate as many varied options for user stories as possible, which adds ultimately to the flexibility of the tool. Part of the agile team is comprised of two kinds of designers, each with a different orientation toward users. One type of designer focuses on *user experience* (UX), the other on *user interface* (UI). The UX designer is responsible for the overall look and feel of a product. The UI designer is responsible for how each particular page is arranged and interacted with, tap by tap or click by click. Just as user stories are important, these two aspects of design, the overall user experience and the well-ordered way of navigating the interface, are essential considerations in a well-designed product. Beyond these two kinds of orientations toward user stories, other important orientations include the view of a user from the perspective of the learning sciences team and from the view of the outreach team. Bringing these diverse user stories together successfully is a critical element of agile approaches.

Two critical elements of the process of agile software development are *sprints* and *scrums*, which set the velocity of the build and help achieve the goal of building an integrated whole. Sprints involve focusing on a portion of the project and then breaking it into smaller segments and short cycles. A sprint is a 1- to 2-week process during which a particular goal or target is reached. A project management team plans the overall development, subject to revision, and then teams are formed to take on sprints. Sprints help teams focus and set achievable short-term goals. A "scrum" is a weekly meeting to plan the activities of the team. During a scrum, tasks are parceled into categories that describe what has been completed, what will happen during the current week, and what is upcoming in the short- and long term. In the next section of the paper (and phase of my work), I discuss how organizing research into sprints with scrums can help a research team to conduct complex projects more efficiently while more fully activating the creativity of various team members.

A final contrast between the corporate and academic setting worth considering rests in views of research. I believe both types of organizations could learn important lessons from each other with regard to this difference. During my sojourn in the corporate setting, market research was routinely confounded with scholarly research, and the result could too often be that only the results that confirmed the value of the product were valued and reported. For example, in the corporate setting, "field" or "pilot" studies were often conducted predominantly (a) to detail the process needed for success in an early release of a product or (b) as a means to recruit early adopters and not to ensure the effectiveness of the product. In academic research, we take for granted that failure in a research effort can stimulate alternatives and provide insights. Rather than accepting and learning from failure, in the corporation, failure was papered over because the product managers worried more about positive product reviews for supervisors and a continuing robust project budget.

Corporate research does have some advantages. By focusing on the users (the ultimate target of sales), you learn to avoid unreasonable expectations of changing users to fit your idealized conception and to consider more carefully how to get a product out of the blocks in order to make the users' work life easier or to satisfy a felt need.

It is also important to help corporations recognize the need to conduct their own design studies, especially early in the process of building innovations. It took me months to get the leadership of my corporate setting to accept the need for studies of the degree to which the curriculum achieved learner-sensitive design *and* leveraged student thinking. A properly balanced agile environment can incorporate many perspectives on research and market research leading to products that can be innovative, effective, scalable, and sustainable.

9.3 Phase 3: Bringing Agility to the Campus

Upon my return to the university after 2 years, I sought to apply an agile approach to research. Along with this process came a heightened sensitivity to a major challenge of engaging deeply in design and development projects in the academy: the resulting stress on the project leader. Part of this stress results from the disconnect between the aims and (necessary) speed necessary in innovative software development, on one hand, and the traditional responsibilities and requirements of the university faculty profession on the other. For instance, faculty are typically evaluated every year primarily on the publication of project research papers. However, researchers who build original software (in education research setting) receive little acknowledgment of the creativity and effort involved in production of the tool itself. All the while, publication of results of research with the software must be delayed until the software is usable by end users—and in my experience, education journals are far less interested in design issues than in publications that contain "results," even though cutting-edge research involves a large component of research into design and implementation. Further, the project leader usually shoulders the primary responsibility for gathering resources to keep all members of the team employed (with often noncompetitive salaries for technical expertise within the university, depending on your geographic location), generate new approaches, and assign responsibilities and timelines. All of these responsibilities are then combined with expectations of service (within and outside the university), teaching and guiding graduate studies. To manage this variety of tensions, especially around publication, one can follow publication of high-impact papers with shorter interpretations of the implications of the research for practitioner journals. Also, it is wise to balance the load of doctoral students. Six is ideal, with two beginners, two intermediate level, and two advanced. Some of the mentoring is then carried on among the cohorts.

I had learned a number of major lessons while working in the corporate sector, and I had a renewed commitment to fold those lessons into my research and build a cutting-edge project that pushes the field forward and solves major problems in student learning and teacher classroom support for student learning. This was my responsibility to my field and to students and teachers and is the kind of charge that we have in the academic, sponsored research. But the return to the academy was not a one-for-one exchange in responsibilities and time commitments. This kind of project, and these goals, means that in many ways, a university professor has to assume the role of project director or entrepreneur (in the corporate sense) while layering on the additional traditional responsibilities and expectations of university education faculty. By working agilely, the leader can share the leadership across the team within a system of self-regulated accountability and decrease the likelihood of his or her own burnout. I illustrate how one can navigate this complicated set of responsibilities by describing a project (the Math-Mapper 6-8 project or MM6-8) I have worked on for 5 years since returning to the academy.

9.3.1 Project Context

During this last phase of engaging with software design, I strived to bring the concepts of agility to the campus. I had rekindled my interest in learning maps, based on the map I built to unpack the Common Core Standards into learning trajectories [LTs].[1] I received a grant from the Bill and Melinda Gates Foundation to build a new map. I had four design goals for the new map:

1. Build the map's foundation on LTs and the associated research.
2. Diminish dependence of the map on the vagaries of standards, and insulate it from the politicization of the Standards in the USA.
3. Strive to make the map's constructs consistent in size across topics (in part to avoid variation in Standards' grainsize).
4. Allow standards to be associated with multiple locations on the map and vice versa.

The map was designed to take the idea of LTs to scale. Hierarchically structured, it comprises nine big ideas within four fields; the big ideas collectively contain 23 relational learning clusters (RLCs), with a total of 62 constructs (each with an LT). It includes a diagnostic assessment and reporting system, built on sound measurement models, to track students' progress along the LTs. It also includes a resource library that illustrates how to leverage free web-based materials related to the LTs and a scheduler to promote customizing a district's scope and sequence to support the use of LTs and the diagnostic assessments (Confrey, 2015; Confrey, Gianopulos, McGowan, Shah, & Belcher, 2017).

In inventing the design and use of a learning map around LTs, my team and I were contributing to a new genre of educational software, a digital learning system (DLS) (Confrey, 2015) that would be an extensible container for the use of online

[1] During my service on the National Validation Committee for the Common Core (which, in part, reviewed the standards in relation to research and international standards), I had built a hexagon map that divided the K-8 standards into 18 LTs, each consisting of a network of standards (see turnonccmath.net). In order to help teachers understand how the standards related to the research underlying the LTs, my team and I had unpacked these LTs to describe the related research on learning (Confrey & Maloney, 2014).

curriculum materials. It would also evaluate their effects and provide a new genre of classroom assessment, formatively aiding teachers in improving instruction as they taught. As the project progressed, we gradually came to understand that it would also support continuous curriculum modification based on data on student learning—a concept we labeled "the agile curriculum" (Confrey et al., 2018). A major goal of the project was to pioneer new forms of assessment, closely connected with the genre of formative assessment but more systematic, diagnostic, and based on both an explicit learning model and feedback of data to students and teachers. This new genre of assessments is being labeled as "classroom assessments" in contrast with interim, benchmark, or high-stakes assessments (Pellegrino, DiBello, & Goldman, 2016). Because we were embedding them in an ongoing DLS, our assessment system required a continuous process of validation of the assessments of LTs (Confrey & Toutkoushian, 2019; Confrey, Toutkoushian, & Shah, 2019). The scope of the project was substantial and daunting.

The team assembled for this project was also complex. On the technical side, at its height, it consisted of three software engineers, a UX designer, and a number of student assistants who supported revision and insertion of items into the system after authoring. The research side consisted of multiple learning scientists, a psychometrician (tasked with leading the establishment of a model for validity of the assessments), and an outreach director.

The final, and perhaps most important, part of the complexity of the project was the ongoing relationship with the field sites. One district approached us because of curricular leaders' knowledge of my prior work on LTs. Another school came on board as the result of our hiring a teacher from that school to assist in item writing. These partners are of fundamental importance to the work in multiple ways. The data to validate the LTs comes from the teachers' use of the assessments at the sites, and we sought to learn from the practices they developed in returning and discussing student data. In addition, interviews with teachers and students provide user data we needed to improve the application. We had considerable access to and buy-in from the school site administrators, who played crucial roles in (a) advocating for the use of MM6-8 and the assessment data in their teachers' instructional decision-making; (b) providing insights into competing pushes and pulls, such as the administration of district benchmark assessments; and (c) adjusting teacher staffing to serve the needs of students, in light of the variations in results in different classrooms.

9.3.2 Examples of Agile Methodology Integration

Building and refining the Math-Mapper project, based on our research on how teachers and students review and act on the data, required us to implement agile methods, using tools and practices that are commonly used in commercial ventures but not widely familiar in the academy. Applying an agile approach has been essential to the success of this project. I will describe how an agile approach was implemented technically and then reflect on how we used its components to address some of the project's challenges.

One critical tool to facilitating our work was Trello (Atlassian, 2017), a web application that helps visually organize work and monitor progress across teams. In Trello, one creates "boards." We have multiple boards for components of the overall project. Our major project board is divided into multiple status columns, each consisting of a set of "cards" labeled "backlog," "ready-to-start," "in-progress," "for review," and "completed." Tasks are migrated to different columns depending on tasks' status. When posted, a task consists of the task name, a checklist of its parts, and a list of responsible members and their roles. Color coding identifies the relevant team(s) (software engineering, metric, learning sciences, work with students and teachers, curriculum, or practical needs). Each project member is typically associated with multiple teams.

Task definitions must be precise enough to define a well-bounded, 1-week task. If tasks are too broad, they tend to need to remain on the board for a long time, and one loses a sense of urgency and progress. If tasks are too narrow, the board gets overwhelmed. Part of the purpose of the scrum, the weekly planning meeting, is to allow groups to set weekly accomplishable task goals. We have learned over time that when we enter into a new area of work, we are often overly ambitious about a task. When that happens, the original task card is archived and replaced with cards with more precise task specifications.

Each week, the associate project director and I meet to review the previous week's Trello board and revise it as needed. Each team head (lead engineer, outreach, learning scientist, and psychometrician) is asked to add proposed tasks to the backlog or the ready-to-start. We examine the review column to be certain that if only a cursory review is needed, it gets completed. We also look over the backlog and the ready-to-start, making adjustments to ensure that sufficient, relevant, and feasible tasks are included in ready-to-start.

The next day we hold a half-hour scrum for all group leaders and graduate students. Team members must learn to be succinct and on message during scrum; the half-hour time limit helps to enforce brevity. A member from each task team reviews the review column and moves tasks to the completed folder; this ensures that each task team's progress is shared with the entire project team. Each team is also asked to identify any obstacles they face due to dependence on work not yet completed by other teams. If necessary, a card is added to signal the need to solve the problem. Scrum provides a means to efficiently coordinate the multiple teams' work. Members know what others are doing and how others' work may affect them and vice versa. It also makes it evident where progress is being made and where things are slowing down or getting mired.

We use several other agile practices. Our engineering team holds daily "stand-ups" where each member reports his previous day's progress and his plans for the current day. The team actually stands up to help focus on brevity and efficiency. The engineering team also uses a systematized agile methodology for software debugging: a software development management system, JIRA (Atlassian, 2018), by which priorities are established and bugs are reported and corrected. Teachers who encounter software behavior problems can immediately contact our outreach coordinator, who then funnels such issues to the engineering team. We monitor daily use of the system through automatically generated reports of the number of tests taken at each of the school sites.

These organization methods are key to managing the complexity of the project, but perhaps even more important to our progress has been the application of the agile principles themselves. Several of the agile manifesto principles (Agile Alliance, 2001) are discussed below with regard to how they helped us solve challenges and build a more responsive product.

9.3.2.1 Example Related to Principle 1

Principle 1: *Satisfy the client and continually develop software.*

In designing MM6-8, we made sure to develop multiple user stories. These stories were an attempt to encompass a myriad of purposes regarding our primary client (teachers). We structured them to begin with statements such as "As a teacher…"

- I will interpret the data on students' progress along LTs to target topics needing further instruction.
- I will elicit diverse students' ideas.
- I will increase student learning.
- I will increase students' awareness of their own learning strengths and weaknesses.
- I will improve my content and pedagogical knowledge.
- I will connect what I learn from the classroom assessments with other sources of information I have about my students.
- I will work collectively, by reviewing data with other teachers, sharing approaches, and planning for subsequent curricular revisions.

Likewise, we articulated a much broader set of claims about *students* as users of MM6-8, including statements of the form, "As a student…"

- I will use the learning map to understand the structure of the mathematics I will be learning.
- I will treat the assessments as a way to become aware of what I know and need to know.
- I will revise and resubmit answers to items I get wrong.
- I will increase my awareness of my own learning strengths and weaknesses.
- I will practice the constructs and levels I find difficult.
- I will participate in the data reviews with the class or a small group.

We subsequently went through a similar process for administrators, albeit much later in the process than we probably should have. Articulating user stories—at the beginning of building MM6-8 and repeatedly throughout the process as we observed the software being used in a variety of settings—has helped us build a better, more responsive tool. When you invent a new form of tool such as MM6-8, you often do not fully know what it is at the outset. These user stories played an important role in continuously and precisely defining what the tool represents.

9.3.2.2 Example Related to Principles 5 and 11

Principle 5: Build projects around motivated individuals. Give them the environment and support they need, and trust them to get the job done.
Principle 11: Self-organizing teams usually create the best architecture, requirements, and designs.

In building MM6-8, it was important to recognize early on that its success would depend on four major centers of activity: learning sciences, psychometrics, engineering, and outreach. Each of these teams had to be self-organized in the sense of taking leadership to conceptualize their "leg" of this four-legged stool.

For instance, the learning science team's realm of expertise included conceptualizing how the map should be organized hierarchically, articulating LTs based on research on learning for the 60-plus constructs, and linking the LTs to the standards. This team also had the responsibility for creating items for each of the LTs' progress levels. The psychometrics team, in parallel, had to design the assessment system and set up the means to create equivalent forms of tests, to sample levels, and to model the data using appropriate measurement theory. It was up to the software engineering team to co-design and to build the software, and all the design had to anticipate (iterative) change, because we knew that many components would be modified and rebuilt as they were used by teachers and students. Finally, from the beginning the outreach director led the effort in recruiting schools, providing professional development, bringing back feedback, and gathering data on tool use.

Each of these teams had to independently work at the cutting edge of their fields and coordinate their timing to make releases work in real time with our school partners. The project faced its most significant challenges when the coordination of timing across components was disrupted or a team had not worked independently enough to identify and address its own challenges. This was a small team of 8–9 people, all working in multiple roles, and there was never a time when team members did not work hard enough.

Two examples, both involving outreach, demonstrate what happens when these agile principles are (and are *not*) enacted successfully. Our outreach team brought back multiple suggestions from our partners that resulted in the addition of critically important features in the tool, illustrating the importance of agile Principles 1 and 11. For example, the teachers asked us to find a way to provide them with more information to scaffold discussions of individual items (Principle 1, pertaining to client needs). They wanted a means to access the correct answer, the percentage of students displaying a misconception, and the analysis of item responses. We had to devote several months of intensive work to build these features into the tool—features that were enthusiastically received and used.

An example of a team not working independently enough (Principle 11, pertaining to self-organizing teams) was when the outreach team overlooking the need to consistently and systematically collect data on classroom patterns of software use. We automatically recorded which classes took which tests, and the outreach team members had paid attention to how teachers returned and discussed the data in professional learning communities. However, we overlooked a systematic means to know when and how often teachers at each site gave the assessments, how those assessments related to grading, and how students viewed those assessments. As a result, the learning science and psychometric teams had insufficient information to help them interpret the data and, perhaps more importantly, to sufficiently inform our partner-school administrators about the use patterns on their campuses so *they* could more effectively use the data. Late in the project, when we recognized this oversight and its effects on other components of the project, we devised means to gather and systematically examine such data on usage, and we are designing and building out a reporting system for administrators based on our newly created user stories. It is important to note that examples of success and failure of the application of these principles could have been generated for any team; agile principles are applied and reapplied in conscious and reflective ways throughout a project.

9.3.3 Of Shelves, Sustainability, and Start-Ups

What if your goal is to develop a software approach that, through widespread use by schools, students, and teachers, makes a sustainable contribution to education? Unfortunately, over the course of my career, I have seen a large number of useful and successful products of research projects (including many of my own) fail to make it into widespread use. They fall by the wayside, get put up on a metaphorical shelf in a dusty closet, or simply wither for want of a viable model of distribution or sustenance. Most recently, for example, Geometer's Sketchpad (Jackiw, 2013) has met such a fate, despite its profound originality. While the primary governmental sponsors of educational research encourage scaling and widespread distribution (including commercialization) of research products where possible, they have never developed clear models for sustaining publicly valuable education products. The acid test of sustainability tends to be commercial viability by whatever means. To a degree, I wonder whether agile Principle 4 (developers and businesspeople must work together throughout the entire project) might contribute to helping designers to invent more enduring (scalable and sustainable) products.

Through my experience working at a corporation that licensed some of my prior work and forged contractual agreements for royalties with the university, and my subsequent experience launching a start-up company, The Math Door (www.themathdoor.com), I have learned a great deal about technology transfer at universities. If you or your team are interested in the process of building a start-up, I strongly advise taking part in a weekend (education) start-up competition, which are hosted in numerous cities around the country. One does not have to have or use one's own idea for a start-up but can instead go, propose a fun alternative idea, or join a team. Five members of my team did this in early 2015[2] and we came in first-place in the competition, winning, to our surprise, best business model. Such an activity can teach the team important skills such as pitching an idea, and, at the same time, help you decide if this is a form of activity that appeals to you and your team.

It is also important to think early and often about how to involve one's institution in the process of taking software to market. Most campuses have some version of an Office of Technology Transfer,[3] and though these offices are typically far more experienced in working with faculty from engineering, science, and medicine, they can offer invaluable advice on how to move a piece of software from research to the market. On the part of the investigator, building a start-up and licensing one's own software back from the university involves a process of disclosure, notice of intent, possibly the formation of a committee to manage possible conflicts of interest, and licensing. My simple advice is to learn about these questions early, before the software has been fully developed, in order to plan accurately, anticipate opportunities, and ensure that you have allowed sufficient time for the overall process.

9.4 Conclusion

Using the principles of agility in software design is the best means I know of working more efficiently and building innovative and interesting educational products and tools. Agility is not a panacea, nor does it eliminate the possibility of failure in a system. But it *does* offer a process in which evidence of impending failure may surface, and be addressed, more quickly.

That said, using agile methods in research can lead to certain challenges. Agile development is designed to be fast; thinking hard is not. No matter how well you organize your team, you must allocate time for the thoughtful consideration of how your work is situated in a field of research. It can be tempting to treat ideas as reducible to tasks on a Trello board, but this will not produce the kind of thoughtful writing and analysis needed in research. Be sure to give enough time to writing and publishing, because it is these works that will secure one's future in the academy and lead to subsequent funding.

Agile development depends on cultivating a strong team with the willingness to work really hard. Most research teams have to run lean, pushing the limits of their budgets with little redundancy among staff specializations. The loss of a team member can leave a big hole to fill. In my experience, I have tried to balance the hiring

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[2] We participated in Triangle Start-Up Weekend EDU held on NC State University's campus in the NC Research Triangle. http://communities.techstars.com/usa/triangle/startup-weekend/3885

[3] NCSU has recently renamed theirs as the "Office of Technology Commercialization and New Ventures," reflecting a university priority of promoting their faculty and research staff to develop and launch socially and economically valuable research products (inventions) and ventures whenever feasible.

of more permanent research staff with support for graduate students or post-docs. Both are critical to a team. The former provide depth of experience as well as team leadership and project continuity. The latter are "term-limited," usually need 2 years to train, and typically provide only 1–2 more years of fully productive contributions. Supporting grad students and post-docs is *very* important, however, because they form the next generation of design-oriented researchers. And having too many research associates can limit your long-term influence on the field, as it is one's graduate students who are most effective as spreading the word when they move into other academic jobs.

Finally, on reflection, it is the combination of the principles I learned in each phase of my career that comprise my final words of advice. Everything in my work drives back to my basic commitment to the learner, who is the ultimate client we serve. Striving to make it possible for young people to pursue their own ideas, to build on their natural curiosity, and express their nascent ideas is the driving force behind my career. Design researchers are not satisfied with *what is*, but are constantly looking toward *what could be*, based on our understanding of learners. Participating in teams devoted to thinking anew about education, and doing it in the company of hardworking and devoted practitioners, is a fantastic way to unearth the potentiality of learners and a fulfilling way to see one's vision realized.

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Chapter 10
Going to Where Your Research Takes You

Brent Davis

Abstract I offer three principles of conducting and designing research—namely pursue a passion, question assumptions, and embrace complexity. These tenets have always been core to my own investigation, and I ensure they are prominently represented in work with new scholars. Each principle is about being attentive to the situation-bound characters of interests and interpretations, and they culminate in the advice of going where your research takes you. That suggestion is not about foregoing agency or making frequent turns but about being steadfast in the obligation to be mindful of our complicity in ever-unfolding possibilities.

Keywords Complexity in mathematics education research · Researcher attitudes · Research as reinterpretation · Contextualizing research sensibilities

In a 1972 radio contest, Peter Gzowski of the Canadian Broadcasting Corporation challenged the nation to "Complete the adage, As Canadian as…." Apparently, most listeners heard the contest as a quest for something quintessentially Canadian—a symbol fitted to an idealized sense of Canadian identity in the ways that *mom* and *apple pie* are invoked to characterize an imagined collective American personality. Most submissions were predictable: hockey, maple syrup, the Mounties. The contest judges, however, were not convinced that what it means to be Canadian could be captured by a single icon; the winner was "As Canadian as possible under the circumstances."

I remember the burst of pleasure when, as a child, I was invited into the paradox of that adage. Our essential Canadian character, it asserted, is that we have no essential Canadian character. And it's not that the nation is trying to skirt the issue. The point is simply that an awareness of the complex, circumstance-dependent nature of self-characterization is one of the defining qualities of Canadian identity.

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The adage has been pinned to my psyche for most of my life. It was especially present for me throughout my graduate studies in mathematics education. In the mathematics education research community of the 1990s, it sometimes seemed as though the needle of the epistemological compass was spinning wildly. In my first months of doctoral study, I found myself learning about not just radical constructivism and sociocultural theory, but a host of disruptive upstarts such as postmodernism, complexity theory, queer theory, and enactivism. Weirdly, however, I never found myself daunted by my program's requirement to locate myself in the flux—and I think it had everything to do with where I was raised and where I was studying. I was immersed in a sensibility of "possible under the circumstances." It was never about seeking a singular truth but about living in the ever-elaborating conversation.

My purpose in opening on this note is neither to assert nor essentialize my citizenship nor to claim some sort of theoretical breadth. It is, rather, to frame some key commitments while I situate myself—tasks that I regard as necessary and foundational to all academic work. One thing that has become abundantly clear to me over my career is that "educational research" tends to have geographical and temporal flavors, influenced by pressing social issues, prevailing ideologies, linguistic nuance, and other elements of cultural ecologies. Further, and especially relevant to this writing, whenever purporting to offer advice to graduate students or colleagues, I make sure that I qualify what I say with some variation of what is "possible under the circumstances." I am regularly surprised how the academic system affords so much space to explore possibilities. The desire to find and learn from the play in the system is the anchor for most of the advice that I might offer a student or colleague struggling with an issue related to designing, conducting, or publishing quality research in mathematics education. Good academic work is not about meeting requirements; it's about expanding the space of the possible.

To that end, as I report in this chapter, I tend to organize my supervisory and mentoring advice around three pieces of received wisdom. That is, what I have to offer here is based on my practice, but it is entirely derivative—so derivative, in fact, that it would feel like plagiarism if I weren't to acknowledge upfront four people: Thomas E. Kieren and Max van Manen, who were especially influential during my graduate work, and Susan E. B. Pirie and David Robitaille, who guided me through my first years in an academic position. Drawing from (or, perhaps, echoing) their counsel, I pass along three principles of academic engagement which, for me, collect into a single guiding metaprinciple, illustrated in Fig. 10.1.

Fig. 10.1 A guiding metaprinciple

pursue a passion

question assumptions

\+ embrace complexity

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go to where your research takes you

I re-emphasize that each of these three principles comes with caveats of doing what is “possible under the circumstances,” coupled to an acknowledgement that circumstances can vary dramatically. I don’t pretend to offer any universally appropriate or context-free insights. But as one who has served as an editor on multiple journals, who has supervised dozens of graduate students, and who has mentored many junior colleagues, I can attest with great confidence that, although the presence of these three elements will not ensure high quality work, the absence of any one will almost certainly compromise quality and insight.

10.1 Regarding Research Topic: Pursue a Passion

When it comes to formal responsibilities of supervising graduate students and mentoring junior colleagues, especially at the start of the relationship, I typically find myself following a fairly routinized script that’s modeled after ones experienced in my own graduate studies. Things begin by getting to know one another. I quiz people on where they’re from, how they got into the field of mathematics education, why they’re interested in graduate studies, what they imagine that to be, and so on. As we chat, I assemble a list of publications in mathematics education that is tailored to the themes of the conversation. Aiming to address historical depth, philosophical variety, and current discussions within the field, at some point I ask my conversation partner to do the same for me. Eventually, we compare those preliminary lists, using them to help lay bare the interests, biases, histories, and expertise that will condition our work together. The co-elaborated list inevitably serves to map out agreements and divergences that set the ground for many conversations to come.

One question that I *don’t* ask in first meetings is, “What would you like to research?” In fact, I actually avoid the topic, especially with new graduate students. I work from the conviction that research in a field as complex and volatile as mathematics education is better guided by passionate interest than well-stated problems. At the risk of revealing too much, very few of the questions I’ve answered through my own research projects have been ones that I asked when I began them. In fact, even the question stated in the opening chapter of my doctoral dissertation (published as Davis, 1996) was something of a cheat. I crafted it in the final stages of editing the document. Only then was I able to express the question I might have answered. I witness the same phenomenon in many of my students’ writings. In fact, I have supervised only one doctoral candidate whose research proposal and dissertation were framed by the exactly the same question. And I have supervised only one doctoral student who was driven by expediency rather than passionate interest. They were the same person.

I thus do what I can to encourage flexible and expansive thinking about what and how one might investigate, especially during the first months of the graduate student experience. I’d like to claim that’s standard practice for me, but I can’t. A few years ago, I broke the pattern—or not so much broke it as permitted the rush of existence to eclipse it. At precisely the busiest time in our academic and teaching year, our Associate Dean of Graduate Studies requested that I take on a doctoral student who had fallen out with her assigned supervisor. Already well into her studies, Monica came with penultimate drafts of scholarship applications that were due in just weeks, a completed sketch of a research proposal and an extensive bibliography of completed and intended readings. Consequently, the getting-to-know-you and what-you-might-want-to-read preliminaries just didn't happen. In fact, the opposite occurred. Our getting-to-know-you discussions revolved around formatting her research interests and describing her personal history in ways that might make them more compelling to scholarship adjudicators.

Cutting a long story short, in the crush of other responsibilities, I didn't notice my lack of familiarity with Monica's deep interests until, many months later, it became apparent that she was struggling to craft a compelling research proposal—in spite of a strong committee, a coherent focus, and an excellent record in course-work. As we met one day to work through small issues on her very-well-crafted methodology section, the conversation shifted abruptly when she confessed that her heart just wasn't in the work. That wasn't a surprise, given our earlier conversations. But it was only when she added that she'd never been particularly interested in the topic that I awakened to the fact that something important had been missed.

One of my research mantras since my own master's study has been to focus on "something that keeps you awake at night." I heard and uttered this phrase hundreds of times during my graduate studies, and I've invoked it many more times since. Yet, it seems, that advice hadn't been part of my conversations with Monica. Realizing this detail, I attempted to introduce passion as an orienting theme in the conversation by asking the questions I should've asked many months earlier.

I was gobsmacked by her responses. Monica, it turned out, was a celebrated educator in her home country. Narrative after narrative of life-changing engagement fell from her lips, some with such subtle emotional potency that I only noticed myself crying when my hand reached to wipe a tear from my cheek. There were stories of not just providing lessons but of opening vast horizons of possibility, of not just transforming lives but saving them, and of, as Monica summarized it, not just *teaching* but *educating*.

Here is not the place to delve into the different meanings of those two words for Monica, but everything is different now in her research life, mostly because it flows in harmony with the rest of her existence. The shifts have demanded tremendous effort—in fact, almost starting over again on matters related to methodologies and elements of the literature. But, while she hasn't kept it secret that shifting her focus has brought on considerable extra work, Monica's "complaints" are currently spoken with a smile.

As it turns out, an antipodal narrative to Monica's unfolded in parallel over the past few years with a junior colleague. Dustin produced an outstanding doctoral dissertation on possible contributions of school mathematics to informing and affecting issues related to sustainability and systemic change. The work was completed at another university, and so I wasn't an official part of his program. Nevertheless, we spent many hours in deep-but-easy conversation on his research.

He was offered an academic position before graduating. Just before he moved to the United States to step into it, we made plans to blend and extend our research interests. Unfortunately, over the past year, it has been made clear to him that his passions and methodological expertise, while generative of interesting arguments, are not well fitted to the perceived needs and approaches in his new context. In our most recent conversation, Dustin confessed that he is giving into pressures to set aside his doctoral foci and to align his work with the research program of a senior scholar at his new institution. We agreed that the plans that we developed together do not fit well within his current situation, and so they have been set aside for the time being. I'm still working with him to hone arguments and craft manuscripts for publication. But, a prodigious thinker and a fluid writer, he confesses that he is struggling to generate even technical pieces of writing at the moment.

I am invigorated by stories like Monica's, and I'm saddened by stories like Dustin's. In one case, I see engagement and productivity massively amplified as passionate interest frames the picture, and in the other I see engagement and productivity drain away as passion declines. To be clear, in both cases, the passion is not articulated as a naïve enthusiasm but as a vitality that is self-aware, intensely analytical, mindful of biases, and generative of possibilities. Small wonder that both feel stifled when it is absent or suppressed.

Such examples notwithstanding, it's important to situate what I'm saying here alongside an important truth: Enthusiasm and personal investment distort perception. But that doesn't mean that passionate interest is antithetical to objective inquiry. We humans simply cannot step outside our biases. There are no neutral stances. It is true that bias steers perception, but it is just as true that perception is impossible without bias (Kahneman, 2011; Willingham, 2010). Indeed, for an interest to be both "academic" and a "passion," there must be a risk of it being proven misdirected or unimportant. An academic passion is not a conviction. It does not seek validation. An academic passion is not a direction, but it does orient. An academic passion is a care; it is a calling to take care.

10.2 Regarding Research Methodology: Question Assumptions

Several years ago, I was asked to be part of a colloquium on research methods. The invitation advised that I should frame my remarks with "the single, most influential quotation in my academic career." Clueless as to what that might be, I decided to go with a paragraph that has found its way into several of my publications, from Dewey's (1910) essay, *The Influence of Darwin on Philosophy*:

> Old ideas give way slowly; for they are more than abstract logical forms and categories. They are habits, predispositions, deeply engrained attitudes of aversion and preference. Moreover, the conviction persists—though history shows it to be a hallucination—that all the questions that the human mind has asked are questions that can be answered in terms of the alternatives that the questions themselves present. But in fact intellectual progress usu-

> ally occurs through sheer abandonment of questions together with both alternatives they assume—an abandonment that results from their decreasing vitality and a change of urgent interest. We do not solve them: we get over them. (pt. III)

This text resonates with me for many reasons. The opening sentences peel back the fallacy that our most confident research claims are somehow free of "attitudes of aversion and preference." The next sentences problematize academia's profound-but-troubling commitment to framing research with well-structured questions. And the final sentence interrupts the project of pursuing solutions.

I realize that not everyone will agree—and, in fact, I acknowledge that I might be an outlier on this issue—but I hear Dewey's remarks as an apt description of the cultural project of educational research. Ours is an evolving domain. I've been active in the field long enough to observe slow transitions from multiple sets of obsessions and frames to entirely different ones. In fact, I've been around long enough to see cutting-edge ideas rise to prevailing orthodoxies and then slip into partial-at-best reminders of where the field used to be. I use such observations to frame one of my favorite activities when teaching graduate courses on research. It involves helping students develop the skill of identifying the decade (and, often, the year) that any given mathematics education research article was published based only on its abstract. It turns out to be really easy when one is aware of prevailing metaphors, epistemologies, and research foci. By the end of the course, accuracy approaches 100% across participants.

My aim in prompting students toward this competence is to afford them access to evolutions in the questions asked, the ways they're posed, the strategies through which they're justified, and the standards implicit in their responses. The global intention is to explore the truth value in the assertion that, following Dewey, it is vastly more important in educational research to be mindful of why we're asking what we're asking than it is to find solutions to whatever those questions may be.

A second exercise that I frequently use when teaching research courses starts by inviting students to select a topic in educational research with a strong thread of quantitative research and/or quasi-experimental methods. In stages, I work with them to push past the mounds of data that typically serve as the foci in published reports, wending to assumptions that reside in the questionnaires, tests, or orienting constructs that are suspended under the counts and statistical analyses. Most often, students react with something that falls between surprise and shock, as publications that on the surface masquerade as objective assessments of verified phenomena turn out to be not especially distant from opinion and riddled with ideological bias. Stated more directly, with very few exceptions, the student-selected quantitative studies we've deconstructed in my grad courses have proven considerably more subjective than nonquantitative articles. Maturana (1987) summed up this issue brilliantly with the concise statement, "Everything said is said by an observer" (p. 65). There are no observerless observations.

On this issue, I would argue on the bases of the analyses we have conducted in graduate research courses that educational research that explicitly aligns with or is situated in interpretive traditions is, in general, more "accurate" than research that sits several layers of data removed from its originating constructs and assumptions. That is, owing to current expectations to be explicit about theoretical commitments and epistemological positionings, most interpretive research is better fitted to the complex cultural project of formal education than most statistics-riddled empirical research. Conversely, having read thousands of publications in the field, the most helpful of the statistics-based empirical studies are the ones that that include explicit and critical accounts of the interpretive acts that constitute their foundations.

That is what drives my conviction that some of the researcher's time should be given to interrogating assumptions, and much more should be given to interrogating assumptions that underpin assumptions. There should be attendance to personal history, context, and other influences on both personal orientations (e.g., convictions, interests, passions) and situational framings (e.g., prevailing discourses, pressing issues). As inevitably highlighted by the grad-class exercises just described, our research passions are never just our own. They arise and persist in an ecosystem of conventions, convictions, norms, and needs. My urging of students and colleagues to question their assumptions' assumptions, then, is an iteration of the advice to follow a passion. It is a call to wonder about "habits, predispositions, deeply engrained attitudes of aversion and preference" (borrowing from Dewey, above) that undergird understandings and frame perceptions. It is an invitation to recognize that "self" and "context" exist in complex-co-implicated relationship, the latter unfolding from and enfolded in the former.

To be clear, I'm not talking here about the commonplace and commonsensical advice that one must be explicit about one's positionality as one goes about designing one's research. Rather, I'm pointing to the fact that every key construct in the previous sentence (and in this one and in the next one) is a metaphor that is rooted in situated experiences and rendered meaningful through a weave of culturally conditioned associations. So, yes, we must be explicit about positionality, but we mustn't mistake statements on positionality as deep or enduring insights into the substrate of our thinking. Returning to Dewey's quote, and reflecting on the mottled landscape of mathematics education, positionality might just as well be understood as something to "get over" as the place from which we speak.

10.3 Regarding Research Attitude: Embrace Complexity

My first explicit encounter with complexity theory was in the autumn of 1994, when I read Waldrop's (1992) account of its emergence as a coherent domain through the last half of the twentieth century. The timing of the read is vivid in my memory, in large part because it happened just months after I'd graduated with my PhD in mathematics education. In my dissertation I had explored many of the defining themes of complexity theory as they might apply to understandings of the structure of mathematics, the nature of learning, and the possibilities for teaching… all while being completely oblivious to the domain.

In and of itself, that coincidence is easily explained. Indeed, as hinted in the previous section of this chapter, I was simply tapping into one of the things that was in the air at the time. Anyone listening intently would have picked up the complexity themes of self-organization, emergence, structural determinism, nested dynamic systems, and so on—at least, anyone who was listening while working at the elbows of someone like Thomas E. Kieren. My memories of our interactions through my doctoral program are peppered with his admonition to "Embrace complexity!" Expressed with frequency and with an infectious enthusiasm by someone with a quick mind and an encyclopedic knowledge, I could not help but embody the advice—even if the "complexity" he encouraged wasn't tethered directly to the emergent academic domain.

Importantly, Tom's advice was not to *avoid simplification*. That would be silly. We humans survive and thrive by reducing complexity. Rather, his caution was but to *avoid oversimplification*, to appreciate that there is no linear relationship between events and whatever they might trigger. It's not an overstatement to say that, since completing graduate work, my research program and my academic career have been all about navigating between the rocks of oversimplification and the rapids of too-entangled-to-be-useful.

I try to bring that attitude of mindful navigation to all my supervisory and mentoring work. Today when I say, "Embrace complexity," I intend it partly as Tom meant it and partly as a suggestion to consider complexity science. While I in no way insist that this domain be employed by everyone with whom I work, I do require that it be considered as a lens for every study. For the most part, I recommend it as a complementary discourse. Complexity thinking is readily partnered with methodologies focused on things (i.e., empirical approaches), those focused on persons (e.g., phenomenology, narrative inquiry), those on peoples (e.g., hermeneutics, ethnography), and/or those on systems (e.g., systems research), and so I've never encountered a situation in which embracing complexity meant rejecting previous thinking or preferred methodologies. That's the case because complexity is as much an attitude as it is an interpretive frame.

A note on the significance of complexity research in the contemporary academic world is in order here. In most other academic domains, complexity science is integrated into sensibilities. For example, all Nobel laureates in both physics and economics over the last quarter century have explicitly aligned their research with complexity science. By contrast, within education, complexity science has had relatively little impact. In fact, it is regarded by many as a fringe discourse. I personally find that alarming. It reveals both an insularity of our field and, possibly, a lingering devotion to oversimplification.

To rephrase in more lighthearted terms, with regard to embracing complexity, I believe that most of educational research is hovering around the second stage of academic argument-making: "Arguments against new ideas generally pass through three distinct stages, from, 'It's not true,' to, 'Well, it may be true, but it's not important,' to, 'It's true and it's important, but it's not new—we knew it all along'" (Barrow, 1995, p. 1). Even among mathematics education researchers, there is little recognition that prevailing theories of learning and emerging accounts of the subject matter are all instances of complex theorizing.

Against this backdrop, I can't help but shake my head when colleagues complain—as they frequently do—that education is a marginalized domain and that colleagues in other disciplines don't listen to us. If we want to be heard, we have an obligation to listen. How else might we hope to frame our insights in terms they can hear. (On this count, my education colleagues are often surprised that I give more talks to mathematicians and physicists than to educationists.) We educational researchers also need to experiment with phrasings intended to communicate insights to colleagues in education, taking care to listen to how we're heard. As frustrating as it can be to receive yet another critical evaluation from reviewers unfamiliar with well-established principles, we are obligated to suppress the urge to respond in kind. If we desire to be heard, we must be open to rephrasing, doing everything we can to forestall misinterpretations, to explicate constructs, to situate assertions, and to ground interpretations. In those respects, I have occasionally run afoul of colleagues who advise that we should always aim for the highest-ranked journals and conferences. When working through the early stages of complex ideas, there's much to be said for engaging the generous expertise of colleagues who devote time to small specialist conferences and less-prominent journals.

On that count, as far as educational research goes, complexity thinking demands that we researchers understand ourselves as implicated in the phenomena we study. Thus, for example, when researching in a classroom, we must be cognizant that the addition of a camera or a person or a new routine irrevocably changes the system—in a manner that might completely transform what would have otherwise unfolded. Further, recognizing that the vast majority of phenomena of interest to educational researchers are associated with learning systems—that is, emergent, adaptive, structurally coupled, self-modifying, self-maintaining phenomena that arise from and that persist within similarly complex phenomena—it's vital to engage with methodologies that include requirements for on-the-fly monitoring and iterative modification. There is limited value in studying a dynamic system through a fixed lens or according to a preplanned agenda.

It is that detail—that is, that educational researchers are always dealing with dynamic systems—that most orients my insistence to embrace complexity. The recommendation arises in the realization that the project of formal education is itself a complex (learning) system, and our role is to participate in its learning. Our responsibility in examining and reporting on different aspects of formal education, then, is never to offer summary conclusions but to participate in making the system more intelligent. Through a lens of complexity, engaging in educational research is about struggling to represent thoughts that are at the edge of current comprehensibility, oriented by an awareness that what we ultimately offer are not final thoughts but scaffolds to more sophisticated thinking. Favorite examples of mine include the now-commonsensical notion of the butterfly effect and the now-commonplace understanding of brain plasticity. These were cutting-edge ideas when I used them in my doctoral research in the early 1990s. When I wrote or spoke about them, reviewers and audience members pushed back, often scornfully. When I invoked them in my teaching, it usually took hours of discussion bolstered by multiple articles from *Science* and *Nature* to convince students even to consider they might be vague possibilities. Today they are uncontested elements of more sophisticated constructs.

10.4 Adding It All Up: Go to Where Your Research Takes You

While typing the first draft of the "Pursue a Passion" section of this chapter, I felt my fingers begin to hack in the oft-heard, "Do what you love, and you'll never work a day in your life." I made it as far as "never" before realizing the lie in the statement. While my career has been regularly punctuated by experiences of the joy of insight and affirmations of impact, pursuing my academic passions has been fraught with unexpected challenges, disheartening resistances, mean-spirited criticisms, and outright condemnations—and not just from others. On occasion, I have been my own worst enemy, rendering myself silent and immobilized at obligations to lay open beliefs and rethink personal commitments. It's what happens when really listening and sincerely embracing complexity.

Pursuing an academic passion, then, is work. It is an obligation to go where your research takes you. The notion of "going where your research takes you" is in no way flippant. It is similar in grammatical structure to the phrase "going with the flow," but "going where your research takes you" means entirely the opposite. It isn't advice. It is consequence. It is an emergent sum of pursuing a passion, questioning assumptions, and embracing complexity. Going where your research takes you can't but happen when attending to and participating in the forces and objects that generate and define the flow of one's academic existence. It is a mindful engagement with difficulties, contradictions, and ambiguities.

I conclude with one final illustrative anecdote. Steven, a former doctoral student and currently an assistant professor at one of Canada's top universities, tagged me in a Facebook post some months ago. "I think I've FINALLY found my focus," he commented at the end of an extended explication of "studying mathematics popularisation as a route to joyful human mathematical experiences." It's been 7 years since Steven graduated from his PhD program. The theme just noted might not be the focus he's been looking for, but it is a clear indication that he is going where his research is taking him. And even if he hasn't actually found his focus, I have every confidence that he eventually will. He's a person attuned to his passions, he's a disarmingly intense listener with an ear constantly to the ground, and he shames me in his commitment to embrace complexity. He cannot help but succeed.

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Chapter 11
Navigating the Self and Engaging with Others in Constructing Visions of Quality in Mathematics Education Research

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Abstract Two factors involved in producing quality research in mathematics education are discussed in this chapter: navigating the role of the self and engaging with others. These factors have the potential to motivate patience and persistence in a quest for quality while building from a researcher's views of others' ideas rather than being subsumed by them. Efforts to produce quality research in mathematics education require carefully managing evolving perspectives of who you are and who you want to be, all in the context of what you want to know and the ideas of others.

Keywords Self as researcher · Researcher growth and development · Researcher engaging with others · Researcher persistence

Behind every mathematics education research study is a collection of stories involving the personal histories of the authors—their experiences, collaborations, and conversations. Such stories reveal processes of becoming a mathematics education researcher. In mathematics, Thurston (1994) called for a view of research representations of mathematics and mathematical proof that communicated *how mathematics was done* in addition to the results of the work of mathematics. The communication of such stories provided ways for others to understand the work of mathematics. In mathematics education, representations of how mathematics education is done are needed to communicate joys and emotional tangles that are part of conducting research. Such representations sustain early career researchers as they confront challenges in their research activities. Without such reports early career researchers may come to view quality mathematics education research as the result of a few gifted authors and creative scientists, rather than as a by-product of navigations of the researcher (self) and engagement with the research community (others).

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Becoming conscious of the role of the self and of others through stories of mathematics educators' development of ideas, design of research, analysis of data, and representation of findings can play a significant role in sustaining interest in and the production of quality research in mathematics education.

In this chapter, I use stories to discuss two factors in the production of quality research in mathematics education: navigating the role of the self and engaging with others. I acknowledge that the stories I have chosen represent my own view and experiences. As such, the stories I have selected are drawn from accounts of established mathematics education researchers who have produced quality research in mathematics education. This selection should not be read as evidence that only a select few can produce such stories or that there is only one way to experience doing mathematics education research. Rather my selection of stories is a function of my experience as a mathematics education researcher and my capability of finding stories. To support your understanding of my experiences and perspective, I share a bit about what informs my thinking—my love of learning. In addition, I encourage early career researchers to construct and publish their own stories, which have the potential to compliment the account in this chapter and to illustrate the dynamic nature of producing quality research in mathematics education.

I love learning, and I love thinking and writing about it. My love of learning is an orienting force in my life and work. My ideas about learning are informed by a radical constructivist epistemology (von Glasersfeld, 1995). As a result, when I think about quality research in mathematics education, the examples I choose come from my view of a constructivist perspective. My view of self as a researcher is inspired by Confrey's (1994) description of the contributions of Piaget and Vygotsky: "These theorists have offered us powerful insights into the human mind and its development, radically transforming our understanding of how children view the world and about how we understand ourselves as individuals within a cultural and historical setting" (p. 2). Of interest for this chapter is how researchers understand *themselves within a cultural and historical setting.* Confrey's description retains the relationship between the self and others. Although I also view the self as always in relation with others in context, I focus on the self and others in separate sections of this chapter to make explicit my view of how the self and others support the development of quality mathematics education research. I begin by describing my view of the self and of the factors involved in navigating the self. I then turn to engaging with others and the role such engagement plays in the construction of quality research in mathematics education.

11.1 Navigating Evolving Perspectives of Self

Producing quality mathematics education research involves navigating the self. In this section I discuss four factors involved in navigating the self while doing research: personal history, who you want to be, what you want to know, and gaining perspective. This inward look highlights navigating the self not as an obstacle in creating quality but as an important part of becoming conscious of positions and directions you might take in creating quality mathematics education research.

11.1.1 Personal History as a Compass

Researchers' personal histories reach forward from the past to inform ideas and actions those researchers take in the present. I suggest that knowledge of one's personal history is salient to the construction of quality mathematics education research. In such research, gaining insight into and from memories can provide an occasion to know about our decisions in new ways. For example, Nolan's (2007, 2010) experiences played an explicit and critical role in her research explorations of prospective teachers' experiences learning mathematics and science. Nolan (2007) describes calling her sister to "reminisce and share school stories" (p. 12). Both women had the same teachers in middle school. Nolan positions the reader to see how her personal history moved her toward some ideas and away from others. Reflections on her own learning of mathematics and science made Nolan aware of new insights about teaching and learning. Nolan's conversations with her sister made her conscious of how her experiences as a learner informed her research and the way she told the story of her research.

Retelling a story from one's personal history allows a researcher to become aware of subjectivities. For example, Lubienski and Gutiérrez (2008) described the way their experiences and personal histories encourage them to take up the notion of a performance gap in mathematics education research. The ways in which they position themselves in conversation and in their research is informed by their histories and has resulted in particular values. Both researchers recognize this personal knowledge and its importance in understanding ideas as informed by history. Each researcher's history and representation of her history reflected the power and limitations of their unique perspective.

11.1.2 Who You Want to Be as a Compass

Researchers' stories suggest views of their future selves and inform the development of their ideas and representation of research. For example, Skovsmose's (2012) description of his desire to become a teacher influenced what he learned:

> I read a variety of books, including many with philosophical topics. Not that I understood what I was reading, but I felt that the authors were talking about something important. I took pleasure in speculating about all kinds of questions in life. (p. 36)

Skovsmose's selection of books and his pleasure in speculating about questions his reading inspired were informed by the future selves he envisioned and perhaps those that were beyond his consciousness at that time. Skovsmose described his view of himself as he completed his doctorate: “I was not really thinking of myself as a researcher per se. Rather, I was more interested in developing ideas and practices, and in this way formulating a critical mathematics education” (p. 40). Skovsmose’s view of himself as a teacher motivated him to represent his ideas in a series of books that were practitioner oriented. Yet when his work was “considered improper for obtaining a doctorate” (p. 41), Skovsmose promptly restructured the work into “proper format for a doctoral thesis” (p. 41). Skovsmose’s view of self and of who he wanted to be enabled him to flexibly represent his ideas for both practitioner and research audiences.

Like Skovesmose, Nolan’s (2007) view of herself as a researcher and a teacher informed her work. Her plans for interacting with her participants were influenced by her view of the “research process”:

> Being rather skeptical of research processes, I envisioned my role as a researcher to be about gathering interview “data” and then cloistering myself off somewhere to produce an interpretive and/or critical and/or feminist narrative piece of work. Therefore, I vowed early on in the design of my study that I would make both the “data collection” and the “interpretation” processes as collaborative and participatory as possible. (p. 22)

Later, Nolan’s research process with participants provided unexpected opportunities for her to reconstruct her view of her teacher self. Nolan recounts coming to consciousness that, as a teacher, when students stopped asking why, she had felt successful. As a researcher, Nolan questioned this view in light of her listening to the stories of participants who had stopped asking “why.” Who Nolan wanted to be as a researcher created an opportunity for her to revisit who she was a teacher.

Skovsmose and Nolan’s stories illustrate that views of self inform the construction of quality in mathematics education. Your consciousness of such views creates opportunities for your research directions, including how you design your inquiry and represent your findings. Another factor that informs the self as researcher is what you want to know.

11.1.3 What You Want to Know as a Compass

Researchers have many ideas, and it can be difficult to focus on just one. Researchers’ stories of their ideas illustrate the joys and challenges of identifying what you want to know. Duckworth’s (1972) description of her own experience with ideas and her observation of children’s experience with having ideas led her to claim that “the having of wonderful ideas is what I consider the essence of intellectual development” (p. 217). The feeling of joy at having a new idea does not mean the idea is well-structured or that possibilities are clear; rather the mental act of producing an idea brings with it other insights you may not have been conscious of. Celebrate this initial excitement and record it if you can. This excitement is a significant source of motivation as you move forward, but there are psychological challenges as an idea is shaped.

Nolan (2007) described one difficulty with the sharing of ideas:

> In my research, it is always difficult to articulate my anticipated results or "answers". Early in my study, I could just say something like "still in process" or "caught in the thick of things" and "can't see the forest for the trees", and other applicable clichés." (p. 16)

Nolan resolved her initial difficulties representing ideas emerging from her data analysis by persisting in pursuing her questions. Sharing an initial idea or a question is made more complex by the dynamic nature of our ideas and the uncertainty of their direction. Possibilities come with the joy of wonderful ideas, but ideas may also introduce ambiguity and confusion as you wrestle with how to communicate the idea to others or even to yourself.

Even as many times as I have begun a study with what I think is a wonderful idea, I can become discouraged. As I work to communicate the idea or face failures in my initial attempts to gain insight, I want to run away from the idea and start over. I have learned to resist this option for myself. The original idea felt satisfying and wonderful. When I have empirical evidence that my approach to an idea is problematic, I turn back and consider how I might think about or explore the idea differently. Knowing what you want to know serves as a touchstone. You may find that the path you are forging is unproductive and feel you have wasted time, yet your journey has resulted in knowing a path not to follow and new ways of thinking about your idea. Turn back with the knowledge that you can explore the idea differently.

11.1.4 Gaining Perspectives on Your Ideas

I have always loved to write. Maybe that is where my journaling habit came from. I consider journaling as a way of talking about my ideas to myself. Journaling helps me represent and gain perspectives on my ideas. I use the term "perspectives" because, while I may take up or experience one perspective at a time, over time I take up many different views. For example, my view of an idea that motivated my doctoral study was represented in a webpage I designed as I began my doctoral study. It is easy to look back at my early research journal and cringe as I share who I was, an idea I was interested in, and my perspective. Yet the initial representation of my ideas created opportunities for new ideas.

> I have worked on a few course development ideas and have fooled around with assessment as part of the packages but, I kept running into brick walls. I really didn't have the know how to put together a study that could prove that the materials I was developing were any better than the text book and methods that other faculty were using. (Grade Distributions don't do it for me!) So here I am. I need to learn quite a bit more before I can do the "work" I dream of doing. (http://jwilson.coe.uga.edu/EMT668/EMAT6680.Folders/Kastberg/bio/skbio.html)

My journaling involves considering objections to my ideas or ways different audiences might make sense of an idea. The process of compiling perspectives allows me to revisit an idea and describe it again and again. Sometimes I describe an idea in the same way multiple times, other times I add new words that I hope create productive images for the reader. I am the primary audience for my journal, and I know that my experience revisiting an idea will provoke questions or new insights about an idea. Journaling is my way of gaining perspectives on my ideas, but there are many other ways to consider. The important thing is to find a way to play with the idea.

Descriptions of the navigation of the self are often left out of mathematics education research papers, and I am not suggesting that quality mathematics education research must or even should include such descriptions. Instead, I am encouraging you to look within and consider ways that your views of these four factors involved in navigating the self might support your inquiry. I further encourage you to consider how your view of self positions you to think about and engage with the views and work of others. This look inward into your history, who you want to be, what you want to know, and your perspectives will help you know why you are interested in an idea, maintain interest in the idea, and grow that idea. Your consciousness of factors involved in navigating the self is a root of quality in your mathematics education research. A second root of quality is a look outward as you engage with others.

11.2 Engaging with Others: Being in Conversation

> *Bringuier*: Do you think a researcher should work alone?
> *Piaget*: Oh, no; you must have contacts, and you must, especially, have people who contradict you. (Bringuier, 1980, p. 18)

In this section, I turn to the idea of engaging with others in ways that are supportive of not only your research but also your emotional and intellectual development. Quality research in mathematics education involves engaging in conversations. To illustrate this point, I begin by describing the diversity of the mathematics education community as a source of strength. I then highlight two possible results of engaging in conversations: the evolution of your ideas and finding a context for your ideas.

You will have relationships with many others in mathematics education. With some others you will have ongoing personal and professional conversations. You will sit and talk, exchanging ideas, and share excitement in your insights. These colleagues can sustain you when you face research challenges and celebrate with you when you feel satisfied by your research. There are also others at a distance. They may be authors whose work you admire or draw upon. They may be anonymous reviewers who support and critique your work. While you do not know these others in the ways you do those you call colleagues, your work is in relation to them just the same. Your relationships with others in mathematics education is how you come to know mathematics education as a community.

11.2.1 Community and Ideas of Others

The community of mathematics education researchers, like any community, contains individuals with a diverse set of perspectives and ways of operating. This is a significant strength in mathematics education. Sometimes belonging to a community can produce internal struggle, while other times there is a feeling of camaraderie and appreciation for the other (Wilson, 2015). The need to belong to communities while simultaneously feeling unique and autonomous is foundational to our intellectual development (Confrey, 1995). We need collegial relationships to explore and develop our ideas as Piaget asserted in the introduction to this section.

Silver and Kilpatrick (1994) pointed to diversity of ideas in mathematics education in their call for "openness, tolerance, and respect for the work and ideas of those colleagues who share neither our culture nor our tradition" (p. 753). This diversity in the mathematics education community offers a rich collection of ideas and perspectives to be learned from and understood. Articles in the *Journal for Research in Mathematics Education* have continued to illustrate different perspectives on ideas in mathematics education. For example, Lerman's (1996) perspective on intersubjectivity in mathematics learning provoked a discussion of radical constructivism that had the potential to excite and inform colleagues in mathematics education. I vividly recall reading the exchange between Steffe and Thompson (1997), Lerman (2000), and the thoughts of Kieren (2000) on this exchange. This discussion informed my views of relevant learning theories in mathematics education. Furthermore, the collection of exchanges provided a model of how mathematics education research was done—how perspectives were shared and how arguments were structured in mathematics education research. Reading these papers suggested to me that discussions in mathematics education about perspective shaped resulting research. I recognized that these authors were not working at the same institutions, and, in retrospect, I wonder if this factor might have made a difference in the way the dialogue was written. A later discussion between Lubienski and Gutiérrez, both in residence at the same institution, was written quite differently. Lubienski and Gutiérrez provided insight into perspectives on performance gaps (Gutiérrez, 2008; Lubienski, 2008; Lubienski & Gutiérrez, 2008). This exchange further informed how I viewed the role of history, perspectives, and arguments in mathematics education research.

Your ideas are informed by the perspectives of others as you construct quality mathematics education research. As the dialogue between Lubienski and Gutiérrez (2008) suggests, mathematics education researchers frame and reframe their questions as a result of personal experiences and consideration of the work of others. Even when perspectives are different, conversations allow for consideration of other's ideas and opportunities for the evolution of one's own ideas.

11.2.2 Evolution of Ideas through Conversations

Belonging to mathematics education research communities involves accepting yourself and others as people in the process of becoming. In the first half of this chapter, you read stories of the process of becoming from the perspective of a researcher. There is no final best idea or research stance since contexts and researchers' ideas are constantly evolving. Engaging in conversations with other researchers requires exploring unfamiliar ideas and perspectives. While approaching an idea with skepticism can help you gain insights into its limits, approaching an idea with acceptance can help you gain insights into the potential of an idea. As Harkness (2009) has noted in her work on believing and doubting, both approaches are needed in building insights in mathematics and mathematics teaching. I suggest that employing both approaches may be useful in the many conversations you will have with colleagues over the years. These conversations create opportunities for you to represent your ideas to others. Being listened to is a critical component of building representations of your ideas (Weissglass, 1990), understanding their meanings to others, and developing confidence in their productivity. In addition, the reactions of others can help you make sense of how to improve the representations of your ideas and the ideas themselves.

The critical role of such conversations is documented in Silver and Keitel-Kreidt's (2015) book of stories about the impact of Jeremy Kilpatrick's work in mathematics education. Jeremy himself told one such story of his experience with Jon Star as an early career scholar:

> I happened to be at Michigan State talking with Jon Star. He told me that he had an idea for a commentary on procedural knowledge and wanted to check whether it would be appropriate. He thought that researchers in mathematics education were not thinking about procedural knowledge in quite the right way. So we talked a little about what he might write. (Kilpatrick, 2013, p. 180)

While it is not clear what Kilpatrick and Star discussed, what resulted was the sort of public conversation that encourages the growth of perspective. Star's (2005) commentary provoked new thinking and alternatives in a response (Baroody, Feil, & Johnson, 2007) followed by a rejoinder from Star (2007). This dialogue began with an idea but became valuable in mathematics education through conversation. Each time I read this dialogue, I take away new insights—my views of Star's ideas shift a bit in relation to my own ideas about learning.

11.2.3 Finding a Context for Your Ideas

I have sometimes felt that an idea I am working on has no referent in mathematics education research, even though I know that conversations about the ideas of others should form a context for my ideas. In 2018, my colleagues and I struggled to situate one of our ideas in the context of the literature (Kastberg, Lishcka, & Hillman, 2018). In an early version of the article, we claimed that there were no research reports relevant to our work. We decided to focus the "conversation" about our ideas in relation to themes in mathematics teacher education. Our conversation was very brief, as published reports of studies of mathematics teacher educators' written feedback were not yet available. A kind and supportive reviewer helped us see that our idea needed a more inclusive context:

> It is claimed (about 3 times) that Buhagiar (2013) is the only published paper on MTEs' [mathematics teacher educator] written feedback. It would be unfortunate if this were not true…. The authors could express [this lack] in terms of their own search. In a broader context, they might take a look at [reference provided]. (personal communication, anonymous reviewer)

This reviewer points toward a need to build readers' trust in our review of the literature by sharing how we searched and identifying a relevant research context. We needed to gain a view of the ideas of others to situate our own idea in a compelling way. Looking more broadly at work in teacher education helped us gain views of the ideas of others and gave dimension to our conversation. This reviewer reminded us that the ideas of others could provide a context for our work. Our ideas evolved as we constructed a relevant research context for them rather than asserting that a conversation was not needed.

This experience once again alerted me to the need for conversations that provide context for an idea but that also highlight the significance of that idea in context. Rather than diminishing the value of either idea, both ideas became more significant. My colleagues and I were reminded that "one critical aspect of conducting and reporting research in mathematics education is building on the work of others" (Leatham, 2015, p. 253). In quality mathematics education research, there is always relevant related research; what varies is how closely related the research is to the focus of the study (Leatham, personal conversation, 2018). Conversations with colleagues and reviewers can help you find a context for your ideas and create a conversation in which your idea fits.

Conversations in the mathematics education research community should support the development and representation of ideas. Whether your conversations appear in print as in Star (2005, 2007) and Baroody, Feil, & Johnson (2007) or over a career as in Kilpatrick (2013) and Wilson (2015), in every case your efforts to contribute quality research to the mathematics education community are supported through your engagement in conversations.

11.3 Conclusions

Throughout this chapter I have worked to draw together sources of inspiration that reflect the development and representations of quality research in mathematics education as a human enterprise that involves self and other in relationship. In addition, I have worked to link evidence of our humanity and relationships as well as reflections on self and others as researchers to reveal the production of mathematics education research as a messy process. The resulting chapter is meant to illustrate some of the factors involved in the human dimension of the construction of quality mathematics education research.

As I undertook this chapter, I feared I would not be able to draw together resources, beyond my own stories, that could reflect the elements of the process of developing and representing quality mathematics education research. Happily, my colleagues and students encouraged me and talked with me about my ideas. Without these others and the chance to represent my ideas here, I seriously doubt I would have persisted in trying to communicate about navigating the role of the self and engaging with others in the production of quality research in mathematics education. So here I am once again in the process of having a wonderful idea, being encouraged to pursue it, believing in my idea, doubting that I could communicate it, and having conversations to further gain insight. These conversations motivated me and informed the meanings I made.

For researchers new to this process, I encourage you to have patience with yourself and your ideas and to be persistent. In addition, seek out others. At times others will not want to have conversations, yet, in this too, be persistent. Create contexts in which ideas can be discussed, whether it be over coffee, at a conference, on video conference, or on social media. As Piaget noted, "you must have contacts" (Bringuier, 1980, p. 18). Cherish and consider the historical roots of your desire to know and the ways those roots, as Lubienski and Gutiérrez (2008) described in their commentary, inform your desire to know and ways of knowing. Have patience with the emergence and development of your ideas. Deadlines in the world can encourage us to create a feeling of panic about our process of doing research and being researchers. Certainly, the pressure is real and the stakes are high. Yet embracing the idea that becoming a mathematics education researcher is a process (Rodgers, 1961) can be useful. The construction of quality research means seeking ways to be productive *and* ways to care for yourself and others. Experience your process, and tell your stories of producing quality research in mathematics education. Such work is critical in sustaining your own work as well as mathematics education as a field of study.

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Part III
Publishing Quality Research in Mathematics Education

Chapter 12
Principles for Effectively Communicating the Theoretical Framing of Our Work

Keith R. Leatham

Abstract Based on my experience as a reviewer and, in particular, as an editorial panel member, I discuss six principles related to the role of theoretical frameworks in research dissemination. Throughout the chapter, I draw on examples from the mathematics education research literature to illustrate what application of the principles looks like in practice. My purpose in articulating and sharing these principles is to help us all think about how we can better communicate the theoretical framing of our work to those who will read it.

Keywords Theoretical framing · Theoretical frameworks · Research dissemination · Publishing quality research

One critical aspect of research activities in mathematics education is the way that work is framed theoretically (Eisenhart, 1991; Mewborn, 2005; Spangler & Williams, 2019). We frame our work theoretically by drawing on, adapting, and developing theories (plausible explanations for phenomena) in order to delineate the scope and nature of the phenomena we study. I use "theoretical framing" and "theoretical framework" interchangeably throughout this chapter and intend for them to capture the collection of theory-related constructs often described using terms such as *theoretical framework*, *conceptual framework*, *analytic framework*, and *theoretical perspective*. Theoretical framing typically has roots in the genesis of a research study, influencing our initial conceptualization and research design. It plays further important roles in guiding our data collection and analysis and continues to influence how we communicate our results. In this chapter I focus on the last stage of this process—on the critical role of theoretical framing in research dissemination.

Having written many reviews over the years (particularly as a member of the JRME editorial panel), I decided to revisit these reviews to examine more closely the nature of the feedback I tended to give with regard to theoretical framing. This

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review of my own reviews coalesced into six principles that seemed to guide my feedback related to the role of theoretical framing in research dissemination. These principles thus represent, in essence, how I have come to understand effective theoretical framing—the kind of framing that convinced me, as a reviewer, of the quality of the research I was reviewing. My purpose in articulating and sharing these principles is to help us as mathematics education researchers consider how we can better communicate the theoretical framing of our work when we write about that work. And, I suggest, we should consider two important audiences with whom we are trying to communicate: the editors and reviewers who are tasked with judging the quality of our written work and the eventual consumers, who will seek to understand, digest, and hopefully build on our work.

12.1 Principle 1: Theoretical Frameworks Should Establish How We Conceptualize and Recognize the Central Phenomena That Are the Focus of Our Work

Theoretical frameworks are lenses through which we identify, define, and interpret the phenomena we seek to understand (cf. Anfara & Mertz, 2015). In order for written reports of our work to make a substantial contribution to the field, we must make these lenses explicit. In doing so we should make clear not just *what* those theoretical frameworks are but *how* those frameworks help us to recognize and make sense of the phenomena we are studying. If a manuscript does not communicate clearly how it was theoretically framed, the reader is left questioning all aspects of the work. For this reason, the strength of our written contributions is determined, to a great extent, by the strength of the theoretical framing of the work. (Not just the strength of the theoretical framework itself but also of our own use of that framing in presenting our research.) That strength matters because an underlying purpose of all research dissemination is to provide not just research results but solid theoretical grounding (for those results) on which the field can build.

Consider, for example, the article *Refusing Mathematics: A Discourse Theory Approach on the Politics of Identity Work* by Chronaki and Kollosche (2019). The very title itself of this article makes clear that the central phenomenon of identity is going to be framed theoretically using a politically focused take on discourse theory. The following statement from the introduction to the article then makes quite explicit just how the authors plan to frame this view of identity:

> We aim in this paper to discuss how the discourse theory of Laclau and Mouffe (1985/2001) can substantiate the study of mathematical identity, not as a fixed but as a contingent meaning-making process that unfolds the political struggles of mathematics education in our contemporary times. (p. 2)

To further illustrate the role of theoretical framing in establishing our view of the central phenomena of our research, consider Munter and Haines (2019). In this article the authors report results from a study that examined "relations between middle and high school students' perceptions of racial differences in opportunity in mathematics and teachers' instructional practice" (p. 2). They then identify the central phenomena they focus on to capture this purpose as "opportunity to learn through cognitively demanding tasks, and perceptions of discrimination and opportunity" (p. 2) and then devote two subsections of the article to describing how they frame these phenomena.

12.2 Principle 2: The Grain Size of a Theoretical Framework Should Align with the Grain Size of the Phenomena Under Consideration

All research in mathematics education is somehow related to mathematical learning and teaching, but not every mathematics education research paper needs to explicate a theory of learning or of teaching. The theoretical frameworks we describe in our papers should frame the particular phenomena we studied, and that means that this framing needs to be specific enough to provide a useful lens for the reader to make sense of our work. A theoretical lens must be just the right distance from a phenomenon in order to meaningfully magnify the phenomenon for the reader; theories that are too close or too far removed only serve to blur the image. For example, the aforementioned article by Munter and Haines (2019) did not (appropriately, I would argue) lay forth a theory of learning.

That said, there is certainly room and value in articulating what we might think of as a theoretical worldview, but such descriptions are typically quite brief. The further the theory is removed from the focus of the paper, the less space should be devoted to it. Although Munter and Haines (2019) did not lay forth a theory of learning, they did preface the theoretical framing of their two central phenomena with a brief description of how their work was influenced by "Martin's (2006) conceptualization of mathematics learning as a *racialized form of experience*" (p. 3, italics in original). This broad perspective provided a theoretical context for the reader as they moved into the theoretical framing of the central phenomena.

Furthermore, in narrowing in on the phenomenon under consideration, we should take care to communicate that our theory likely has wider use than for just the particular study we conducted. Thus, we should frame our theoretical frameworks such that their relevance to the field is apparent. For example, consider the way Izsák, Jacobson, and Bradshaw (2019) framed their theoretical framing. In order to scale up their exploration of their phenomenon of focus—teachers' "reasoning with measured quantities" (p. 162)—the authors developed "a grounded framework for psychometric measurement" (p. 162). They described in some detail the process they used in this theoretical development and made explicit note of how the process might contribute "more broadly to the measurement of mathematical knowledge" (p. 190). This detail thus allowed them to frame their current work and also to frame this framing in such a way that the rest of the field might benefit more broadly from applying or adapting their process.

12.3 Principle 3: We Should Distinguish Our Theoretical Voices

In general, as stated previously, our work should build on the theoretical work of others. At times this building involves adopting particular theoretical frames; at times it involves adapting those frames; and at times it involves creating new theoretical lenses. Regardless of the chosen variation, readers need to know which variation they are experiencing and thus to what extent the theoretical framing is drawn from others or originates with us. Part of the issue here is one of ownership—we should not claim, even unintentionally, that theory is our own when it is not (Leatham, 2015). Yet another part of the issue is one of giving the reader confidence in the researcher—confidence that the author is aware of what the field has to offer theoretically and how that theory is related to what they are offering in this manuscript. In order to establish credibility with the reader, we should explicitly communicate the origin of adopted or adapted theories that contribute to our theoretical framing. And, just as importantly, we should explicitly communicate the adaptations that represent our own contributions to this framing. The more integral theoretical frameworks are to our research activities, the more those lenses become second nature. And because of this familiarity, the distinctions between these origins and adaptations can sometimes remain implicit in the writing of our manuscripts. Thus, it is important that we take a step back and explicitly state both how we view things and the origins and evolutions of those viewpoints.

For example, consider the theoretical framing in Darragh (2018). Darragh adapted positioning theory to frame an exploration of how young adult fiction portrays mathematics. Having clearly attributed positioning theory to the originators (Harré & van Langenhove, 1999), and described her interpretation of that work, she then proceeded to explain how she "understood the notion of positioning in a broader context than that of conversation" (Darragh, 2018, p. 118). In a similar way she distinguished her particular way of viewing "storyline" in the context of her study from how it is typically used. Throughout these descriptions the author clearly delineated her descriptions of how the field views positioning theory from her own adaptations.

With respect to building on the theoretical ideas of others, sometimes these ideas get presented in manuscripts in what amounts to just a list of theories. Such lists typically leave the reader wondering, "Okay, so, which one of these theories, if any, are you adopting?" or "Why are you telling me about all of these theories?" Typically, such collections of theories are only useful to the extent to which we use them to describe our theoretical framing. Although it is valuable to present varying viewpoints for the sake of comparison and contrast, such a purpose is not accomplished unless those similarities and differences are made explicit for the reader. In addition, those similarities and differences need to feel useful to the reader, helping them to better understand where we are coming from.

For example, consider how Voutsina (2016) discussed various theories in the context of presenting her exploration of knowledge explication in children's counting development, as framed by representational rediscription theory:

> There is a significant difference between previous references to knowledge explicitation and Karmiloff-Smith's representational redescription (RR) theory. Karmiloff-Smith observes, positions and analyses knowledge explicitation in the part of learning and development that takes place following successful performance in a given task rather than in the part of an individual's learning and developmental trajectory that leads to initial procedural mastery and successful performance in completing particular tasks. This feature of the theory differentiates it from other theorisations of mathematical thinking and development that depict the developmental trajectory from initial levels of weaker conceptualisation and also limited in some aspects, or erroneous task performance, to levels of increased abstraction and understanding (see for example, Resnick's (1992) developmental model of mathematical thinking or from the field of mathematics education Mulligan and Mitchelmore's (2009) levels of mathematical structure). (p. 181)

Notice how other theoretical framing options are described in terms of how they contrast to what Voutsina (2016) sees as a critical characteristic of her chosen framing. The writing makes clear that these other theories are mentioned for the purpose of providing contrast to the chosen theoretical framing for the study.

Furthermore, having distinguished our theoretical voices from among the various ways we might have chosen to theoretically frame our work, we need to help readers understand *why* we chose the frame we did. In a sense, quality manuscripts not only make their theoretical framing explicit, they also make explicit a rationale for that theoretical choice. Such a rationale goes a long way toward convincing the reader that the chosen theory is sensible, particularly when our theoretical choices involve introducing a new theory when current theories exist or substantially altering these extant theories. There is definitely a need for both research that builds on extant theoretical frames as well as research that moves beyond or diverges from those frames. The direction we take should depend very much on the nature of the research. We do not always need to claim to be introducing a new way to view things in order to contribute in meaningful ways to the field. There are many ways to make significant theoretical contributions by drawing on existing theory and showing its application in new or novel ways. Regardless of the theoretical direction, quality research in mathematics education communicates to the reader why that direction was taken.

As an example of providing a rationale for theoretical choices, consider DeJarnette's (2018) study of how secondary students reason about trigonometric functions. DeJarnette chose to view this reasoning through the lens of Balacheff's (2013) cK¢ (conception, knowing, concept) model of conceptions. Having described the model and how it applied to the study at hand, DeJarnette (2018) went on to provide a rationale for this particular choice of framing:

> The cK¢ model offers several advantages for examining students' conceptions of trigonometric functions. Beyond inferring the state of students' knowledge through their observable work, the cK¢ model prioritizes attention to what prior knowledge students draw on and how they use it for completing a task. Additionally, this model of students' conceptions makes explicit the situated nature of student thinking, specifically how students' thinking is

> likely to shift with changes to the environment, to the problem, or in their interpretation of the problem. Finally, the cK¢ model allows for explicit attention to how students make connections between symbolic and graphical representations of function and what motivates these choices. (p. 396)

In providing this rationale, DeJarnette gave the reader insight into *why* this framework was chosen—how the choice of framework allowed the author to focus on particular aspects of student reasoning in this particular context.

12.4 Principle 4: In Order to Be Believable, Theoretical Frameworks Should Be Coherent, Comprehensive, and Consentient

In this principle I pull together three notions related to the extent to which theoretical framing stands up to the reader's scrutiny—the extent to which theoretical framing is believable, giving the reader confidence that the theoretical framing of the study adequately frames the work. The presented theoretical framing needs to be coherent (presenting no apparent inconsistencies), comprehensive (covering essential aspects of the phenomena under consideration), and consentient (accurately representing extant theory). In essence, the first two notions are issues from within the manuscript, whereas the third is an issue that reaches beyond the manuscript.

12.4.1 Theoretical Coherence

Theoretical frameworks often require pulling together views of a number of related phenomena. When doing so, the various pieces of the framework need to cohere. The reader needs to see how the pieces relate to each other, and they want any apparent or potential inconsistencies or contradictions to be addressed. If multiple ideas related to a given phenomenon are presented, the reader wants to see evidence that these ideas have been synthesized into one coherent whole.

One common roadblock to theoretical coherence is when we are sloppy with our vocabulary, using different language across the paper to refer to a given phenomenon without making it clear to the reader whether this difference in language is intended to communicate important subtle differences in ideas or rather just to provide linguistic variation so as not to sound overly repetitive. Such coherence in language is particularly important at the dissemination stage, and the absence of coherence makes theoretical frameworks less believable. For example, consider research on teacher beliefs. There are many terms potentially related to beliefs, such as *view*, *preference, attitude*, *perspective*, and *conception*. When reporting research on teacher beliefs, it would be important to theoretically frame the phenomenon of *belief*. In addition, however, if other terms such as those listed above are used, the reader needs to know whether they are intended to be synonymous with belief.

12.4.2 Theoretical Comprehensiveness

As argued in Principle 1 (Sect. 12.1), our manuscripts need to theoretically frame the central phenomena under consideration. A comprehensive framework would thus explain how we view *all* of the primary phenomena under investigation. In addition, within a particular phenomenon, theoretical frameworks should be comprehensive in the sense that they address all relevant situations. Readers should not be left to wonder how critical aspects of a phenomenon are being viewed. Such insufficiency leads to unbelievability because readers are left suspecting that, viewed through this lens (as presented in the manuscript), there are important aspects of the phenomenon that the researcher is likely to miss. For example, drawing again on the area of research on teacher beliefs, if a research study frames beliefs as predispositions to act in particular ways, but then presumes that teachers are capable of articulating all of their beliefs, the reader is likely to question whether the theory will sufficiently capture the phenomenon. Theoretical framing might address this inadequacy by explicitly limiting the study to a particular subset of teacher beliefs, such as their stated or *espoused beliefs*, or by expanding their theory to allow them to infer beliefs beyond teachers' abilities to articulate them.

That said, sometimes comprehensiveness is compromised because the theoretical framework covers *more* than is necessary, leaving the reader to wonder whether they have accurately understood the nature of the phenomena under consideration. Making the theoretical focus of a paper clear and concise increases the likelihood of presenting a comprehensive theoretical framework. We do not need to write about every theoretical idea we considered or even used throughout the course of the study. We should focus on the theory that fitly frames every aspect of the portion of the study the current paper reports—no more and no less.

12.4.3 Theoretical Consentience

It must be clear in our writing that we understand the way the field views the theories on which we are building. For example, we can appeal theoretically to radical constructivism (von Glasersfeld, 1995) without being in total agreement with that theory, but what we say about radical constructivism ought to be consentient with how radical constructivism is characterized in the field of mathematics education. Thus, although *we* may not agree, our characterization needs to agree with how the field as a whole has come to view this theory. Disagreements with and critiques of theories can help to move the field forward, but such arguments will not convince others unless we demonstrate an understanding of extant views of those theories.

Once we have described extant theory sufficient to demonstrate understanding, we can move on to delineate how our views might differ. If instead we present a description or an argument that is incomplete or that misrepresents a commonly used theory, then the reader's believability in that theory as a meaningful way of grounding our work is diminished. Furthermore, this lack of consentience casts doubt on the rest of the manuscript, particularly the methods, results, and conclusions. As a field we do not need to have consensus; reasoned, impassioned, articulate differences strengthen us all. That said, we do need our characterization of the field to be consistent with the field's view.

12.5 Principle 5: Situating a Study Empirically Is Not the Same as Situating It Theoretically

There is no consensus in mathematics education research dissemination with regard to the terms we use to describe the various reasons we appeal to the literature. This lack of consensus causes the most confusion to me as the reader when I come across the section that we often call "literature review." What the reader gets in this section varies a great deal from paper to paper. I argue here that there are two particularly important, but substantially different, purposes for drawing on the literature in our research papers and that we could better communicate with readers by more explicitly delineating these purposes in our writing.

For me, the purpose of a literature review is to draw on the literature in order to situate our studies within related research *results*. In other words, literature reviews are intended to help the reader understand what we know or what the field already knows about the phenomenon we are studying. We do so, at least in part, in order to make a case that we have something to add to what is already known. Thus, a literature review synthesizes the results of extant research and does so while foreshadowing the results we are about to share. When done well, literature reviews both provide relevant background knowledge and demonstrate the need for the knowledge the current paper contributes.

Whereas the literature review is situated within the *results* of extant literature, our theoretical frameworks are situated within the *theoretical framing* of extant literature (as described in Principle 1, Sect. 12.1). The purpose in describing a theoretical framework is to help the reader understand our way of viewing a particular phenomenon and how that view compares to how others view that phenomenon, not to report what we do and do not know *about* that phenomenon.

Although the theoretical framework can either precede or follow the literature review, that order should be purposeful. We might choose to present our theoretical framework first so as to provide language and organizational structure to the literature review. Alternatively, we might begin by reviewing the literature, illustrating some important variations or limitations in extant results, and then use these variations or limitations to motivate the theoretical framing we present next (presumably one that tries to account for these variations or limitations).

Regardless of the ordering of these two ways of situating our work in the literature (or whether they are integrated into a single section, an approach that is difficult to do well and that I do not recommend for the novice researcher), our manuscripts are more convincing when we help the reader to see how we are accomplishing both purposes—how we are situating our work with respect to both results and theories. The more explicit we can be about how we are accomplishing these two purposes, the more convincing our arguments will be to the reader.

To illustrate the use of a theoretical framework to frame the literature review, consider Lew and Mejía-Ramos (2019). The authors viewed the language of mathematical proof through the lens of the mathematics register (Halliday, 1978). The authors first described this theoretical framework for proof, then used this theoretical lens in order to frame their discussion of the results of related proof literature. This framing allowed them to argue, for example, after reviewing several studies related to semantic contamination (studies that were not framed using registers), that such studies "have illustrated the relationship between the mathematical and the everyday registers and how such relationships may influence the learning of certain concepts in advanced mathematics" (p. 123). Thus, the authors use their choice of theoretical framework to interpret the literature and situate their study within it.

To illustrate the use of a literature review in order to motivate the use of a particular theoretical framework, consider Lewis (2014). In her review of research on mathematical learning disabilities (MLDs), Lewis established that a preponderance of these studies "focused on elementary-aged students' speed and accuracy on written assessments of basic arithmetic calculation," resulting in conclusions that students with MLDs make errors because of "insufficient automaticity of arithmetic number facts" (p. 352). She further argued that "the few studies that have begun examining MLDs in more complex mathematical domains have found conceptual and representational issues—not difficulties with number facts—to be central to the errors made by students with MLDs" (p. 352). Lewis then used these observations to provide a rationale for the way she chose to theoretically frame MLDs, a framing that sought to account for aspects of MLDs (such as persistent understandings) that prior studies had not been able to see because of their chosen frameworks.

12.6 Principle 6: Theoretical Frameworks Should Have Pervasive Explanatory Power

Although it is important for theoretical frameworks to be made explicit (often through presenting them in their own section), the influence of this framing on the study as a whole should be evidenced throughout the entire paper. To elaborate on this principle, I briefly discuss each of the main elements of a research paper. In doing so I describe how a theoretical framework might provide explanatory power to that element and then illustrate such usage from a contemporary publication in mathematics education.

A theoretical framework can bolster a paper's *rationale*. At times, as mentioned in the discussion of Principle 3 (Sect. 12.3), there might even be a theoretical rationale for the work. Consider, for example, Rott and Leuders (2016). After briefly establishing that research on teachers' beliefs is an established and valued area of inquiry in mathematics education, the authors home in on a particular subset of beliefs—epistemological beliefs (i.e., beliefs about the nature of knowledge). They then argue that "there is a growing body of empirical evidence that epistemological beliefs are less coherent, more domain-specific, and more context-dependent than previously assumed" (p. 272) and propose a way of theoretically framing epistemological beliefs (wherein they draw distinctions between epistemological beliefs and epistemological judgments) that is designed to address these previous limitations. In essence, their way of framing epistemological beliefs is put forward as a way to address a problem in this line of research and thus as a good reason for conducting their study.

A theoretical framework can help us to articulate and clarify our *research questions*, providing a specificity of language and construct that could not happen otherwise. Furthermore, as argued by Cai et al. (2019), "the theoretical framework shapes the researcher's conception of the phenomenon of interest, provides insight into it, and defines the kinds of questions that can be asked about it" (p. 119). Consider, for example, Nachlieli and Tabach (2019), which reported on a study designed to understand the "resilience of traditional teaching" (p. 3). Drawing on Sfard's (2007) notion of commognition, the authors view teaching and learning as routines—where similar situations call for similar actions. They further home in on ritual routines, the performance of which usually "includes imitating someone else's former performance" (Nachlieli & Tabach, 2019, p. 255). The authors then operationalize "teachers' actions that provide students with tasks that could be successfully performed by rigid application of a procedure that had been previously learned" (p. 257) as ritual-enabling opportunities-to-learn (OTLs). Having presented their way of framing traditional teaching, the authors are able to provide the following research questions: "Are ritual-enabling OTLs prevalent in the mathematics classrooms? And if so, what could be gained by ritual-enabling OTLs?" The authors' theoretical framing provided critical specificity of language to articulate these questions as well as insight into what it might look like to try to answer them.

As argued and illustrated already in Principle 5 (Sect. 12.5), a theoretical framework can guide how we present a *review of the literature*, providing a lens through which we view and interpret extant research findings. This theoretical framing illustrates again how our choice of framework defines what we see—in this case what we see in the body of related literature.

Theoretical frameworks should influence how we describe our *data collection*, particularly in the sense that, having described how we recognize a particular phenomenon when we see it, the theoretical framework provides the means for arguing that the data we collected indeed allowed us to see what we wanted to see (and in the ways we indicated we would see it). For example, consider again the study of mathematical learning disabilities reported in Lewis (2014), where she argued that “methodologies used to study individuals with MLDs should capture… alternative understandings” (p. 354) and then described collecting data that “provided a context in which the difficulties that arose for the students with MLDs could be analyzed” (p. 360).

The theoretical framework should be evident in the ways we describe our *data analysis*. Readers tend to become leery when they arrive in the results or, worse yet, in the discussion section and find themselves thinking, “I don’t think I’ve seen the theoretical framework since the theoretical framework section.” The reader wants to get a sense of *how* the theoretical framing helped us to see the phenomenon in our data. By way of illustration, I return to the cK¢-framed study of students’ trigonometric reasoning presented in DeJarnette (2018). The chosen framework models students’ conceptions through four components. In the analysis section of the paper, DeJarnette described the general process by which the data were analyzed in order to identify evidence of these components and then illustrated this process with a detailed example. As such, the reader was provided with ample evidence of the influence of the theoretical framing on the data analysis in the study.

Our theoretical framing should be evident in the way we present our *results*. Again, because our theoretical frameworks can become second nature to us, although *we* see how our theoretical grounding influenced or is related to our results, too often these connections are left implicit in our manuscripts. Thus, when presenting what we saw in our data, we should take care to highlight ways that our theoretical framing influenced that view. Sometimes (but certainly not always) our theoretical framing suggests an organizational structure to our findings, as was the case in the results of the just-mentioned DeJarnette (2018) study of trigonometric reasoning. More generally, however (and certainly with the structural influence just described as well), the theoretical framing provides critical vocabulary for describing our results. Illustrations of such linguistic influence can be seen in the following excerpts from the results sections of articles I have discussed in this chapter:

- “The findings from this section also suggest that some students may not necessarily understand mathematical language to be a subregister of academic English.” (Lew & Mejía-Ramos, 2019, p. 136)
- “This deco[mposition] of tasks to sub-tasks provides students with ritual-enabling OTLs rather than exploration-requiring ones.” (Nachlieli & Tabach, 2019, p. 268)
- “On the basis of the RR model predictions of learners’ behaviour at the ‘procedural’ phase, Sara’s observed behaviour could be interpreted as being supported by well-functioning procedures that needed though to be run afresh every time because they consisted of successful but non-well connected behavioural units.” (Voutsina, 2016, p. 184)

Such explicit usage of the descriptive language of the theoretical framework provides evidence to the reader of the integral nature of the framework.

Finally, theoretical frameworks carry more explanatory power if we use them to help to qualify our *conclusions and implications*. In this section of the paper, we can acknowledge and discuss the fact that the results we have reported were very much dependent on the way we were viewing the phenomena we studied. To illustrate how theoretical framing might frame conclusions and implications, I return to the discourse theory-framed study of mathematical identity presented in Chronaki and Kollosche (2019). Having presented the results of their analysis of Anja's mathematical identity, the authors concluded that

> discourse theory allowed us to appreciate Anja's struggle to discursively articulate her relation to mathematics around the nodal points of dignity, togetherness, relevance and bodily activity…. Anja's discursive articulation of mathematics needs to be appreciated, with Laclau and Mouffe's discourse theory, as a contingent and temporal assemblage of discursive and non-discursive elements from her socio-material reality that features very specific moments in the central position of nodal points, and it has to be expected that hers or other students' experiences might result in different articulations at different spaces and times. (p. 466)

Note how the authors appealed to their theoretical framing to qualify their results, both with respect to what the analysis through this framework allowed them to see and with respect to what it may *not* have allowed them to see.

12.7 Final Thoughts

Theoretical frameworks should play a critical role in all aspects of research in mathematics education, including in research dissemination. The six principles outlined in this paper could serve at least three purposes: (a) to provide novice mathematics education researchers with tools for learning about the role of theoretical frameworks in research dissemination, (b) to remind established mathematics education researchers of the theoretical aspects of their work that may have become transparent and thus gone unarticulated in the dissemination of their research, and (c) to guide reviewers and editors as they judge the quality of the written products of our theoretical work. Toward these ends, Fig. 12.1 brings together the six principles and associates questions one might ask as a means of exploring the extent to which any given manuscript aligns with the principles. As I have argued elsewhere (Leatham, 2015), if we all attend to this particular aspect of our scholarship, then our aggregate contributions will increase the power of our mutual efforts to build a knowledge base in mathematics education and, in the end, improve the learning of mathematics and the facilitation of that learning.

Principle 1: *Theoretical frameworks should establish how we conceptualize and recognize the central phenomena that are the focus of our work.*

Q1: What are the central phenomena this study explored?

Q2: To what extent does the manuscript describe how the authors conceptualize and recognize these phenomena?

Principle 2: *The grain size of a theoretical framework should align with the grain size of the phenomena under consideration.*

Q1: To what extent does the theoretical framework provide a meaningful way to view the phenomena under consideration at the level those phenomena were studied?

Principle 3: *We should distinguish our theoretical voices.*

Q1: Is it clear which aspects of the theoretical framework originate with the authors and which originate with others?

Q2: Is it clear why various theories are presented?

Q3: Is it clear why the authors chose to frame their work the way they did?

Principle 4: *In order to be believable, theoretical frameworks should be coherent, comprehensive and consentient.*

Q1: To what extent is it clear how the various pieces of the theoretical framework fit together and what meaning linguistic variation is intended to convey?

Q2: To what extent does the manuscript make clear how the theoretical framework frames all relevant aspects of the phenomena under consideration, and no more?

Q3: To what extent does the manuscript demonstrate an understanding of extant theories?

Principle 5: *Situating a study empirically is not the same as situating it theoretically.*

Q1: To what extent is the manuscript situated in related research results?

Q2: To what extent is the manuscript situated in related theories?

Q3: To what extent do the literature review and theoretical framework sections build on each other in meaningful ways?

Principle 6: *Theoretical frameworks should have pervasive explanatory power*.

Q1: To what extent is the theoretical framework both apparent and useful throughout the entire manuscript?

Fig. 12.1 Six principles related to the role of theoretical frameworks in research dissemination and associated reflective questions

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Chapter 13
Writing as Communicating with Reviewers: Strategies for Anticipating and Addressing Insightful and Skeptical Reviews

Sandra Crespo and Jinfa Cai

Abstract Communicating research insights is challenging and is often work that is underestimated. In this chapter, readers are invited to take the stance of writing as communicating with reviewers. After sharing how reviewers are assigned to manuscripts, the authors (experienced journal editors) discuss three of the common issues reviewers usually raise when recommending that manuscripts not be accepted for publication—coherence, claims, and contribution. They also share strategies that prospective authors can use to anticipate and address these issues when preparing and revising their manuscripts.

Keywords Writing as communicating with reviewers · Anticipating reviewers' feedback · Strategies for improving manuscripts · Claims and evidence · Contribution of manuscript · Manuscript's coherence

Research studies in general, and mathematics education research studies in particular, do not contribute to a larger body of knowledge until they are subjected to professional scrutiny by peers and are widely disseminated. The peer review process is crucial to the growth of knowledge in all academic fields, and, as researchers in mathematics education, we engage in all aspects of the production and dissemination of knowledge. However, to those who are newcomers, the nature of writing for publication in and undergoing the review process for peer-review journals is not always clear.

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This chapter focuses on the process of collaborative and public critique that occurs for articles submitted to peer-reviewed journals. Using our experience as editors of two different mathematics education research journals—the *Journal for Research in Mathematics Education* (JRME) and the *Mathematics Teacher Educator* (MTE) journal—we approach this chapter with the goal of demystifying the review process by highlighting common issues that reviewers raise about unsuccessful manuscripts. We begin with a brief discussion of how reviewers are assigned to submitted manuscripts. We then discuss three common criteria that reviewers often attend to and that often determine the fate of a submitted manuscript—coherence, claims, and contribution. (Although researchers need to consider more than these three issues when writing their manuscripts, other issues have been discussed in more detail in other chapters in this volume.) After illustrating these issues, we share strategies that prospective authors can use to anticipate and address reviewers' feedback when preparing or revising their manuscripts. Our hope is that this chapter will help readers to develop the habit of keeping reviewers in mind when they write their manuscripts.

Although much has been written about what can go wrong with the review process and about insensitive and unhelpful reviewers (Brown, 2015), our aim for this chapter is to operate under the assumption that the review process has worked as intended and that the feedback has been offered by an insightful and appropriately skeptical reviewer. Indeed, in our roles as editors, we can attest to the overwhelming generosity of reviewers who invest their time and energy to provide what can be called "educative feedback" to prospective authors and meaningful advice to the editors of the journal (see Cai, Hwang, & Robinson, 2019; Crespo, 2016).

13.1 How Reviewers Are Selected and Assigned to Submitted Manuscripts

As the title of this chapter indicates, it is important to conceptualize the writing process as communicating with reviewers and to carefully consider who might review a submitted manuscript. It may seem as though authors have little say in deciding who reviews their manuscript; however, it is important to realize that authors' writing and framing of the manuscript can provide important information to editors about areas of scholarship and expertise that they should seek when searching for reviewers. Although we cannot speak for all editors, in this section we share our thoughts on and process for matching manuscripts to potential reviewers. Anticipating what happens at this stage of the manuscript review can be helpful to authors when they are writing their manuscripts and when considering what and how to communicate to editors regarding ideal reviewers for their manuscript.

When a manuscript is submitted to a journal's manuscript processing system, the editorial team first determines whether it passes quality control (QC), in other words, whether the manuscript matches the journal's mission and has the appropriate format and standards for blind review. Once the manuscript passes QC, the editor reads the manuscript to determine whether the manuscript is or is not ready for the review process. Manuscripts that are not ready for the review process are missing one or more essential elements of the journal's review criteria. At *JRME*, for example (see Cai, Hwang, & Robinson, 2019), the editor reads the manuscript, discusses it with the editorial team, and assigns it one of the following four categories:

Inappropriate (I): Manuscripts pertaining to topics that do not fit the purposes of *JRME* (e.g., the presentation of a mathematical proof) are considered inappropriate for the journal and are returned to the author without further consideration.

Desk Reject (DR): Manuscripts for which the quality of the research does not meet the standards of *JRME* or manuscripts that do not meet the journal's technical or stylistic requirements (e.g., a verbatim chapter of a thesis) are desk rejected and returned to the author without undergoing external review. Typically, these reports have serious flaws or the work does not move the field of research in mathematics education forward in significant ways.

Editorial Review (ER): Manuscripts designated for editorial review show promise but are unlikely to be accepted for publication in their current form. As part of the educative mission of *JRME*, manuscripts from dissertation work often receive an editorial review rather than a desk reject. For an editorial review, a single member of the *JRME* Editorial Panel is chosen to evaluate and provide feedback on the manuscript.

Full Review (FR): Manuscripts designated for full review are sent to three to five reviewers. Typically, one reviewer is a member of the JRME Editorial Panel and the other reviewers are selected for their expertise relative to various aspects of the manuscript.

Figure 13.1 shows the flowchart used by the JRME editorial team to process submitted manuscripts. *MTE* and other peer-review journals follow a similar process.

Although a desk rejection may appear to be a harsh outcome for a submitted manuscript, it can also turn out to be a positive and productive one for many authors. The turnaround time is shorter and the editor makes explicit to the authors what aspects of their manuscript precipitated the decision. The following excerpt of a desk reject letter from *MTE* serves to illustrate:

> Our editorial office has reviewed your manuscript "[title of manuscript]" and determined that it is not yet ready to be sent out for full review. We have carefully read your manuscript in relation to the MTE review criteria and determined that this version of your manuscript does not explicitly address several of the journal's criteria. We have decided not to send it out for review at this time, but encourage you to review the comments below and the resources attached to help you more fully develop a manuscript that meets the journal's review criteria.

It is therefore important for authors of manuscripts to consider the intended journal's review criteria prior to submitting the manuscript. By self-reviewing their manuscript using these criteria, authors can strengthen their manuscript with regard to the criteria that their manuscript falls short on. Both *MTE* and *JRME*, as well as

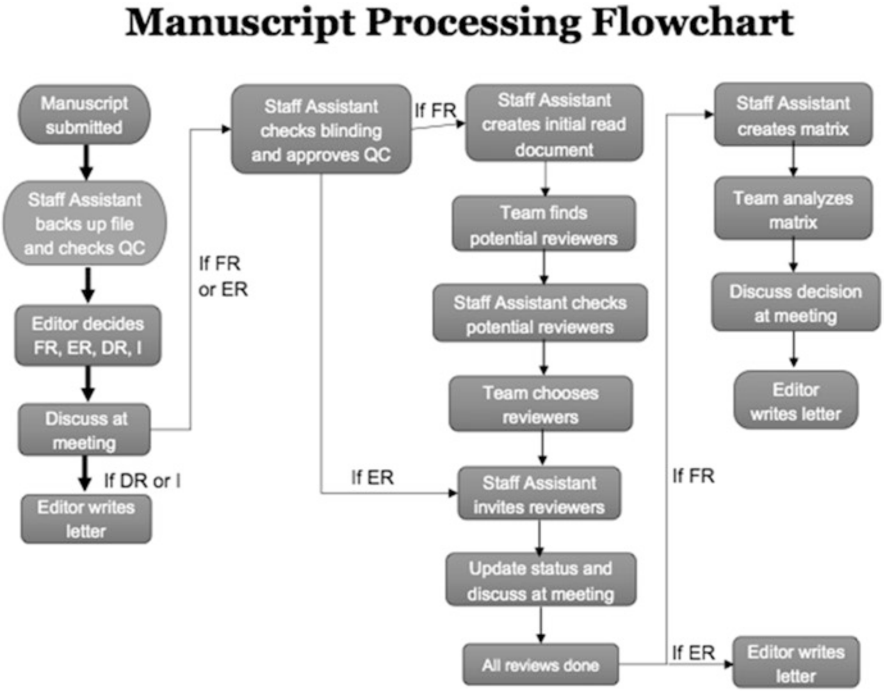

Fig. 13.1 JRME manuscript processing flowchart

many other journals, have resources intended to help authors to self-assess their manuscript with regard to the journal's review criteria. Becoming a reader and reviewer of the journal is another important step that authors can take to learn how to write for that journal. Upon completion of the review process, all of the reviewers are sent the editor's decision letter along with the other reviewers' comments. By examining these materials, reviewers can gain important insights into how other reviewers and the editor of the journal offer feedback to authors of manuscripts and how aspects of manuscript can meet or fall short of meeting the journal's review criteria. Appendix 1 lists the characteristics of high-quality manuscripts developed for authors submitting manuscripts to *JRME*. It is important to note that the author guidelines are closely related to the review criteria journals then use to determine the quality of submitted manuscripts.

Once the editor has determined that a submitted manuscript qualifies for full review, the manuscript is moved to the next stage during which editors invite potential reviewers for the manuscript. For both *JRME* and *MTE*, one editorial board member is assigned to review the submitted manuscript to ensure equitable access to quality feedback for all authors. The other reviewers are selected from the journal's reviewer database. For *MTE*, using the keywords submitted by the author and created by the editorial office's initial review of the manuscript, the editor generates a list of potential reviewers (using the *MTE* reviewer database) that best match the

(a) Does the research extend or deepen our understanding of important issues in mathematics education? Does it have the potential to lead the field in new directions?
(b) Do the research questions pertain to issues of significant theoretical or pragmatic concern? Are they well-grounded in theory or in prior research?
(c) Is there an appropriate match between the research question(s) and the methods and analyses employed to answer the question(s)?
(d) Does the conduct of the study include the effective application of appropriate data collection, analysis, and interpretation techniques?
(e) Are the claims and conclusions in the manuscript justified in some acceptable way, and do they logically follow from the data or information presented?
(f) Is the writing lucid, clear, and well organized?

Fig. 13.2 Guiding questions for reviewers of JRME manuscripts

manuscript's area of research. Reviewers with a 60% or higher match to the manuscript are then considered as potential reviewers. For *JRME*, a minimum of three reviewers are required, though each manuscript typically needs four or five reviewers to cover various areas of expertise. The *JRME* editorial team first identifies three to five areas of expertise needed for a manuscript based on an initial review of the manuscript. For each area of expertise, five or six potential reviewers are identified using the *JRME* reviewer database. At least one reviewer from each area of expertise is invited to review the manuscript. A major takeaway from this discussion of the reviewer selection process is that the selection of keywords for a submitted manuscript should not be an afterthought, as these keywords provide editors with important information about the expertise they should seek in their selection of potential reviewers.

In general, journals provide specific instructions to reviewers when they are asked to review manuscripts. For *JRME*, the list of questions shared in Fig. 13.2 are provided to reviewers who accept the request to review a manuscript. Thus we encourage authors to think through and anticipate how reviewers might respond to these questions before formally submitting their manuscript for review. Considering questions such as these can help authors to approach the writing of their manuscript as communicating with reviewers and to improve the quality of their manuscript.

13.2 Common Issues Raised by Reviewers of Manuscripts

Once the reviews are completed, the editor reads the reviewers' feedback and prepares a decision letter for the author that highlights common themes across the reviewers' comments that justify their decision to reject, encourage revisions, or accept the manuscript. In our roles as editors, we have found that unsuccessful manuscripts elicit consistent reviewers' feedback related to shortcomings in the manuscript's coherence, claims, and contribution. Before proceeding, we offer a short description of what we mean by a manuscript's coherence, claims, and contribution.

With respect to *coherence*, there is an expectation that manuscripts exhibit clear focus in their content and structure. This coherence could be reflected in the alignment between different sections of the manuscript as well as in the language or the theoretical constructs and conceptual tools used throughout the manuscript. With respect to *claims*, there is an expectation that there are clear connections between the claims and the evidence shared in the manuscript. Reviewers also raise issues about claims when the evidence seems stretched, when the claims seem overgeneralized, and when claims are presented as uncomplicated and without accounting for possible counterclaims and limitations of the study. With respect to *contribution*, there is an expectation that manuscripts explicitly establish how they contribute to the field by situating within, and establishing connections to, the existing literature as well as discussing the nature of the contribution in relation to ongoing conversations in the field about important and enduring challenges for the research community.

These three themes are often implicitly embedded within the reviewing guidelines provided to reviewers. Notice, for example, that in the list of questions used by JRME (see Fig. 13.2), questions (a) and (b) are related to the contribution of a manuscript, questions (c) and (d) connect with the theme of coherence, and questions (e) and (f) draw attention to the soundness and communication of the manuscript's claims.

We now invite readers to test their understanding of these three expectations. Consider the following excerpts from across three reviews of a manuscript submitted to *MTE* and look for these reviewers' expectations for coherence, claims, and contribution. Do this before moving on to read our subsequent analysis of these excerpts.

> The focus of the manuscript is unclear at times. At the beginning, the paper seemed focused on using [a particular instructional activity is mentioned] for motivational purposes. Later in the paper the manuscript focused on how students perceived the experience. The paper ended with an implication that [said instructional activity] should be used, but there were not clear connections between these claims and the evidence in the manuscript. As a result, while the concept of using [said instructional activity] in a teacher education program may have merit, this article does not clearly articulate how and what preservice teachers learned from taking part in this process. Further clarification would strengthen the manuscript. (Reviewer 1)
>
> There is a lack of alignment between the introduction and what actually transpires in the article. Broadly, the introduction sets the reader up to engage in a discussion about how [featured instructional activity] can help assess mathematical reasoning but I felt that the article diverged from this path in the following ways: The examples given, in my opinion, did not illustrate mathematical reasoning in an obvious way. Maybe I just missed it, but then I think that the author should discuss the specifics regarding how their work in [said instructional activity] demonstrated mathematical reasoning for the students. The report of the data is very scattered, with a brief discussion of each example. I think it would be more meaningful if the author focused on one or two examples and then described what they saw as illustration of mathematical reasoning within the [instructional activity]. As it is written, the examples seem shallow and not at all describe mathematical reasoning. (Reviewer 2)
>
> Despite the argument that is presented, the focus of the use of [instructional activity] with preservice teachers for this purpose misses the mark. In particular, the provided examples do not make the case for nor illustrate how to use [said instructional activity] effectively

> to enhance or assess students' mathematical understanding. In addition, this manuscript falls short of the standard for evidence recommended for manuscripts to be published in MTE. Other issues include the use of colloquial tone rather than a scholarly one. After reading the manuscript, I was not convinced that [said instructional activity] has a place in preservice teacher education. (Reviewer 3)

Notice that all three reviewers communicated the expectation that the focus of the manuscript be clear and coherent. In this case, Reviewers 1 and 2 explicitly took issue with the manuscript's shift in focus, specifically, that it begins with a focus on student motivation before switching to focusing on the participants' perception and understanding of mathematics. The reviewers also disclosed their expectation to see clear connections between the claims and evidence in the manuscript. Reviewer 3 suggested that the chosen examples neither made the case nor illustrated the claims made by the authors. Reviewer 2 disapproved of the emphasis on the quantity of examples and pushed for greater focus on the quality and depth of analysis of those examples as a way to provide further explanation and convincing evidence for the claims made in the manuscript. All of the reviewers expressed skepticism about the potential contribution of the manuscript when making statements such as "I was not convinced that [said instructional activity] has a place," "it would be more meaningful if," and "the concept of using [said instructional activity] may have merit... but further clarification would strengthen the manuscript."

At JRME, several editorials have been written to address these issues. Readers are encouraged to read these editorials. With respect to coherence, Cai, Morris, et al. (2019c) described how the conceptual framework provides a connecting thread that ties together all of the parts of a research report into a coherent whole. With respect to claims, Cai, Morris, et al. (2019a) discussed the methodological choices that arise when one has articulated research questions and constructed at least a rudimentary theoretical framework. Just as the researcher must justify the significance of research questions and the appropriateness of the conceptual framework, they argue that the researcher must thoroughly justify the selection of methods. With respect to contribution, Cai, Morris, et al. (2019b) discussed what counts as a significant research question in mathematics education research, where significant research questions come from, and how researchers can develop their manuscripts to make the case for the significance of their research questions.

13.3 Strategies for Anticipating and Addressing Reviewers' Feedback

Framing manuscript writing as communicating with reviewers requires becoming aware of how the review process works and of editors' and reviewers' expectations for manuscripts, which we have discussed above. It also entails becoming proactive about developing strategies to address these expectations. We now turn to offering some strategies that we have found useful when mentoring and supporting the scholarly writing of junior colleagues.

13.3.1 Improving a Manuscript's Coherence

It is a challenge for manuscript writers to maintain a clear focus throughout the paper and not lose sight of the overall flow of the main argument. Thinking through reviewer criteria (such as the set of questions in Fig. 13.2) can help authors improve the coherence of their manuscripts. The key is to have a chain of connected arguments throughout the manuscript, from the research question(s) to the literature review to an appropriate framing to the design of the study to the data analysis to the presentation and interpretation of the results (Cai, Morris, et al., 2019b).

The *MTE* journal has worked to help prospective authors with the challenge of maintaining the coherence of their manuscript by designing a writing template (Crespo & Bieda, 2017; see Appendix 2). Prospective authors can use this writing template to outline their manuscript without losing sight of how different parts of the manuscript connect with each other. By outlining the manuscript within the writing template, authors can keep an eye on the flow of the storyline of the manuscript as well as the connective tissue that glues together the different segments of their manuscript.

Another useful strategy for maintaining coherence is to construct an abstract based on the completed draft of the manuscript. This is an exercise discussed in Goodson (2016) as a way of not only creating an abstract for a written manuscript but for assessing the overall flow and coherence of a manuscript. The basic instructions for this strategy are as follows:

- Highlight each paragraph's key sentence
- Choose several of these key sentences and copy/paste them to make a first draft of the abstract
- Edit for flow and connections and remove unnecessary text

Once these steps are completed and all of the key sentences are lined up on a page, examine the flow of the argument and consider what needs to happen to the text to tell a coherent story of the manuscript that you have written (or want to write). What will it take to improve the flow and focus of the abstract? This exercise aids in revising the manuscript by applying the edits and revisions made to the abstract to the corresponding sections of the manuscript.

13.3.2 Connecting Claims and Evidence

Different studies may make different claims based on their type of evidence—anecdotal, descriptive, correlational, or causal. As Chojnacki, Resch, Vigil, Martinez, and Bates (2016) suggested, understanding the nature of the evidence is a first step to creating appropriate and persuasive claims. Which claims to highlight is both an art and a science in the sense that the highlighted claims need to demonstrate the significance of the contribution. Based on our experience processing a number of manuscripts for both *JRME* and *MTE*, a common error we see with claims and evidence that are not connected is the making of presumed claims that do not have the support of the data (Cai, Morris, et al., 2019a). We recommend three strategies for connecting claims with the evidence.

The first strategy is to take a step back and search for unexpected evidence. For any study, all authors have some hypotheses about what to expect. Sometimes, however, claims based on unexpected evidence will strengthen the study's contribution. For example, in an early study, Cai (2003) examined parental roles in Chinese and US students' learning based on a Parental Involvement Questionnaire (PIQ) that examined parental roles in students' learning of mathematics. Unexpectedly, Cai (2003) found that students whose parents completed the PIQ had significantly higher mean scores on both routine problem-solving and nonroutine problem-solving than those students whose parents did not complete the PIQ. Thus, Cai crafted a claim that highlighted this particular piece of unexpected evidence by arguing that not completing the PIQ survey could be used as another indicator of parental reluctance to become involved in their children's learning.

The second strategy we recommend is to use the Toulmin model of argumentation (Toulmin, 1958) to analyze whether a claim is established based on evidence. This model has been successfully used to investigate students' mathematical arguments by mathematics education researchers (e.g., Hollebrands, Conner, & Smith, 2010; Wagner, Smith, Conner, Singletary, & Francisco, 2014). It has also been helpful in the teaching and learning of the research process (e.g., Booth, Colomb, & Williams, 1995) by highlighting the basic components of a good argument—claims, evidence, warrants, and qualifications:

Claims: What the reader is asked to accept (believe).
Evidence: Why this is a reasonable claim to accept/believe (considering source, quality, and type of evidence).
Warrants: Why the evidence is (necessary and) sufficient (making visible the connection between the claims and evidence).
Qualifications: Limits on the claims.

Consider, for example, the following claim and evidence in a recently published manuscript abstract in the *MTE* journal: "Through qualitative analysis of instructor field notes, teacher generated mathematical models, and teacher survey responses, we found that teachers who participated in the Flint Water Task (FWT) engaged in mathematical modeling and critical discussions about social and environmental justice" (Aguirre, Anhalt, Cortez, Turner, & Simic-Muller, 2019, p. 7). The claim here is that "teachers who participated in the Flint Water Task (FWT) engaged in mathematical modeling and critical discussions about social and environmental justice"; the evidence comes from "qualitative analysis of instructor field notes, teacher generated mathematical models, and teacher survey responses."

Obviously, it is not possible to include all of the elements of an argument in an abstract of a manuscript, but for the most part, the main claim and the type of evidence are often present. The warrants and qualifications are found in the results section of the manuscript where the discussion of the analysis and the implication of the findings are shared. In the case of the example above, the authors include a whole section titled “A Closer Look at Teachers’ Work in Modeling” with subsections detailing the evidence of the impact of the FWT task in their analysis of the teacher-generated models and in the teacher survey. In terms of qualifications, the authors discussed the contextualized nature of these kinds of tasks and turned these limitations into a call for future research focused on better understanding how and whether similar tasks would produce similar kinds of outcomes.

Given space limitations, we cannot expand on Toulmin’s model of argumentation here, but we invite readers to identify arguments in an article of interest and to identify how the argument is made and how it contains claims, evidence, warrants, and qualifications. We also refer readers to Booth et al. (1995) book *The Craft of Research* for more specifics and to Bieda and Crespo’s (2018) editorial titled *What’s your Evidence: Making Evidence-based Claims and Why This Matters*.

The third strategy is to make an audit of the claims and the quality of the evidence used to support those claims. This entails making a list of all the claims and the supporting evidence and then taking a harsh look at both how the claims are articulated and the strength of the evidence that is used to support them. This is where it is helpful to use what Elbow (2000) calls playing the doubting and believing game. A doubting stance provides counterexamples and reasons for not believing the claim or the evidence, whereas a believing stance provides suggestions for how to strengthen the claim or the evidence.

To illustrate this strategy, let’s consider again the example of Aguirre et al.’s (2019) claim that “teachers who participated in the Flint Water Task (FWT) engaged in mathematical modeling and critical discussions about social and environmental justice” and the associated evidence of “qualitative analysis of instructor field notes, teacher generated mathematical models, and teacher survey responses” (p. 7). A doubting stance might question whether a single task alone could produce the kind of outcome the authors claim; a believing stance might consider whether the claim could be strengthened by emphasizing the multifaceted analysis of data that produced evidence for their claim.

13.3.3 Articulating the Contribution of the Manuscript

Choosing an appropriate framing of the study is extremely important for clearly articulating the contribution of a manuscript. In this volume, Spangler & Williams (Chap. 1) and Leatham (Chap. 12) have discussed issues related to appropriate framing. Readers may also look into a JRME editorial on the topic (Cai, Morris, et al., 2019c). In this section, we focus on specific strategies related to the process of writing as communicating your manuscripts’ contribution to reviewers.

Reviewer feedback about manuscripts that are not clear about their contribution often touches on the need to be clear about the “so what” and “who cares” of the work. Although this feedback is useful, it does not provide us with strategies for addressing the issue. In a recent editorial (Crespo, Bieda, & Dubbs, 2018), *MTE* focused on the challenge of articulating and making explicit the contributions that a manuscript makes to the field. We have found Graff and Birkenstein's (2010) discussions of the writing process as one of entering and relating to ongoing conversations to be very helpful in thinking about strategies for addressing this concern. The bad news is that there are no shortcuts to addressing this issue, and authors need to invest in developing a serious reading habit.

Contributing to an ongoing conversation means investing time in listening to a conversation before entering it. As discussed in Crespo et al.'s (2018) editorial, there are different ways to approach reading for ongoing conversations; one way to approach it is by reading intentionally to learn how authors of published manuscripts position their work as contributing to an ongoing conversation of mathematics education research. This way of reading the research literature is different from instrumental approaches, which most typically focus on collecting and gathering information. This approach focuses instead on the writing moves that successful authors make.

Another strategy is to read the editorials of journals. As editors, we have devoted considerable time to learning about ongoing conversations in our journals and how these conversations relate to and connect to ongoing conversations of educational researchers more broadly. Our editorials tend to reflect on where the conversation has been and where it is going, what the major accomplishments are, what the continued challenges for the field are, and so on. Thus, developing a manuscript for publication for a specific journal requires some investment in reading what has been discussed in that journal in the past 2–3 years and considering how a new manuscript submission engages and contributes to an ongoing conversation in that journal.

13.4 Concluding Thoughts

Scientific writing is a craft. One important aspect of this craft is viewing this writing as communicating with reviewers and asking questions such as the following: Have I persuaded reviewers of the merits of the arguments in the manuscript? Have I communicated ideas to reviewers clearly? What possible questions might reviewers raise with respect to the study and the reporting of the study? In this chapter, we have shared some reflections and advice regarding these questions based on our experience as the editors of *JRME* and *MTE*.

To close, we invite readers to be intentional about improving their academic writing by considering reviewers as the first set of readers and to anticipate their feedback. Becoming a mathematics education scholar who communicates their research insights to multiple audiences requires mastering the review and revision process. Similar to what we believe as mathematics educators—that learning mathematics requires productive struggle—learning to write with reviewers in mind requires a similar disposition. This chapter, as well as others in this volume, provides a good start to becoming an intentional writer. We also suggest engaging in this work within a supportive community of colleagues that help us anticipate and address the feedback of insightful and skeptical reviewers.

Appendix 1: *JRME*'s Characteristics of High-Quality Manuscripts

Retrieved from www.nctm.org/Publications/write-review-referee/journals/Characteristics-of-a-High-Quality-JRME-Manuscript/

Inclusion of Appropriate Purpose and Rationale

- Describe a clear purpose for the study.
- Establish why the general area of study is important and how this particular study can contribute important information to the field. (One should not conduct a study simply because no such study has ever been done.)
- If examining a second context for an existing study, explain why the second study is useful. (This is not intended to suggest that replication studies are not appropriate.)

Clear Research Questions

- State research questions or research hypotheses explicitly and clearly in the manuscript. (The reader should not have to guess what the research questions were.)
- Clear research questions are guided by the theoretical framework and are addressed by the data collected and analysis performed on that data.

An Informative Literature Review

- Provide a basis for doing the study that is reported.
- Synthesize studies, creating more than a listing or summary of existing studies.
- Include credible sources (e.g., peer-reviewed journal articles) rather than drawing exclusively on project reports and unpublished works. Address results of previous research along with pertinent policy documents.
- Cite from a source accurately and reflect what was published in the original source.
- Include pertinent international research literature rather than limiting the review to that of a single country.
- Cite a variety of pertinent studies, not just your own work or that of your colleagues and collaborators.
- Include important works that support and ground the research such as current research in mathematics education, foundational research that is the basis for the study, and potential works outside of mathematics education as appropriate.

A Coherent Theoretical Framework

- The study is guided by a theoretical framework that influences the study's design; its instrumentation, data collection, and data analysis; and the interpretation of its findings.

- The literature review connects to and supports the theoretical framework.
- Make it clear to the reader how the theoretical framework influenced decisions about the design and conduct of the study.

Clearly Described Research Methods*
Include key elements of research methodology such as:

- From what population the subjects were drawn, how and why they were selected, and how many were included
- Information on the instructors and their backgrounds
- When and how often the subjects were interviewed or tested
- How many classrooms were included in the study
- How each variable was measured
- How research instruments were adapted or developed
- Examples of items from research instruments
- Descriptions of instructional approaches
- Examples from instructional materials
- Protocols used for classroom observation or interviews
- Details of the procedures used to analyze qualitative data

Sound Research Design and Methods*
Employ research design and methods appropriate for answering the study's research questions:

- Give validity and reliability data for the instruments used.
- Use appropriate statistical procedures and meet their assumptions.
- Use instruments appropriate to the study's subjects to measure outcome variables.
- Address threats to trustworthiness.
- Describe discrepant events.
- Use member checking when appropriate.

Claims About Results and Implications That Are Supported by Data*
- Provide supporting data for each claim that is made.
- Do not draw conclusions or suggest implications that inappropriately extend beyond what is reasonable based on the data.
- Interpret and contextualize the study's results.

Contribution to the Field of Mathematics Education
- The study examines some aspect of the teaching and learning of mathematics and offers new results or new insights to mathematics education that extend beyond what has been reported in prior studies.
- The study moves the field beyond current methods, instruments, and/or theories.

- Focus goals on understanding a phenomenon deeply rather than investigating any particular classroom, student, lesson, or content.

Clearly Explained and Appropriately Used Terms

- Clearly define terms that are likely not to be understood by many readers (e.g., educational terminology unique to a particular country or region).
- If using familiar terms in nonstandard ways, provide explanations for doing so.
- When using terms that have several possible interpretations, clearly identify which interpretation is intended.
- Avoid using terms interchangeably that have different meanings (e.g., proof, reasoning, argumentation, and justification).
- Do not treat multidimensional entities as if they were one-dimensional (e.g., "reform curricula" are not a singular entity and "reform" involves changes in curriculum, pedagogy, and assessment, not just in curriculum).

High-Quality Writing

- Provide helpful transitions so the manuscript flows well from one section to another.
- Develop ideas rather than listing collections of thoughts in paragraph form.
- Ask colleagues or employ editors to correct errors in grammar, spelling, and sentence structure.
- One additional issue is that international authors may have language issues; although we do thorough copy editing, in the early stages when we are sending for review, we recommend that international authors for whom English is a second language use a native speaker as editor (use language from decision letters).

Mathematical Accuracy

- Use mathematical terms correctly in conceptualizing their research.
- Use correct mathematics content in instructional materials, interview protocols, and written instruments.

*These items may not be applicable to manuscripts that primarily address theoretical issues.

Appendix 2: *MTE*'s Manuscript Writing Template

Identify shared *MTE* problem	**Situating problem in literature**
What important problem or issue in the practice of mathematics teacher educators does the manuscript describe?	*To which existing knowledge base in mathematics teacher education does the manuscript connect?* *In which theory and/or on which previously published articles is the manuscript grounded?*
Description and argument for the innovation (solution/intervention/tool)	
What argument does the manuscript make for the innovation that addresses the identified problem? *What details does the manuscript provide to allow for replication or modification of the innovation by subsequent authors?*	
Details of the research on the innovation (solution/intervention/tool)	
What description of how the results of the innovation were studied and documented does the manuscript contain? *What details does the manuscript provide to allow for verification of how the innovation was researched?*	
Provide evidence for claims (and consider limitations)	
Beyond simply describing an innovation, what evidence does the manuscript provide of the effectiveness of the solution/intervention/tool? *What warrants does the manuscript provide so that recommendations for policy and practice can be constructed or justified?*	
New contribution to knowledge and practices of MTEs	
What specific new contribution to our knowledge does the manuscript make explicit? *What discussion does the manuscript contain about how this study can inform or influence the shared problem of MTEs' practice?*	

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Chapter 14
Removing Obstacles to Quality Research Publishing

Lyn D. English

Abstract This chapter provides guidance on quality publishing in mathematics education and STEM education journals and books. Consideration is given first to the selection of an appropriate journal outlet and some of the features and requirements that need to be taken into account. Next, a selection of core components of a research journal article in mathematics and STEM education are examined, with suggested ways of addressing these components to avoid manuscript rejection. These components include title and abstract, problem statement, research questions, contribution to the field, research design and methodology, results and discussion, conclusions and limitations, and references and appendices. Responding to reviewers' concerns is also briefly addressed. Finally, a few points to keep in mind when preparing books and book chapters for mathematics and STEM education volumes are presented.

Keywords Quality academic publishing · Journal publication requirements · Core components of a research journal article · Editing academic books · Writing edited book chapters

Writing journal articles is rarely easy, for either beginning or experienced researchers. Book chapters likewise can be challenging, especially those for an international handbook. My aim in writing this chapter is to provide some guidance on quality publishing, drawing on my experiences over several decades both as an author and as a journal editor (*Mathematical Thinking and Learning: An International Journal*). In this chapter, I first consider choosing an appropriate publication outlet, primarily research journals in mathematics education and STEM education more broadly. Next, I examine some of the core components of a research journal article in mathematics education and STEM education, and how these components might be

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addressed to avoid manuscript rejection. Finally, I offer a few points to keep in mind when preparing books and book chapters for these fields.

14.1 Before Submission: Choosing an Appropriate Journal

The quickest way to prevent the publication of an article is to choose an inappropriate journal. I consider it imperative that one or perhaps two reputable journals (i.e., not a "predatory" journal, namely, one that is published by an unscrupulous organization with little or no actual review) be selected for potential submission prior to writing and submitting a manuscript. Viewing articles that have been published in a targeted journal is essential to ascertain the nature and scope of studies featured. In my role as journal editor, I periodically receive inappropriate manuscripts, that is, manuscripts that do not meet the aims and scope of articles published in *Mathematical Thinking and Learning*. In these cases, I have to reject the paper outright and sometimes suggest an alternative journal.

Assuming a publishing outlet is appropriate, it is important to take note of the style guidelines such as the nature of section headings, line spacing, font type, referencing format, line spacing, table and figure requirements, and word/page length. It is easy for a journal editor to gauge whether an author has studied these guidelines. Such an omission can detract substantially from a submission. From my experience, some of the more common errors include incorrect line spacing (e.g., single instead of double), incorrect section headings (e.g., numbered instead of non-numbered), and exceeding a journal's page or word limits (if this criterion applies). Many editors will return manuscripts to authors for these errors to be corrected prior to processing for external review; this is especially the case for manuscripts that exceed a journal's page or word limit.

Targeting a professional or practitioner journal (e.g., *Mathematics Teacher: Learning and Teaching PreK-12*) as opposed to a research journal also requires a careful examination of the type of articles published. Usually, the first feature one observes in professional journal articles is the difference in genre and the brevity of the articles. Professional journals, as the name implies, are targeted primarily to practices in the profession. They are usually more "reader friendly" or "less dense" than research journal articles and are designed for direct classroom application. While it is important to cite research in supporting one's ideas and recommendations in a professional journal manuscript, detailed research reports are inappropriate. The task for authors is to select educational implications from their research and translate these into appealing, creative, and readily implementable classroom activities. Professional journal articles usually present "novel" ideas and activities, that is, those that vary from typical, routine classroom tasks. At the same time, the recommended activities should target important mathematical curriculum concepts and processes. Although transforming a comprehensive research study into a brief, classroom-based manuscript for a professional journal can be challenging, it is nevertheless imperative that educational research be translated into effective classroom practice (Cai et al., 2018).

14.2 Addressing Core Components of a Research Journal Article

Composing a research journal manuscript is complex and challenging, especially when one has to address several interacting components within a word or page limit. Typically, these components include (a) a concise title and appropriate abstract; (b) the nature and significance of the problem being investigated; (c) a review of existing research, including where the gaps in the literature lie, and thus how the study is making a significant contribution to the field; (d) the research questions investigated; (e) the conceptual or theoretical framework guiding the study; (f) the research design adopted; (g) the methodology, including the study context, participants, and the nature of the intervention or program(s) implemented (if applicable); (h) methods of data collection (how evidence was gathered to answer the research questions); (i) how the data were analyzed; (j) the results and interpretation of the analysis; (k) a discussion of the findings; (l) conclusions and limitations of the study; and (m) appropriate use of tables, figures, and references.

A detailed discussion of each of these components is not feasible within the confines of this chapter. Other chapters in this volume give a more in-depth coverage of these features, as do other sources such as guidelines put forth by the *American Educational Research Association* (AERA, 2006) and the *Journal for Research in Mathematics Education* (JRME, 2015). These sources are in addition to the foundational reference, the *Publication Manual of the American Psychological Association* (APA, 2013). I now consider some of the more common pitfalls to avoid with respect to these core components. I do so from the perspective of a journal editor as well as an author of journal articles.

14.2.1 Title and Abstract

A title is usually what captures the reader's attention and should thus be clear, concise, accurate, and present appropriate expectations for both readers and electronic indexing services (Saracho, 2013). Overly long titles are not recommended, as they can detract from the overall appeal of the article and can also be misleading for indexing (APA, 2013). Sometimes deciding on a title can be difficult. I recommend using a title that is befitting to a research journal, rather than one that would be more suitable for a practitioner journal. Titles for the latter journal type frequently involve a turn of phrase with a double meaning, such as "Hands Together: An Analog Clock Problem" (Earnest, Radtke, & Scott, 2017).

Abstracts can be difficult to write and can take longer than anticipated; however, they form a key component of journal articles and other research publications. Abstracts should convey within 150–250 words (depending on the journal) the study's purpose, methodology, key findings, and conclusions or implications. In the words of Saracho (2013), an abstract needs to provide a "complete but concise description of the study" (p. 48) as well as incorporate keywords that can be used for indexing and databases. To illustrate the importance of constructing an appropriate abstract, the following is an excerpt from one reviewer's feedback on a journal manuscript. The reviewer's feedback reflects the foregoing points on the need to clearly and concisely convey the core aspects of a study.

> The abstract is currently written in vague language that does not communicate a summary of findings, assertions, or implications very clearly or precisely. Since many readers only look at the abstract, it is essential that it be written in a way that clearly communicates key details about the study. In a revision, it would be useful to clearly state 1–2 key findings, 1–2 central assertions about how/why the findings happened, and 1–2 conclusions or implications about what these findings mean for scholarship and for practice.

Although they appear up front, abstracts are probably best composed fully as a last step. A draft abstract can comprise one or two sentences that capture the essence of each of the manuscript's main sections (personal communication, Keith Leatham, 2018).

14.2.2 Problem Statement and Contribution to the Field

One of the initial aspects a journal editor observes is whether a submitted manuscript has a clear statement of the problem and whether an investigation of this problem makes an important contribution to the field. A manuscript without a clear problem statement or a comprehensive review of the related literature is unlikely to receive favorable reviews. A limited literature review might be characterized by brevity, omission of important studies in the field, dated literature that has been superseded by more recent and pertinent research, literature that is confined to just one nation, inappropriate/irrelevant studies, and failure to indicate where the gaps lie in the field. Furthermore, if a manuscript is submitted to a mathematics education or a STEM education journal, it is expected that the literature review comprises primarily studies in these domains. A focus on these studies, however, does not exclude reference to articles from other fields in the social and learning sciences provided they support and enhance research in mathematics/STEM education. Often, interesting and innovative manuscripts will draw on other disciplines (e.g., neuroscience, philosophy, social psychology) to enhance a study in mathematics education and STEM education.

Simply reporting on research in a given field is inadequate. Literature reviews need to critically analyze existing research, identify areas that remain under-researched, and expose gaps in knowledge that the reported study intends closing. In so doing, the study's contributions to the field can be highlighted and the groundwork laid for the theoretical or conceptual framework. This framework is important in guiding the research questions or hypotheses, the subsequent conduct of the study, the data collected, the analysis performed, and the discussion of results. Manuscripts that lack such a framework are likely to have reduced impact, with reviewer requests to address this omission.

It is worth mentioning that if an aspect of a study, such as key components of the theoretical framework, has been reported in another research outlet, the unique contributions of the new, submitted manuscript must be clearly indicated. That is, submitted manuscripts should be significantly different from the author's other works that share several components of the new submission. Reviewers frequently source the author's other articles that have emanated from one study and will question how the new manuscript differs from these published works, if this has not been made clear in the new submission.

The nature of the research questions or hypotheses posed (or not posed) is also one of the initial aspects an editor and reviewers will scrutinize. As JRME (2015) emphasizes, such questions or hypotheses should be explicit and clear and not leave the reader guessing what issues are being investigated. Research questions emanate from the theoretical framework and are addressed by the data collected and the analysis performed on these data. Posing appropriate research questions can be challenging, as many postgraduate research students can attest to. Yet these questions are the essence of a study—they indicate what captured one's interest, what enduring problem needed further investigation, whether different perspectives or approaches might resolve longstanding issues, and how new findings might advance the field.

As an example of a reviewer highlighting some of the above points, one reviewer's feedback on a submission is presented below. Aspects have been omitted in reproducing this excerpt to avoid any author identification.

> Revisit the literature in the literature review to ensure that you are accurately framing the current state of scholarship about … Use this literature to help you highlight a problem or question that is central to your study, that shows how your study makes a contribution to a growing body of research that is building better knowledge and understanding of … [Use this literature in] building stronger evidence for theories about … For example, in the literature review, the author cites scholarship from the 1980s asserting an often-repeated idea about … Is this assertion still true? Does the current body of scholarship in mathematics and science teacher preparation uphold this assertion or is there now counter-evidence for this claim? Similarly, the author draws upon early work from … (2002) to make note of gaps in research. Are these gaps still a problem or have researchers begun to answer the call put forth by …? Finally, what are some of the big questions or puzzlements emerging from the body of scholarship reviewed in this paper?

14.2.3 Research Design and Methodology

Describing an appropriate research design and methodology can likewise be challenging, even for the experienced researcher and author. The theoretical framework, together with the research questions, should inform decisions about the design and conduct of a mathematics education or STEM education study. Authors often omit, or at least address minimally, an account of how they selected their research design and their rationale for doing so. With many journals having strict length limits, giving adequate attention to design and methodology can be difficult. Yet reviewers will usually request justification for research design selection if this is missing. For example, design research methodology (e.g., Cobb, Jackson, & Dunlap, 2016; Design-Based Research Collaborative, 2003) is popular in mathematics/STEM education and in other domains, often because of the diverse range of settings in which design studies can be conducted, the insights generated into student and teacher learning, and the affordances for theoretical and empirical developments. Because of the many roles of design research methodology, it is important to indicate how it was applied in a study and why.

Detailing the nature, number, and background of the participants, the population from which they were drawn, and how they were selected should appear in the methodology section of an empirical study. Describing instruments administered, including their appropriateness for addressing the research questions, how they were developed or sourced, their reliability for yielding the required data, and samples of items (or the entire instrument) should be included in the methodology, where appropriate. Sometimes authors omit aspects or all of this information, which makes it difficult for reviewers to determine the validity of the results and of any subsequent claims made.

Studies that report on treatments or classroom interventions should describe them in such detail that their key features can be determined and applied in interpreting the results (AERA, 2006). The types of approaches adopted, examples of instructional materials or treatments implemented, and the duration and frequency of implementation or administration should also be indicated. Although such requirements are well known, it is easy to overlook some aspects, especially when a journal has tight submission lengths. Furthermore, the nature of the intervention should be guided/supported by the theoretical framework, which enables the reader to see how the study emanated from its conceptual foundation/s. This aspect again highlights the importance of establishing an appropriate theoretical framework. It is not uncommon for an author to advocate a particular perspective (e.g., constructivism) but then describe a study that does not reflect its core philosophy or ideas. Reviewers invariably question this failing.

Descriptions of the data analysis undertaken should give the reader confidence in trusting any claims made in the results and conclusions sections. A number of approaches to providing such a warrant are cited by AERA (2006), including triangulation of data, having data coded by other researchers, and a critical examination of how the researchers' pre-existing perspectives or beliefs might have influenced the data collection and analysis. Triangulation is frequently used in mathematics/STEM education research, such as supporting quantitative data by including examples of specific participant responses from classroom or group discussions. If, for example, a claim is subsequently made that students were engaging in metacognitive activity, then concrete examples of students' actions in this regard can further support the reported data.

Manuscripts that omit or have inadequate information on how a study's data were obtained and analyzed, including justification for the data analysis methods used, will be questioned by a reviewer. As emphasized by AERA (2006), data analysis procedures should be

> precisely and transparently described from the beginning of the study through presentation of the outcomes. Reporting should make clear how the analysis procedures address the research question or problem and lead to the outcomes reported. The relevance of the analysis procedures to the problem formulation should be made clear. (p. 37)

14.2.4 Results

In reporting the results of data analysis, it is important to keep in mind the research questions being investigated. It should be made clear how the analysis addresses the research questions and leads to the outcomes (AERA, 2006). It is not uncommon for a submitted manuscript to overlook the questions posed and to report on results that address other issues. One approach to avoiding this problem is to organize the results according to each research question, that is, revisit each question in turn (assuming more than one question). Only those results that actually answer the questions should be included. Other pertinent findings might emerge from the data analysis, in which case they could be incorporated within the discussion and cited as unanticipated outcomes. Such results could serve as one area for further research.

Ensuring that all claims and conclusions made are supported by the data is especially important (AERA, 2006) and is an aspect that can be easily overlooked even by experienced researchers. It is easy to over-interpret a study's findings, that is, making claims that are not supported by the data produced. As previously noted, the reader needs to be able to trust the claims made.

In documenting data outcomes, it is recommended that tables should only be used when they clarify or summarize outcomes involving multiple data points (Saracho, 2013). APA (2013) provides examples of appropriate table layouts. Generally, the fewer the tables the better, as too many can detract from a manuscript and extend its length, especially for those journals that have a strict page limit (e.g., 30 pages in total for the *International Journal of Science and Mathematics Education*). While tables should be readily interpreted, the messages they convey need to be summarized in the related text. One of the problems with several of the manuscripts submitted to *Mathematical Thinking and Learning* is their overuse of tables and figures. Furthermore, sometimes these tables and figures, especially figures, can be so dense, coupled with small font size, that they are barely legible and more importantly will not reproduce well in the printed journal issue. Conversely, figures and tables can be overly simplistic, leaving the reader wondering why they were even included in the manuscript.

The use of color is also problematic for printed issues of journals (but not the online format), as color is costly for the publisher; authors are thus required to meet the costs of color if this is desired. Problems can arise when color is essential for some graphs or other figures where it is used as a distinguishing feature. In this instance, the use of shading or other such effects will need to serve as a substitute.

14.2.5 Discussion

There are a number of ways to approach the discussion section, but typically these approaches include a summary of the outcomes, together with an "interpretive commentary" (AERA, 2006, p. 38) providing a more in-depth understanding of the claims made—claims that are supported by the data. Such a commentary would indicate how each research question was addressed, offer possible reasons for how and why particular outcomes occurred, the context/s in which the outcomes took place, how they support or challenge existing theory and previous research, and possible alternative interpretations. Importantly, the discussion should indicate how the outcomes and conclusions drawn from the study connect to and support (or perhaps challenge) the theoretical framework, how they support or refute existing research, and the implications that follow. Such implications might refer to theoretical, practical, or methodological considerations (AERA, 2006). One of the problems I frequently see in manuscripts submitted to *Mathematical Thinking and Learning* is a failure to revisit the theoretical framework in light of the study's findings.

Limited reference, if at all, to existing research in discussing the study outcomes is also a weakness of some submissions. It is important that researchers indicate how their study has extended current work in the field, thus advancing the existing knowledge base. This aspect links to the earlier point regarding the importance of a study's problem statement and contribution to the field. One of the more common reasons for a reviewer to reject a manuscript is that it does not make a significant contribution to the field, rather, it simply reinforces well-established research; the reader thus comes away questioning why the study did not progress beyond this point. Although studies that duplicate the findings of earlier research can still contribute, the nature of any such contribution should be well argued, with implications for further studies clearly indicated, such as how a task or context variation might generate new insights.

14.2.6 Limitations and Conclusions

Acknowledging the limitations of a study is an aspect that can also be overlooked by authors. Suggestions for reducing these limitations should be indicated, such as the need for additional research. Reviewers invariably comment on an omission of limitations.

The inclusion of a concluding section is not always followed in mathematics/STEM education journal articles but can provide a valuable summation statement. Conclusions are normally brief and succinct, with Saracho (2013) recommending that they include a clear statement on the key outcomes and justification for their significance with reference to related studies and a few core conclusions from the study results. Recommendations for future research are sometimes included in this section.

14.2.7 References and Appendices

Many mathematics/STEM education journals follow APA (2013) guidelines in citing references, with some minor variations in a few journals. In addition to adhering to a journal's referencing requirements, it is important to ensure all references cited within the text also appear in the reference list. The converse applies. Although researchers are usually aware of this standard requirement, it is easy to miss some references. Furthermore, some journal editors or editorial offices can return a manuscript to those authors who do not follow the journal's referencing guidelines.

Appendices are valuable for including important information that can be distracting or difficult to incorporate within the text, such as questionnaires and tests administered, or excerpts from these. APA (2013) provides detailed instructions on the inclusion of appendices together with other supplemental materials that might be included in the electronic version of an article.

14.3 Reviewing Prior to Submitting

On completion of a manuscript, undertaking a review of its overall structure and readability is essential. A manuscript ready for submission should read well, be cohesive, address the required components, and be free of typographical errors and awkward expression. This last aspect can be difficult when English is not an author's primary language, in which case some editorial assistance will need to be sought. One of the many obstacles to achieving journal publication is to submit a manuscript that is not "reader friendly." Although it is time consuming and bothersome to conduct this refinement prior to submission, it is nevertheless essential—it is just as time consuming and annoying for reviewers to read through poorly written manuscripts, and poor expression can be extremely off-putting for reviewers. Trying to interpret a manuscript for meaning, prior to undertaking a review of its contents, adds an extra layer of unwelcome work for reviewers.

14.4 Responding to Reviewers' Comments

A final point in manuscript preparation and submission pertains to dealing with reviewers' feedback. As this aspect is explored in other chapters in this book, it is not considered in depth here. It is worth commenting, however, that responding to reviewers' concerns can present another set of challenges, which can be both frustrating and rewarding. Invariably a much-improved manuscript results after reviewers' queries and recommendations have been attended to.

Of particular importance in submitting a revised manuscript is a detailed indication of how each reviewer's points were handled. It is not sufficient to simply indicate that each reviewer's concerns have been considered; rather a clear indication of how each point has been responded to in the revised manuscript is normally required. Subsequent acceptance, even after a few revisions, is rewarding after all the time and thought that has been devoted to the manuscript. In contrast, a rejection is extremely disappointing, especially after revision has been made. From my perspective as both an author and a journal editor, I know the upset a rejection can cause. For those journal editors who take the time to provide feedback on the reviews and the reasons for the subsequent editorial decision, their words of advice and/or encouragement can be reassuring. Taking time to reflect on one's manuscript and the reasons for rejection is invaluable in moving forward, as is seeking advice from research colleagues.

14.5 Undertaking Books and Book Chapters

In this section, I first offer some suggestions for editors who are comparatively inexperienced in book editing, and then turn to advice for chapter authors. Books and chapters in mathematics education and in STEM education more broadly, naturally vary in their requirements and procedures for submission and publication. Any suggestions I offer in this section are of a general nature and have been garnered from my many years in editing books mostly with Springer and Routledge (Taylor & Francis). Edited books are especially popular in our field today, probably more so than authored books. As an editor, choosing an appropriate and reputable book publisher (again, not a "predatory" one) is as important as it is for journal submissions. Some publishers have special series on selected topics, such as *Early Mathematics Learning and Development* (Springer) with dedicated series editors. Submitting a book proposal, whether for an authored or edited book, requires completing a comprehensive publisher submission form. These forms vary in nature but typically request author/editor details and curriculum vitae; the title and subject of the book; whether substantial aspects have already been published; the major contribution of the book to the field; methods, results, or topics that will be of particular interest to readers, and why; benefits of the book for the reader; a proposed table of contents and approximate book length; a projected timeline; other competing books in the field; and supporting references. Examples or abstracts of chapters to be included in the book are usually requested. Some publishers request that whole chapters be submitted, especially for authored book proposals.

Many of the points I have made regarding journal manuscript submission apply to book proposals, especially presenting a clear and concise case for the book indicating how it will advance the field. Of course, a comprehensive, well-written proposal (without expression weaknesses and typos) has a better chance of acceptance than a hastily constructed one that is poorly presented. The latter type of proposal does not exactly engender an enthusiastic response from a potential publisher. As a frequent reviewer of book proposals, I find it difficult to strongly support those where the authors or editors have not paid careful attention to presenting a professional and convincing case.

Editing a book, especially a handbook, is a comprehensive and time-consuming task. There appear no specific procedures for securing authors for an edited book, but often an expression-of-interest flyer is sent to a range of organizational and networked groups. Specific authors who have done worthy research in the targeted area may also be approached, as well as some authors who have contributed to previous editions of a book. Potential authors are normally required to submit a detailed abstract of their proposed chapter, addressing the aims and requirements of the intended book. Any calls for chapters from editors thus need to clearly specify what is required of authors. It can sometimes be difficult for editors to choose chapter abstracts, especially when many are submitted. At the other end of the spectrum, securing potential authors for an edited book in a new, under-researched field (e.g., early engineering learning) can likewise be problematic. Once authors are secured, the next challenge for an editor is to maintain the proposed timeline, or at least as closely as possible. Tardy authors often need many reminders, especially when their chapters are delaying a book's production. Maintaining regular contact with authors to check on their progress can avoid excessive delays. Tardy editors, on the other hand, are equally problematic if they take a long time to finalize a book before it is sent for final review. As Stephen Brown, a professor of political science at the University of Ottawa, commented in 2016, "Just as editors can pave the way for publication, they can also inadvertently block it."

To increase the cohesion of an edited book, cross-chapter referencing is recommended to enable common threads to be carried through. Usually an editor will alert authors to other chapters where shared ideas are addressed (e.g., equity issues in STEM education). For a large book, such as a handbook, that comprises many sections, such cross-referencing might not be needed or might be restricted to those chapters within a given section.

Book and book chapter manuscripts require reviews from experts in the field and, in the case of edited books, sometimes from other contributing authors. At least two reviews of each chapter manuscript for an edited book should be sought, with reviewers (hopefully) providing valuable advice on chapter improvements. Chapters might undergo a number of revisions or perhaps end up being rejected if requested improvements cannot be made or if an author is unable to meet the required timelines. Such a situation may require careful negotiation between the editor and author, especially if the chapter's removal would leave a gap in the book.

Finally, editing chapters to ensure a publisher's style guidelines are adhered to (e.g., numbered headings/no numbering, font type/size, single/double line spacing) is usually expected, although some publishers will undertake this editorial work. Nevertheless, editors need to remind authors to ensure their chapter manuscripts align with publisher requirements as closely as possible.

In the final stages of a book's production, chapter proofs with a number of queries will be sent to authors and editor/s. This stage is usually a rewarding one and it behoves all authors and the editor to ensure a prompt turnaround of corrections to the publisher's editorial staff. Late returns of proofs only delay the book's production. Timely book production and its subsequent release are of the essence.

With respect to authors of book chapters, many of the foregoing points also apply. For example, cohesion within chapters, both in content presentation and organization, is required, as is the case with journal submissions. When there are multiple chapter authors, ensuring this cohesion can at times be difficult and may require the lead author to make necessary adjustments. It is helpful for chapter authors to have the potential Table of Contents to facilitate cross-referencing where appropriate.

Requirements of book chapter authors vary with the nature of the book and topic being examined. Handbook chapters in particular are worthy of mention. Authors of these chapters are usually expected to undertake a wide survey and review of international research in the field, a statement of what has been achieved to date internationally, and how their chapter will address and significantly advance a key aspect of the handbook's subject area/s. Handbook abstract proposals that are narrowly confined to research in an author/s' country or institution and/or report on an individual study are usually inappropriate for a handbook chapter. Handbook chapters are usually challenging to write because of the extensive preliminary research required, as well as ongoing monitoring of latest research in the field.

Finally, it is worthwhile commenting on an issue that researchers commonly pose or debate: Are edited books and book chapters worth the effort in the current climate of journal metrics and article citations? From my experience, I would say most definitely. Books published by reputable publishers, especially handbooks, provide a rich source of research conducted and a diverse collection of ideas on significant topics. Such a collection of research reports that explore a given topic, especially an emerging or underrepresented one, can be invaluable in advancing the field as well as increasing recognition and appreciation of a researcher's contribution. A case in point is James Hiebert's foundational, edited book on conceptual and procedural knowledge in mathematics education (Hiebert, 1986). His book is still frequently cited, three decades on. There are numerous other such cases in our field.

14.6 Concluding Points

In undertaking this chapter, I have attempted to provide suggestions for removing obstacles to quality publishing and improving the publication of journal articles, books, and book chapters. My recommendations are by no means exhaustive. Other chapters in this volume will add to my points, perhaps refuting some of them. I have drawn on several decades of researching, publishing, and editing, where I have experienced and continue to experience the bumpy road toward publication. From my literature searches, there appears to be little in the way of advice on publishing for authors, especially with respect to books and book chapters. This dearth of published advice perhaps reflects the diverse expectations of journal and book editors, together with those of the various publishing houses. Nevertheless, it is hoped that the advice I have offered here is of some assistance and encouragement to scholars.

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Chapter 15
Publishing for International Impact in Mathematics Education Research Journals

Merrilyn Goos

Abstract The conduct and publishing of mathematics education research is an international concern. Yet new researchers in our field often struggle to frame and communicate their research so that it is relevant and accessible to an international audience. This chapter shares an international journal editor's expectations for publishable manuscripts, elaborating in particular on what is required to show that the manuscript makes an original and significant contribution to knowledge. The chapter explores ways in which researchers can situate their work within a broader international landscape without losing sight of the local context that motivates the study.

Keywords Mathematics education · International research journals · Contribution to knowledge · International impact

For new researchers in mathematics education, the motivation to enter the field often comes from their own experiences as learners or teachers of mathematics and their desire to understand and ultimately improve the mathematics education experiences of others in their immediate environment. While one's local context—whether this is a classroom, school, school system, or education policy environment—can be a rich source of problems for investigation, mathematics education research is an international enterprise and researchers are expected to show how their findings produce new knowledge beyond local or national interest. Working out how to meet this requirement is especially important when submitting manuscripts to international journals in mathematics education.

My aim in this chapter is to help new researchers to conceptualize and communicate their research so that the contribution to knowledge is clear, relevant, and accessible to a research journal's international audience. This work is not simply a

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matter of conforming to English language norms in academic writing, even though research communicated in English language journals and conferences dominates the literature in our field. In fact, the dominance of English language publications makes it even more important for all researchers—no matter their linguistic or cultural background—to develop sensitivity to culturally inflected ways of framing and communicating research.

The chapter is structured in two main parts. The first part examines expectations of international research journals in mathematics education, especially in relation to making an original contribution to knowledge that is relevant to an international audience. The second part of the chapter addresses how to respond to these requirements, with examples and advice drawn from my experience as editor-in-chief of *Educational Studies in Mathematics* (ESM).

15.1 Expectations of International Research Journals

First, it's worth thinking about what makes a research journal "international." One way to answer this question is to look at the extent to which the editors and editorial board members are broadly representative of the international research community rather than being concentrated in just one country or one geographical region. Information about the names and affiliations of journal editors and editorial board members can usually be found on the journal website and in the front matter of the print version of the journal. It is also useful to get a sense of authors' countries of origin as an indicator of a journal's international reach. A search of the last 5 years of a journal's online or print issues will yield this kind of information about authors.

More formally, a journal's expectations are communicated by a statement of aims and scope. For example, the aims and scope of ESM (Springer Nature, 2019) indicate a preference for articles that would be of interest to readers beyond the context in which the research was conducted:

> *Educational Studies in Mathematics* presents new ideas and developments of major importance to those working in the field of mathematical education. It seeks to reflect both the variety of research concerns within this field and the range of methods used to study them. It deals with didactical, methodological and pedagogical subjects, rather than with specific programmes for teaching mathematics. The emphasis is on high-level articles which are of more than local or national interest.

A journal's expectations are perhaps most powerfully represented by the criteria against which submitted manuscripts are reviewed. The review criteria for ESM and the *Journal for Research in Mathematics Education* (JRME) are shown in Figs. 15.1 and 15.2, respectively.

1. Is this article clearly an educational study in mathematics?
2. Does it make an original contribution to mathematics education?
3. Are the aims of the article made clear, and are they formulated sufficiently early in the article?
4. Are the aims of the article fulfilled?
5. If applicable, are the aims, hypotheses and methodology of the research, reported in the article, clear and reasonable?
6. Does the article provide a well founded and cogently argued analysis?
7. Do the conclusions follow from the data and/or the argument?
8. Does the article take appropriate account of previous work?
9. Is it accessible and interesting to an international readership?

Fig. 15.1 ESM review criteria

1. Does the research extend or deepen our understanding of important issues in mathematics education? Does it have the potential to lead the field in new directions?
2. Do the research questions pertain to issues of significant theoretical or pragmatic concern? Are they well grounded in theory or in prior research?
3. Is there an appropriate match between the research question(s) and the methods and analyses employed to answer the question(s)?
4. Does the conduct of the study include the effective application of appropriate data collection, analysis, and interpretation techniques?
5. Are the claims and conclusions in the manuscript justified in some acceptable way, and do they logically follow from the data or information presented?
6. Is the writing lucid, clear, and well organised?

Fig. 15.2 JRME review criteria

Most of these review criteria would be common to many mathematics education research journals, for example, expectations concerning clear aims and research questions, a literature review that builds on previous research, a coherent theoretical framework that guides the design of the study, clearly described and well-justified research methods, claims supported by evidence and argument, and good-quality writing. These are important aspects of publishing research that are addressed elsewhere in this book. In this chapter, I am concerned with the criteria that typically distinguish the most respected international journals in mathematics education: an *original and significant contribution to knowledge* (expressed via criterion 2 for ESM and criterion 1 for JRME) and *accessibility to an international readership* (expressed via criterion 9 for ESM but not explicitly mentioned in the JRME review criteria). Meeting these criteria in particular is part of achieving international impact in mathematics education research.

15.2 Achieving International Impact

15.2.1 Making Research Accessible to an International Audience

I explained earlier that one measure of a journal's international reach is the profile of author countries of origin. For example, ESM receives around 300 manuscripts per year from authors in more than 50 different countries and publishes 60–70 articles per year representing authors from around 20 countries. Despite this broad geographical representation, every year there is a clear majority of authors from the English-speaking countries of the USA, the United Kingdom, and Canada. This pattern might lead one to conclude that lack of proficiency in writing in the English language is a barrier to making research accessible to an international audience. However, the literature on academic research, writing, and publishing in mathematics education identifies more profound challenges that have implications for all authors. Beyond technical difficulties with the English language, barriers can arise from subtle differences between semantic fields in different languages and differences in the significance of research questions across cultural contexts (Bartolini Bussi & Martignone, 2013; Geiger & Straesser, 2015; Meaney, 2013). I address these barriers in turn in the next two sections.

15.2.1.1 Understanding Differences in Semantic Fields

Geiger and Straesser (2015) discussed some of the challenges experienced by English non-dominant-language researchers who engage with a publishing landscape dominated by English language journals and conferences. They drew on Barwell's (2014) notion of centripetal forces to argue that authors might feel pressured to "adopt a single standardised linguistic code—an 'official' language which must be adopted to gain full acceptance into a professional community or other social system" (p. 38). They suggested that this press for uniformity is at odds with the subtle variations in meaning that emerge when one attempts to translate words from one language into another. They attributed this phenomenon to the existence of overlapping *semantic fields* in different languages: so a word situated in a particular semantic field in one language, when translated literally, might invoke a different semantic field in another language and thus lose some of the intended meaning.

An example will illustrate the linguistic choices that English non-dominant-language authors must sometimes make when writing in English. French mathematics education researchers Dominique Passelaigue and Valérie Munier published an article in ESM on the difficulties experienced by pre-service elementary school teachers in understanding the concepts of "attribute" and "measurement" (Passelaigue & Munier, 2015). After establishing the significance of these concepts in mathematics education, they went on to examine the meanings of the concepts and to justify their choice of the English word "attribute" as a translation of the French word *grandeur*.

> There are a number of linguistic difficulties with the use of the term "*grandeur*" in French (…), so finding an English term that refers to the same concept is not easy. In the literature, we find three different English words as translations of the word "*grandeur*" as it is employed in "*Grandeur et mesure*" lessons in French elementary school: magnitude, quantity, and attribute. The term "magnitude" is used by Douady and Perrin-Glorian (1989) in a paper on teaching the concept of area (…). Many studies dealing with mathematics education (see for example Sarama & Clements, 2009), as well as many standards, e.g., NCTM standards on measurement (online at http://www.nctm.org/standards/content.aspx?id=316) and the Mathematics Australian Curriculum (online at http://www.australiancurriculum.edu.au/mathematics/curriculum/f-10), use the term "attribute". Finally, in the national curriculum of England (online at https://www.gov.uk/government/publications/national-curriculum-inengland-mathematics-programmes-of-study) and in the International Vocabulary for Metrology (Bureau International des Poids et Mesures, 2008), the English word employed for the French word "*grandeur*" is "quantity". A priori, all of these documents are relevant, but we had to make a choice. Because we are dealing here with the teaching of attributes and measurement, we chose "attribute", which is consistent with many English language standards and with research on mathematics education. (pp. 308–309)

Towards the end of the article, Passelaigue and Munier (2015) identified as one of the limitations of their study that it was conducted in France, and so their findings were situated "in a particular culture and language" (p. 333). They noted that "a linguistic specificity, for example, is the French word *grandeur*, which may carry some extra baggage not implied by its English counterpart" (p. 333). This is another way of saying that the French word *grandeur* and the English translation "attribute" invoke different semantic fields and convey different meanings.

It is difficult to provide straightforward advice to authors for addressing differences in semantic fields. Geiger and Straesser (2015) maintain that journal editors have an important role to play in developing awareness of the challenges involved and suggest that editors could offer tailored support to authors who do not have English as their dominant language. They also point to the need for greater recognition of bilingual and non-English-language journals and encourage research and publishing collaborations between academics from English and non-English-speaking backgrounds. However, the example presented above illustrates another fruitful approach to dealing with the press for linguistic uniformity often experienced by English non-dominant-language authors. By exposing and discussing the subtle differences in meanings of key terms, Passelaigue and Munier (2015) provided readers with access to ideas that exist outside the culture of English-language-dominated mathematics education, thus generating a more diverse linguistic repertoire.

15.2.1.2 Avoiding Monoculturalism

It is perhaps unfortunate that Passelaigue and Munier (2015), as French authors, would consider the cultural context of their study a limitation. Meaney (2013) argued that one of the consequences of the privileging of the English language in mathematics education research "is that this research is becoming monocultural" (p. 5). She expressed concern that researchers from non-Anglophone countries might feel obliged to use terminology more common to English-speaking countries. For example, authors might describe the organization of schooling using terms such as "elementary school" even when schools are not organized in this way in their countries. Assumptions about what needs to be explained and what is accepted as "normal" can work against the development of cross-cultural understandings about mathematics education research and reinforce a tendency towards monoculturalism at the expense of cultivating diversity.

Meaney's (2013) arguments concerning the monocultural construction of mathematics education research should remind all researchers to take care in explaining the context in which they work, making no assumptions about what is "normal" and what an international audience should be expected to know and understand. Researchers from English-dominant-language backgrounds are not always fully aware of this need. For example, I regularly receive manuscripts that use terms specific to the US context such as "freshman," "Title I schools," and "middle school," that cite only US literature and curriculum documents, or that assume that readers will know a "midwestern university" refers to the midwest of the USA. My feedback to authors of these manuscripts typically includes a paragraph like the one shown in Fig. 15.3 (the topic of the study has been blinded to preserve the author's anonymity).

A good example of how a US author succeeded in making her research accessible to an international audience is provided by Andrea McCloskey, who published an article in ESM on ritual as a lens for understanding persistent practices in mathematics classrooms (McCloskey, 2014). McCloskey highlighted the cultural nature of teaching and learning mathematics in schools, explaining that her study was motivated by "the well-documented resistance to reform of schools in general and mathematics education in particular" (p. 19). She went on to list practices in contemporary US

> A second important requirement for publication in ESM is that the article must be accessible and interesting to an international audience. A research study is always conducted in a particular geographical, cultural, and educational context, and so it is up to the authors to show how their study is relevant to a larger concern that is shared across national contexts. This would certainly be possible when investigating XXXX, as it is an issue of interest in many countries and there is already a body of literature on this topic. However, your own study is narrowly grounded in the US context, with all its curriculum structures and terminology that are unique to thiscountry, and so such a study would be of limited relevance to readers outside the US.

Fig. 15.3 ESM editor's feedback on a manuscript that is not accessible to an international audience classrooms that have remained the same for many years, such as *timed tests*. Because this practice is not common in all countries, she used a footnote to explain:

> Timed tests or speed drills are assessments in which students, as young as six years old, are asked to complete basic fact computations (simple operations on single-digit numbers) in a certain amount of time. For example, first graders may be asked to solve 50 addition problems in three minutes. (p. 19)

In the Introduction section to her article, McCloskey referred to Stigler and Hiebert's (2009) analysis of the *Trends in International Mathematics and Science Study* (TIMSS) that used the idea of cultural scripts to explain differences in teaching practices across countries, in order to situate her study of ritual in a broader international landscape. Having established the significance of her investigation in this way, McCloskey then drew on examples of persistent classroom practices in the US context and explained her choices rather than assuming they were "normal" practices in other cultural contexts. Her first example involved the practice of *assigning grades*:

> In US K-12 classrooms and university teacher preparation courses, it is typical for instructors to assign numerical or categorical grades (typically using the "A, B, C, D, F" scale) throughout the course on assignments and assessments and then to assign a final grade at the conclusion. The practice of assigning grades in contemporary US classrooms feels so natural and normal that it is difficult to imagine alternatives. (p. 31)

Later in the article she introduced her discussion of several "iconic" mathematics classroom practices in this way:

> There are several common practices in the types of settings with which I am most familiar that suggest ritual aspects. Most adults in the USA participated in these practices as students during their own schooling, and they feel familiar and "normal" to many who have spent time in US mathematics classrooms. Some examples of these practices are timed tests (as described earlier, and as recently critiqued by Boaler, 2012), board races (in which children compete with one another to perform calculations in front of the class on the blackboard), homework (which include outside-of-classroom experiences, and often involve parents and other family members, sometimes leading to "emotional and mathematical trauma," as described in Lange and Meaney, 2011), and the subsequent practice of "going over" the homework the next day, but there are many other practices that could be added to the list. (pp. 32–33)

By acknowledging the local context on which she drew for her analysis while situating her study in the broader cultural landscape, McCloskey succeeded in making her research accessible and relevant to the journal's international audience.

15.2.2 Making an Original and Significant Contribution to Knowledge

For a manuscript to be accepted for publication in the most prestigious international research journals, it is not enough to report on a competently conducted study; editors will be looking for manuscripts that additionally make an original and significant contribution to knowledge that advances the field by deepening our understanding of important issues in mathematics education. These new insights should go beyond what has already been reported in previous studies and should yield deeper understanding of a significant phenomenon rather than focusing only on a particular research context (student, lesson, classroom, etc.).

The framing of a research study that makes an original contribution to knowledge will have two key characteristics: a research question that targets an issue of significant theoretical or practical concern and a purpose or rationale that is relevant to contexts beyond the local context in which the study was conducted. These characteristics are interrelated, and they demonstrate why the study is *needed*, and not just novel.

15.2.2.1 Research Question

Apart from studies that are purely theoretical in nature, research questions in mathematics education mostly arise from curiosity about one's local context or observations of a local phemonenon. How, then, can research questions be formulated that transcend this specificity to context? Silver (1994) reflected on this issue in a background paper for the 8th ICMI Study on the topic of *What Is Research in Mathematics Education and What Are Its Results?*

> Are all mathematics education research questions able to be considered within the international community? […] If the research questions cannot be disembedded from their local contexts, then how is it possible to convey to those who wish to understand the research important background information about the context when it is vital to the research question under consideration? Is it possible that, despite their obvious relevance to mathematics education within a particular country, some very important research questions may never be considered more generally within the international community? (p. 335)

Bartolini Bussi and Martignone (2013) aimed to stimulate debate about these issues by highlighting the cultural framing of their research on mathematics machine laboratories in Italy and the potential for this research to draw on, and enrich, international literature. They provided a comprehensive description of the mathematical laboratory tradition in Italian schools, the significant presence of geometry in the Italian school mathematics curriculum, and the ways in which teachers learned to create classroom tasks using mathematical machines—working reconstructions of historical mathematical instruments. Bartolini Bussi and Martignone puzzled over why this project had such little impact on the international research community, despite widespread international reporting. They concluded that elements of the cultural background in Italy that afforded success in the project might not be present in other contexts, but they also acknowledged their need to better explain the cultural dimensions by means of a "cultural dialogue" (p. 6) between themselves as "insider" researchers and the international audience, as "outsiders." To do so requires a degree of awareness that enables the researcher to take the perspective of both the insider and the outsider in formulating the research questions that frame a study.

There has already been a great deal of research on gender differences in mathematics achievement, and so you need to make a stronger case for why this particular study could add to what is already known. You describe a little about the schools' cultural context, but this would need to be further developed to explain how this context is similar to and different from other school contexts in the US (and why these similarities and differences would be of interest to an international readership). What makes this particular context a fruitful site for research on gender differences?

Fig. 15.4 ESM editor's feedback on manuscript aim

To better understand the nature of this "insider-outsider" cultural dialogue, consider the following examples of manuscripts submitted to ESM that reported on studies of gender differences in mathematics achievement of school students. The first study was conducted in a school system serving a particular cultural group in a large US city. The author described the aim of the study as comparing the mathematics proficiency scores of girls and boys in this coeducational school system. Part of my decision letter rejecting the manuscript, shown in Fig. 15.4, explains that a stronger case needed to be made for why this aim is significant.

The second study was carried out in Lebanon and investigated gender differences in student achievement and attitudes as well as differences in teachers' gender-related beliefs and classroom practices (Sarouphim & Chartouny, 2017). The authors listed the following research questions:

1. Do significant gender differences exist in the mathematics achievement of middle school Lebanese students?
2. Do gender differences exist in the students' attitudes toward mathematics?
3. What are the teachers' beliefs concerning the characteristics, abilities, and mathematics performance of their male and female students?
4. Do mathematics teachers show differential behavior in the classroom with male and female students? (p. 56)

At first glance, this manuscript seems to be addressing context-specific aims similar to those in the previous example. However, in the introduction to the article Sarouphim and Chartouny explained the significance of these questions to their "outsider" international audience: "First, there is a lack of literature on gender differences in mathematics and, secondly, the Lebanese culture remains fairly traditional with prevailing patriarchal values" (p. 55). They went on to provide an account of cultural and educational influences in Lebanon, followed by a review of the international literature on gender differences in mathematics that highlighted surprising differences in student achievement and attitudes across Western and Arabic-speaking countries in the Middle East. The study found that "male and female students' mathematics achievement was equally high and that their attitudes toward this subject matter were equally positive as well" (p. 66). The authors claimed that their study "contributed to the literature by challenging a widespread belief about the existence of a large gender gap in mathematics achievement among Lebanese students" (p. 66). They elaborated on the broader significance of these findings in the final paragraph of the article:

> In sum, this study has highlighted a topic much investigated in the Western literature but barely addressed in the Arab world. The results expand the pool of research by adding evidence that support the recent trends of a narrowing of the gender gap in mathematics education. This is significant given the prevalent belief that females in traditional societies have negative attitudes toward mathematics and do not achieve as well as males in this field. The results of this study provide empirical evidence to the contrary, paving the way to further research on this topic and inciting changes in practice to promote equity among the genders in mathematics education. (p. 66)

The article by Sarouphim and Chartouny (2017) also illustrates how authors can make explicit the contribution to knowledge made by their study. They did this in three places. The first was in the introduction to the manuscript, where they stated the problem under investigation and argued for its significance. The second place was in the literature review section, where they not only selected key works and discussed their contribution to the field, but also identified both the *gap* and *need* that their own study intended to address. The third place where they reinforced the contribution to knowledge was in the concluding discussion, where they connected the study's findings to the literature reviewed earlier in the manuscript.

15.2.2.2 Purpose and Rationale

Research questions flow from the study's purpose and rationale, and it is up to the author to argue why the study can contribute important new knowledge to the international research field. It is not enough to claim that such a study has not been done before, as this author tried to do: "The purpose of this study was to investigate and improve the knowledge of XXXX of end-of-fifth-grade students, which is different from previous research on XXXX" (research topic deleted to preserve the author's anonymity). My response, shown in Fig. 15.5, was to explain some of the reasons why this rationale was insufficient, together with other flaws in the study that led me to decide that the manuscript was not acceptable for publication.

One of the main problems with the manuscript is that it does not make a significant and original contribution to knowledge in mathematics education—this is the most important question considered by reviewers and editors for ESM. To make a case that demonstrates such a contribution, you would first need to provide a more thorough and up to date review of literature, and show how your study adds something new to what is already known. The design of the study also needs to be considered—let me highlight the confusion amongst the reviewers over why six different populations are identified and why twelve hypotheses need to be tested. In relation to the findings, I can see evidence that students in Groups 1 and 2 had similar performance before the intervention (so they are comparable groups), and that in both groups performance was better immediately after the intervention and also some time later. What these findings seem to say is that the students learned what was taught in the intervention, and they retained this learning. While this is good news, it's difficult to see how the findings are substantial enough to warrant publication in ESM, especially when there are few details on the nature of the intervention and its theoretical justification.

Fig. 15.5 ESM editor's feedback on manuscript rationale

A second common problem with manuscripts lacking a convincing purpose and rationale is that they remain anchored to the context in which the study was conducted and are unable to demonstrate relevance beyond this context. This happens, for example, when an author attempts to justify the study by pointing out that research in their country has not yet addressed the issue under consideration (issues such as mathematics anxiety, gender differences in mathematics performance, or the relationship between attitudes and mathematics achievement). What an international journal looks for is ideas that "travel," that is, ideas that speak to the interests and concerns of researchers across the world while contributing new knowledge to the field. A key characteristic of manuscripts that demonstrate this quality is the relationship that the author creates between *context* and *theory* through the processes of contextualization and decontextualization, which in turn enables readers to recontextualize the study's purpose and findings to their own local situations.

Contextualization refers to the cultural framing of the study, as explained by Bartolini Bussi and Martignone (2013), and works to avoid constructing mathematics education as a monocultural enterprise, as advocated by Meaney (2013). Readers need to understand the cultural context and how this has shaped the study. *Decontextualization* requires the researcher to interpret the problem under investigation in terms of theory and to identify the broader class of problems of which this is a specific example. This process allows readers from an international audience to recognize features of the study that might be pertinent to their own contexts and thus to *recontextualize* the findings so they are relevant to this context.

An example will illustrate how authors can establish this relationship between context and theory. In this study, conducted in South Africa, Hamsa Venkat and Mike Askew developed a framework for considering the quality of teachers' mediation of primary mathematics (Venkat & Askew, 2018). An excerpt from my decision letter requesting revisions to the first version of the manuscript is reproduced in Fig. 15.6 to illustrate how I tried to explain the need for both contextualization (by the authors) and then recontextualization (by readers).

> A stronger argument needs to be presented, and early in the paper, as to why this work is relevant and interesting for ESM's international readership. I think it most certainly is, but then I'm fairly familiar with the South African context whereas very many ESM readers are not. In one sense this is about being sensitive to one's audience and what they could be expected to know, or not. In another sense, it means thinking about what is the bigger question or concept or need of which your South African question/concept/need is but one (very interesting) example. Every empirical study collects data in a specific context, usually in one country. The challenge is to imagine a broader problem that encompasses the local issue being researched, but without losing the sharpness and immediacy and urgency of the local problem that has motivated you towards action. This is a difficult balancing act, I know – perhaps it might help to ask yourselves what would a reader in, say, Peru, or Mexico, or Indonesia, or Cyprus find valuable in your paper (these are all countries of origin of recently published ESM articles).

Fig. 15.6 ESM editor's feedback on a manuscript seeking contextualization

The final, published version of the paper deftly referred to the specific South African context as part of the motivation for the study but additionally pointed to its relevance to other countries:

> The motivation for devising a framework for considering the quality of mathematics instruction began in longitudinal development work in primary mathematics teaching in South Africa in the Wits Maths Connect-Primary (WMC-P) project. The South African context has particularities linked to the legacy of apartheid on the preparation and development of teachers, but shares, with many contexts in the developing and developed world features that include: evidence of low performance in mathematics at all levels; problems with equitable access to resources; gaps in primary teachers' mathematical knowledge; widespread evidence of teacher-centered forms of instruction. (Venkat & Askew, 2018, p. 73)

The connection between context and theory in this study was achieved by creating a framework for examining the quality of mathematics instruction that yielded insights into mathematical structure and generality. The authors argued for the distinctiveness and broad relevance of this approach in the first paragraph of the article:

> In this paper, we discuss the theoretical antecedents, literature bases, and empirical motivations underpinning a framework we have developed for considering the quality of teachers' mediation of primary mathematics. Our focus is on how mathematics can be taught in ways that pay attention to, and help develop, understanding of the underlying structures of mathematics, and number in particular, at the primary school level, ways that lead learners to consider generality and to understand number as a scientific concept in the Vygotskian sense. Such attention to generality and structure is in contrast to the continued emphasis in many primary classrooms on teaching calculation procedures as the main purpose or goal, a stance that is particularly prevalent in early primary schooling in South Africa, but not unique to there. (Venkat & Askew, 2018, p. 72)

In taking up the challenge to contextualize and then decontextualize their study, linking the specifics of context with the generality of theory, Venkat and Askew made an original contribution to knowledge that could be recognized and used by researchers in other countries who recontextualized this contribution to their own contexts.

15.3 Concluding Comments

In this chapter I have attempted to organize and communicate ideas about publishing for international impact that have been evolving as a result of gradually gaining experience as the editor of an international mathematics education research journal. I chose to focus on requirements that are common to the most highly regarded journals in our field: making an original contribution to knowledge that is relevant and accessible to an international audience. Finding that these requirements have not been met is a prime reason for rejecting manuscript submissions. While there are no simple recipes for writing a publishable manuscript, the advice offered in this chapter is intended to uncover the meanings behind editor's decisions and to develop new researchers' understanding of how language and culture, research questions and purposes, and theory and contexts are implicated in designing, conducting, and publishing quality research.

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Chapter 16
Revising and Resubmitting: Building on Rejection

Gwendolyn M. Lloyd

Abstract Publishing articles in peer-reviewed journals involves, at times, receiving rejections. Although rejection can be difficult to handle, it also provides an opportunity for authors to improve their manuscripts. In this chapter, I offer advice about how authors of rejected manuscripts can move forward with their work. The chapter includes suggestions for developing an understanding of the overall message of a rejection letter, making sense of the editor's concerns and reviewers' comments, and weighing options as an author. Both resubmitting a revised manuscript and finding a new home for a manuscript are explored as possible next steps. Following the guidance of this chapter will increase the likelihood of manuscripts becoming published articles and contributing to scholarship in mathematics education.

Keywords Revising research manuscripts · Responding to reviews · Submitting revised manuscripts · Understanding the publication process

When you flip through the pages of the latest issue of the *Journal for Research in Mathematics Education* (*JRME*) or scan the "online first" articles of *Educational Studies in Mathematics* (*ESM*), you encounter published research articles—the final products of a lengthy editorial and peer review process. What you cannot see is how those manuscripts got to the point of being accepted, sometimes after being rejected from a different journal and revised multiple times for publication in that journal. Rejected papers may be hidden from sight, but every faculty member has a favorite personal anecdote about a rejected manuscript and its eventual fate. Here's one of mine: In reading the reviews of a rejected manuscript, I learned that I was "trying to do too much" in my densely written manuscript pages. When I split the original manuscript into two, I was able to improve my writing by focusing more and going into greater depth. Those two reports were accepted (after revisions, of course) by

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https://doi.org/10.1007/978-3-030-23505-5_16

ESM and *Journal of Mathematics Teacher Education* (*JMTE*). The initial rejection hurt, but the eventual outcome was wonderful!

Because rejection is an inevitable part of academic publishing, it is important to learn how to handle it and continue to move forward with your work. This does not mean that rejections won't sting—they will. Rejections invite feelings of disappointment, frustration, and doubt (Schuman, 2014). At such times, you have two choices: to give up or to keep trying. Persistence is almost always the better path. A rejection provides an opportunity for you to improve your writing and research skills and develop a better manuscript to share with the scholarly community.

During my career, I have experienced my share of ups and downs as an author of research manuscripts and have learned from other authors' experiences by reviewing for major journals. In addition, I have gained firsthand knowledge of the editorial decision-making process by serving as a member and chair of the *JRME* Editorial Panel, associate editor for *JMTE*, associate editor of the *Review of Educational Research* (*RER*), and coeditor of the *Journal of Teacher Education* (*JTE*). In this chapter, I draw on my experiences and perspectives to offer advice to authors about how to move forward from a decision letter that communicates rejection. I suggest how you can come to understand the overall message of a rejection letter and the intentions of the editor, process reviewers' comments, and weigh your options as an author. I also address important aspects of resubmitting a revised manuscript or finding a new home for your manuscript if the original journal is no longer an option.

16.1 Rejection Letters

There are many different kinds of decision letters, and the possibilities vary somewhat from journal to journal. Acceptance letters typically take the form of an *outright* or *final accept* or an *accept pending minor revisions*, also known as a *conditional acceptance*. All other decision letters are considered rejection letters, although they vary in the nature of the rejection.

16.1.1 Desk Reject or Immediate Reject

One type of rejection letter is known as a *desk reject* or an *immediate reject*. This kind of decision letter typically arrives within a few weeks of submission to a journal after an editor, editorial assistant, or editorial team reads the manuscript and determines that it should not be sent out for peer review. Desk reject decisions are more common at some journals than others. Major journals with large numbers of submissions may desk reject 40% of the manuscripts they receive. There are two primary reasons for this type of decision: either your manuscript does not seem like a good "fit" for the journal you submitted it to, or your manuscript is not considered to be of sufficient quality for the journal. In both cases, an editor's decision to desk reject a manuscript reflects consideration of reviewers' time; the editor prefers not to ask reviewers to read and evaluate a manuscript that has extremely limited chances for publication in the journal. This decision also saves you, the author, time. Without having to wait for the results of a peer review, you can move forward and submit your manuscript to another journal.

Problems of *fit* occur when a manuscript falls outside the scope of the journal. This might happen if an author submits a mathematics paper containing proofs to a mathematics education research journal or submits a study of students' learning to a mathematics teacher education journal. When I was on the editorial team for *JTE*, we desk rejected a fair number of manuscripts that dealt with teaching or teachers but not with teachers' learning or development. Many of these manuscripts were excellent reports of high-quality studies, but they were out of *JTE*'s scope. Similarly, at *RER*, we desk reject empirical studies, regardless of quality, because the journal specializes in systematic literature reviews. Another problem of fit can involve the audience for your manuscript. For example, if the audience for your manuscript is practitioners, it is unlikely to gain acceptance in a research journal. Fit matters to editors because they want the articles published in the journal to be relevant, appeal to its readership, and make an impact.

An immediate rejection based on *quality* may indicate that your manuscript is poorly written or that the argument is difficult to follow. Often a desk reject decision occurs due to serious quality issues with the conceptualization or design of your study or with the nature of your results (e.g., descriptive or surface level rather than analytic or critical). Your manuscript may not appear to contribute something new to the literature or to advance the field, at least not at the level expected for articles in this journal. If the editor shares his or her concerns about your manuscript in the decision letter, you should take them seriously.

16.1.2 Reject Following Review

Another type of rejection letter is one that the editor writes after several carefully chosen peer reviewers have submitted commentaries about your manuscript. A rejection letter after review will usually indicate that there are problems or concerns with the research you have reported or with how you have presented the research in the manuscript, or some combination of the two. In many cases, the editor, drawing on reviewers' input, determines that the study has issues that cannot be resolved or addressed in a clear way through revision. Or, the manuscript may need such extensive rewriting that it would take multiple rounds of revision and review to develop a publishable manuscript.

A rejection letter after review typically communicates that you are encouraged to pursue other outlets for your work. In some cases, however, an editor might offer a *soft reject*, meaning that the journal is willing to consider a revised version of your manuscript, most likely as a new submission. A soft reject is sometimes offered when the editor sees potential in the manuscript but is unsure whether the needed revisions can be accomplished. This decision, while still a rejection, opens the door for the author to take the feedback from the review, make extensive revisions, and submit the revised manuscript again for peer review. As an associate editor for *JMTE*, I sometimes included a sentence like this one in such a rejection letter: “If you find that you are able to address the reviewers’ concerns to develop a new manuscript with a deeper analysis and clearer contribution, you are welcome to submit a revised manuscript to *JMTE*, and it will be treated as a new submission.” In contrast, some journals, like *RER*, for instance, have a strict policy that revisions of previously rejected manuscripts will not be reviewed again. Such policies are usually due to the high volume of manuscripts received by certain journals.

16.1.3 Revise and Resubmit

A third type of rejection letter invites you to *revise and resubmit* your manuscript to the journal. This decision usually indicates that the editor is cautiously optimistic that you can make revisions that will result in a publishable article. Smith (2013), in a *Mathematics Teacher Educator* editorial, described that a revise and resubmit decision is “only given when fixing the weight of what needs to be revised for the manuscript to be accepted seems doable (i.e., there is a clear pathway to get there) and a complete rewrite of the manuscript is not needed” (p. 3). Within this type of decision letter, you will be able to discern that the editor recognizes strong potential in your manuscript. Martin and Miller (2014) described that, in the case of a *JRME* revise and resubmit decision, “your manuscript has the potential to make a significant contribution to the literature. There may be aspects that need rewriting and rethinking, [but] you have a compelling argument as well as the data or evidence to support your claims” (p. 286).

Usually, with a revise and resubmit decision, you are reminded that submitting a revised manuscript does not guarantee eventual publication in the journal. The manuscript will probably be sent out for peer review again, and you may, unfortunately, face a rejection at that point. More likely, however, you will be asked to make some additional revisions based on the second round of review. Although you may dread the extensive changes you are asked to make, a decision of revise and resubmit is generally considered a positive step in the publishing process. If you respond to the feedback by making careful revisions, you will improve substantially your chances of (eventually) receiving a letter of acceptance.

16.2 Reading and Processing a Rejection Letter

When you receive a rejection letter, the first step in moving forward is to acknowledge that you have received a rejection and allow yourself time to cope with the feelings associated with it. After that, you can begin to develop a more nuanced understanding of the decision and shift to problem-solving about what actions to take. Figure 16.1 offers a visual overview of the process I describe in this section.

16.2.1 Taking Time to Process the Rejection

When authors read rejection letters from editors, their initial reactions are often quite emotional. Different types of rejection letters, received at different times, may evoke different reactions. A letter containing a *revise and resubmit* decision may bring out anything from excitement to mixed feelings to disappointment, especially when time is of the essence (e.g., just before a tenure review). Upon receiving a *desk reject* letter, you may feel your heart sink into your stomach (and many other physical and emotional reactions). It is normal to feel this way; rejection really does hurt (see, e.g., Eisenberger, 2013).

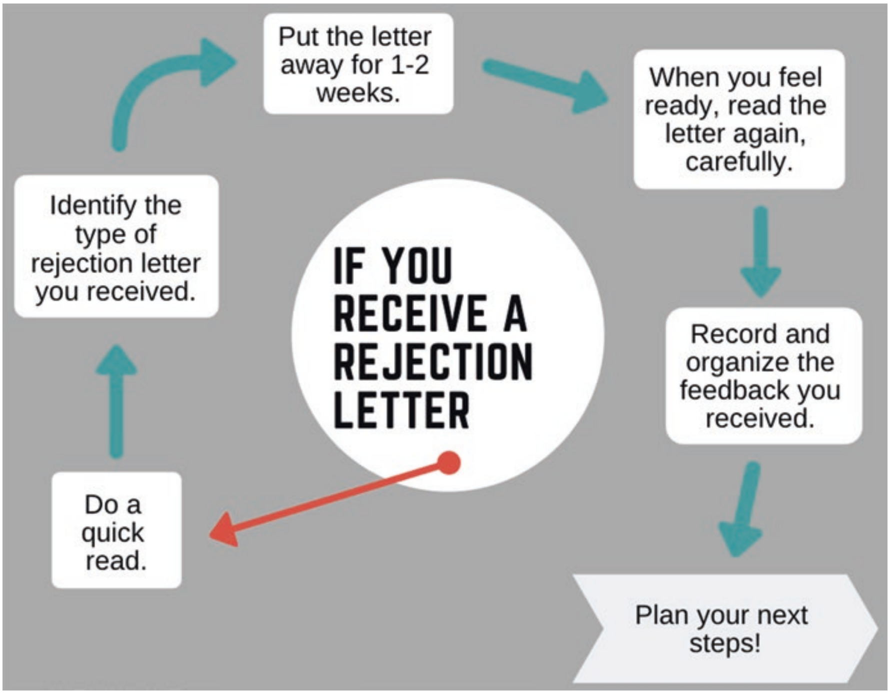

Fig. 16.1 Process for moving forward from a rejection letter

Give yourself time to cope with the rejection, regardless of your immediate reaction. Put the letter away for a little while. In addition to continuing with "business as usual," make a point to enjoy an extra scoop of ice cream, watch a favorite movie, strap on your roller skates, blast a tune at high volume, or talk with friends. Let time do its work. Whatever you do, do not discard the letter or the manuscript! You have already done the hardest work of conducting your study and writing the first version of your manuscript. You were brave enough to submit your manuscript once, and you can do it again. Respect the time and effort you and others have invested in this work—and persist! Remember that your manuscript will improve and your chances of acceptance will increase manyfold as you take action in response to the editor's feedback (and reviewers' feedback, if the paper was reviewed).

When faced with a rejection letter, it is easy to imagine the editor who sent it as a demon with devil horns and with little concern for your condition. Be aware that the editor took no pleasure in writing the rejection letter (see, e.g., Silver, 2001). The editors of mathematics education journals are colleagues who you see at professional conferences and who have received many rejection letters themselves. Editors are aware of the hours of work that went into conducting the study and writing the manuscript, and they understand the hopes that are riding on the decision. When I write negative decision letters, I often include something like, "I know this decision will be disappointing to you," because I truly do know how terrible rejection feels.

After 1 or 2 weeks (or when you are ready), reopen the decision letter and read it again. This time, your goal is to read for the deeper messages contained within the letter. This experience can still be very difficult, from an emotional standpoint, because you are going to find out why your manuscript was not accepted. As an author, I have used two different strategies for reading rejection letters, and each has worked for me at different times. The first involves reading the letter as if it were written to a colleague or student. This strategy creates some distance and allows me to react less emotionally to what I am reading. The second strategy takes the opposite approach; I try to indulge, rather than avoid, my emotions. I underline each of the main reasons the paper was rejected and label it with how I feel when reading it: "Ouch!" or "Ugh, yes, probably true." This process seems to allow me to move forward from those emotions and feel more prepared to take action. There are certainly many other strategies for reading difficult letters; what is important is to find a personal strategy that will help you identify the editor's and reviewers' main concerns.

16.2.2 Identifying the Editor's Concerns

Whatever type of rejection letter you receive, it is necessary to identify what the editor sees as the biggest weaknesses (and the potential contribution, if that is expressed) in your manuscript. You may disagree with the editor entirely, but it is still important to understand the editor's perspective. Read the decision letter and write down for yourself what the editor is telling you. Likely, the editor will refer to elements of the reviewers' comments. In some cases, however, the editor may not be specific and will simply refer to the reviews generally.

An editor's decision is based on his or her own reading of the manuscript as well as a synthesis of the reviews. Sometimes there is consensus in reviewers' comments, but the reviews can also seem quite disparate. Authors can be baffled by a rejection that does not appear to reflect an "averaging" of the reviewers' recommendations. Why might an editor decide to reject a manuscript when two or three reviewers recommend *revise and resubmit*? There are many possibilities. One is that, across the reviews, there are so many major concerns in such a wide range of areas (conceptual, methodological, etc.) that the editor is not sufficiently confident that the needed revisions can be accomplished without a complete rewrite. Another possibility is that one of the reviewers made a particularly strong argument or identified a serious, seemingly unresolvable issue within the study. Rejection in this scenario can be tough for authors to process, especially when one negative review is accompanied by two positive reviews. At times, but not always, the recommendations of particular reviewers may receive greater weight than others. For example, journal editorial board members are typically top researchers in the field, have seen many decision letters, and are familiar with the standards of quality for the journal, so their recommendations tend to be more closely calibrated with the editor's decisions.

As an author, you will typically not know who the reviewers are in a double-blind review, and it is best not to try to determine who they are. You might think you know (e.g., if a reviewer recommends citing a certain paper, you might suspect the reviewer is the author of that paper), but it is wise not to assume anything. Trying to guess reviewers' identities (so that you know who to be mad at, especially if the editor seemed to favor one negative review) will only lead to frustration! Moreover, assumptions about reviewers could negatively impact your future interactions with colleagues in your field. Try to remind yourself that reviewers are fellow mathematics education researchers who are performing an essential service to the field, and focus your attention on the reviews rather than the reviewers.

16.2.3 Reading Comments from Reviewers

Most reviewers in mathematics education write their comments with an orientation towards helping authors to improve the manuscript. Nevertheless, some reviewers' comments can feel quite harsh and cutting when they focus primarily on identifying flaws in the work (Schneiderhan, 2013). I suspect that most of these reviews are not actually written with the mean intentions that authors perceive; some reviewers approach the job of reviewing from an evaluative rather than educative perspective (Crespo, 2016).

Regardless of how reviewers share their concerns with you, your job is to extract the main message from their comments. If you receive a harshly worded or primarily evaluative comment, you may need to work harder to understand the problem and how to address it. Consider these two reviewers' comments about the same concern:

> Reviewer A: *The analysis fails to draw on the theoretical framework presented in the introduction of the manuscript.*
> Reviewer B: *The theoretical framework is appropriate for this study and is presented clearly. It would be helpful for readers to gain a better sense of how the framework influenced your data analysis. I suspect that it did, based on my reading of your findings, but I would like to see this made more explicit in the analysis section so that I can better understand how the findings emerged, given your theoretical perspectives.*

Each of these comments refers to a concern about the study's theoretical framework and its relationship with other parts of the manuscript. Whereas Reviewer A identifies a flaw, Reviewer B shares a concern and why it matters and provides some sense of how to address it. Regardless of the tone, however, both reviews suggest that the author needs to revise the data analysis section to connect it more explicitly to the theoretical framework. The author may also need to do some reanalysis of data and rewriting of the findings and discussion sections.

The important thing is for you, as the author, to make your own notes, essentially rewriting and synthesizing the reviews for yourself. On her blog, *Get a Life*, *PhD*, Golash-Boza (2011) suggested that authors make their notes in a spreadsheet with four columns: *Reviewer*, *Suggestions*, *Response*, *Done?* She points out that you can record the reviewers' suggestions in the spreadsheet in your own words, removing any harsh language in the process. Once you have recorded all of the suggestions in the spreadsheet, you can reorganize the rows of the file according to the sections of the manuscript (i.e., Introduction, Theoretical Framework, etc.). This will allow you to identify similarities (and contrasts) in different reviewers' comments about each component of your manuscript. Creating your notes in a spreadsheet (or other organized file) will help you to feel more grounded and in touch with the process, even when the feedback you receive is extensive.

16.3 Deciding on Next Steps for Your Manuscript

Now that you have a spreadsheet or organized file of notes outlining concerns with your manuscript, it is time to think about next steps. You will need to make decisions about where to submit the next version of your manuscript and how extensively you will revise the manuscript. I strongly recommend that you talk through your options with coauthors (if you have some), peers, and mentors. You may think that you are bothering more established researchers with conversations of this sort, but you will find that they are happy to offer guidance.

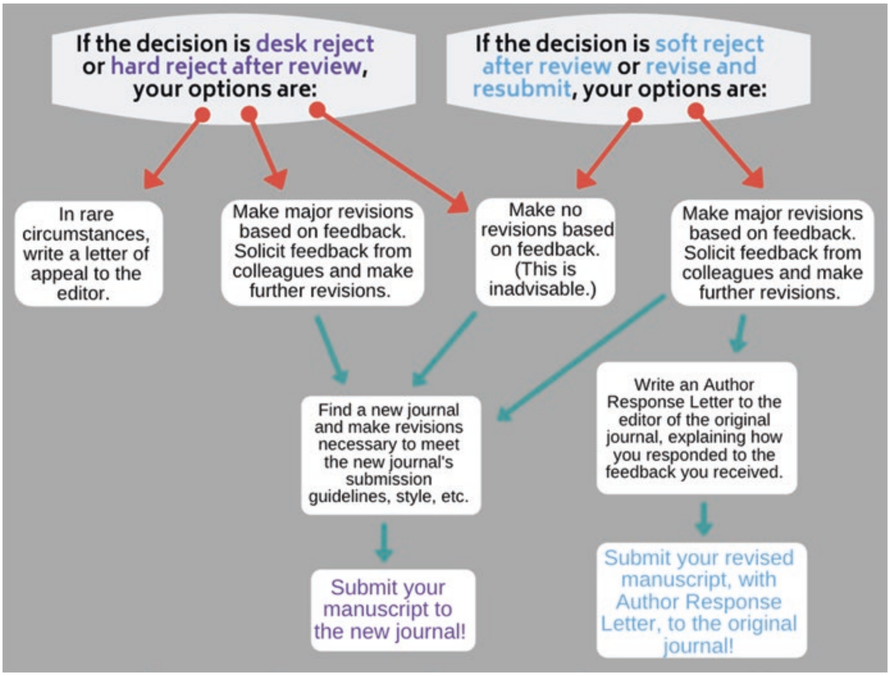

Fig. 16.2 Options for moving a manuscript forward depending on type of rejection

16.3.1 Options Following Different Types of Rejection

Depending on the type of decision letter you received, you have various options for moving the manuscript forward, as illustrated in Fig. 16.2. Notice that I did not give you the option of abandoning your manuscript! If you follow one of the paths in Fig. 16.2, you will have a very good chance of eventually publishing this work.

16.3.1.1 Desk Reject or Hard Reject Following Review

One of the most difficult things about receiving a *desk reject* or *hard reject* decision is that you must let go of your hopes of publishing in this journal at this time. You will need to submit your manuscript to another journal. Moving forward can be particularly hard when you disagree strongly with the editor's decision. In that case, it can be tempting to make an appeal, essentially asking the editor to reconsider the decision. My sense is that appeals seldom result in a different decision by the editor. However, if you truly believe that the editor or one or more reviewers misunderstood a critical aspect of your work and that the rejection hinges on that misunderstanding, it may be worth a try. If your manuscript reports research that is on the periphery of the field, or you draw on controversial or esoteric frameworks or methods, you may face unique challenges in publishing in mainstream journals, and this could be cause for a letter of appeal. If you choose to write a letter of appeal, be sure to wait a sufficient amount of time after receiving the decision letter so that your request is respectful and does not have an angry or bitter tone. Ask a trusted friend or colleague to read your letter before you send it.

The more likely scenario, however, is that you will be submitting your manuscript to a new journal. Therefore, you will need to decide whether or not (or how much) to revise your manuscript based on any feedback received. Some authors will simply submit their original manuscript "as is" to another journal. This might make sense if your manuscript simply did not fall within the scope of the original journal. However, even in the case of a desk reject, there might be feedback in the decision letter that you can use to make revisions. If you received a hard reject after review, some of the reviewer feedback might be specific to the original journal, but the majority of feedback will be worth considering even for another journal. Subsequent reviewers are likely to identify the same or similar concerns as the original reviewers. Why not address them now? Valuable time and effort—yours, the reviewers', and the editor's—will be wasted if you do not take advantage of the feedback you have received. Moreover, your next set of reviewers might even include one of the original reviewers. In a small, specialized field like ours, this happens more often than you might think.

16.3.1.2 Revise and Resubmit or Soft Reject Following Review

If you received a *soft reject* or a *revise and resubmit* decision, you are equipped with several reviews and an editor's letter. You have a choice to submit a revised manuscript to the same journal or to a new journal. This gives you multiple possibilities, as illustrated in Fig. 16.2. Your decision will likely hinge on a few factors, including how doable the suggested revisions seem and your timeline for publication. Whether you decide to continue with the original journal or find a new one, your chances of successfully publishing your piece will increase when you take into account the feedback you received; it rarely makes sense to completely ignore feedback from a peer review.

Resubmitting a manuscript that received a soft reject to the same journal may not be wise for some faculty members (e.g., when the tenure clock is ticking). For others, it may be worth the gamble. With a soft reject, you will have to think carefully about whether you are able and willing to respond fully to the requested revisions, which are likely to be extensive. It sometimes makes better sense to revise the manuscript as best you can and then submit to another journal. However, if you think you can significantly revise the manuscript in response to reviewers' comments and you are convinced that this journal is the right home for your paper, then go for it! Keep in mind that since the revised manuscript will be treated as a new submission, it is unlikely to go to the original reviewers and may not even be assigned to the same editor or associate editor. So, in a sense, you are starting over as if you sent the manuscript to a new journal.

In the case of a revise and resubmit, however, I strongly encourage you to respond to the reviewers' and editor's feedback and resubmit to the original journal. The editor and reviewers have identified potential in your piece and it is worthwhile to pursue their recommendations in your revisions. They, and you, are invested in the improvement of this manuscript. Nonetheless, there are some situations involving a revise and resubmit decision in which you might decide to submit your revised manuscript to a different journal. When you do not feel you are able to respond to one or more of the *major* concerns of the editor or reviewers, this is usually a good idea. For example, you may be asked to make a change that requires analysis of data that you simply do not have. In cases like this, you can still incorporate some feedback into a revision and then find another journal. Turning away from a journal that gave you a revise and resubmit decision is a bold move, one that you should definitely discuss with colleagues and possibly even the editor who sent you the decision letter.

16.3.2 Finding a New Home for Your Manuscript

Early in my career, a manuscript based on my dissertation was published in *JRME*. Although that article represented a very exciting start to academic publishing for me, my next submission to *JRME* was firmly rejected. In the decision letter, the editor encouraged me to resubmit the manuscript elsewhere, expressing her confidence that I would find a suitable home for this manuscript—and I did. In subsequent years, I have not forgotten her encouragement. You, the author, need to persist in finding a journal where your manuscript can make an impact.

Finding a new journal home can be challenging, particularly because you initially wrote the manuscript with a specific journal in mind. When faced with finding a new journal for a rejected manuscript, you will need to consider all the factors that you would normally consider when choosing a journal (see, e.g., chapters by English and Goos, this volume), including journal readership, scope or focus, and reputation. For the sake of efficiency, I suggest choosing the journal that is closest in readership and scope to the original journal so that your revisions will consist primarily of responding to feedback (if any) on your original submission and making changes in response to the new journal's submission and formatting guidelines.

16.3.3 Making Major Revisions

Use your notes about the reviewers' and editor's feedback as your guide in revising your manuscript. If you made a Golash-Boza (2011) spreadsheet, you can include an action plan for each reviewer comment in the "Response" column. Start making revisions one by one. Some authors like to start with the big issues and work their way down to the more fine-grained suggestions. Others prefer to take on the easy changes first. Whatever your approach, as you address each revision, mark it as "Done" in your spreadsheet.

In the "Response" column of the spreadsheet, explain and justify your response to each comment from reviewers. Every concern or suggestion warrants a response, even if you disagree with it. (See Sect. 16.3.5, for more on this.) If a point is made by multiple reviewers, you should seriously consider making discernible changes in response to that point. Sometimes, you will encounter conflicting suggestions about the same concern or contradictory commentary about the same part of the manuscript. One reviewer loves your research design, another sees it as fitting poorly with your research questions. Contradictory elements of reviews are bound to occur, given the diversity in perspectives and expertise among reviewers. Some editors provide guidance about how to handle conflicting reviews; others do not. As the author of the manuscript, you are uniquely positioned to identify ways to respond productively. In the example above, a slight rewording of the research questions might create stronger ties between the questions and the design, satisfying both reviewers. When you search for solutions of this sort, you will often find that reviewers' comments are less contradictory than you initially perceived.

As you revise, you will probably need to rewrite certain sections of your manuscript. You may need to alter your framework, conduct a literature review in a specific area, reanalyze some of your data, or identify some implications of your work for future research. In almost all cases of revisions, you will need to do more than simply add new information or move parts of the manuscript around. Rewriting is almost always needed when your main argument, or the main contribution of your piece, is not coming through in a clear and logical manner. I suggest tracking changes electronically to monitor what you have accomplished. Expect the manuscript file to look very messy as you make your revisions; this is a good sign that you are making more than superficial changes. At times, while revising, you will feel that the work is tedious, but it is very rare for the author of an accepted paper to express that the revised and published article is worse than the original manuscript. Authors are nearly always grateful that they had the opportunity to improve the manuscript based on constructive feedback.

16.3.4 Colleagues as First Reviewers

One of the best things you can do with your newly revised manuscript is ask a colleague to read it for you. It can be helpful to choose a colleague who is familiar with the journal you are submitting to, but unfamiliar with the research study you are reporting. Take their feedback seriously. Darley, Zanna, and Roediger (2004) advised, "If your colleagues find something unclear, do not argue with them.… Their suggestions for correcting the unclarities may be wrong, even dumb. But as unclarity detectors, readers are never wrong" (p. 205). Your colleagues' reactions are a good indicator of how future journal reviewers will react.

16.3.5 Resubmitting Your Revised Manuscript

If you received a revise and resubmit decision, you will be asked by most journals to include an *Author Response Letter* with your revised manuscript. Even if the journal does not ask for this, write one and attach it to your submission. This letter of response should be blind and separate from your cover letter to the editor, so that it can be shared with reviewers. The Author Response Letter is extremely valuable to the editor and reviewers because it shows them how you responded to the feedback your received. To ensure that you are thorough in responding to feedback, use your notes or Golash-Boza (2011) spreadsheet as a starting point for this letter. Some authors choose to begin with Reviewer 1's first comment and explain how it was addressed (or not), then list Reviewer 1's second comment and explain how it was addressed, etc. Other authors group together comments that are closely related and then explain how those concerns were addressed.

Be aware of the tone of your Author Response Letter. Even if you do not make a change based on a reviewer's comment, you can still respond to the comment diplomatically and justify that decision in your letter. Here are a few examples of respectful, appreciative language adapted from response letters I have seen as an editor:

- *I appreciated the suggested references about students' conceptions of limit provided by Reviewer A. I found that three of the eight articles connected closely with my findings and I integrated these studies into my Discussion.*
- *While Reviewer 2 thought our conceptual framework was comprehensive and thorough, Reviewer 1 considered it to be overly detailed and lengthy. We worked through this section to tighten our presentation of the framework so that we retained major ideas but expressed them more concisely. We are pleased with the improvements to this section.*
- *Reviewer B indicated that missing from our study context is a description of the 65 school districts' approaches to professional development for mathematics instruction. Although we agree that this information would be valuable, we do not have these data. We have included a statement to this effect in the Limitations section.*

Rather than pointing out that several references provided by Reviewer A were not closely related to the study, that Reviewers 1 and 2 contradicted each other, or that Reviewer B's request was unreasonable, these examples instead respectfully explain how the authors responded to the suggestions. This approach is more productive than stating that reviewers are misinformed or ignorant! Again, remember that reviewers and editors are doing this work as a professional service, and the best approach is to demonstrate your appreciation for their efforts.

You may wonder why the tone and content of the Author Response Letter matters so much. In an immediate sense, the letter may impact how reviewers respond to your revised manuscript. For instance, reviewers may be taken aback if you argue with their suggestions and seem unwilling to make changes. By contrast, reviewers will be impressed with the effort you made to learn from their suggestions and improve your manuscript. You could find yourself at a professional conference years later and meet someone who lets you know that they enjoyed reviewing your *ESM* piece. As an editor, I feel a lot of pride in the published articles that I have seen go through multiple rounds of revision and review, and I look forward to meeting those authors and following their scholarship.

16.4 Closing Thoughts: From Rejection to Publication

My advice in this chapter is unlikely to relieve the disappointment and frustration of rejection, but I am hopeful that you now have a better understanding of the editorial process around rejection and some specific ideas for moving forward: when you receive a negative decision, give yourself time to recover following the blow of rejection before deciding on next steps. Do not give up on a manuscript! Instead, put concerted effort into identifying and pursuing alternative homes for it, if needed. Take seriously any feedback that you receive on your writing—if you do not, you will likely receive it again from another source. Respond politely and with gratitude to editors and reviewers who have invested in reading and thinking about your work. All of these actions will greatly increase the likelihood of your manuscript becoming a published article. Moreover, they will enhance your ability to make a meaningful contribution to the body of scholarship in mathematics education.

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Chapter 17
Getting Published: Perspectives from Early-Career Scholars

Nicole L. Louie, Daniel L. Reinholz, and Niral Shah

Abstract The process of publishing research in academic journals can be mystifying. In this chapter, the authors offer insights from their experiences publishing as early-career scholars. In addition to discussing nuts and bolts (e.g., what a response letter to a "revise and resubmit" decision needs to accomplish, and various ways of structuring one), the authors attend to the emotional work of navigating the publishing process, emphasizing the value of ongoing learning and a supportive community.

Keywords Getting published · Manuscript revision and resubmission · Early-career scholarship · Working with editors

Unlike other contributions to this volume, the chapter you are just beginning to read is written by three mathematics education scholars who are all pre-tenure. In this chapter, we will describe what we have learned about publishing our research in a range of venues over the past several years. The perspectives we have to offer vary, but all reflect our embeddedness in our current experiences as early-career researchers, at institutions that prioritize research productivity.

Spoiler alert: we are not going to present 12 foolproof steps for getting your manuscripts published or for getting tenure. Instead, our goal is to shed some light on the often mystifying process of publishing peer-reviewed papers and the options

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you have along the way, with particular attention to navigating "revise and resubmit" (R&R) decisions. We will describe various choices we have made and what we have learned from them.

17.1 The Emotional Work of Navigating the Publishing Process

Before we dive into details, we would like to give some explicit attention to the emotional work that comes along with engaging in the publishing process. First, we acknowledge that it can be challenging. Working in a field that is perpetually ready to shut us out can push us into some pretty negative spaces, from doubting our professional worth to feeling angry at colleagues who don't understand or care about what we're trying to do. (There is a Facebook group with more than 13,000 members called "Reviewer 2 must be stopped!" where members share horror stories about obnoxious reviews they have received.) And we are acutely aware that our racialized and gendered identities, the nature of our research, and the emotional labor of working to publish and otherwise legitimize our research all intersect; two of us identify as people of color, one as a woman of color, and all of us study issues linked to race and hierarchy in mathematics education (which some reviewers—and readers—treat as inappropriate or worse).

When we are faced with critiques that feel personal, it is important for us to give ourselves space to process our feelings (whether by talking about them, exercising, listening to loud music, or looking at pictures of kittens). When we return to work on developing and communicating our ideas, we have to keep our eyes on the prize: getting our ideas out there and developing personally as thinkers and scholars. Feeling angry or bitter and snapping back at reviewers does not help. Set aside your ego; it will not help you here.

At the same time, we find it important to remember that our work has value. *We* have value. Period. The nature of our profession means that at various points, we will all get signals that what we are doing is not good enough or does not matter. Rejection often reflects not on us but on things like fit (e.g., between a manuscript and a journal), timing (e.g., how backlogged an editor is when you submit your manuscript), and interpersonal dynamics (e.g., Reviewer 2!), which are difficult to control and even understand, especially as new scholars. It is essential that we not take these things personally, even while acknowledging that for many of us, our work is very personal. We need to cultivate relationships with people who remind us of what makes our work powerful and who fuel our motivation to carry it forward, and we should not hesitate to lean on those relationships in times of need.

In addition, we find it useful to take a learning stance. Many of our successes have arisen from applying what we learned from failures. Niral published an article in *Teachers College Record* after revising a manuscript in response to a rejection from another prestigious journal (Shah, 2017). Nicole had a similar experience with an article she ultimately published in the *Journal for Research in Mathematics Education (JRME;* Louie, 2017*)*. Daniel received a rejection from *JRME* that was later published in the *International Journal of Research in Undergraduate Mathematics* (Reinholz, 2015) and highlighted in *Science* magazine (Mueller & Smith, 2016). Recalling these experiences helps us find opportunities for learning and even gratitude in critical feedback. In turn, focusing on learning supports us not only to be more productive but also to experience a greater sense of agency. We can all take challenging reviews and decisions and turn them into successes.

In what follows, we discuss how we have engaged with the publication process as human beings (with feelings and lives outside of academia), not pretending that we're just article-generating machines. We begin by providing an overview of the publication process, which is targeted at early-career scholars just beginning to think about getting published. We then outline the range of options for responding to various kinds of reviews. We provide stories and insights from our own experiences along the way.

17.2 Overview of the Publication Process

Our brief overview of the publication process has three subsections: (1) selecting a journal, (2) interacting with the editorial team, and (3) getting a decision (see also Cai, Hwang, & Robison, 2019, for a detailed account specific to publishing in *JRME*).

17.2.1 Selecting a Journal

Where should you submit your manuscript? Beyond the usual issue of topical fit, this decision depends on your academic position and immediate professional needs. At certain points in your career, you may benefit from submitting your work to a less competitive journal or a journal with shorter review times. This approach might be useful when you are a graduate student getting ready to go on the job market, or when you are about to go up for tenure at an institution where the number of publications really matters. On the other hand, when major professional evaluations are less pressing, you might be more able to manage the longer review times and more critical reviews that typically come with submitting to more highly ranked journals. Most journals post review timelines on the front page of each issue they publish; asking colleagues and advisors can also help in gauging a journal's efficiency in the review process.

You should also consider the type of manuscript being submitted. A few basic types are empirical studies, theoretical/conceptual pieces, and practitioner articles. Make sure that you send your manuscript to the right type of journal. By reading a journal's "scope and aims" (often on its website), as well as articles that have appeared in it recently, you can get a sense of what types of articles a journal accepts and whether yours would be a good fit. Some journals are unlikely to publish ethnographic research, while others routinely do; some publish *only* empirical studies, while in others, conceptual papers frequently appear. You might also consider where pieces you have cited have been published as an indication of potentially suitable outlets for your manuscript. The research that you do to find an appropriate journal should make its way into the cover letter you submit with your manuscript. In our experience, cover letters for manuscript submissions tend to be short (fitting on one single-spaced page) and focus on explaining both what the manuscript is about and why you think the journal's readership would find it interesting. For example, if you feature a particular article in your literature review, you might find out where that article was published and send your manuscript to the same journal, with a cover letter highlighting the connection and briefly describing how your manuscript builds on the article they previously published.

You may have heard the saying: "every article has a home." We agree. It's just a matter of finding it. Articles of different scope and depth have different homes. You should be deliberate about finding a good match (ideally *before* you're ready to submit, so that you can tailor your manuscript to the journal as you write). It can be a great asset to write manuscripts of different types, both to broaden your authorial repertoire and to reach different audiences and increase your impact.

17.2.2 Interacting with Editors

You've submitted your work; now what? Hurry up and wait. Remember that editors and reviewers are scholars just like you: busy people volunteering their time. This means that the whole process can take a while. One way to mitigate frustration with waiting is to work on multiple manuscripts simultaneously, so that you can advance other aspects of your research while some pieces are under review.

If you are concerned that your manuscript has gotten buried in the review process, it might be a good idea to contact the editor. Many journals publish average review times; if you have waited more than a month past that, it is reasonable for you to take action. Or, if no time is posted on the journal's website, we have found that 6 months is a reasonable expectation in our field. You may find it useful to keep a spreadsheet listing your active submissions, important dates for each one (when you submitted, when you received an initial decision, when you resubmitted, etc.), and other information such as your username for journal websites, so you don't have to wonder whether it's been half a year yet (and to save yourself the trouble of combing through your email for that information).

If an appropriate amount of time has passed, you can send a short and respectful email to the editor asking if they have any updates (perhaps mentioning the reason you need the process to move along if you have one, e.g., you're going up for review). Remember, editors are your allies, not your adversaries. It does not help to get angry with them. It may be that they have lost track of your manuscript, but it is much more likely that they are waiting for reviews to come in, and they may be just as frustrated as you are about the situation. While an editor may not be able to give you an exact timeline as to when you will receive a decision, they can likely update you on where your paper is in the process. (Some journals also have online submission systems that provide this information.)

It can also save both you and editors time if you inquire about fit before beginning the submission process. For instance, if you are thinking about submitting a manuscript to a journal but you are unsure whether it's a good fit, sending a brief description (e.g., the abstract) to the editor and asking for clarification would be appropriate. If your manuscript is not a good fit and you skip this step, you will still have to spend time waiting for a decision, and the editor will have to spend time reading it and issuing a "desk reject" (without sending it out for review). You might also contact the editor during the revision process, as we describe below.

17.2.3 Getting a Decision

There are essentially three kinds of decisions you can receive on a manuscript. In very rare cases, a manuscript may be accepted outright, typically pending some kind of revisions (whether major or minor). It can be rejected, either because your research is not appropriate for the journal or because the revisions necessary to bring the manuscript in line with the journal's expectations are so substantial that they would result in a new paper. Finally, you can receive a "revise and resubmit" or an "R&R." This decision means that the editor has not committed to accepting your paper, but will send it out for review again (sometimes to the same reviewers and sometimes to some additional ones, depending on the journal) *if* you respond effectively to the reviews provided (i.e., if you thoroughly revise your paper and resubmit it; if you resubmit without responding substantively to much of the feedback provided, your revision may be rejected without going out for another round of review). Depending on the journal and the nature of the revisions, you may get more than one R&R on the same manuscript; even if you successfully address all of the comments in the first round of revisions, your changes may bring new issues to the fore. In the next section, we describe some of your options for responding to "R&R" and "reject" decisions.

17.3 Responding to an R&R

If you have received a revise and resubmit decision, congratulations! It is extremely uncommon to have an article accepted without going through this phase, even for senior scholars, and getting an R&R is a big step toward getting published.

The first question to ask yourself with an R&R is: do I *want* to revise and resubmit? For us, the answer has almost always been yes. The editor and reviewers have often invested substantial time and effort in our work, and it generally makes sense to capitalize on that investment by responding to their feedback and resubmitting our revisions to the same journal. But there are situations where you might decline. For instance, if a journal took a year to review your paper and you are at a crucial juncture in your career, you may want to try to submit the paper somewhere that has a reputation for a faster turnaround. Or, the editor may outline the need for substantial changes that go well beyond what you are willing to do. In these cases, it may be appropriate to decline to revise. A short and polite note is sufficient (but no note is technically required, and some people advise against sending one; it is a courtesy to the editor to send one, but could be awkward if you change your mind). Still, when thinking about the next steps for your paper, you should use the reviews you received to revise your paper. It is very possible that another journal could ask the same reviewer to look at your paper, and if they do and no changes have been made, their review will probably not be very sympathetic. Additionally, responding to at least some of the feedback you have received could substantially improve your chances of success with another journal.

If you do decide to take up an R&R, there is no magic formula for turning it into an acceptance. But in the next few sections, we lay out the space and describe some of the decision points we have faced and the options we have exercised, looking at four categories of reviewer feedback: (1) comments you agree with, (2) comments you agree with but are not willing to take up, (3) comments you disagree with, and (4) comments that leave you stumped.

17.3.1 Comments You Agree with

When you receive feedback that you immediately agree with, your response is simple: make the suggested revisions (and acknowledge the contribution of the reviewer in your response letter, which we discuss in more detail below). It can take some time to realize that you do in fact agree with a comment, however. For example, if a reviewer says that a section of your manuscript was unclear, your first reaction might be, "Oh yeah, I knew that was a problem," or it might be, "What are you talking about? I spelled it out so plainly. I have no idea how to make it any clearer." But if you step back to let an outsider's perspective on your work sink in, you might see some merit in their critique and move from a space of irritation to one of agreement. (All of this has happened to us; a more detailed example from Niral's experience is below.)

It can also happen that reviewers suggest solutions that you don't like while pointing to underlying problems that you can find agreement with. For example, if a reviewer asks why you aren't using some particular theoretical framework, you might not agree that that is what you should do, while you can agree that the theoretical framework you *are* using needs further development in your manuscript.

Teasing apart what you do and do not agree with isn't always completely straightforward, but it is an important part of responding productively to the feedback you have received.

17.3.2 Comments You Agree with but Are Not Willing to Take Up

Sometimes reviewers describe the study they think you should have conducted and the paper they think you should have written. In some cases, you may agree that your work would be stronger if you had more data, or if you had analyzed additional constructs. And in a subset of those cases, it may be possible for you to go ahead and conduct another interview or two, or to run some additional analyses. But in other cases, it's simply frustrating, because what the reviewers are suggesting is not feasible, not interesting to you, or not even clearly relevant to the study you *did* conduct and the paper you *did* write.

Again, you are not required to take up all reviewer feedback, but it is important to try to identify the problems behind the solutions they suggest (although they are not always explicit about what those problems are) and make a solid effort to address those problems. For example, one way to respond to a request for more data is to adjust your research questions or scale back the claims you're making. In fact, Nicole was able to rescue a paper that one reviewer had recommended rejecting for making "a circular argument"—finding what she had already assumed to be true—by clarifying her research questions. (Thankfully, this was a solution that the editor had also spotted, so the manuscript was given an R&R instead of a rejection.) If you are very clear about what your paper can do (and what it can't), then you may be able to justify having a paper published without additional data or analyses.

17.3.3 Comments You Disagree with

As we have suggested, you do not need to implement every piece of advice you receive from reviewers. In fact, doing so is often challenging because different reviewers may give conflicting feedback. (You can ask the editor to help you make sense of such situations if they have not already provided clear guidance in their summary of the reviews.) And certainly, there are times when you simply don't agree with a reviewer's comment or suggestion. But ignoring what you disagree with is not an option. A better approach is to explicitly say that you disagree, respectfully and with justification (again, in your response letter). If you find yourself disagreeing with *everything* the reviewers are saying, that may be a problem on your end that requires some reflection, but editors do not expect that you will implement every change suggested to you.

A particular type of disagreement we want to address is the “no-you-didn’t, yes-I-did” disagreement. A reviewer might say something like, “You didn’t provide your definition of equity,” or “This article has no research questions,” while you can point to specific places in your manuscript where you explicitly defined equity or stated your research questions. While you may want to mention these places in your response letter, it is likely that the reviewer missed them because of other issues with your writing that can be improved. Rather than getting angry at the imperceptive reviewer, work to make your points clearer to the reader, keeping in mind that many of the people you hope to reach with your work may not be specialists in your exact area. Also remember that many reviewers (and readers) are busy and will not mull over every word in your manuscript, so one important goal in your writing should be that your manuscript gets its primary points across to someone who is skimming your work.

17.3.4 Comments That Leave You Stumped

The final type of comment we will discuss is the comment that you do not know what to do with. You have options here. One is to reach out to the editor and ask for guidance. Another option is to reach out to other scholars in your network and ask for their advice. (Soliciting the perspectives of trusted colleagues is a great strategy any time you feel confused or annoyed by reviews; they can help you interpret unclear prose and excavate grains of useful feedback.) Yet another option is to explain (in your response letter) what you think the reviewer means, acknowledging that you aren’t entirely sure, and describe how your revisions are responsive to your interpretation. In any case, do your best to address the comment.

Keep in mind that you don’t have to have a perfect fix for every problem in order to get your manuscript accepted. You just need to address *most* of the issues that have been raised by reviewers in a satisfactory fashion. For example, in Nicole’s experience with *JRME* (Louie, 2017), she was aware that she had struggled to write a solid discussion (end-of-paper burnout, maybe), and the editor, Jinfa Cai, confirmed that this was the case in his initial decision letter:

> Currently the discussion reads more like a concluding/summative statement than a discussion of ideas. I agree with the reviewers that more connections need to be made in the discussion with the literature review and theoretical framing of the article.… Also, it would be helpful if you explained what you learned from this study, how the research question was answered, what the significance of the findings are for the field, what limitations this study has and how that should impact our interpretation of the findings, what new questions you have from conducting this study, what recommendations you would make, etc.

Nicole completely rewrote the discussion. In the decision letter for the revised manuscript, Jinfa responded, “Although I asked you in my previous decision letter not to restrict your discussion to a summary of the results, it seems that you have gone the opposite direction and removed the entire summary." He had five additional points about how to improve the discussion (let alone the rest of the paper). And yet, the manuscript was accepted, pending revisions—and Nicole gained the above guidance for writing solid discussions, useful not only for the paper in question but for future ones as well.

17.3.5 Writing a Response Letter

As we have hinted, the response letter is a critical part of addressing any requests for revision. The letter helps the editor and reviewers understand how the revisions you made responded to their feedback. The more detail you can provide, the better. Some people organize their revisions into a table, with each reviewer comment in a cell on the left, and descriptions of their responses in corresponding cells on the right, with references to relevant locations in the text such as page numbers (bookended by an overview of the changes at the beginning and thanks at the end). Others go through the decision letter line by line using prose. Either way, your letter is likely to be long (10 single-spaced pages is not an unusual length). The point is to make it easy for the editor and reviewers to see how you addressed their feedback. They should not have to guess. Note that simply turning on the "track changes" feature in your word processing software is not adequate; you are likely to make so many changes that tracking all of them this way will make your document much harder to read, and even if that isn't the case, tracked changes still do not necessarily clarify for an editor or reviewers how your revisions respond to their feedback.

It is helpful to keep good records of the changes you make to your manuscript as you go. You might do this in a separate notes document, in a spreadsheet, or even in a working draft of the response letter itself. "Track changes" can also be useful here, especially if you add comments narrating your changes along the way. Without these records, it is easy to lose track of what changes you have made and why, whereas with them you have given yourself a head start on writing your response letter (which, as we have suggested, can be its own non-trivial writing task).

If you are feeling particularly emotional as you begin writing the letter, it may be best to draft it and set it aside for a few days. Later, you can come back and edit with an eye toward ensuring that your tone communicates respect for the reviewers' opinions and gratitude for the time and energy they put into reading your work. You can also ask a critical friend to help you edit your letter if you are struggling to find the distance you need to write a respectful response. Just remember that most reviewers are trying to help you improve your manuscript and, in the process, to help you become a better scholar.

17.4 Responding to Rejection

It is possible to take all the best advice and still got a “reject” decision on your manuscript. Now what? We lay out two paths forward. One way forward is to learn what you can and move on with your life. The other path is to follow up with the editor and see if it is possible to “reject the rejection.”

17.4.1 Learning from Rejection

Earlier, we mentioned that taking a “learning stance” has helped us turn rejections into successes. If you’ve received a rejection, try to find space to identify the positive things you can take away from this negative experience. This can be a difficult task, but it is generally possible. We provide two examples here, written in first person by Niral and Nicole.

17.4.1.1 Niral’s Story

I do research on race and various forms of racism in education. Part of this work has involved interviewing students of color about their racialized experiences and how they make sense of and navigate racist narratives about their academic ability. In my first year on the tenure track, I wrote a manuscript on the content and structure of racial discourse in mathematics education based on data from my dissertation. Because I had originally submitted the manuscript to a prestigious journal, I was not surprised that it was ultimately rejected. Many of the reviews made legitimate critiques of my analytical approach and conceptual frame. What upset me, though, was one of the reviews I received. The reviewer had written only a short paragraph chastising the journal for not immediately desk rejecting a piece of research that was so thoroughly racist.

At first, I was shocked and hurt: how could the reviewer say that about my article (and me)? How could they say that my work was racist, when promoting racism is the exact opposite of what I’m trying to do as a scholar and person? But then I realized the problem: the reviewer had somehow interpreted the data I was reporting (i.e., the racist narratives that students hear and experience) as my endorsement of the validity of those narratives. Clearly, they had not actually read my article. I was angry. I also felt like I had let that reviewer down because they came away thinking that one of the top journals in the field had propagated racism just by sending my manuscript out for review.

I let the review and my feelings about it simmer for some time, but ultimately I came to the following conclusion: it’s on me, not the reviewer. It is up to me to shore up my writing. Especially when writing about consequential, sensitive topics like race, my writing has to be as clear as possible to limit misinterpretation. I spent the next year rewriting the manuscript from the ground up (and also addressing the other reviewers' concerns by reanalyzing all of my data and elaborating my conceptual frame). I resubmitted to another prestigious journal, and the article was accepted with minor revisions on the first round of reviews (Shah, 2017).

Overall, this was a formative experience. I learned that when you're in graduate school, faculty and student colleagues know you and what you're about—thus, they are more likely to read your work with generous eyes and give you the benefit of the doubt. However, when you put your work into the world and it's under blind review at a journal, reviewers only have your written words in front of them. That's why it is imperative we take care to anticipate ways our manuscripts might be read (or misinterpreted) and to accordingly aim for clarity in our writing.

17.4.1.2 Nicole's Story

The first manuscript I ever submitted for publication was rejected (as other manuscripts have been since then; this is not a one-time experience!). Initially, it bothered me that the first reviewer (of three) did not seem to have read my manuscript very carefully (e.g., the review kept referencing "urgency," when I had written about "agency"). It bothered me even more that the editor had rejected the piece with a boilerplate email, even though the other two reviews were positive. But in time, I came to focus on the second review. The reviewer had engaged seriously with my ideas, affirming that I was in interesting territory and suggesting some directions for improving the paper. I ended up feeling grateful for the process, and a few years later, after many conversations and revisions, a much-improved version of the rejected manuscript was published in *JRME* (Louie, 2017). One of my takeaways is that rejection might mean that we have more work to do than we thought, but it does *not* mean that we are not good enough or that our work is not good enough. We shouldn't let the voices that seem to say the opposite stop us from doing the work that matters to us.

17.4.2 Rejecting Rejection: Dan's Stories

The idea of rejecting a rejection is somewhat tongue-in-cheek. We do not mean to promote arguing with editors or communicating that they're idiots for missing the brilliance of your work. But a rejection from a particular journal *may* not mean that there is no future for that piece with that particular journal. In this section, Dan provides some examples from his own experiences.

My first experience "rejecting rejection" was with one of the very first manuscripts I had written, which described a theoretical model for learning through peer assessment (Reinholz, 2016). With this manuscript, I had already gone through one round of revisions and had submitted my response letter to the editor. The editor came back to me with a rejection decision, rather than sending my revision out for further review. Although the reviews I was responding to were not uniformly positive, they generally acknowledged that there were aspects of the manuscript that were worth pursuing. After I dealt with my frustration (I'm a drummer, so there was a lot of heavy metal drumming involved), I reached out to a mentor of mine (Noah Finkelstein) to ask for advice. My mentor suggested that I write to the editor to follow up. In doing so, I explained to the editor that I had considered two potential routes for my revision, and I asked for the opportunity to have a short phone call to discuss these alternatives and the piece more generally. Ultimately, the editor allowed me to resubmit the revised article to the journal, and this time around it was accepted and published.

In another instance, I had written a manuscript that was rejected because there was a change in editorial staff and the new editor didn't have the time and bandwidth to follow the submission to possible publication. I wrote to the outgoing editor, who was kind enough to have a 1-h phone call with me to discuss the manuscript and possible ways to move the work forward in submitting to a new journal. This piece was also published eventually, in a journal different from the one I initially submitted to (Reinholz, 2017).

We want to be clear: our advice is not to belligerently argue with an editor's decision, which does not help anyone. However, in some circumstances it may be appropriate for you to have a follow-up discussion with the editor, because they have spent a lot of time considering your work and may have valuable insights. Especially if you are writing an article that you really feel needs to reach the audience of a particular journal, and you have some relatively positive reviews, it may be reasonable (if not always successful) to revisit a rejection.

17.5 Conclusion

In this chapter, we have described some options, rooted in our experiences as early-career scholars, for responding to reviews. To reiterate some of the big ideas: take care of yourself emotionally, adopt a learning stance, and use reviews strategically to push your thinking forward. Receiving a revise and resubmit decision is a big and often necessary step toward publication, and as such, it is a reason to celebrate. And even if the outcome of a review process is rejection, that is not the end of your manuscript. In the best cases, the feedback we receive—irrespective of the decision or how hard it may be to swallow when we first read it—presents learning opportunities that ultimately make our work better and, thus, make the field better.

As a final comment, we want to highlight a subtly recurring theme in this chapter: it takes a village to develop and publish the scholarship that we are most proud of. We are not alone in the process. There are people who are there to lend sympathetic ears, read drafts, give formal and informal feedback, bounce ideas, offer perspective, and cheer us on. They include friends and family members outside of academia, journal editors we have never met, anonymous reviewers, and close colleagues. Don't be afraid or ashamed to ask for help when you need it! Just remember the generosity you receive and give as much or better when your turn comes to provide feedback to others, and together, we will keep moving the world of mathematics education forward.

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Author Index

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