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Overview of the Assumptions

To apply the common methods for pricing and risk management we need to make assumptions which are necessary for the construction of the associated models. A complete list of all model assumptions made in this book are summarized here in order to provide an overview of the numerous conditions and assumptions arising in the various methods. For each pricing and risk management method discussed in the following chapters, we will specify which of these assumptions are needed for their application. Beforehand, it should be noted that all these assumptions can at best approximate reality more or less accurate and in times of a crisis often not even that.

1. Given identical costs and risks, investors always prefer the strategy earning the greatest profit.
   This is one of the fundamental motivations for trading. No one deliberately forgoes a profit.
2. There are no arbitrage opportunities.
   Should opportunities exist allowing a profit to be made without risk and additional costs, they would be exploited by arbitrageurs until an "equilibrium" is reestablished in the market in short term.
3. The markets have infinite liquidity.
   Unlimited quantities of financial instruments are available for purchase and sale at any time without affecting their price. As is indicated in its the formulation, this assumption becomes increasingly questionable the more illiquid the market for the instrument becomes. In practice, liquidity risks

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are reflected in broad bid/ask spreads. Thus, Assumption 5 usually implies Assumption 3.

4. There is no counterparty risk.
   The price of an instrument is solely a function of market parameters (prices of the risk factors, underlyings, volatility) and is in no way dependent on the instrument's seller. This is a good approximation for products traded on an exchange which also acts as a clearing house. This, of course, is not the case when dealing with OTC transactions. The quantitative treatment of credit and counterparty risks has in recent years rapidly developed. Nowadays, it is market best practice to include default risk of derivatives in valuation and risk management (see Chap. 20).
5. There is no "friction" in the market.
   Market friction is a general term referring to all costs involved in trading. These include transaction costs, bid/ask spreads, (opportunity) costs of margins, taxes, etc. All such costs not belonging to the actual *investment* are neglected. This is by no means a bad approximation for large institutions.
6. Continuous trading is possible.
   This means that the time between two trades can be arbitrarily small as can the differences in price and the number of instruments traded. The assumption that trading can take place in arbitrarily short time intervals is an approximation for the simple reason that no exchange is *always* open for trade (weekends, bank holidays). Furthermore, infinitely many adjustments in a continuous hedging strategy would cause infinitely high transaction costs if Assumption 5 did not hold. This means that Assumption 5 must hold if Assumption 6 is to make sense. The assumption that arbitrarily small price differences are possible can be problematic for options which are well out of the money. For example, the smallest possible price change of bond futures ("tick size") usually is 0.01 which is small in comparison to the futures quote, say 99.8. Therefore, the assumption of continuously changing prices is good for futures. However, for an *option* on the future which is far out of the money, a change in the futures price by one tick can result in a change in the price of the option of 50% of its value. Thus, such options do not have continuous price changes.
7. The logarithm of the relative price change of a risk factor is a random walk.
   In light of Sect. 2.3, this implies that the price of a risk factor is lognormally distributed.
8. Interest rates are not stochastic.
   Intuitively, this means that the evolution of interest rates over time does not involve a random component but is completely deterministic. In such a world, future interest rates are known today.

9. Interest rates are constant.
   This means that future interest rates are not only known today, they have today's value as well. Interest rates do not change in the course of time.
10. The volatilities of risk factors are not stochastic.
    Intuitively, this means that the volatilities as a function of time do not involve a random component but are completely deterministic. Future volatilities are known today.
11. The volatilities of risk factors are constant.
    This means that future volatilities are not only known today, they have the same value as today as well. Volatilities do not change in the course of time.
12. Dividend yields of risk factors are constant.
    In many pricing models for derivatives on equities, the dividend (yield) of an underlying is a parameter which is assumed to be constant throughout the term of the derivative contract.
13. Correlations between the risk factors are not stochastic.
    As with volatilities, the evolution of the correlations over time has no random component. Future correlations are known today. As in the case of volatilities, this is not often the case in reality.
14. Dividends are deterministic.
    In reality, dividends depend on the p&l progression of the company and the decision of the board of directors resp. the shareholder assembly and will be fixed at least once a year. Regardless, most models assume that time and amount of all future dividend payments are known in advance until maturity of the derivative.