Role of Hall Effect in Magnetotelluric Sounding

Valery V. Plotkin, Vladimir S. Mogilatov and Vladimir V. Potapov

Abstract Many minerals belong to the semiconductor class from the point of view of the mechanism of electrical conductivity. One of the experimental methods for determining the electrical conductivity of semiconductor minerals is based on the Hall effect. Due to the presence of the Earth's magnetic field it is possible to expect the manifestation of this effect in natural conditions during electromagnetic soundings. To detect manifestations of the Hall effect during magnetotelluric (MT) sounding, a polarization analysis method is proposed, based on a data processing algorithm that divides the time spectrum of the MT field into normal mode spectra with right and left circular polarization. Numerical estimates and results of a special field experiment are presented.

Keywords Magnetotelluric sounding · Hall effect · Normal modes · Right and left circular polarizations

1 Introduction

Host rocks overlaying oil and gas deposits penetrated by a stream of hydrocarbon fluids, represent a semiconductor medium in which manifestation of the Hall effect is possible [1]. Interest in this phenomenon arose from the results of electromagnetic soundings with controlled sources in hydrocarbon-rich areas [2]. Similar phenomena may be observed during electromagnetic soundings of porous geological media due to the effect of the Earth's magnetic field on currents in fluids. Here the role of viscosity is large, and viscosity coefficient can be determined from the experiment.

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2 The Polarization Analysis Method

So far, for geological media the characteristics of microprocesses are insufficiently known, therefore it is better to address an experiment. For its planning, it is possible to consider the influence of the Earth's magnetic field by introducing the Hall conductivity σ_H into the electrical conductivity tensor. In an anisotropic medium, the field splits into two components, called normal modes, which differ in attenuation coefficients and phase velocities. The difference between modes is connected with its polarization and direction of rotation of the field vector; in the first mode the field rotates clockwise, in the second counterclockwise. For physical reasons, it is clear that due to Hall effect the response of the medium may be different in the case of excitation of the medium by only one of the normal waves.

Analysis of MTS data shows that generally the direction of rotation of MT-field vectors (right or left polarization) is defined by characteristics of frequency spectrum. It is well known that polarization is generally elliptic, and at different frequencies, the direction of rotation of field vector is determined by the phase difference of spectral elements of horizontal components at different frequencies. So, if we assume the field time dependence in the form $e^{i\omega t}$ and introduce the designations $H_x(\omega) = A_x(\omega)e^{i\varphi_x}$ and $H_y(\omega) = A_y(\omega)e^{i\varphi_y}$, then the horizontal component of magnetic field vector rotates clockwise when $\sin(\varphi_y - \varphi_x) > 0$, and counterclockwise when $\sin(\varphi_y - \varphi_x) < 0$. This can be explained by the ratio of amplitudes of normal modes with circular polarization representing the MT-field.

The mode amplitudes can be determined using a polarization analysis of time spectrum of MT-field. For this purpose, it is necessary to know the polarization coefficients for a given mode. The calculations carried out in [3] for the horizontally layered medium with the parameters $\sigma_n, h_n, n = 1, \ldots, N, h_N \to \infty$ showed that in middle latitudes mode polarizations coefficients in all layers are close to $\pm i$ when $\sigma_H \ll \sigma_n$. It means that under these conditions modes with right or left circular polarization are normal. Then, for any elliptic polarization, the decomposition of MT-field into modes with right and left circular polarizations can be performed using the following representation for the real part of horizontal components:

$$Re\left\{H_x(\omega)e^{i\omega t}\right\} = Re\left\{A_x(\omega)e^{i(\varphi_x+\omega t)}\right\} = A_1\cos(\varphi_1 + \omega t) + A_2\cos(\varphi_2 + \omega t),$$

$$Re\left\{H_y(\omega)e^{i\omega t}\right\} = Re\left\{A_y(\omega)e^{i(\varphi_y+\omega t)}\right\} = A_1\sin(\varphi_1 + \omega t) - A_2\sin(\varphi_2 + \omega t),$$

where the mode with index 1 is characterized by the counterclockwise rotation of magnetic field horizontal components, and the mode with index 2 by the clockwise rotation. Amplitudes and phases of modes are then calculated using the polarization analysis formulas:

$$H_{1,2}(\omega) = A_{1,2}e^{\varphi_{1,2}},$$

$$A_1^2 = \frac{\left[A_x^2 + A_y^2 + 2A_x A_y \sin(\varphi_x - \varphi_y)\right]}{4}, \quad A_2^2 = \frac{\left[A_x^2 + A_y^2 + 2A_x A_y \sin(\varphi_y - \varphi_x)\right]}{4},$$

$$\tan\varphi_1 = \frac{A_x \sin\varphi_x + A_y \cos\varphi_y}{A_x \cos\varphi_x - A_y \sin\varphi_y}, \quad \tan\varphi_2 = \frac{A_x \sin\varphi_x - A_y \cos\varphi_y}{A_x \cos\varphi_x + A_y \sin\varphi_y}$$

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According to the similar formulas, a transition to the modes is carried out for the horizontal components of the electric field. Further on, for any time harmonic, instead of standard impedance ratios for horizontal MT-field components, impedance ratios are introduced for the modes with right and left circular polarizations:

$$E_1(\omega) = Z_{11}(\omega)H_1(\omega) + Z_{12}(\omega)H_2(\omega), \quad E_2(\omega) = Z_{21}(\omega)H_1(\omega) + Z_{22}(\omega)H_2(\omega).$$

The difference between mode impedances $Z_{11}(\omega)$ and $Z_{22}(\omega)$ will indicate a possible occurrence of the Hall effect. This difference will also manifest itself in the values of apparent resistivities and impedance phases introduced by conventional formulas.

3 A Numerical Model of the Hall Effect in MTS Data

We will confirm the above by numerical calculations using the synthetic MTS data obtained as follows. First, using the normal distributions of random values, initial time spectra for the horizontal magnetic field components $H_{x,y}(\omega)$ were synthesized. The parameters of distributions were chosen to achieve similarity with examples of available experimental data. Such approach allows a multiple repetition of numerical experiments without carrying out real MTS. Then, for the given model of horizontally layered medium, the impedance tensor was calculated using the method described in [4]. The time spectra of horizontal components of electric field are determined using the standard impedance ratios for the magnetic field time spectra:

$$E_x(\omega) = Z_{xy}(\omega)H_y(\omega) + Z_{xx}(\omega)H_x(\omega), \quad E_y(\omega) = Z_{yy}(\omega)H_y(\omega) + Z_{yx}(\omega)H_x(\omega).$$

Using such a scheme for the preparation of synthetic MTS data, we next consider the results of their processing using the proposed version of the MT field polarization analysis.

Numerical calculations were carried out for the horizontally layered medium model consisting of four layers with thicknesses from top to down of 0.7, 5, 2 and 9 km and specific electric resistivities (SER) 100, 1000, 300 and 100 of Ω m respectively, resistivity of underlying homogeneous subsurface was 20 Ω m.

It is interesting to compare the calculation results for the cases when the Hall conductivity of the medium is zero, and when it is present in all layers of the medium.

We consider that $\sigma_H = 1/1000$ S/m, the Earth's magnetic field is deflected from the vertical by an angle of 25°, the *OX* axis is set to the plane of magnetic meridian and directed northward, the *OZ* axis is directed down to the medium. The MTS curves obtained by modal impedances $Z_{11}(\omega)$ and $Z_{22}(\omega)$ are presented in Fig. 1 (on the left when the Hall conductivity is absent, on the right when it is not equal to zero). Naturally, the observed difference in the modal curves increases with increasing Hall conductivity. It should be noted that this difference practically does not depend on the ratio of amplitudes $A_{x,y}(\omega)$ in the original spectra of horizontal components of magnetic field (Fig. 2, $\sigma_H = 1/1000$ S/m).

The question arises whether the observed effect is due to the influence of the Hall conductivity. Would not the manifestations be similar, for example, in the case of apparent anisotropy arising in a thin-layer medium?

In order to reveal differences in the results of processing data by the proposed method for a medium with the Hall conductivity and for a thin-layer medium, respectively, calculations were also carried out for a transversal anisotropic medium. We assume that in this case in the coordinate system where the tensor $\hat{\sigma}'$ is diagonal (in the main axes) alternating thin layers with different SERs, the normal to which is directed along the axis OZ', lead to an anisotropy, and it was considered that $\sigma'_{xx} = \sigma'_{yy}$, $\sigma'_{zz} = \sigma'_{xx}/k_a$, where k_a is the anisotropy coefficient.

Note that when taking into account the Hall conductivity, it was assumed that the geomagnetic field is directed along the axis OZ' in the same coordinate system, and the tensor $\hat{\sigma}'$ has identical diagonal components $\sigma'_{xx} = \sigma'_{yy} = \sigma'_{zz}$, but additional antisymmetric components $\sigma'_{xy} = -\sigma'_{yx} = \sigma_H$ appear. Differences of tensors $\hat{\sigma}'$ are

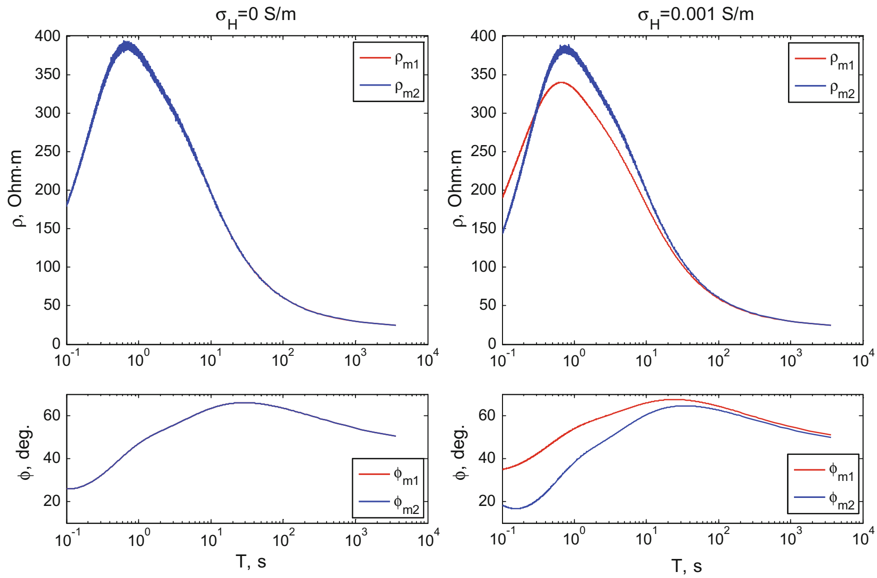

Fig. 1 The MTS modal curves in the case when the Hall conductivity is presented in the medium

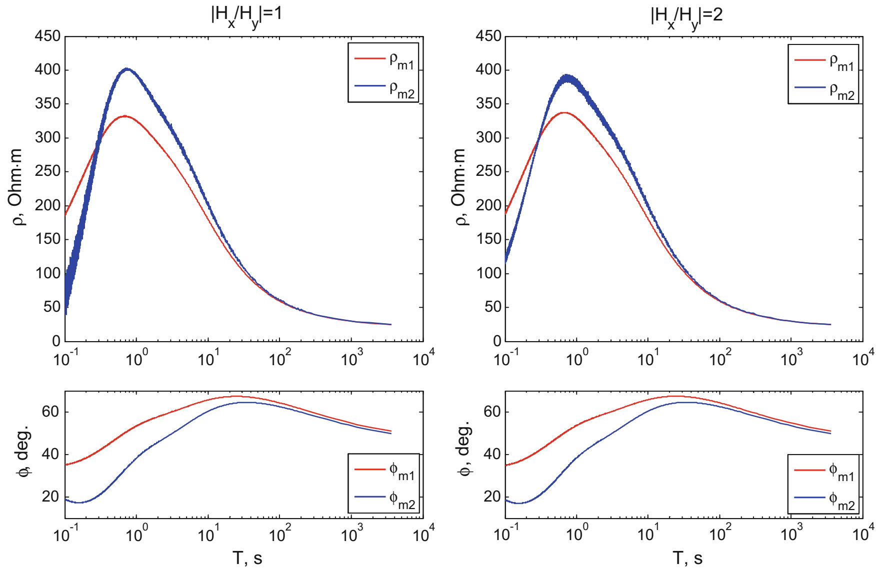

Fig. 2 The MTS modal curves for different ratios of $H_{x,y}(\omega)$ amplitudes

associated with these circumstances when studying the Hall effect and the apparent anisotropy of a thin-layer medium. The same transition matrix was used in the calculations of tensors σ′ in the laboratory coordinate system in both cases.

As an example of differences in conductivity tensors, we present the values of its components in the laboratory coordinate system at the same depth (on the left—in case of the Hall's effect, on the right—for the thin-layer medium):

$$\hat{\sigma} = \begin{pmatrix} 0.05 & 0.0009 & 0.0004 \\ -0.0009 & 0.05 & 0 \\ -0.0004 & 0 & 0.05 \end{pmatrix}, \qquad \hat{\sigma} = \begin{pmatrix} 0.05 & 0 & 0 \\ 0 & 0.0485 & 0.0032 \\ 0 & 0.0032 & 0.0432 \end{pmatrix}.$$

Certainly, the calculations were performed for the same selected model of horizontally layered medium consisting of four layers described above. The results are presented in Fig. 3. It can be seen that for the thin-layer medium (even with the bigger coefficient of anisotropy $k_a = 1.2$) the difference of modal curves is significantly less than in the case of presence of the Hall effect.

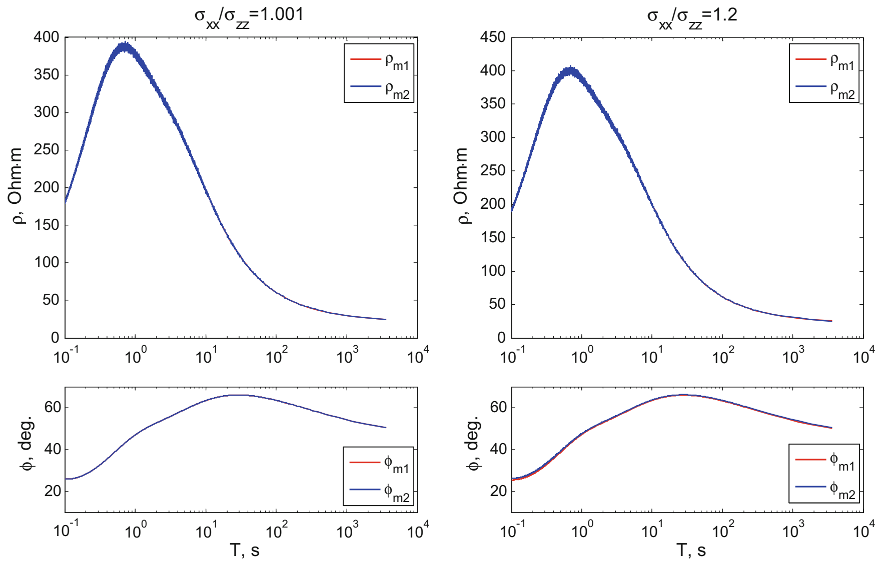

Fig. 3 The MTS modal curves in the case of a thin layer anisotropic medium

4 The Results of Experiment to Detect the Hall Effect in Real MTS Data

The experiment to detect the Hall effect is easiest to carry out in the case of a horizontally layered medium. The suitable site for such an experiment is available in the Tatar area of West-Siberian lowland, near Orlovka village (54°51′ N, 76°02′ E). MTS was performed in July 2018 using the MTU-5 station (Phoenix Geophysics). Signal registration lasted 40 h. The results obtained by a standard processing of experimental data are presented in Fig. 4.

On the short periods, the selected area quite corresponds to a horizontally layered medium. Therefore, the polarization analysis was performed for these periods (Fig. 5) with various selections of time windows lasting 10^4 s and sample rate of 15 Hz. For each window the time spectra of horizontal component variations of the field were transformed to the spectra of amplitudes of normal modes with right and left circular polarization. The similar procedure was carried out in addition to the processing of spectra. In Fig. 5, dots show the estimates of mode apparent resistivities obtained from different data samples; the solid lines are their approximations by polynomials. The dotted lines present the modal curves calculated for two medium models (shown with dotted lines in the left part of the figure) and the specified values σ_H. The differences of modal curves are also shown for clarity. The search of medium models and σ_H allows approximately bring together all curves. The estimates show that the σ_H values do not exceed ~10^{-3} S/m.

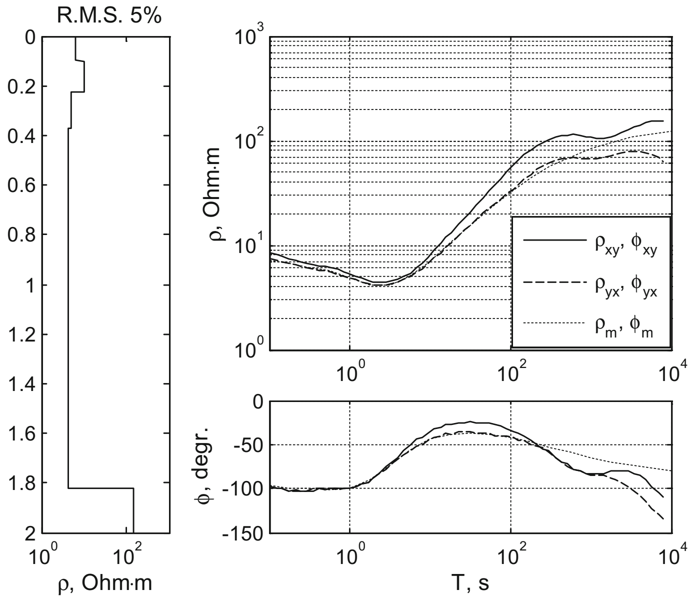

Fig. 4 MTS curves and the model of medium obtained by a 1D inversion

It should be noted that a possibility exists that differences of modal curves are connected with the lateral inhomogeneity of the medium. To estimate its role, it is convenient to construct polar diagrams of impedances. In case of a 2D heterogeneity the polar diagrams of additional impedances taking into account the Hall effect and without it considerably differ. An example of polar diagram for the discussed experimental data is shown in Fig. 6. Apparently, the role of lateral inhomogeneity in this case is small. It is yet another argument favoring a possibility of manifestation of the Hall effect.

5 Conclusion

To find manifestations of the Hall effect in the MTS data, it is possible to apply the method of polarization analysis based on the data processing algorithm with the division of time spectra of MT-field into spectra of normal modes with right and left circular polarization. It is established that with the use of the proposed method it is possible to distinguish effects for a medium with the Hall conductivity and for a thin-layer medium, respectively. First experimental estimates showed that σ_H values do not exceed ~10^{-3} S/m.

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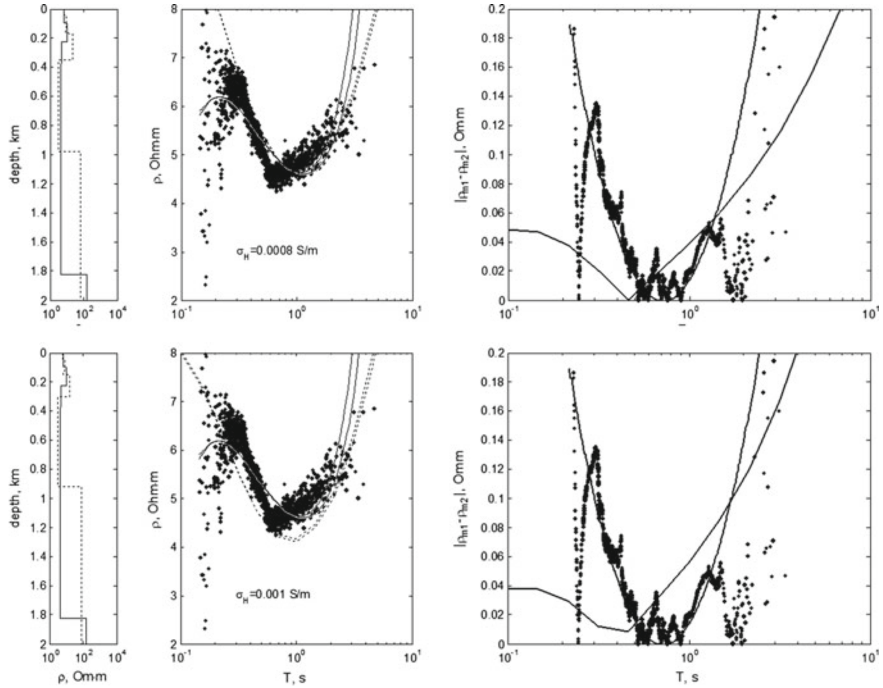
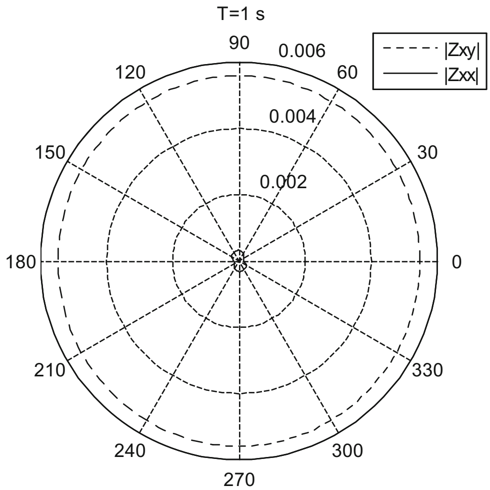


Fig. 5 The models of medium (left), the modal curves (center) and their distinction (right). Dots are the experimental estimates, solid lines are the approximations by polynomials, dotted lines are the results of calculation of modal curves for the model shown by dotted line and the given Hall conductivity

Fig. 6 The polar diagram of impedances for the time period T = 1 s



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