Mechanics of Elastic Wheel Rolling on Rigid Drum

M. Yu. Karelina, T. A. Balabina and A. N. Mamaev

Abstract At present, evaluation of rolling resistance of automobile tires, as well as the determination of the coefficient of resistance to lateral diversion, is carried out on drum stands. Testing cars on drum stands of various designs is also becoming increasingly common. However, there is no research in the literature considering the mechanics of interaction of an elastic wheel with a drum and its kinematic and force characteristics, which has defined the tasks of this study. The research is based on the fact that with steady rolling, the wheel surface elements entering the contact zone are not yet "prepared" to perceive the tangential force and at the same time pressed to the base by the normal force, start moving without slipping, while obtaining tangential displacement. As the coupled elements of the wheel and the support base move in the reversed mechanism in the contact zone, their tangential displacements increase, and therefore, the tangential friction force between the coupled elements also increases. In the place of contact, where the increased friction force reaches the ultimate in adhesion, there happens a breakdown, and on the entire part of the contact located behind the point of breakdown, a slip occurs. Based on this, the authors determine the coordinate of the areas of adhesion and slip in the contact of a wheel with a drum, relative loss of speed of the wheel, the tangential force acting in the contact, the torque on the wheel, power of friction loss in the contact, and the hysteresis loss of the wheel.

Keywords Wheel · Drum · Resistance · Friction · Contact · Power

M. Yu. Karelina (✉)
Moscow State Automobile and Road Technical University (MADI), 64, Leningradsky prospect, Moscow 125319, Russia
e-mail: karelinamu06@gmail.com

T. A. Balabina · A. N. Mamaev
Moscow Polytechnic University, 38, Bolshaya Semyonovskaya str., Moscow 107023, Russia

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1 Formulation of the Problem

Currently, issues related to the mechanics of rolling wheels on a flat surface are considered quite widely [1–23]. However, the wheel rolling process on the drum remains poorly understood, although this requires close study, as rolling and research tests of cars on drum stands are widely used today, which necessitates obtaining dependencies that determine the forces at the wheel's contact with the drum, friction power losses in the contact, wheel slippage, etc.

2 The Main Part

The mechanics of rolling an elastic wheel on a drum is the same as when rolling a wheel on a flat rigid supporting surface.

When the driven wheel is rolling, loaded only with a normal load, due to the imperfect elasticity of the material, there are losses due to internal friction in the wheel material (hysteresis), which cause the occurrence of the moment of resistance M_f and the appearance of a rolling resistance force F_f—the longitudinal tangential force acting in contact of the wheel with the base in the direction opposite to the movement of the wheel. A similar rolling resistance force arises for a brake wheel which is loaded, in comparison with a driven wheel, with an additional braking torque M_{T}. The presence of this force leads to the slippage of the elements of its treadmill relative to the base in the contact zone and to the loss of the angular velocity of the wheel.

During the rolling of the drive wheel, the movement of which occurs under the action of torque M_K, in contact, a driving (traction) force arises, directed along with the wheel. As in the previous case, this force causes slippage of the treadmill elements in the contact zone with the base and loss of the linear speed of the wheel axis.

The mechanism of occurrence of sliding elements of the wheel surface relative to the base is considered in detail in works [1–5]. Using the scheme of the inverted mechanism "elastic wheel—rigid foundation" based on the theory of preliminary displacement, it was shown that with steady rolling, the wheel surface elements entering the contact zone, not being "prepared" for the perception of tangential force and at the same time pressed to the base normal force, begin to move without sliding, while receiving tangential displacement (directed opposite to rolling for the brake and driven wheels, and in the direction of rolling—for the driving wheel). As the coupled elements of the wheel and the base move in the reversed mechanism in the contact zone, their tangential displacements increase, and therefore, the tangential friction force between the coupled elements also increases. In the place of contact, where the increased friction force reaches the ultimate in adhesion, a breakdown occurs and on the whole part of the contact located beyond the point of failure, regardless of whether it is in the zone of decreasing or increasing normal pressures, slip occurs (Fig. 1).

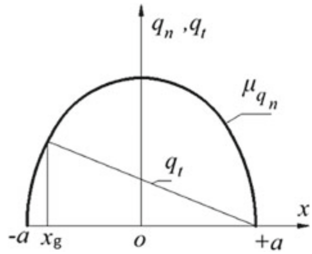


Fig. 1 Normal q_n and tangential q_t stresses in contact with rolling wheels, x_g—coordinate of boundary between grip and slip areas

With an increase in wheel speed loss and a corresponding increase in the tangential force acting in the contact, the slip zone increases, as well as the power of friction loss in the contact, which characterizes the wear intensity of the treadmill and partly the wheel rolling resistance.

Tangential displacements of treadmill points in the contact zone can be represented as a sum of two terms, one of which is due to the realization of the tangential force in the contact, and the second one—to the wheel geometry (its circular shape in cross section to the axis).

Neglecting the displacements due to the geometry of the wheel does not lead to a significant error in determining the kinematic parameters of the wheel as a function of the realized tangential force. In this regard, when solving the tasks, we will take into account only the tangential displacements of the points of the treadmill wheel, due to the implementation of the tangential force [1, 2]:

$$U = \xi(a - x) = \left(\frac{r_k^c}{r_k} - 1\right)(a - x) \tag{1}$$

In the formula, ξ is the relative loss of speed, a is the half-length of the pad wheel with a rigid support surface, x is the distance from the beginning of the contact area to the considered point of the wheel in the contact area, r_k is the wheel rolling radius, and r_k^c is the free rolling radius.

As $\omega_k r_k = V$, where V is the reversed wheel speed, therefore

$$U = (a - x)\left(\frac{\omega_k r_k^c}{V} - 1\right) \tag{2}$$

Applied to wheel rolling on a hard drum $V = V_\delta = \omega_\delta r_\delta$, where ω_δ and r_δ are angular velocity and radius of the drum. As a result, tangential displacements of points on the surface of an elastic wheel, due to the implementation of a tangential force in contact with the drum, can be represented in the grip section by the expression:

$$U = (a - x)\left(\frac{\omega_k r_k^c}{\omega_\delta r_\delta} - 1\right) = \xi(a - x), \tag{3}$$

where is the relative velocity difference.

$$\xi = \frac{\omega_k r_k^c}{\omega_\delta r_\delta} - 1 \tag{4}$$

With a known value ξ, the ratio of the angular velocities of the wheel and the drum will be equal to:

$$\frac{\omega_k}{\omega_z} = (1 + \xi)\frac{r_\delta}{r_k^c} \tag{5}$$

Proceeding from the proportionality of tangential stresses (specific tangential forces) to tangential displacements, we can state that tangential stresses caused by the realization of a tangential force in contact are as follows:

$$q_{\mathrm{t}} = \lambda U = \lambda\xi(a - x), \tag{6}$$

where λ is the wheel tangential stiffness coefficient, determined [3] as:

$$\lambda = \lambda_k \frac{r_\delta}{r_\delta + r} = \frac{1.5qr}{a^3}\frac{1}{1 + r/r_\delta} \tag{7}$$

With a parabolic law, the distribution of normal pressures along the length of the contact area the coordinate of the boundary between grip and slip areas (Fig. 1), determined from the equality $q_{\mathrm{t}} = \mu q_n$, can be shown as the following relationship [1, 2]:

$$x_g = -a \pm \frac{\lambda\xi}{\mu q_{n_0}} \tag{8}$$

The moment on the drum, due to the action of tangential force, is equal to:

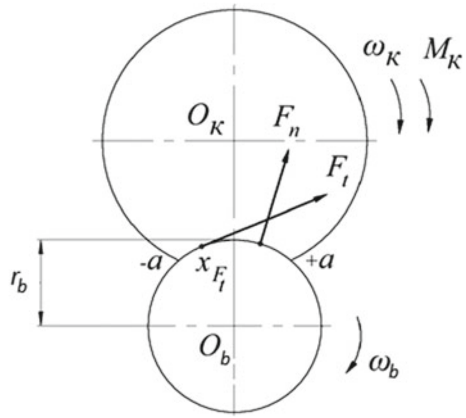
$$M_t = 2b\int_{-a}^{x_\partial} r_\delta q_t^{-\hbar\cdot} dx + 2b\int_{x_\partial}^{+a} r_\delta q_t dx = \left[2b\int_{-a}^{x_\partial} q_t^{-\hbar\cdot} dx + 2b\int_{x_\partial}^{+a} q_t dx\right] r_\delta = F_t r_\delta \tag{9}$$

The value in square brackets, equal to the algebraic sum of all specific tangential forces in the contact, we call the circumferential force of thrust (Fig. 2):

$$F_t = 2b\left[\frac{\lambda\xi}{2}(a - x_\partial)^2 \pm \frac{1}{3}\mu q_{n_0}(2a^3 + 3a^2 x_\partial - x_\partial^3)\right] \tag{10}$$

Putting into (10), the expression $lx = \pm m q_{n_0}(a + xg)$, obtained from (8), after transformations, we arrive at an equation, the solution of which gives the dependence for finding the coordinate of the boundary of the cohesion and slip sections:

Fig. 2 Forces in contact with the wheel drum



$$x_{\mathrm{g}} = a\left(1 - 2\sqrt[3]{1 - \frac{F_{\mathrm{t}}}{\mu F_z}}\right) \tag{11}$$

As a result

$$\xi = \frac{\pm 1}{\lambda} 2\mu q_{n_0}\left(1 - \sqrt[3]{1 - \frac{F_{\mathrm{t}}}{\mu F_z}}\right) \tag{12}$$

or considering expressions for q_{no} (2) and (7)

$$\xi = \pm\frac{\mu a}{s}\left(\frac{1}{r} - \frac{1}{r_\delta}\right)\left(1 - \sqrt[3]{1 - \frac{F_{\mathrm{t}}}{\mu F_z}}\right) \tag{13}$$

The last expression, provided that we do not take into account the saturation coefficient of the treadmill pattern s (for wheels without treadmill pattern $s = 1$), matches a similar formula acquired by Fromm [6, 7] (the difference is only in the degree of the radical: H. Fromm suggests a square root), and then by Vyrabov [3] for a friction gear, consisting of two cylinders.

Given the known relationship for ξ, the ratio of the angular velocities of the elastic wheel and the rigid drum as a function of the thrust force F_{t} and the normal load in accordance with Formulas (4), (12), and (13) can be represented as:

$$\begin{aligned}\frac{\omega_k}{\omega_\delta} &= \frac{r_\delta}{r_k^c}\left[1 \pm \frac{2\mu q_{n_0}}{\lambda}\left(1 - \sqrt[3]{1 - \frac{F_{\mathrm{t}}}{\mu F_z}}\right)\right] \\ &= \frac{r_\delta}{r}\left[1 \pm \frac{\mu a}{s}\left(\frac{1}{r} - \frac{1}{r_\delta}\right)\left(1 - \sqrt[3]{1 - \frac{F_{\mathrm{t}}}{\mu F_z}}\right)\right]\end{aligned} \tag{14}$$

In case of small tangential forces the last formulas can be simplified if the expression $\sqrt[3]{1 - F_{\mathrm{t}}/\mu F_z}$ is expanded in a power series, then discarding the values of the second infinitesimal order:

$$x_g = a\left(-1 + \frac{2}{3}\frac{F_{\mathrm{t}}}{\mu F_z}\right) \tag{15}$$

$$\xi = \frac{a}{3s}\left(\frac{1}{r} + \frac{1}{r_\delta}\right)\frac{F_{\mathrm{t}}}{F_z} \tag{16}$$

$$\frac{\omega_k}{\omega_\delta} = \frac{r_\delta}{r}\left[1 - \frac{a}{3s}\left(\frac{1}{r} + \frac{1}{r_\delta}\right)\frac{F_{\mathrm{t}}}{F_z}\right] \tag{17}$$

In the Expressions (15)–(17), the force F_t is positive for the drive wheel and negative both for the driven and brake ones.

Due to the realization of a thrust force at the contact, the power loss caused by friction in the contact of an elastic wheel with a rigid drum is determined by the same relationship as for the case of a wheel rolling on a flat bearing surface:

$$P_{\mathrm{fr.}} = F_{\mathrm{t}} x V = F_{\mathrm{t}} x w_d r_d \tag{18}$$

To determine the power loss to the hysteresis in the material of an elastic wheel rolling on a rigid drum, as before, we will take into account only the normal deformation of the wheel, which can be represented as a sum of (Fig. 3): $W = W' + W''$.

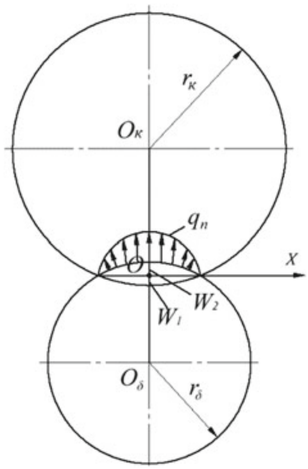


Fig. 3 Normal stresses q_{n} when the wheel is pressed against the drum

As $W' = \frac{a^2 - x^2}{2r}$ and $W'' = \frac{a^2 - x^2}{2r_\delta}$," then

$$W = \frac{a^2 - x^2}{2}\left(\frac{1}{r} + \frac{1}{r_\delta}\right) \tag{19}$$

The hysteresis power loss can be found using the following relationship:

$$P_{\mathrm{h}} = \beta_{\mathrm{h}} \int_0^{\mathrm{a}} \mathrm{q}_n \left|\frac{\mathrm{dW}}{\mathrm{dt}}\right| \mathrm{d}x 2b, \tag{20}$$

where β_{h} is the hysteresis loss coefficient; dW/dt—wheel warping speed:

$$\frac{dW}{dt} = \frac{dW}{dx}\frac{dx}{dt} = \frac{dW}{dx}V_{\text{,,}\hbar p.} = \frac{dW}{dx}\omega_k r \tag{21}$$

Here, the x coordinate lies on the axis OX (Fig. 3.); $\frac{\mathrm{d}x}{\mathrm{d}t} = V_c = \omega_k r$ since the change in the normal deformation dW/dt occurs at a speed equal to the peripheral speed of the wheel as the wheel tread element moves into the contact depth. Considering (19)

$$\frac{\mathrm{d}W}{\mathrm{d}t} = -x\left(\frac{1}{r} + \frac{1}{r_\delta}\right)\omega_k r \tag{22}$$

As a result,

$$P_{\mathrm{h}} = \frac{3}{16}\beta_{\mathrm{h}} F_n a \omega_k r \left(\frac{1}{r} + \frac{1}{r_\delta}\right) \tag{23}$$

With the dependence found for the P_{h}, the moment of hysteresis in the tire material can be represented as:

$$M_{\mathrm{h}} = \frac{P_{\mathrm{h}}}{\omega_k} = \frac{3}{16}\beta_{\mathrm{h}} a F_n \left(1 + \frac{r}{r_\delta}\right) \tag{24}$$

Then, the shift shoulder of the normal drum reaction will be equal to:

$$h_0 = \frac{M_{\mathrm{h}}}{F_n} = \frac{3}{16}\beta_{\mathrm{h}} a \left(1 + \frac{r}{r_\delta}\right) \tag{25}$$

According to [4, 5],

$$3\beta_{\mathrm{h}} a^{sh}/16 = f_0 r_k^c \approx f_0 r. \tag{26}$$

Then, $h_0 = f_0 r\left(1 + \frac{r}{r_\delta}\right)\frac{a}{a^{sh}}$

Here, a^{sh} is the half-length of the wheel contact with a flat rigid bearing surface with the same load F_n.

Knowing the shoulder h_0, it is possible to find [4, 5] the dependence for the tangential force (it is also the rolling resistance force of the driven wheel) due to the hysteresis:

$$F_{\tau_0} = F_n f_0 \left(1 + \frac{r}{r_\delta}\right) \frac{a}{a^{sh}} \tag{27}$$

Since the ratio of the rolling resistance force to the normal force $F_{\tau_0}/F_n = f_0$ is the wheel rolling resistance coefficient, then for the considered case of an elastic wheel rolling on a rigid drum

$$f_0^\delta = f_0 \left(\frac{r}{r_\delta} + 1\right) \frac{a}{a^{sh}} \tag{28}$$

When $r_\delta \to \infty$, the dependences (24), (27), and (28) lead to expressions derived for the case of rolling an elastic wheel on a flat rigid supporting surface. Comparison of these expressions with the above dependencies leads to the conclusion that both the moment from the hysteresis and the force and the rolling resistance coefficient of the driven elastic wheel along with a rigid drum, caused by the hysteresis, increase $a(1 + r/r_\delta)/a^{sh}$ times compared to rolling the same wheel on a flat hard surface.

3 Conclusion

Based on the considered mechanics of rolling an elastic wheel over a rigid drum, the dependences are obtained for calculating the tangential force at the wheel's contact with the drum, the torque on the drum, the power of friction loss in contact, and the amount of slip (relative velocity loss).

An increase in rolling resistance on a drum leads to a difference in lateral drag coefficients determined on the drum and when the wheel moves on a flat supporting surface.

When using lateral drag and rolling resistance coefficients, obtained experimentally on a drum stand, for the case of a wheel moving on a flat support surface, appropriate correction factors should be introduced.

Next, one should determine the points of application of tangential and normal forces in the contact, wheel rolling on two drums, and also the coefficient of resistance to lateral drift when the wheel is rolling on the drum.

The issues of interaction of an elastic wheel with a rigid support surface are set forth in more detail in the list of references given at the end of this chapter [1–23].

References

1. Vyrabov RV (1982) Traction properties of friction gears. Mach Eng
2. Vyrabov RV, Mamaev AN (1980) Analysis of kinematic and force relations in the rolling of elastic wheels on rigid base. Mech Mach, 101–106
3. Vyrabov RV, Mamaev AN (1978) Determination of power losses to friction in the contact of friction pair of wheel with pneumatic tyre-rigid base. Interuniversity collection of scientific papers "Continuously variable adjustable transmission", pp 61–67
4. Vyrabov RV, Mamaev AN, Balabina TA (2010) General questions of interaction between elastic wheels with a rigid support surface. In: Materials of international scientific and technical conference AAI "Car and tractor construction in Russia: development priorities and training", dedicated to the 145th anniversary of MGTU "MAMI"
5. Balabina TA, Mamaev AN (2014) The mechanics of rolling elastic wheels on rigid support surface. In the book "Engineering science: trends, prospects and technology development"
6. Fromm H (1927) Berechnung des Schlupfes beim Rollen deformierbaren Scheiben. Zf Angew Math und Mech Bd 7, H.I
7. Fromm H (1928) Arbeitsverlust, Formänderungen und Schlupf beim Rollen von treibenden und gebremsten Rädern oder Scheiben. Beitrag zur Analyse der Reibungsgesetze. Z.f
8. Vyrabov RV, Mamaev AN (1983) The effect of toroidally elastic wheels for uneven wear across the width of the treadmill—Izv. Universities, Mechanical Engineering, pp 94–97
9. Vyrabov RV, Mamaev AN (1987) To the question on the least value of the coefficient of rolling resistance of elastic wheels on rigid horizontal surface. Izv. Universities, Mechanical Engineering, pp 85–88
10. Mamaev AN (1982) Determination of the coefficient of tangential elasticity of the wheels with the toroidal shape of the treadmill
11. Mamaev AN, Vukolova GS, Dmitrieva LN (1999) The effect of the view taken of the law of distribution of normal pressure in the wheels in contact with the rigid base on the calculated force and kinematic parameters of the wheel. Collection of scientific papers dedicated to the 60th anniversary recreation of MAMI. MSTU "MAMI", Moscow
12. Vyrabov RV, Mamaev AN, Marinkin AP, Yuriev Yu M (1986) Influence of the mode of rolling elastic wheels on the magnitude of lateral forces during a side slip. J Mech Eng, 33–35
13. Vyrabov RV, Mamaev AN (1980) Analysis of power relations in the rolling of the slave elastic cylindrical wheel along a curvilinear trajectory. Mech Mach, 105–112
14. Vyrabov RV, Mamaev AN (1980) Investigation of contact phenomena in nonlinear toroidal rolling wheels. Izv. Universities, Mechanical Engineering, pp 33–38
15. Vyrabov RV, Mamaev AN (1980) Determination of forces and moments acting on the toroidal wheel in nonlinear rolling. Izv. Universities, Mechanical Engineering, pp 30–34
16. Mamaev AN, Sazanov IV, Nazarov Yu P (1990) Determination of the force characteristics of the elastic wheel rolling with a slip along a curved path. In: Proceedings of II all-union symposium "Problems of tires and rubber-cord materials. Strength and durability"
17. Mamaev AN (1999) The influence of the operating conditions of the tyres for wear their treadmill. Truck
18. Mamaev AN, Vyrabov RV, Portugalsky VM, Chepurnoy SI (2010) Determination of power and kinematic characteristics of an elastic wheel during rolling on a rigid drum. In: Proceedings of the international scientific and technical conference dedicated to the 145th anniversary of MSTU "MAMI"
19. Mamaev AN (2010) Driving wheel rolling resistance on a hard drum. In: Proceedings of the international scientific and technical conference, dedicated to the 145th anniversary of MSTU "MAMI"
20. Mamaev AN (1982) Influence of design parameters of elastic wheels on the value of their deflection and dimensions of the contact area with a rigid base. Interuniversity collection of scientific works "Safety and reliability of an automobile", pp 203–211

21. Mamaev AN, Alepin EA (1980) Determining the size of the contact area and deflection of the wheel with a rubber tire with a static pressure wheel to a rigid base. Collection of scientific works “Machine Science”, pp 82–85
22. Mamaev AN (1982) On the determination of the hysteresis loss coefficient of highly elastic rolling elements
23. Balabina TA, Mamaev AN, Chepurnoy SI (2013) Determination of the ratio of camber and toe angles of the elastic wheels, ensuring the lowest rolling resistance. News of MSTU “MAMI”